String Theory lessons for Higher Spins

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Based on:

Master Thesis (2009) [arXiv:1005.3061] (M.T. advisor: Prof. A. Sagnotti)

&

to appear: A.Sagnotti and M.T.

What is String Theory?

✓It is a scheme based on the mechanical model of the vibrating relativistic string

✓Although very natural a large number of questions remain unsolved: Background (in)dependence(?) ....

Key ingredient for consistency: Infinite tower of HS excitations

Soft UV behavior, open-closed duality, modular invariance, etc...

The difficulty → The mechanical model hides the systematic

A possible way out → Probe String Theory within a regime where HS excitations are relevant

String Theory is a consistent Higher-Spin Theory!

Higher Spins and String Theory

In the limit where the string tension tends to zero or the energy tends to infinity all HS excitations must be treated on the same footing

$$s \to \infty$$

Gross, Mende (1988),
Amati, Ciafaloni, Veneziano (1989)
Moeller, West (2005)

$$T = \frac{1}{2\pi\alpha'\hbar c} \to 0$$

Bengtsson (1986)
Henneaux, Teitelboim (1987)
Pashnev, Tsulaia (1998)
Francia, Sagnotti (2002)
Bonelli (2003)
Sagnotti, Tsulaia (2004)
Fotopoulos, Tsulaia (2007)

String Ward identities and linear relations between scattering amplitudes involving HS excitations

Lee, et al. (1991-2008)

La Grande Bouffe

Sundborg (2001)
Sezgin, Sundell (2002)
Bianchi, Morales, Samtleben et al (2003)

What is the Grande Bouffe?

Coleman, Mandula (1967);
Weinberg (1964);
Velo Zwanziger (1969);
Haag, Lopuszanski, Sohnius (1975);
Aragone, Deser (1979-1980);
Weinberg, Witten (1980);
Porrati (2008)

Several no-go results:

Two different approaches on the field theory side:

- ✓ Frame-like (Vasiliev et al, 1980-; Boulanger, Iazzeolla, Sundell, 2008)
- ✓ Metric-like (Singh, Hagen, 1974; Fronsdal, Fang, 1978; Buchbinder, Tsulaia, Pashnev et al, 1998-; Francia, Sagnotti, Mourad, Campoleoni, 2002-)

Using String Theory as a theoretical laboratory one can extract a lot of information about consistent interactions of HS states

- ✓ Massive scattering amplitudes and couplings do not suffer from the usual problems encountered in the massless case
- ✓ In the massless limit ordinary gauge invariance emerges!

Is String Theory unique as a Higher-Spin Theory? → **No!**

Plan

- Open Bosonic String Tree-level S-matrix Amplitudes
  - Review of the operator formalism
  - String symbol calculus
  - Generating function for 3-point correlation functions
  - 3-point Amplitudes
  - 4-point Amplitudes
  - String Currents
- String Higher-Spin Couplings
  - Couplings 0-0-s
  - Couplings 1-1-s
  - General case
  - Higher-Spin Conserved Currents
- Field Theory Scattering Amplitudes
  - Generating function of current exchanges
  - Scattering amplitudes

(Open) Bosonic-String S-matrix

Chan-Paton factors

The starting point is the gauge fixed version of the Polyakov path integral

$$S^{open}_{j_1 \cdots j_n} = \int_{\mathbb{R}^{n-3}} dy_4 \cdots dy_n \, |y_{12} y_{13} y_{23}| \times \langle \mathcal{V}_{j_1}(\hat{y}_1) \mathcal{V}_{j_2}(\hat{y}_2) \mathcal{V}_{j_3}(\hat{y}_3) \cdots \mathcal{V}_{j_n}(y_n) \rangle Tr(\Lambda^{a_1} \cdots \Lambda^{a_n}) + (1 \leftrightarrow 2)$$

$y_{ij} = y_i - y_j$

One needs vertex operators associated to asymptotic states via the state-operator isomorphism

$$(L_0 - 1) \, |\phi\rangle = 0 \qquad L_1 \, |\phi\rangle = 0 \qquad L_2 \, |\phi\rangle = 0$$

Very difficult to recover the complete covariant solution in closed form

Open-String Vertex Operators

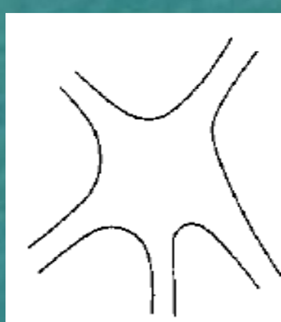
The key open-string oscillators are codified by the mode expansion

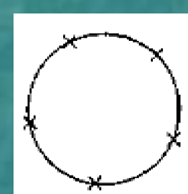
$$X^{\mu}(\tau,\sigma) \;=\; x^{\mu} \;+\; 2\alpha' p^{\mu}\tau \;+\; i\sqrt{2\alpha'}\,\sum_{n\neq 0}\frac{1}{n}\,\frac{\alpha^{\mu}_{n}}{z^{n}}$$

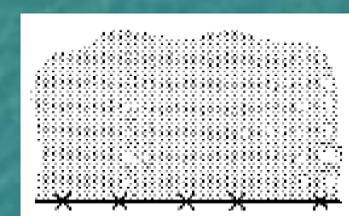
From it one recovers the state operator isomorphism

$$|0;p\rangle \;\sim\; :e^{ip\cdot X(0,0)}:$$

$$\alpha^{\mu}_{-m}|0\rangle \;\sim\; -\frac{i}{\sqrt{2\alpha'}}\,\partial^{m}X^{\mu}(0)\;, \quad m\geq 1$$







General polynomial in the oscillators:

$$|\phi\rangle \;=\; \sum \phi_{\mu_1\ldots\mu_s}\,\alpha^{\mu_1}_{-n_1}\cdots\alpha^{\mu_s}_{-n_s}\,|0;\,p\rangle \;\longrightarrow\; \phi(y) \;=\; \sum \phi_{\mu_1\ldots\mu_s}\,\partial^{n_1}X^{\mu_1}\cdots\partial^{n_s}X^{\mu_s}\,e^{ik\cdot X(y)}$$

Symbol Calculus

It is convenient to consider the symbols of the usual string oscillators

$$\alpha^{\mu}_{-n} \rightarrow \xi^{(n)}_{\mu}$$

In terms of symbols the usual vertex operator correlation function that enters into the definition of the S-matrix becomes

Symbol of the vertex operator

Generating Function

$$\left\langle \phi_1(y_1;p_1)\ldots\phi_n(y_n;p_n)\right\rangle = [\phi_1(p_1)\ldots\phi_n(p_n)] \star Z$$

We have defined the contraction

$$\star:\ \phi(\xi_1),\psi(\xi_2) \rightarrow \phi\star\psi = \exp\left(\frac{\partial}{\partial\xi_1}\cdot\frac{\partial}{\partial\xi_2}\right)\phi(\xi_1)\psi(\xi_2)\Big|_{\xi_i=0}$$

A useful integral representation is available

$$\phi\star\psi = \int\frac{d^d\xi}{(2\pi)^{d/2}}\,\tilde{\phi}(\xi)\,\psi(i\xi)$$

Generating Functions

$$\xi_i^{(1)\mu} \to \xi_i^{\mu}$$

Standard two-dimensional field theory generating function

$$Z[J] = i(2\pi)^d \pm^{(d)} (J_0)^{\mathcal{C}} \exp\left(-\frac{1}{2} \int d^2\sigma d^2\sigma' \, J(\sigma) \cdot J(\sigma') \, G(\sigma, \sigma') \right)$$

$$Z(\xi_i^{(n)}) \sim exp\Big(\sum \xi_i^{(n)} A_{ij}^{nm}(y_l) \, \xi_j^{(m)} + \xi_i^{(n)} \cdot B_i^n(y_l; p_l) + \alpha' \, p_i \cdot p_j \ln |y_{ij}|$$

For external states in the first Regge trajectory of the open string

$$\phi_i(p_i, \xi_i) = \frac{1}{n!} \, \phi_{i\,\mu_1 \cdots \mu_n} \, \xi_i^{\mu_1} \ldots \xi_i^{\mu_n}$$

$$Z = i(2\pi)^d \pm^{(d)} (J_0) \, \mathcal{C} \, exp \left[-\frac{1}{2} \sum_{i \neq j}^{n} \alpha' \, p_i \cdot p_j \ln |y_{ij}| - \sqrt{2a'} \, \frac{\xi_i \cdot p_j}{y_{ij}} + \frac{1}{2} \frac{\xi_i \cdot \xi_j}{y_{ij}^2} \right]$$

Three-point Amplitudes

The issue is now to impose the Virasoro constraints at the level of the generating function in order to select the physical information

L_0 constraint: mass parameterization for the first Regge trajectory

$$-p_1^2 = \frac{s_1 - 1}{\alpha'} \qquad -p_2^2 = \frac{s_2 - 1}{\alpha'} \qquad -p_3^2 = \frac{s_3 - 1}{\alpha'}$$

L_1 constraint: transversality

$$p_i \cdot \xi_i = 0$$

L_2 constraint: tracelessness

There is no term proportional to $\xi_i \cdot \xi_i$

At the end one obtains the physical generating function for three-point amplitudes

$$\mathbf{Z}_{phys} = i g_o \frac{(2\pi)^d}{\alpha'} \pm^{(d)}(p_1+p_2+p_3) \exp\left\{ \sqrt{\frac{\alpha'}{2}} \left(\xi_1 \cdot p_{23} \left\langle \frac{y_{23}}{y_{12} y_{13}} \right\rangle + \xi_2 \cdot p_{31} \left\langle \frac{y_{13}}{y_{12} y_{23}} \right\rangle + \xi_3 \cdot p_{12} \left\langle \frac{y_{12}}{y_{13} y_{23}} \right\rangle \right) + (\xi_1 \cdot \xi_2 + \xi_1 \cdot \xi_3 + \xi_2 \cdot \xi_3) \right\}$$

$$\langle x \rangle = \mathrm{sign}(x)$$

Three-point Amplitudes

The amplitudes are given by the star product of the physical generating function with the symbols of the vertex operators

$$[\phi_1(\xi_1)\,\phi_2(\xi_2)\,\phi_3(\xi_3)] \star \mathbf{Z}_{phys}$$

$$\phi_i(p_i;\xi_i) = \sum_{n=0}^{\infty} \frac{1}{n!}\ \phi_{i\,\mu_1\cdots\mu_n}\,\xi_i^{\mu_1}\ldots\xi_i^{\mu_n}$$

The result is

$$\mathcal{A} = i\,\frac{g_o}{\alpha'}\,(2\pi)^d \pm^{(d)}(p_1+p_2+p_3)\,\{\mathcal{A}_+(p_1,p_2,p_3)Tr\,[\Lambda^{a_1}\Lambda^{a_2}\Lambda^{a_3}] + \mathcal{A}_-(p_1,p_2,p_3)Tr\,[\Lambda^{a_2}\Lambda^{a_1}\Lambda^{a_3}]\}$$

Fourier transform with respect to the symbols

$$\mathcal{A}_\S\,(p_1,p_2,p_3) = \int \prod_{i=1}^{3} \frac{d^d\xi_i}{(2\pi)^{d/2}}\,\tilde{Z}_\S\,(p_1,p_2,p_3;\xi_1,\xi_2,\xi_3)\,\phi_1(p_1,i\xi_1)\,\phi_2(p_2,i\xi_2)\,\phi_3(p_3,i\xi_3)$$

Performing the integrations one arrives at

$$\mathcal{A}_\S = \phi_1\left(p_1,\ \frac{\partial}{\partial\xi}\ \S\ \sqrt{\frac{\alpha'}{2}}\,p_{31}\right)\phi_2\left(p_2,\ \xi + \frac{\partial}{\partial\xi}\ \S\ \sqrt{\frac{\alpha'}{2}}\,p_{23}\right)\phi_3\left(p_3,\ \xi\ \S\ \sqrt{\frac{\alpha'}{2}}\,p_{12}\right)\Bigg|_{\xi=0}$$

Four-point Amplitudes

In this case the amplitude is given by

$$[\phi_1(\xi_1)\,\phi_2(\xi_2)\,\phi_3(\xi_3)\,\phi_4(\xi_4)] \star \mathbf{Z}_{phys}$$

Again, leaving arbitrary the three fixed insertion positions, one arrives to a Mobius invariant result

$$\begin{aligned}\mathcal{A}^{(4)} &= i\frac{g_o}{\alpha'}(2\pi)^d \pm (p_1+p_2+p_3+p_4)\int_0^1 d\lambda \\ &\Big[(1-\lambda)^{-\alpha' t-2}\lambda^{-\alpha' u-2}\Big(\mathcal{A}^{(1)}_{+}(\lambda)\,Tr[\Lambda^{a_1}\Lambda^{a_4}\Lambda^{a_2}\Lambda^{a_3}] + \mathcal{A}^{(1)}_{-}(\lambda)\,Tr[\Lambda^{a_2}\Lambda^{a_4}\Lambda^{a_1}\Lambda^{a_3}]\Big) \\ &+(1-\lambda)^{-\alpha' s-2}\lambda^{-\alpha' t-2}\Big(\mathcal{A}^{(2)}_{+}(\lambda)\,Tr[\Lambda^{a_1}\Lambda^{a_2}\Lambda^{a_4}\Lambda^{a_3}] + \mathcal{A}^{(2)}_{-}(\lambda)\,Tr[\Lambda^{a_2}\Lambda^{a_1}\Lambda^{a_4}\Lambda^{a_3}]\Big) \\ &+(1-\lambda)^{-\alpha' u-2}\lambda^{-\alpha' s-2}\Big(\mathcal{A}^{(3)}_{+}(\lambda)\,Tr[\Lambda^{a_4}\Lambda^{a_1}\Lambda^{a_2}\Lambda^{a_3}] + \mathcal{A}^{(3)}_{-}(\lambda)\,Tr[\Lambda^{a_4}\Lambda^{a_2}\Lambda^{a_1}\Lambda^{a_3}]\Big)\Big]\end{aligned}$$

$$\mathcal{A}^{(i)}_{\S}(\lambda) = [\phi_1(\xi_1)\,\phi_2(\xi_2)\,\phi_3(\xi_3)\,\phi_4(\xi_4)] \star \mathbf{Z}^{(i)}_{\S}(\xi_1,\xi_2,\xi_3,\xi_4;\, p_1,p_2,p_3,p_4)$$

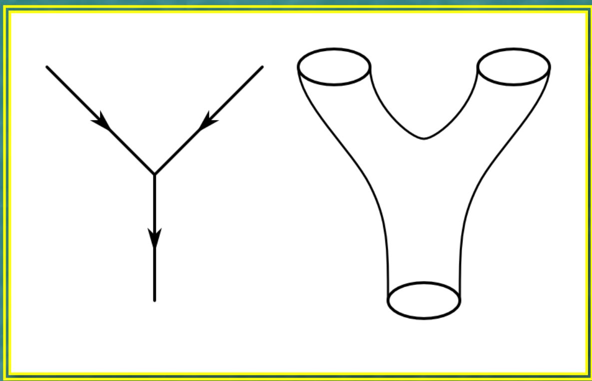
Label the various non cyclic orderings

One can proceed in analogy with the three-point case

$$\begin{aligned}\mathcal{A}^{(1)}_{\S}(\lambda) &= \exp\Big\{\Big[\lambda(1-\lambda)\,(\partial_{\xi_1}\cdot\partial_{\xi_2}+\partial_{\xi_3}\cdot\partial_{\xi_4}) + \lambda\,(\partial_{\xi_1}\cdot\partial_{\xi_3}+\partial_{\xi_2}\cdot\partial_{\xi_4}) + (1-\lambda)\,(\partial_{\xi_1}\cdot\partial_{\xi_4}+\partial_{\xi_2}\cdot\partial_{\xi_3})\Big]\Big\} \\ &\times \phi_1\Big(p_1,\ \xi_1\ \S\ \sqrt{2\alpha'}\,(-p_3\,\lambda + p_4\,(1-\lambda)\Big)\ \phi_2\Big(p_2,\ \xi_2\ \S\ \sqrt{2\alpha'}\,(+p_3\,(1-\lambda) - p_4\,\lambda)\Big) \\ &\times \phi_3\Big(p_3,\ \xi_3\ \S\ \sqrt{2\alpha'}\,(+p_1\,\lambda - p_2\,(1-\lambda)\Big)\ \phi_4\Big(p_4,\ \xi_4\ \S\ \sqrt{2\alpha'}\,(-p_1\,(1-\lambda) + p_2\,\lambda)\Big)\Big|_{\xi_i=0}\end{aligned}$$

Currents

The three-point amplitudes computed can be easily turned into couplings ➡ $\mathcal{J} \star \phi$



The general expression for the current is

$$\mathcal{J} = i\frac{g_o}{\alpha'}\left\{J_+\, Tr[\,\cdot\,\Lambda^{a_1}\Lambda^{a_2}] + J_-\, Tr[\,\cdot\,\Lambda^{a_2}\Lambda^{a_1}]\right\}$$

$$j_\S = \int \frac{d^d\xi_2}{(2\pi)^{d/2}}\frac{d^d\xi_3}{(2\pi)^{d/2}}\; \tilde{\mathbf{Z}}_\S\,(p_1,p_2,p_3;\xi,\xi_2,\xi_3)\,\phi_2(p_2,i\xi_2)\,\phi_3(p_3,i\xi_3)$$

$$J_\S\,(\xi) = \phi_1\left(x\,\ddot{}\;\; i\sqrt{\frac{\alpha'}{2}}\,\xi,\ \partial_\zeta + \xi\,\ddot{}\;\;\sqrt{2\alpha'}\;\overrightarrow{\partial}_2\right)\phi_2\left(x\,\S\; i\sqrt{\frac{\alpha'}{2}}\,\xi,\ \zeta + \xi\,\S\;\sqrt{2\alpha'}\;\overleftarrow{\partial}_1\right)$$

These currents are NOT conserved! But ...

Perturbative Noether Procedure

$$S[\phi] = \sum S^{(0)}[\phi_{\mu_1 \dots \mu_s}] + \epsilon S^{(1)}[\phi_{\mu_1 \dots \mu_s}] + O\left(\epsilon^2\right)$$

At least one spin greater then 2!

$$\pm_\xi \, \phi_{\mu_1 \dots \mu_s} = \pm_\xi^{(0)} \phi_{\mu_1 \dots \mu_s} + \epsilon \pm_\xi^{(1)} \phi_{\mu_1 \dots \mu_s} + O\left(\epsilon^2\right)$$

Redefinitions of fields and gauge parameters

$$\begin{aligned} \phi_{\mu_1 \dots \mu_s} &\rightarrow \phi_{\mu_1 \dots \mu_s} + \epsilon f(\phi)_{\mu_1 \dots \mu_s} + O\left(\epsilon^2\right) , \\ \xi_{\mu_1 \dots \mu_{s-1}} &\rightarrow \xi_{\mu_1 \dots \mu_{s-1}} + \epsilon \zeta(\phi, \xi)_{\mu_1 \dots \mu_{s-1}} + O\left(\epsilon^2\right) , \end{aligned}$$

$$A_\mu \, \partial_\nu \, (\bar{\psi} \circ^{\mu\nu} \psi)$$

It does not deform the abelian gauge symmetry

$$A_\mu \, \bar{\psi} \circ^{\mu} \psi$$

It deforms the abelian gauge symmetry

0-0-s Couplings

In this case the current generating function is conserved and is given by

$$\mathcal{J} = i\frac{g_o}{\alpha'}\left\{J_+(x,\xi)\,Tr[\,\cdot\,\Lambda^{a_1}\Lambda^{a_2}] + J_-(x,\xi)\,Tr[\,\cdot\,\Lambda^{a_2}\Lambda^{a_1}]\right\}$$

coordinate space

$$J_{\S}(x,\xi) = \Phi\left(x\ \S\ i\sqrt{\frac{\alpha'}{2}}\,\xi\right)\Phi\left(x\ \ddot{}\ \ i\sqrt{\frac{\alpha'}{2}}\,\xi\right)$$

Berends, Burgers and Van Dam, (1986)

Bekaert, Joung, Mourad, (2009)

In this case it is possible to compute the coupling for every physical state in the spectrum showing that only the totally symmetric states couple with the tachyon

1-1-s Couplings

Simplest non trivial example!

This coupling can be extracted from our general result and is given by

$$\mathcal{A}^{\S}_{s-1-1} = \left(\S\sqrt{\frac{\alpha'}{2}}\right)^{s-2} s(s-1)\, A_{1\,\mu}\, A_{2\,\nu}\, \phi^{\mu\nu\cdots} p_{12}^{\,s-2}$$
$$+ \left(\S\sqrt{\frac{\alpha'}{2}}\right)^{s} \left[A_1 \cdot A_2\, \phi \cdot p_{12}^{\,s} + s A_1 \cdot p_{23}\, A_{2\,\nu}\, \phi^{\nu\cdots} p_{12}^{\,s-1} + s A_2 \cdot p_{31}\, A_{1\,\nu}\, \phi^{\nu\cdots} p_{12}^{\,s-1}\right]$$
$$+ \left(\S\sqrt{\frac{\alpha'}{2}}\right)^{s+2} A_1 \cdot p_{23}\, A_2 \cdot p_{31}\, \phi \cdot p_{12}^{\,s}$$

The on-shell gauge variation with respect to the spin-1 external legs vanishes identically

On the contrary the gauge variation with respect to the spin-s external leg gives

$$\pm H = p_3\, \Lambda$$

$$\pm\mathcal{A}^{\S}_{s-1-1} = \left(\S\sqrt{\frac{\alpha'}{2}}\right)^{s-2} \frac{s(s-1)}{2} \left[A_2 \cdot p_{31}\, A_{1\,\mu}\, \Lambda^{\mu\cdots}\, p_{12}^{\,s-2} - A_1 \cdot p_{23}\, A_{2\,\mu}\, \Lambda^{\mu\cdots} \cdot p_{12}^{\,s-2}\right]$$

$$\sim\; p_i \cdot p_j \; ?$$

To recover the massless limit one should take care of such hidden subleading terms

1-1-s Couplings

In this case we can try a simple route! ⇒ Parameterize the amplitude!

The result is

$$\mathcal{A}^{\S}_{s-1-1} = 2\left(\S\sqrt{\frac{\alpha'}{2}}\right)^{s+2}\left(F^{2}\right)\,\phi\cdot p_{12}^{\,s} - 4s\left(\S\sqrt{\frac{\alpha'}{2}}\right)^{s}\left(F^{2}\right)_{\mu\nu}\phi^{\mu\nu\cdots}\,p_{12}^{\,s-2}$$

Part of the spin-1 energy momentum tensor

Traceless on-shell

It is gauge invariant on-shell with respect to the spin-s leg in the massless limit!

This term gives rise to the non zero gauge variation with respect to the spin-s leg

$$\begin{aligned}\mathcal{A}^{\S}_{s-1-1} = &\left(\S\sqrt{\frac{\alpha'}{2}}\right)^{s-2} s\,(-\alpha'\,p_3)\,A_{1\,\mu}A_{2\,\nu}\,\phi^{\mu\nu\cdots}\,p_{12}^{\,s-2}\\ &+\left(\S\sqrt{\frac{\alpha'}{2}}\right)^{s}\Big[(s+\alpha' p_3)A_1\cdot A_2\,\phi\cdot p_{12}^{\,s} + sA_1\cdot p_{23}\,A_{2\,\nu}\,\phi^{\nu\cdots}\,p_{12}^{\,s-1} + sA_2\cdot p_{31}\,A_{1\,\nu}\,\phi^{\nu\cdots}\,p_{12}^{\,s-1}\Big]\\ &+\left(\S\sqrt{\frac{\alpha'}{2}}\right)^{s+2} A_1\cdot p_{23}\,A_2\cdot p_{31}\,\phi\cdot p_{12}^{\,s}\end{aligned}$$

$$-p_1^3 = \frac{1}{\alpha'}(s-1)$$

$$-p_1^2 = -p_2^2 = 0$$

1-1-s Couplings

The same result could be recovered without any explicit reference to some parameterization!

Reconstruct the tail of the couplings starting from the highest derivative one!

Start from

$$\left(\S\sqrt{\frac{\alpha'}{2}}\right)^{s+2} A_1 \cdot p_{23}\, A_2 \cdot p_{31}\, \phi \cdot p_{12}^{s}$$

Compute the gauge variation

$$\left(\S\sqrt{\frac{\alpha'}{2}}\right)^{s+3} \left[p_1 \cdot p_{23}\, \Lambda_1\, A_2 \cdot p_{31}\, \phi \cdot p_{12}^{s} + \ldots\right]$$

Find a counter term that cancels this contribution

$$\left(\S\sqrt{\frac{\alpha'}{2}}\right)^{s} \left[\alpha' p_3^2\, A_1 \cdot A_2\, \phi \cdot p_{12}^{s} + \ldots\right]$$

Continue until no more contributions arise

Disentangle this tail from the string coupling

$$m^2 \to \Box$$

Massless limit of the Theory

The procedure outlined in the previous example can be implemented for the general s_1-s_2-s_3 coupling!

$$m^2 \rightarrow \Box$$

The result is:

$$\begin{aligned}\mathcal{A}_{\S} \; &= \exp\left\{\sqrt{\frac{\alpha'}{2}}\Big[(\partial_{\xi_1}\cdot\partial_{\xi_2})(\partial_{\xi_3}\cdot p_{12}) + (\partial_{\xi_2}\cdot\partial_{\xi_3})(\partial_{\xi_1}\cdot p_{23}) + (\partial_{\xi_3}\cdot\partial_{\xi_1})(\partial_{\xi_2}\cdot p_{31})\Big]\right\} \\ &\times\, \phi_1\left(p_1;\, \xi_1 + \sqrt{\frac{\alpha'}{2}}\,p_{23}\right)\phi_2\left(p_2;\, \xi_2 + \sqrt{\frac{\alpha'}{2}}\,p_{31}\right)\phi_3\left(p_3;\, \xi_3 + \sqrt{\frac{\alpha'}{2}}\,p_{12}\right)\Bigg|_{\xi_i=0}\end{aligned}$$

This amplitude is gauge invariant up to linearized massless equations of motion and uniquely determines its off-shell completion up to partial integrations and field redefinitions

Simplest non trivial operator commuting with: $p_i \cdot \xi_i$

Higher-Spin Conserved Currents

The couplings so far obtained are induced by noether currents

$$J \star \phi$$

$$J(x\,;\,\xi) \;=\; \exp\left(-i\sqrt{\frac{\alpha'}{2}}\,\xi_{\alpha}\left[\partial_{\zeta_1}\cdot\partial_{\zeta_2}\,\partial^{\alpha}_{12} - 2\,\partial^{\alpha}_{\zeta_1}\,\partial_{\zeta_2}\cdot\partial_1 + 2\,\partial^{\alpha}_{\zeta_2}\,\partial_{\zeta_1}\cdot\partial_2\right]\right)$$
$$\times\;\phi_1\left(x - i\sqrt{\frac{\alpha'}{2}}\,\xi\,,\,\zeta_1 - i\sqrt{2\alpha'}\,\partial_2\right)\,\phi_2\left(x + i\sqrt{\frac{\alpha'}{2}}\,\xi\,,\,\zeta_2 + i\sqrt{2\alpha'}\,\partial_1\right)\Bigg|_{\zeta_i=0}$$

Again they are conserved up to massless Klein-Gordon equation, divergences and traces, but their completion turns to be completely fixed!

$$\mathcal{A}_{\S} \;=\; \exp\left(\sqrt{\frac{\alpha'}{2}}\,\left[(\partial_{\xi_1^1}+\ldots+\partial_{\xi_1^n})\cdot(\partial_{\xi_2^1}+\ldots+\partial_{\xi_2^n})\right]\left[(\partial_{\xi_3^1}+\ldots+\partial_{\xi_3^n})\cdot p_{12}\right]\;+\;\text{cyclic}\right)$$
$$\phi\,1\left(p_1;\,\xi_1^k+\sqrt{\frac{\alpha'}{2}}\,p_{23}\right)\phi_2\left(p_2;\,\xi_2^k+\sqrt{\frac{\alpha'}{2}}\,p_{31}\right)\phi_3\left(p_3;\,\xi_3^k+\sqrt{\frac{\alpha'}{2}}\,p_{12}\right)\Bigg|_{\xi_i=0}$$

Current Exchanges

The propagators propagate only irreducible degrees of freedom

$$\mathcal{P}^{(s)}_{\mu_1\cdots\mu_s;\nu_1\cdots\nu_s} = -\frac{1}{p^2+M^2} P_{\mu_1\cdots\mu_s;\nu_1\cdots\nu_s}$$

It must project onto the traceless-transverse part

same trick: use the symbols

$$\mathcal{P}^{(s)}(\xi^2\lambda^2, \xi\cdot\lambda)$$

Homogeneous symmetric polynomial of degree m for totally symmetric tensors

Massless case: only traceless projector

Trace operator ⟷ $\partial_\xi \cdot \partial_\xi$

(Laplace Operator)

The propagator is associated to a harmonic polynomial

The result for massless totally symmetric fields is

$$\mathcal{P}^{(s)}(\xi,\lambda) = -\frac{1}{p^2}\left\{K \left(\xi^2\lambda^2\right)^{s/2} f_s^{[\frac{d}{2}-2]}\left(\frac{\xi\cdot\lambda}{\sqrt{\xi^2\lambda^2}}\right)\right\}$$

$$f_s^{[\frac{d}{2}-2]}(x) = G_s^{[\frac{d}{2}-2]}(x) \quad d>4$$

$$f_s^{[\frac{d}{2}-2]}(x) = T_s(x) \quad d=4$$

Generating functions of exchanges

Generating function techniques give the possibility to sum an infinite number of propagators with arbitrary coupling constants

$$\widehat{\mathcal{P}} = \alpha' \int_0^1 dy\, y^{-\alpha' s-2} \left[a\left(\frac{y}{2}\, \xi \cdot \lambda + \frac{y}{2} \sqrt{(\xi \cdot \lambda)^2 - \xi^2 \lambda^2}\right) + a\left(\frac{y}{2}\, \xi \cdot \lambda - \frac{y}{2} \sqrt{(\xi \cdot \lambda)^2 - \xi^2 \lambda^2}\right) - a_0 \right]$$

Massless HS in D=4 if y=1

Massive HS in D=3 (First Regge trajectory multiplet)

The result in arbitrary D is very complicated and simplifies only for some particular choices of the coupling constants

$$a(t) = \frac{1}{(1-t)^\alpha} = \sum_{n=0}^{\infty} \frac{1}{n!} \frac{\Gamma(\alpha+n)}{\Gamma(\alpha)}\, t^n$$

$$\mathcal{P} = -\frac{1}{p^2}\left(1 - \xi \cdot \lambda + \frac{\xi^2 \lambda^2}{4}\right)^{-\alpha}$$

Massless HS in D=2 α+4

$$\mathcal{P} = \alpha' \int_0^1 dy\, y^{-\alpha' s-2}\left(1 - \xi \cdot \lambda\, y + \frac{\xi^2 \lambda^2 y^2}{4}\right)^{-\alpha}$$

$$a(z) = \sum_{n=0}^{\infty} \frac{1}{n!}\, a_n z^n$$

Coupling function (Bekaert, Joung, Mourad, 2009)

Massive HS in D=2 α+3 (First Regge trajectory multiplet)

Scattering Amplitudes

One can use the string currents to compute scattering amplitudes with exchanges of infinitely many higher-spin particles

$$J_{\S} = \exp\left(\S\sqrt{\frac{\alpha'}{2}}\,p_{12}\cdot\xi\right)\phi_1\left(p_1,\S\sqrt{2\alpha'}\,p_2\right)\phi_2\left(p_2,\ddot{}\sqrt{2\alpha'}\,p_1\right)$$

s-channel

$$\mathcal{A}^{(s)} = \alpha'\int_0^1 dy\, y^{-\alpha' s-2}\left[a\left(\frac{\alpha' y}{4}(u-t)+\frac{\alpha' y}{2}\sqrt{-ut}\right)+a\left(\frac{\alpha' y}{4}(u-t)-\frac{\alpha' y}{2}\sqrt{-ut}\right)-a_0\right]$$
$$\times\,\phi_1\left(p_1,\S\sqrt{2\alpha'}\,p_2\right)\phi_2\left(p_2,\ddot{}\sqrt{2\alpha'}\,p_1\right)\phi_3\left(p_1,\S\sqrt{2\alpha'}\,p_4\right)\phi_4\left(p_2,\ddot{}\sqrt{2\alpha'}\,p_3\right)$$

Similar results for the other currents, in higher dimensions and for massive particles

Outlook

We considered totally symmetric spin-s fields belonging to the first Regge trajectory of the open bosonic string:

- All three-point couplings have been found (both abelian and non-abelian!)
- Signals of non-abelian gauge symmetry at the cubic order
- All four-point amplitudes have been computed
- Higher-spin four-point amplitude computation on the Field theory side (infinitely many particles propagating)
- Any reference to the mechanical model has been completely eliminated

Very enticing: dissect the Veneziano amplitude and confront with the field theory result

Quartic coupling computation

Moreover:

- Similar analysis for mixed-symmetry fields of the spectrum (work in progress)

$$|\phi\rangle_{(s_1,\ldots,s_n)} = \phi_{\mu^1_1\ldots\mu^1_{s_1},\ldots,\mu^1_n\ldots\mu^n_{s_n}}\,\alpha^{\mu^1_1}_{-1}\ldots\alpha^{\mu^1_{s_1}}_{-1}\ \ldots\alpha^{\mu^n_1}_{-n}\ldots\alpha^{\mu^n_{s_n}}_{-n}\,|0;\,p\rangle$$