Stringy Instanton Corrections
To Gauge Couplings

Alberto Lerda

U.P.O. *"A. Avogadro"* & I.N.F.N. - Alessandria

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Foreword

- This talk builds over a rather vast (± recent) literature (apologies for missing references). For a review, see

  R. Blumenhagen, M. Cvetic, S. Kachru and T. Weigand, "*D-brane Instantons in Type II String Theory*," arXiv:0902.3251 [hep-th].

- The results presented here mostly come from

  M. Billò, M. Frau, F. Fucito, A. L., J. F. Morales and R. Poghosyan, "*Stringy instanton corrections to $\mathcal{N} = 2$ gauge couplings*," JHEP **1005** (2010) 107 , arXiv:1002.4322 [hep-th]

- Previous results are contained in

  M. Billò, L. Ferro, M. Frau, L. Gallot, A. L. and I. Pesando, "*Exotic instanton counting and heterotic/type I' duality*," JHEP **0907** (2009) 092 , arXiv:0905.4586 [hep-th]

Plan of the talk

1. Introduction and motivations
2. The set-up
3. D-instanton effects
4. Conclusions and perspectives

Introduction and motivations

- The possibility of acquiring control over non-perturbative effects has been a unifying theme behind many developments in string theory.
- Recently, there has been a growing interest in the effects induced by D-instantons or, more generally, by Euclidean D-branes (E-branes):
  - They allow to reproduce the (standard) instanton calculus in string theory

    Polchinski, 1994; ...; Green+Gutperle, 2000; ...; Billò et al. 2002; ...
  - They may give rise to non-perturbative couplings that are forbidden in perturbation theory but necessary for phenomenological applications (neutrino masses, Yukawa couplings, ...)

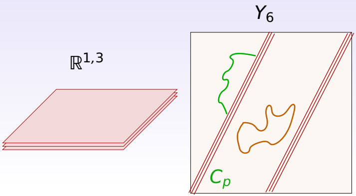
    Blumenhagen et al, 2006; Ibanez + Uranga, 2006; ...
- Like instantons in gauge theories, also the instantonic branes lead to a particularly tractable class of non-perturbative phenomena in string models.

D-brane worlds

Our set-up will be that of "D-brane worlds":

- SM-like gauge sector from open strings on stacks of space-filling D($3+p$) branes wrapped on some internal p-cycle C_p

- Gravitational sector from closed strings propagating in the bulk



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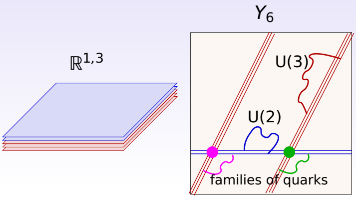


- Gravitational sector from closed strings propagating in the bulk

- Gauge groups from multiple branes, bifundamental chiral matter from “twisted” strings, replicas from multiple intersections

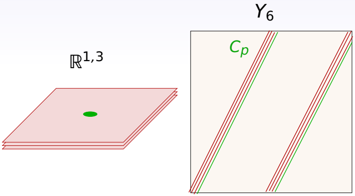
see, e.g., Uranga, 2003; Kiritsis, 2004; Lust, 2004; Blumenhagen et al., 2005;...

- (String) topology of the internal space + choice of branes lead to very rich model building scenarios

Instanton effects in brane-worlds

- In brane-world models, instantons are engineered with E(uclidean) branes, *i.e.* D-branes that are point-like in the 4d space-time and are totally wrapped in the internal Y_6 space.

- First possibility:
  - The gauge and the instantonic branes wrap the SAME internal cycle in Y_6.



  - These are the usual gauge instantons.

    Their effects are suppressed by

$$e^{-\frac{V(C_p)}{g_s}} = e^{-\frac{8\pi^2}{g_{YM}^2}} .$$

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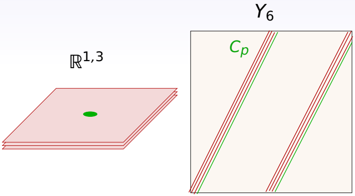

  - These are the usual gauge instantons.
    - ADHM moduli from strings attached to the instantonic branes
      Witten, 1995; Douglas, 1995-1996; ...
    - Non-trivial instanton profile of the gauge field
      Billo et al., 2002
    - Techniques to embed the instanton calculus in string theory have been developed
      Polchinksi, 1994; Green-Gutperle, 2000...; Turin/Rome/Münich/UPenn/Madrid groups...

Instanton effects in brane-worlds

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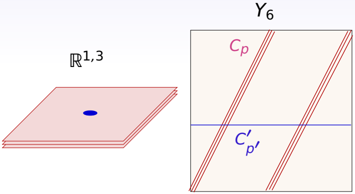
- Second possibility:
  - The gauge and the instantonic branes wrap the DIFFERENT cycles in Y_6.



  - These are the so-called exotic or stringy instantons.

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$$e^{-\frac{V'(C'_p)}{g_s}} = e^{-\frac{V(C_p)}{g_s}\frac{V'(C'_p)}{V(C_p)}} = e^{-\frac{8\pi^2}{g^2_{YM}}\frac{V'(C'_p)}{V(C_p)}} \neq e^{-\frac{8\pi^2}{g^2_{YM}}} .$$

Instanton effects in brane-worlds

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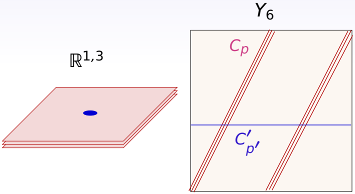


  - These are the so-called exotic or stringy instantons.
    - ⋆ Exotic instantons may lead to interactions that are perturbatively forbidden but of great phenomenological relevance (neutrino Majorana masses, Yukawas in certain GUT models,. . .).
    - ⋆ Need to understand their status in 4d gauge theories and to construct precise rules for the "exotic" instanton calculus.

Our strategy

- Stringy computational techniques for ordinary instantonic branes reproduce the standard instanton calculus

  Billò et al. 2002; ...

- Same kind of techniques techniques should extend to exotic instantonic branes

- Our strategy to test this assumption is to select a set-up such that:

  - exotic instantonic branes contribute to the gauge effective action

  - all exotic instanton numbers contribute to gauge couplings (like for ordinary instantons in $\mathcal{N} = 2$ SYM)

  - the theory possesses a computable heterotic dual, so that the results of the exotic calculus can be tested against it

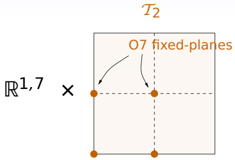
A 4d model

- We start from Type I', namely Type IIB on a two-torus $\mathcal{T}_2$ modded out by

$$\Omega = \omega\,(-1)^{F_L}\,I_2$$

ω = w.-s. parity, F_L = left-moving fermion #, I_2 = inversion on $\mathcal{T}_2$

- There are 4 O7 fixed-planes



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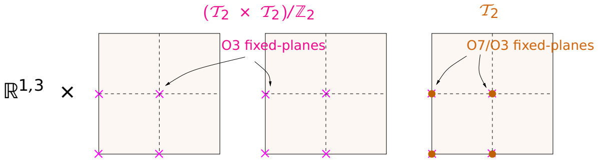
- Then we compactify Type I' on $K3 \sim (\mathcal{T}_2 \times \mathcal{T}_2)/\mathbb{Z}_2$



- There are 64 O3 fixed-planes + 4 O7 fixed-planes.

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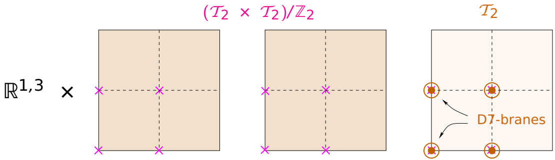
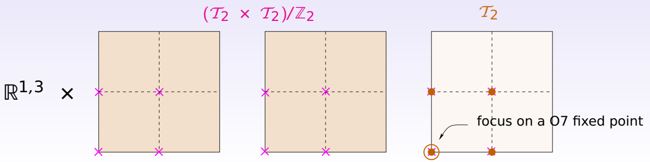


- Local tadpole cancellation requires to put 4 D7-branes plus their images at each O7 fixed-plane.

A 4d model

- Let us focus on one of the O7 planes where we have 4 D7-branes + their orientifold images

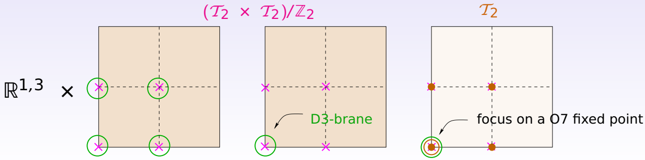


- The orientifold action on the Chan-Paton factors implies that the gauge group on the D7's is U(4) ($\hookrightarrow$ SO(8))
- The D7/D7 strings organize in 1 adjoint $\mathcal{N} = 2$ vector multiplet + 2 antisymm. hypers of U(4)
- The U(4) gauge coupling is

$$g_{\rm YM}^2 \sim \frac{g_s}{Vol(\mathcal{T}_2 \times \mathcal{T}_2)}$$

A 4d model

- O3 tadpole cacellation requires 4 D3-branes plus their images, to be distributed on the various O3 fixed points.



- We place 4 D3's at 4 distinct O3 fixed points on top of the chosen D7 stack
- The D7/D3 strings give rise to 4 hypers in the fundamental of U(4).
- This theory is conformal: for the SU(4) part, the 1-loop β-function coefficient is

$$b_1 \propto (4 - m) \ = 0 \quad \text{with } m = 4 \text{ fundamental hypers}$$

A 4d model

Summary:

- On each stack of D7's we have a conformal $\mathcal{N} = 2$ U(4) gauge theory containing:
  - 1 adjoint vector mult. + 2 antisymm hypers (→ D7/D7 strings)
  - 4 fundamental hypers (→ D7/D3 strings)

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Summary:

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  - 1 adjoint vector mult. + 2 antisymm hypers (→ D7/D7 strings)
  - 4 fundamental hypers (→ D7/D3 strings)
- The quadratic effective action for the gauge fields can be written using holomorphic couplings f_{ab} as

$$S = \int d^4x \left\{ \mathrm{Re}\, f_{ab} \mathrm{Tr}\left(F^a_{\mu\nu} F^{b\mu\nu}\right) + \mathrm{Im}\, f_{ab} \mathrm{Tr}\left(F^a_{\mu\nu} {}^* F^{b\mu\nu}\right)\right\}$$

- In terms of the $\mathcal{N}=2$ U(4) vector multiplet Φ

$$\Phi(x,\theta) = \varphi(x) + \theta^\alpha \Lambda_\alpha(x) + (\theta\gamma^{\mu\nu}\theta) F_{\mu\nu}(x) + \dots ,$$

we have two color structures

$$S = \int d^4x d^4\theta \left\{ f\, \mathrm{Tr}\left(\Phi^2\right) + f' \left(\mathrm{Tr}\, \Phi\right)^2 \right\} + \mathrm{c.c.}$$

Effective action

- The holomorphic couplings f and f' have tree-level, 1-loop and non-perturbative terms:

$$\text{Single trace} \quad : \quad f = f_{(0)} + f_{(1)} + f_{\text{n.p.}}$$

$$\text{Double trace} \quad : \quad f' = f'_{(0)} + f'_{(1)} + f'_{\text{n.p.}}$$

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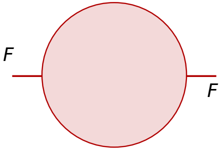
- At tree level:



$\Rightarrow$

- $$f_{(0)} = \frac{4\pi}{g^2_{\text{YM}}} - \mathrm{i}\,\frac{\theta_{\text{YM}}}{2\pi} \; = \; -\mathrm{i}\,t$$
$$= \frac{Vol(\mathcal{T}_2 \times \mathcal{T}_2)}{g_s} - \mathrm{i} \int_{\mathcal{T}_2 \times \mathcal{T}_2} C_4$$
- $$f'_{(0)} = 0$$

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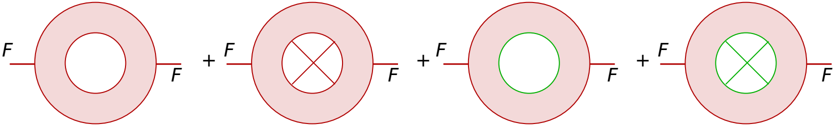
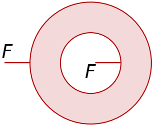
- At 1-loop:



$= 0$



$\neq 0 \quad \rightarrow (\text{Tr}(F)^2)$

- $f_{(1)} = 0$
- $f'_{(1)} = -8 \log \left(\eta(u)\right)^2$

  u : complex structure of $\mathcal{T}_2$

Effective action

- The holomorphic couplings f and f' have tree-level, 1-loop and non-perturbative terms:

$$\text{Single trace} \quad : \quad f = f_{(0)} + f_{(1)} + f_{\text{n.p.}}$$

$$\text{Double trace} \quad : \quad f' = f'_{(0)} + f'_{(1)} + f'_{\text{n.p.}}$$

- At the non-perturbative level:

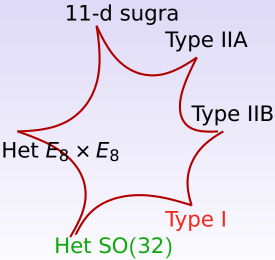
How do we compute

$f_{\text{n.p.}}$ and $f'_{\text{n.p.}}$?

Intermezzo

Heterotic dual

- Our type I' model has a computable heterotic dual: the U(16) compactification of the SO(32) heterotic string on $\mathcal{T}_2 \times (\mathcal{T}_2 \times \mathcal{T}_2)/\mathbb{Z}_2$ plus Wilson lines on $\mathcal{T}_2$ breaking U(16) to $U(4)^4$



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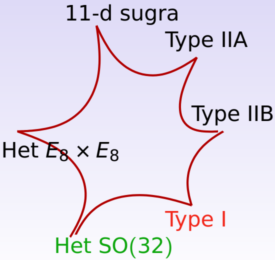

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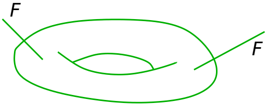

- The holomorphic gauge couplings for each U(4) factor can be derived from a protected 1-loop threshold computation. This was not present in the literature, so we carried it out finding:

$$f^H_{(1)} = 8\log\left(\frac{\eta\left(\frac{T}{4}\right)^2}{\eta\left(\frac{T}{2}\right)^2}\right) \quad , \quad f'^H_{(1)} = 8\log\left(\frac{\eta\left(\frac{T}{2}\right)^2}{\eta(U)^2\eta\left(\frac{T}{4}\right)^4}\right)$$

where T and U are the Kähler and complex structures of $\mathcal{T}_2$.

Type I' / Heterotic duality

- Type I' variables : $\big(\tau\ ,\ u\big)$ $\quad(\tau = C_0 + i/g_s$ = axio-dilaton)
- Heterotic variables : $\big(T\ ,\ U\big)$
- Type I' / Heterotic map: $\quad \tau \longleftrightarrow T/4 \quad , \quad u \longleftrightarrow U$

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- Type I' / Heterotic map: $\quad \tau \longleftrightarrow T/4 \quad , \quad u \longleftrightarrow U$
- Expanding the heterotic threshold coefficients for $T \to \infty$, we have

$$f^{H}_{(1)} = 8\log\left(\frac{\eta\left(\frac{T}{4}\right)^2}{\eta\left(\frac{T}{2}\right)^2}\right) \simeq \cdots - 16\sum_{k=1}^{\infty}\sum_{d|k}\frac{1}{d}\left(e^{2\pi i k\frac{T}{4}} - e^{2\pi i k\frac{T}{2}}\right)$$

$$f'^{H}_{(1)} = 8\log\left(\frac{\eta\left(\frac{T}{2}\right)^2}{\eta(U)^2\eta\left(\frac{T}{4}\right)^4}\right) \simeq \cdots - 16\sum_{k=1}^{\infty}\sum_{d|k}\frac{1}{d}\left(e^{2\pi i k\frac{T}{2}} - 2\,e^{2\pi i k\frac{T}{4}}\right)$$

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- Heterotic variables : $\left(T\,,\ U\right)$
- Type I' / Heterotic map: $\tau \longleftrightarrow T/4$, $u \longleftrightarrow U$
- In Type I' language we have

$$f^{H}_{(1)} = 8\log\left(\frac{\eta\left(\tau\right)^2}{\eta\left(2\tau\right)^2}\right) \simeq \cdots - 16\sum_{k=1}^{\infty}\sum_{d|k}\frac{1}{d}\left(\mathrm{e}^{2\pi i k\tau} - \mathrm{e}^{4\pi i k\tau}\right)$$

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- In Type I' language we have

$$f^{H}_{(1)} = \cdots - 16\,q^2 - 8\,q^4 + \cdots$$

$$f'^{H}_{(1)} = \cdots + 32\,q^2 + 32q^4 + \cdots$$

where $q = \mathrm{e}^{i\pi\tau} \neq \mathrm{e}^{-\frac{8\pi^2}{g_{\mathrm{YM}}}+i\theta_{\mathrm{YM}}}$ $\longrightarrow$ exotic instanton expansion

Back to non-perturbative corrections

Type I' non-perturbative corrections
from D-instantons

- We focus on the D-instanton contributions

Billo et al 2010

  - Work in progress on the E3 sectors

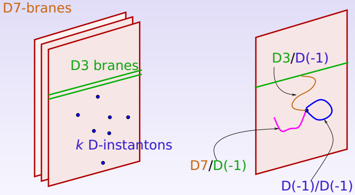
- The D(-1)'s correspond to exotic instantons w.r.t. to the D7 gauge theory. Their effects are weighted by

$$q^k = e^{-k\, S_{D(-1)}} \sim e^{-k\,\frac{\pi}{g_s}} \sim e^{-k\,\frac{8\pi^2}{g_{\rm YM}^2\, Vol(\mathcal{T}_2\times\mathcal{T}_2)}}$$

which is not the usual gauge istanton factor $e^{-k\,\frac{8\pi^2}{g_{\rm YM}^2}}$.

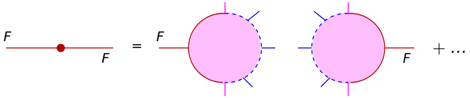
This is the right weight expected from the duality with the heterotic theory.

Effective action from D-instantons



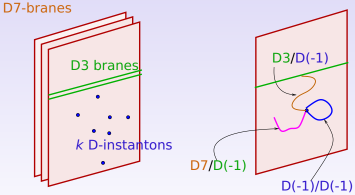
- Open strings with at least one end on a D(-1) carry no momentum: they are moduli rather than dynamical fields.

- Effective interactions between gauge fields (encoded in Φ) can be mediated by D-instanton moduli through mixed disks



connected by integration over the instanton moduli $\mathcal{M}_{(k)}$

Effective action from D-instantons



- Open strings with at least one end on a D(-1) carry no momentum: they are moduli rather than dynamical fields.

- We must sum over D(-1) conf.s and instanton #'s k and compute

$$\sum_{conf.s} \sum_{k} q^k \int d\mathcal{M}_{(k)}\, e^{-\mathcal{S}(\mathcal{M}_{(k)},\Phi)}$$

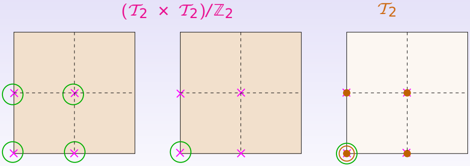
  - $\mathcal{S}(\mathcal{M}_{(k)}, \Phi)$ arises from (mixed) disk diagrams describing interactions of the moduli among themselves and with the gauge fields

- From this we should extract the n.p. effective action in the form

$$S_{n.p.}(\Phi) = \int d^4x\, d^4\theta\, \mathcal{F}_{n.p.}(\Phi)$$

Adding D-instantons

- Let us add k D-instantons:



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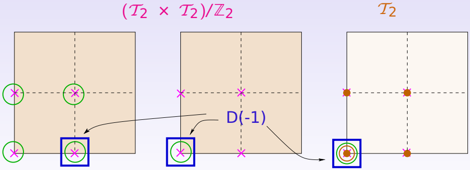

- In this case the D(-1)'s are both on top of the D7's and of the D3's.

$\Rightarrow$ both D(-1)/D7 and D(-1)/D3 strings have massless modes.

Adding D-instantons

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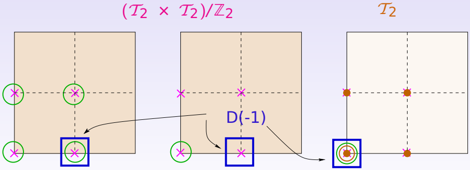

- In this case D(-1)'s are on top of the D7's but not on the D3's.

$\Rightarrow$ only D(-1)/D7 strings have massless modes.

- The fermionic D(-1)/D7 mixed moduli are always present (only fermionic: typical of exotic instantons)
- The bosonic and fermionic D(-1)/D3 mixed moduli can or cannot be present.

Computing D-instanton effects

- We face a very complicated matrix integral:

$$\sum_{conf.s} \sum_{k} q^k \int d\mathcal{M}_{(k)}\, e^{-\mathcal{S}(\mathcal{M}_{(k)},\Phi)}$$

  - the moduli spectrum contains bosonic and fermionic moduli with transformation properties under the Chan-Paton groups

$$U(k) \times U(4) \times U(m)$$

    corresponding to strings ending on the k D(-1)'s, the 4 D7's and the m D3's (with $m = 0, 1$)

  - The moduli action $\mathcal{S}(\mathcal{M}_{(k)}, \Phi)$ contains many moduli interactions

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- However,...

Computing D-instanton effects

- Substantial progress can be made by exploiting the SUSY properties of the moduli action, which lead to:
  - an equivariant cohomological BRST structure
  - a localization of the moduli integrals (after suitable closed string deformations)
- Similar techniques have been successfully used to
  - compute the YM integrals in $d = 10, 6, 4$ and the D-instanton partition function

    Moore+Nekrasov+Shatashvili, 1998
  - compute multi-instanton effects in $\mathcal{N} = 2$ SYM in $d = 4$ and compare with the Seiberg-Witten solution

    Nekrasov, 2002; + ...
  - derive the multi-instanton calculus using D3/D(–1) brane systems

    Fucito et al, 2004; Billò et al, 2006; ...

General idea

- Our brane system is BPS.
- There are SUSY transformations under which $\mathcal{S}(\mathcal{M}_{(k)}, \Phi)$ is invariant.

General idea

- Our brane system is BPS.
- There are SUSY transformations under which $\mathcal{S}(\mathcal{M}_{(k)}, \Phi)$ is invariant.
- Select a particular component of the SUSY charges as BRST charge Q:
  - The "Lorentz" symmetry SO(4)×SO(4)×SO(2) is restricted to the $SU(2)^3$ subgroup that leaves Q invariant
  - Organize the D-instanton moduli in BRST doublets $(\mathcal{M}_0, \mathcal{M}_1)$ such that
$$Q\mathcal{M}_0 = \mathcal{M}_1$$
  - The moduli action $\mathcal{S}(\mathcal{M}_{(k)}, \Phi)$ is BRST exact:
$$\mathcal{S}(\mathcal{M}_{(k)}, \Phi) = Q\,\Xi$$

Moduli spectrum

sector	$(\mathcal{M}_0, \mathcal{M}_1)$	$U(k) \times U(4) \times U(m)$	$SU(2)^3$
D(-1)/D(-1)	(B_ℓ, M_ℓ)	$(\text{adj}, \mathbf{1}, \mathbf{1})$	$(\mathbf{2}, \mathbf{1}, \mathbf{2})$
	$(B_{\dot\ell}, M_{\dot\ell})$	$(\square\!\square, \mathbf{1}, \mathbf{1}) + \text{h.c.}$	$(\mathbf{1}, \mathbf{2}, \mathbf{2})$
	$(N_{\dot\alpha\dot a}, D_{\dot\alpha\dot a})$	$(\begin{smallmatrix}\square\\\square\end{smallmatrix}, \mathbf{1}, \mathbf{1}) + \text{h.c.}$	$(\mathbf{2}, \mathbf{2}, \mathbf{1})$
	(N_m, d_m)	$(\text{adj}, \mathbf{1}, \mathbf{1})$	$(\mathbf{1}, \mathbf{1}, \mathbf{3})$
	$(\bar\chi, \eta)$	$(\text{adj}, \mathbf{1}, \mathbf{1})$	$(\mathbf{1}, \mathbf{1}, \mathbf{1})$
	χ	$(\text{adj}, \mathbf{1}, \mathbf{1})$	$(\mathbf{1}, \mathbf{1}, \mathbf{1})$
D(-1)/D7	(μ', h')	$(\square, \overline{\square}, \mathbf{1}) + \text{h.c.}$	$(\mathbf{1}, \mathbf{1}, \mathbf{1})$
D(-1)/D3	(w_α, μ_α)	$(\square, \mathbf{1}, \overline{\square}) + \text{h.c.}$	$(\mathbf{1}, \mathbf{1}, \mathbf{2})$
$(m = 0$ or $1)$	$(\mu_{\dot a}, h_{\dot a})$	$(\square, \mathbf{1}, \square) + \text{h.c.}$	$(\mathbf{1}, \mathbf{2}, \mathbf{1})$

- $B_\ell \sim$ positions of the D(-1)'s in spacetime; M_ℓ superpartner
- Component along the identity $\sim$ Goldstone modes of broken (super)-translations $\sim$ supercoordinates (x, θ).

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- $(B_{\dot\ell}, M_{\dot\ell}) \sim$ positions of the D(-1)'s in $(\mathcal{T}_2 \times \mathcal{T}_2)/\mathbb{Z}_2$.

Moduli spectrum

sector	$(\mathcal{M}_0, \mathcal{M}_1)$	$U(k) \times U(4) \times U(m)$	$SU(2)^3$
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	$(B_{\dot\ell}, M_{\dot\ell})$	$\big(\square\square, \mathbf{1}, \mathbf{1}\big) + \text{h.c.}$	$(\mathbf{1}, \mathbf{2}, \mathbf{2})$
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- χ, $\bar\chi$ ~ positions on $\mathcal{T}_2$
- χ has a particular rôle and does not belong to a BRST doublet.

Moduli spectrum

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- The physical D(-1)/D7 moduli μ' are only fermionic: typical of exotic instantons (h' are auxiliary)

BRST structure

- The components of each doublet are connected by BRST transformations

$$Q\mathcal{M}_0 = \mathcal{M}_1$$

such that

$$Q^2 \bullet = \left[T_{U(k)}(\chi) + T_{U(4)}(\Phi) \right] \bullet$$

where

  - $T_{U(k)}(\chi) =$ inf.mal U(k) rotation parametrized by χ (D(-1)/D(-1) scalar)
  - $T_{U(4)}(\Phi) =$ inf.mal U(4) rotation parametrized by Φ (D7/D7 scalar)

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  - $T_{U(4)}(\Phi) =$ inf.mal U(4) rotation parametrized by Φ (D7/D7 scalar)
- However, the moduli action $\mathcal{S}(\mathcal{M}_{(k)}, \Phi)$ is invariant also under
  - U(m) (associated to the $m = 0, 1$ D3's)
  - the residual "Lorentz" group $SU(2)^3$

BRST structure

- We could replace Q with a modified BRST charge $\widetilde{Q}$, nilpotent also up to U(m) and $SU(2)^3$ transformations, *i.e.* such that

$$\widetilde{Q}^2 \bullet = \Big[T_{\mathrm{U}(k)}(\chi) + T_{U(4)}(\phi) + T_{U(m)}(\pi) + T_{\mathrm{SU}(2)^3}(\epsilon) \Big] \bullet$$

  where

  - $T_{U(k)}(\chi) =$ inf.mal U(k) rotation parametrized by χ (D(-1)/D(-1) scalar)
  - $T_{U(4)}(\Phi) =$ inf.mal U(4) rotation parametrized by Φ (D7/D7 scalar)
  - $T_{\mathrm{U}(k)}(\pi) =$ inf.mal U(m) rotation parametrized by π (D3/D3 scalar)
  - $T_{\mathrm{SU}(2)^3}(\epsilon) =$ inf.mal $SU(2)^3$ rotation parametrized by ϵ (R-R fields)

$\widetilde{Q}$ is equivariantly closed

- This equivariant BRST structure allows to suitably rescale the integration variables, and show that the the semiclassical approximation is exact.

Scaling to localization

- The integrals over all moduli (except χ) become quadratic and yield in the end
$$\prod_{\mathcal{M}_0} det_{\mathcal{M}_0}^{\pm\frac{1}{2}}(\widetilde{Q}^2)$$
where $\mathcal{M}_0$ = first components of BRS doublets in the spectrum and ± depend on their fermionic or bosonic nature.

- The action of $\widetilde{Q}^2$ on $\mathcal{M}_0$ is completely determined by the symmetry properties of $\mathcal{M}_0$

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- By taking the parameters χ, Φ, π and ϵ in the Cartan directions, we get a rational function determined by the weights of the rep.s to which $\mathcal{M}_0$ belongs

D-instanton partition function

- At instanton # k we get

$$Z_k^{(m)}(\Phi,\pi,\epsilon) = \left(\frac{s_3}{\epsilon_1\epsilon_2}\right)^k \int \prod_{i=1}^{k} \frac{d\chi_i}{2\pi \mathrm{i}} \prod_{i<j}^{k} \left(\chi_i-\chi_j\right)^2 \left((\chi_i-\chi_j)^2 - s_3^2\right)$$

$$\times \prod_{i<j}^{k} \prod_{\ell=1}^{2} \frac{\left((\chi_i+\chi_j)^2 - s_\ell^2\right)}{\left((\chi_i-\chi_j)^2-\epsilon_\ell^2\right)\left((\chi_i+\chi_j)^2-\epsilon_{\ell+2}^2\right)}$$

$$\times \prod_{i=1}^{k} \left[\prod_{\ell=1}^{2} \frac{1}{\left(4{\chi_i}^2 - \epsilon_{\ell+2}^2\right)} \prod_{r=1}^{m} \frac{\left((\chi_i+\pi_r)^2 - \frac{(\epsilon_3-\epsilon_4)^2}{4}\right)}{\left((\chi_i-\pi_r)^2 - \frac{(\epsilon_1+\epsilon_2)^2}{4}\right)} \prod_{u=1}^{4} \left(\chi_i - \Phi_u\right) \right]$$

(here $\{\epsilon_A\}$ with $\sum_{A=1}^4 \epsilon_A = 0$ are the Cartan param.s of $SU(2)^3$ embedded in SO(4)× SO(4) rot.s and $s_1=\epsilon_2+\epsilon_3$, $s_2=\epsilon_1+\epsilon_3$, $s_3=\epsilon_1+\epsilon_2$)

- The χ integrals can be done as contour integrals and the final result for $Z_k(\Phi,\pi,\epsilon)$ comes from a sum over residues

Moore+Nekrasov+Shatashvili, 1998

D-instanton partition function (continued)

- Once the integrals are done, we can obtain the non-perturbative effective action.
- But there are some caveats:
  - At instanton number k, there are disconnected contributions from smaller instantons k_i (with $\sum_i k_i = k$). To isolate the connected components we have to take the log of the "grand-canonical" partition function:

$$\mathcal{Z}^{(m)}(\Phi, \pi, \epsilon) \equiv \sum_k Z_k^{(m)}(\Phi, \pi, \epsilon)\, q^k \;\rightarrow\; \log \mathcal{Z}^{(m)}(\Phi, \pi, \epsilon)$$

  - In obtaining $Z_k^{(m)}(\Phi, \pi, \epsilon)$ we integrated over all moduli, and hence also over the superspace coordinates x and θ.

$$\Downarrow$$

We expect a divergence $1/(\epsilon_1 \epsilon_2)$ representing the (regularized) super-volume factor.

An 8-dimensional contribution

- However, $\log \mathcal{Z}^{(m)}(\Phi, \pi, \epsilon)$ is divergent as $1/(\epsilon_1\epsilon_2\epsilon_3\epsilon_4)$.
- This factor arises from the integral over the moduli corresponding to the (super)coordinates in the first 8 directions
- To remove this contribution we make the replacement

$$\log \mathcal{Z}^{(m)}(\Phi, \pi, \epsilon) \;\rightarrow\; (\epsilon_1\epsilon_2\epsilon_3\epsilon_4)\; \log \mathcal{Z}^{(m)}(\Phi, \pi, \epsilon)$$

and then turn-off the ϵ-deformations, obtaining

$$\mathcal{F}_{IV}(\Phi) = \lim_{\epsilon\to 0} \left[\epsilon_1\epsilon_2\epsilon_3\epsilon_4 \; \log \mathcal{Z}^{(m)}(\Phi, \pi, \epsilon)\right]$$

- $\mathcal{F}_{IV}(\Phi)$ is finite and has an 8d interpretation as a quartic prepotential for Φ. It agrees with the one computed in the D7/D(-1) system in type I'

Billo et al, 2009

- It does not depend on the D3 d.o.f. π (hence not on m)

The 4d prepotential

- $\log \mathcal{Z}^{(m)}(\Phi, \pi, \epsilon)$ has also a subleading divergence in $1/(\epsilon_1\epsilon_2)$ corresponding to the 4d (super)coordinates.
- To isolate this 4d term, we define

$$\mathcal{F}_{II}^{(m)}(\Phi, \pi) = \lim_{\epsilon\to 0}\Big(\epsilon_1\epsilon_2 \log \mathcal{Z}^{(m)}(\Phi, \pi, \epsilon) - \frac{1}{\epsilon_3\epsilon_4}\,\mathcal{F}_{IV}(\Phi)\Big)$$

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- Explicitly, up to 3 instantons and neglecting the π-dependence to focus on the D7 d.o.f., we find the quadratic prepotentials

$$\mathcal{F}_{II}^{(m=0)}(\Phi) = \Big(-\sum_{i<j}\Phi_i\Phi_j\Big)\,q + \Big(\sum_{i<j}\Phi_i\Phi_j - \frac{1}{4}\sum_i \Phi_i^2\Big)\,q^2 + \Big(-\frac{4}{3}\sum_{i<j}\Phi_i\Phi_j\Big)\,q^3 + \cdots\,,$$

$$\mathcal{F}_{II}^{(m=1)}(\Phi) = \Big(3\sum_{i<j}\Phi_i\Phi_j\Big)\,q + \Big(\sum_{i<j}\Phi_i\Phi_j + \frac{7}{4}\sum_i \Phi_i^2\Big)\,q^2 + \Big(4\sum_{i<j}\Phi_i\Phi_j\Big)\,q^3 + \cdots\,.$$

The 4d prepotential (continued)

- We still have to sum over configurations with $m = 0$ and $m = 1$, with the correct combinatorial factors, namely

$$\mathcal{F}_{n.p.}(\Phi) = 48\,\mathcal{F}_{II}^{(m=0)}(\Phi) + 16\,\mathcal{F}_{II}^{(m=1)}(\Phi)$$

$$= 0\,q + \left[-16\mathrm{Tr}\Phi^2 + 32(\mathrm{Tr}\Phi)^2\right] q^2 + 0\,q^3 + O(q^4)$$

- Thus, the non-perturbative corrections to the quadratic gauge couplings are

$$\boxed{f_{n.p.} = -16\,q^2 + O(q^4)} \quad \text{and} \quad \boxed{f'_{n.p.} = +32\,q^2 + O(q^4)}$$

  - No contributions in q and q^3 (as effect of sum over conf.s)
  - At order q^2, a ratio of -2 between f and f' in perfect agreement with the dual heterotic calculation!!

Conclusions and perspectives

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- We have considered a consistent string set-up where the 4d gauge theory living on a D-brane stack receives non-perturbative corrections from “exotic” brane instantons
- We explicitly computed such corrections by integrating over exotic instanton moduli space by means of localization techniques
- We successfully checked the result against a dual heterotic computation

Conclusions

- We have considered a consistent string set-up where the 4d gauge theory living on a D-brane stack receives non-perturbative corrections from "exotic" brane instantons
- We explicitly computed such corrections by integrating over exotic instanton moduli space by means of localization techniques
- We successfully checked the result against a dual heterotic computation
- The string instanton calculus is on solid ground also for the "exotic" configurations which have a very different spectrum of moduli as compared to ordinary gauge instantons

Perspectives

- In our set-up, there are other possible non-perturbative corrections from E3 branes wrapped on $(\mathcal{T}_2 \times \mathcal{T}_2)/\mathbb{Z}_2$. They correspond to usual gauge instantons for the D7 theory, and would be n.p. on the heterotic side. We're investigating them.

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- The non-perturbative description of D7 backgrounds should be geometrized by F-theory. D7/D3/D(-1) systems are a testing ground to link directly F-theory curves to non-perturbative prepotentials both in 8d and 4d. Work in progress.

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Annecy-Torino
Torino-Roma Tor Vergata

- Most important, the exotic instanton calculus might be applied in different set-ups and to diferent kind of couplings, possibly of more direct (string)-phenomenological interest

Grazie !