$\mathcal{N} = 2$ CHERN-SIMONS MATTER THEORIES: RG FLOWS AND IR BEHAVIOR

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Motivations

$\mathrm{AdS}_4/\mathrm{CFT}_3$ correspondence states that the strong coupling dynamics of a $\mathcal{N} = 6$ Chern-Simons theory for $U(N)_k \times U(N)_{-k}$ with bifundamental matter is described by M–theory on $\mathrm{AdS}_4 \times S^7/\mathcal{Z}_k$

Aharony, Bergman, Jafferis, Maldacena

- The field theory is at its superconformal fixed point
- According to the dual supergravity description the IR fixed point should be stable

Questions

- Isolated fixed points?
- If not, how are they connected to other fixed points?
- Are the fixed points IR stable against marginal deformations?
- Generalizations of $\mathrm{AdS}_4/\mathrm{CFT}_3$ correspondence: tests of predictions done from the dual sugra side

PERTURBATIVE INVESTIGATION

Plan of the talk

- Introduction to the $\mathrm{AdS}_4/\mathrm{CFT}_3$ correspondence
- $\mathcal{N} = 2$ Chern–Simons matter theories. Quantization in superspace
- Spectrum of fixed points (superconformal field theories with different degrees of susy)
- Study of RG flows and IR stability of fixed points

- ABJM/ABJ–like theories
- ABJM/ABJ–like theories with flavors
- $\mathcal{N} = 2, 3$ Chern–Simons theories with different CS levels

- Summary and conclusions

Marco S. Bianchi, S.P., M. Siani, JHEP 1001:080,2010

JHEP 1005:106,2010

Naively CFT_3 is the low energy dynamics of D2–branes (Type IIA string theory) and lives at the boundary of AdS_4.

- AdS_4 arises from $\mathrm{AdS}_4 \times S^7$ compactifications in M–theory
- We expect CFT_3 to describe the low energy dynamics of a stack of M2–branes
- S^7 has SO(8) isometry group. We expect a $\mathcal{N} = 8$ 3D theory

Which kind of theory do we obtain?
Is it a 3D SYM ?

3D peculiarities

- M2–branes are described by 8 scalar dof. No dynamical gauge fields.
- In M–theory there is no dilaton, so no free parameter to be interpreted as coupling constant
- In 3D the gauge coupling is dimensionful

Possible solution

CHERN–SIMONS THEORY

$$S_{CS} = K \int d^3x \epsilon^{\mu\nu\rho} \, \mathrm{Tr} \left(A_\mu \partial_\nu A_\rho + \frac{2}{3} A_\mu A_\nu A_\rho \right)$$

$$+$$

MATTER (8 real scalars)

BLG and ABJM theories

Bagger, Lambert, PRD75:045020 (2007), PRD77:065008, JHEP 0802:105 (2008)
Gustavsson, NPB807:315-333 (2009)
Aharony, Bergman, Jafferis, Maldacena, JHEP0810 (2008)
Van Raamsdonk, JHEP0805 (2008)
Benna, Klebanov, Klose, Smedback, JHEP0809 (2008)

CS theory for $U(N)_k \times U(N)_{-k}$, (A^I, B_I) matter

$$A^I \in (N, \bar{N}) \qquad B_I \in (\bar{N}, N) \qquad I = 1, 2$$

and maximal symmetric superpotential ($SU(2)_A \times SU(2)_B$ global symmetry)

$$W = \epsilon_{IJ}\epsilon^{KL}\mathrm{Tr}\left(A^I B_K A^J B_L\right)$$

$$\mathcal{S} = \frac{k}{2\pi}\int d^3x\, d^4\theta \int_0^1 dt\ \mathrm{Tr}\Big[V\bar{D}^\alpha\left(e^{tV}D_\alpha e^{-tV}\right) - \hat{V}\bar{D}^\alpha\left(e^{t\hat{V}}D_\alpha e^{-t\hat{V}}\right)\Big]$$

$$+\int d^3x\, d^4\theta\ \mathrm{Tr}\left(\bar{A}_A e^V A^A e^{-\hat{V}} + \bar{B}^A e^{\hat{V}} B_A e^{-V}\right)$$

$$+\frac{2\pi}{k}\int d^3x\, d^2\theta\ \mathrm{Tr}\left[(A^1B_1A^2B_2) - (A^2B_1A^1B_2)\right]$$

$$\mathrm{SU}(2)_R \times \mathrm{SU}(2)_A \times \mathrm{SU}(2)_B \to \mathrm{SU}(4)_R$$

$\mathcal{N} = 6$ supersymmetry

($\mathcal{N} = 8$ supersymmetry when $N = 2$)

$U(N)_k \times U(N)_{-k}$ CS THEORY WITH (A^I, B_I) MATTER IN THE BIFUNDAMENTAL DESCRIBES THE LOW–ENERGY DYNAMICS OF N M2–BRANES IN M–THEORY PROBING A $\mathcal{C}^4/\mathcal{Z}_k$ SINGULARITY

THE CORRESPONDENCE

$U(N)_k \times U(N)_{-k}$ CS THEORY WITH (A^I, B_I) MATTER IN THE BIFUNDAMENTAL

DUAL TO

M–THEORY ON $\mathrm{AdS}_4 \times S^7/\mathcal{Z}_k$

AdS_4 radius $\quad \frac{R^2}{\alpha'} \sim \sqrt{\frac{N}{k}} \equiv \sqrt{\lambda}$

$S^1 \subset S^7$ radius $\quad \frac{R_{S^1}^2}{\alpha'} \sim \left(\frac{N}{k^5}\right)^{1/6}$

$N \gg k$, $N \gg k^5$	$\longrightarrow$	M–theory description
$k \ll N \ll k^5$	$\longrightarrow$	Type IIA supergravity on $\mathrm{AdS}_4 \times CP^3$
$k \gg N$	$\longrightarrow$	Perturbative field theory

GENERALIZATIONS

- $U(N)_k \times U(M)_{-k}$ CS theory with the same matter content (Aharony, Bergman, Jafferis, JHEP0811 (2008))
- $U(N)_{k_1} \times U(M)_{k_2}$ with $k_1 \neq -k_2$. This corresponds to a 0–form flux $F_0 = k_1 + k_2$ in the dual description (Romans mass) (Schwarz; Gaiotto, Tomasiello)
- Addition of flavor matter in the fundamental of the gauge groups (Hohenegger, Kirsch; Gaiotto, Jafferis; Hikida, Li, Takayanagi)

Adding flavors (probe D6–branes in the $\mathrm{AdS}_4 \times CP^3$ geometry)

$$Q_1^i \in (N, 1) \qquad Q_2^j \in (1, M) \qquad i = 1, \cdots, N_f$$

$$\tilde{Q}_1^i \in (\bar{N}, 1) \qquad \tilde{Q}_2^j \in (1, \bar{M}) \qquad j = 1, \cdots, N_f'$$

Maximal supersymmetric superpotential ($\mathcal{N} = 3$)

$$W_{flav} = \mathrm{Tr}\Big[\frac{2\pi}{k_1}(Q_1\tilde{Q}_1)^2 + \frac{2\pi}{k_2}(Q_2\tilde{Q}_2)^2$$

$$+\frac{4\pi}{k_1}\tilde{Q}_1(A^1B_1 + A^2B_2)Q_1 + \frac{4\pi}{k_2}\tilde{Q}_2(B_1A^1 + B_2A^2)Q_2\Big]$$

Most general $\mathcal{N}=2$ action, $U(N)\times U(M)$ group

$$S_{CS} = \int d^3x\, d^4\theta \int_0^1 dt \Big[K_1 \mathrm{Tr} V \bar{D}^\alpha \left(e^{-tV} D_\alpha e^{tV}\right) + K_2 \mathrm{Tr} \hat{V} \bar{D}^\alpha \left(e^{-t\hat{V}} D_\alpha e^{t\hat{V}}\right)\Big]$$

$$+ \int d^3x\, d^4\theta\ \mathrm{Tr}\left(\bar{A}_i e^V A^i e^{-\hat{V}} + \bar{B}^i e^{\hat{V}} B_i e^{-V}\right)$$
$$+ \int d^3x\, d^4\theta\ \mathrm{Tr}\left(\bar{Q}^1_r e^V Q^r_1 + \bar{\tilde{Q}}^{1,r} \tilde{Q}_{1,r} e^{-V} + \bar{Q}^2_{r'} e^{\hat{V}} Q^{r'}_2 + \bar{\tilde{Q}}^{2,r'} \tilde{Q}_{2,r'} e^{-\hat{V}}\right)$$

In general $\quad K_1 \neq -K_2 \quad (2\pi K_i =$ integers)

$+\,\mathcal{N}=2$ superpotential

$$\int \mathrm{Tr}\Big[h_1(A^1B_1)^2+h_2(A^2B_2)^2+h_3(A^1B_1A^2B_2)+h_4(A^2B_1A^1B_2)$$

$$+\lambda_1(Q_1\tilde{Q}_1)^2+\lambda_2(Q_2\tilde{Q}_2)^2+\lambda_3Q_1\tilde{Q}_1Q_2\tilde{Q}_2$$

$$+\alpha_1\tilde{Q}_1A^1B_1Q_1+\alpha_2\tilde{Q}_1A^2B_2Q_1+\alpha_3\tilde{Q}_2B_1A^1Q_2+\alpha_4\tilde{Q}_2B_2A^2Q_2\Big]$$

QUANTIZATION IN $\mathcal{N} = 2$ SUPERSPACE

$$\langle V^A(1)\, V^B(2)\rangle = -\frac{1}{K_1}\frac{1}{\Box}\left(\bar{D}^\alpha D_\alpha + \alpha D^2 + \alpha \bar{D}^2\right)\, \delta^4(\theta_1 - \theta_2)\, \delta^{AB}$$

$$\langle \hat{V}^A(1)\, \hat{V}^B(2)\rangle = -\frac{1}{K_2}\frac{1}{\Box}\left(\bar{D}^\alpha D_\alpha + \alpha D^2 + \alpha \bar{D}^2\right)\, \delta^4(\theta_1 - \theta_2)\, \delta^{AB}$$

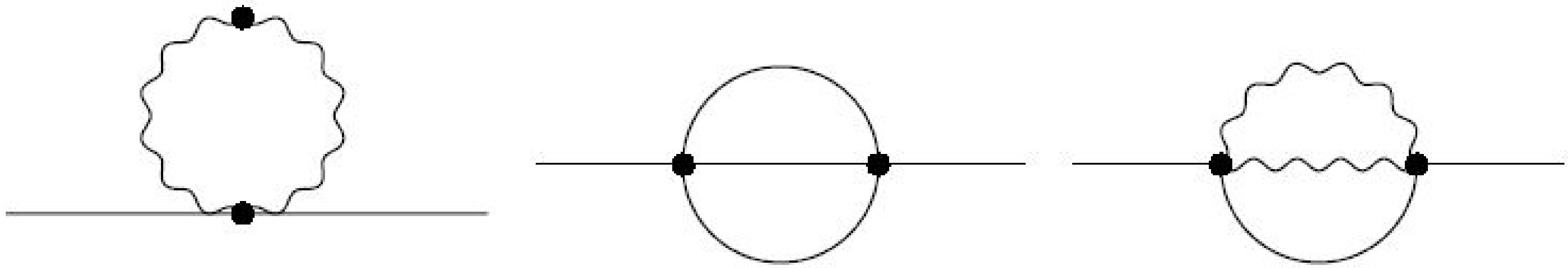
$$\langle \overline{c}' c\rangle = -\frac{1}{\Box} \qquad \langle c' \overline{c}\rangle = -\frac{1}{\Box}$$

$$\langle \bar{A}^{\hat{a}}{}_a(1)\, A^b{}_{\hat{b}}(2)\rangle = -\frac{1}{\Box}\delta^4(\theta_1 - \theta_2)\, \delta^{\hat{a}}_{\hat{b}}\, \delta_a{}^b$$

Perturbative evaluation of γ and β–functions $(K_{1,2} \gg M, N)$

- UV divergences appear only at even orders in loops, so the first nontrivial order is two loops
- $\mathcal{N} = 2$ CS action does not receive perturbative corrections
- A non–renormalization theorem for the superpotential is still valid

UV DIVERGENCES ONLY FROM MATTER SELF–ENERGY DIAGRAMS



We work in dimensional regularization $D = 3 - 2\epsilon$

Renormalizing the fields

$$\Phi = Z_\Phi^{-\frac{1}{2}} \Phi_B \qquad , \qquad \bar{\Phi} = \bar{Z}_{\bar{\Phi}}^{-\frac{1}{2}} \bar{\Phi}_B \qquad (\Phi = A^I, B_I, Q_A, \tilde{Q}_A)$$

and the couplings

$$h_j = \mu^{-2\epsilon} Z_{h_j}^{-1} h_{j\,B} \qquad \lambda_j = \mu^{-2\epsilon} Z_{\lambda_j}^{-1} \lambda_{j\,B}$$

$$\alpha_j = \mu^{-2\epsilon} Z_{\alpha_j}^{-1} \alpha_{j\,B} \qquad K_{1,2} = \mu^{2\epsilon} K_{1,2}^{(B)}$$

$$\gamma_{A^1} = \gamma_{B_1} = \frac{1}{32\pi^2}\Big[-\frac{2NM + NN_f + 1}{K_1^2} - \frac{2NM + MN_f' + 1}{K_2^2}$$

$$-\frac{2NM + 4}{K_1 K_2}$$

$$+4|h_1|^2(MN+1) + (|h_3|^2 + |h_4|^2)MN + (h_3\bar{h}_4 + h_4\bar{h}_3)$$

$$+(|\alpha_1|^2 NN_f + |\alpha_3|^2 MN_f')\Big]$$

$$\gamma_{A^2} = \gamma_{B_2} = \frac{1}{32\pi^2}\Big[-\frac{2NM + NN_f + 1}{K_1^2} - \frac{2NM + MN'_f + 1}{K_2^2}$$

$$-\frac{2NM + 4}{K_1 K_2}$$

$$+4|h_2|^2(MN+1) + (|h_3|^2 + |h_4|^2)MN + (h_3\bar{h}_4 + h_4\bar{h}_3)$$

$$+(|\alpha_2|^2 NN_f + |\alpha_4|^2 MN'_f)\Big]$$

$$\gamma_{Q^1} = \gamma_{\tilde{Q}_1} = \frac{1}{32\pi^2}\Big[- \frac{2NM + NN_f + 1}{K_1^2}$$

$$+4|\lambda_1|^2 \, (NN_f+1) + |\lambda_3|^2 MN'_f + (|\alpha_1|^2 + |\alpha_2|^2) MN \Big]$$

$$\gamma_{Q^2} = \gamma_{\tilde{Q}_2} = \frac{1}{32\pi^2}\Big[- \frac{2NM + MN'_f + 1}{K_2^2}$$

$$+4|\lambda_2|^2 \, (MN'_f+1) + |\lambda_3|^2 NN_f + (|\alpha_3|^2 + |\alpha_4|^2) MN \Big]$$

$$\beta_{h_1} = 4h_1\gamma_{A^1} \qquad \beta_{h_2} = 4h_2\gamma_{A^2}$$

$$\beta_{h_3} = 2h_3(\gamma_{A^1} + \gamma_{A^2}) \qquad \beta_{h_4} = 2h_4(\gamma_{A^1} + \gamma_{A^2})$$

$$\beta_{\lambda_1} = 4\lambda_1\gamma_{Q_1} \qquad \beta_{\lambda_2} = 4\lambda_2\gamma_{Q_2} \qquad \beta_{\lambda_3} = 2\lambda_3(\gamma_{Q_1} + \gamma_{Q_2})$$

$$\beta_{\alpha_1} = 2\alpha_1(\gamma_{A_1} + \gamma_{Q_1}) \qquad \beta_{\alpha_2} = 2\alpha_2(\gamma_{A_2} + \gamma_{Q_1})$$

$$\beta_{\alpha_3} = 2\alpha_3(\gamma_{A_1} + \gamma_{Q_2}) \qquad \beta_{\alpha_4} = 2\alpha_4(\gamma_{A_2} + \gamma_{Q_2})$$

In general $\beta_i = 0 \Rightarrow \gamma_i = 0$. Theory is finite on a 7-dim hypersurface of couplings.

ABJ–LIKE THEORIES

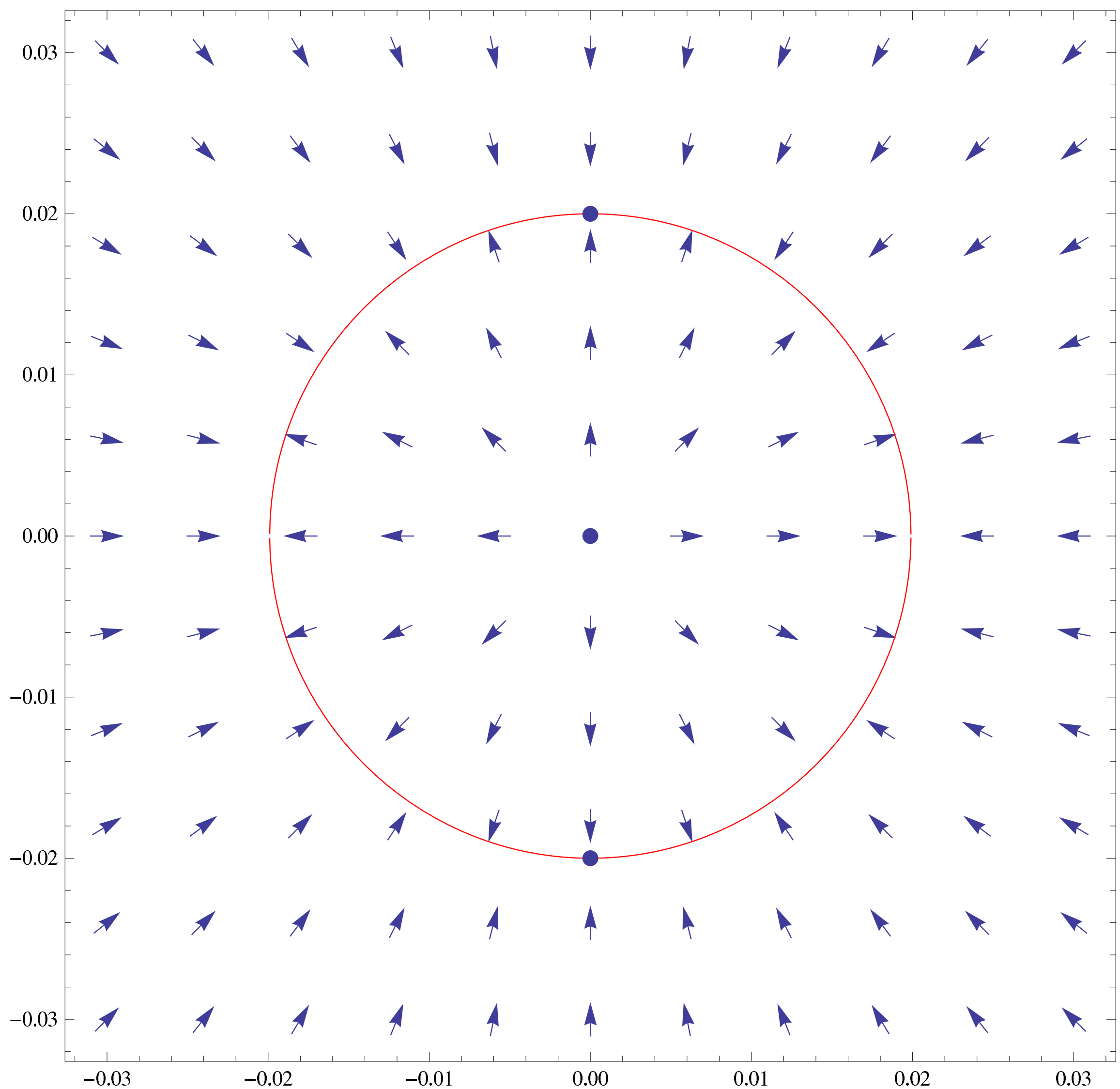
$$K_1 = -K_2 \equiv K \quad , \quad h_1 = h_2 = 0$$

(1) NO FLAVORS Then $\gamma_{A_1} = \gamma_{A_2}$

Define $y_1 = h_3 + h_4$ $y_2 = h_3 - h_4$

$$\int \left[\frac{y_1}{2}\mathrm{Tr}\left(A^1 B_1 A^2 B_2 + A^2 B_1 A^1 B_2\right) + \frac{y_2}{4}\epsilon_{ij}\epsilon^{kl}\mathrm{Tr}\left(A^i B_k A^j B_l\right)\right]$$

- Fixed points $y_1^2(MN+1) + y_2^2(MN-1) = \frac{4}{K^2}(MN-1)$
- RG trajectories $\frac{dy_2}{dy_1} = \frac{y_2}{y_1}$


0.03
0.02
0.01
0.00
−0.01
−0.02
−0.03
−0.03
−0.02
−0.01
0.00
0.01
0.02
0.03

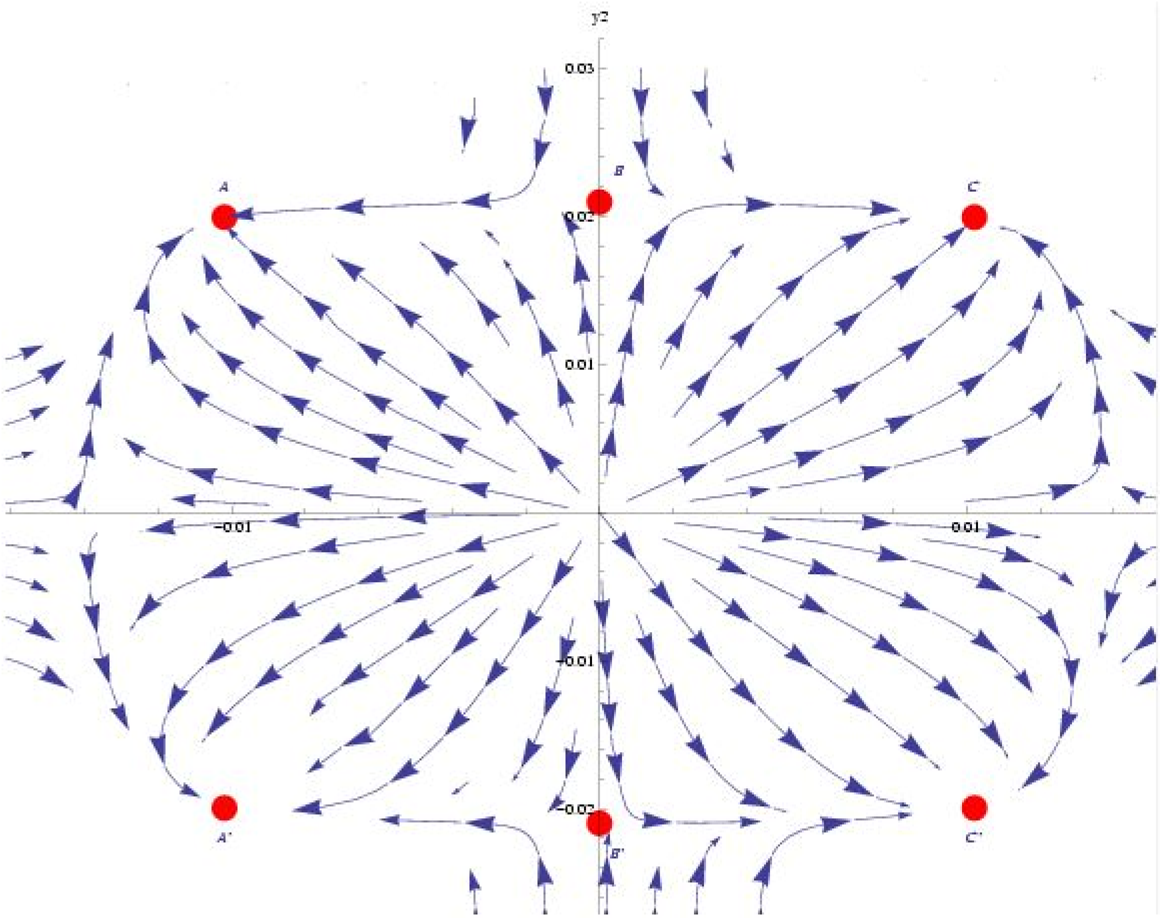
(2) We add FLAVORS with

$$\lambda_j = 0, \qquad \alpha_1 = \alpha_2 = -\alpha_3 = -\alpha_4 \equiv \alpha$$

$$\gamma_{Q^1}, \gamma_{Q^2} = 0 \quad \Rightarrow \quad \alpha = 0, \pm\frac{1}{K}\sqrt{1 + (NN_f + 1)/2N^2}$$

$$y_1^2(MN+1) + y_2^2(MN-1) = \frac{4}{K^2}(N^2 + NN_f - 1) - 4\alpha^2 NN_f$$

$y_1 = 0$ plane



GENERALIZATION TO $K_1 \neq -K_2$

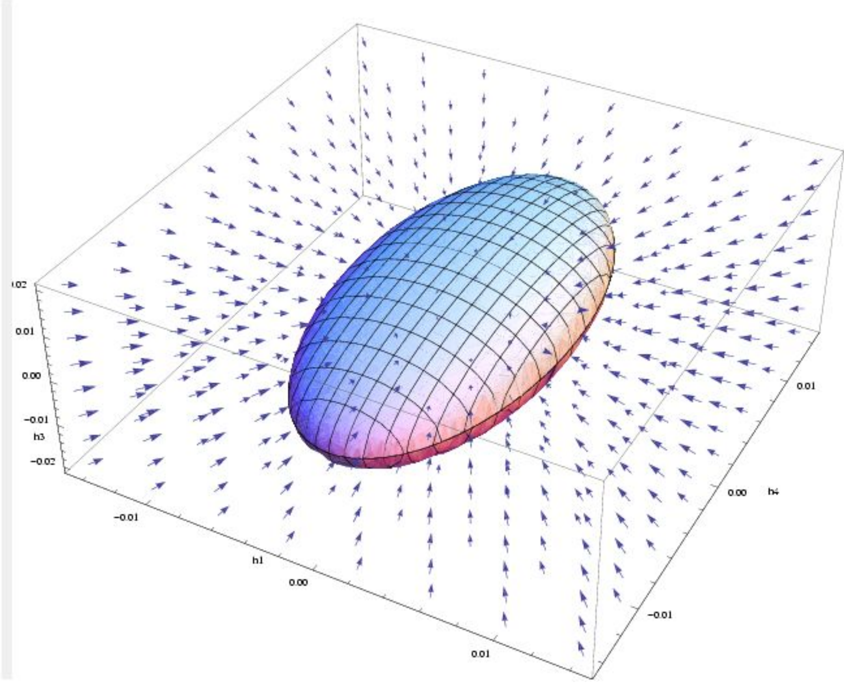
NO FLAVORS

- $h_1 = h_2 = 0$

$$y_1^2(MN+1)+y_2^2(MN-1) = 2(2MN+1)\left(\frac{1}{K_1^2}+\frac{1}{K_2^2}\right)+2\frac{2MN+4}{K_1K_2}$$

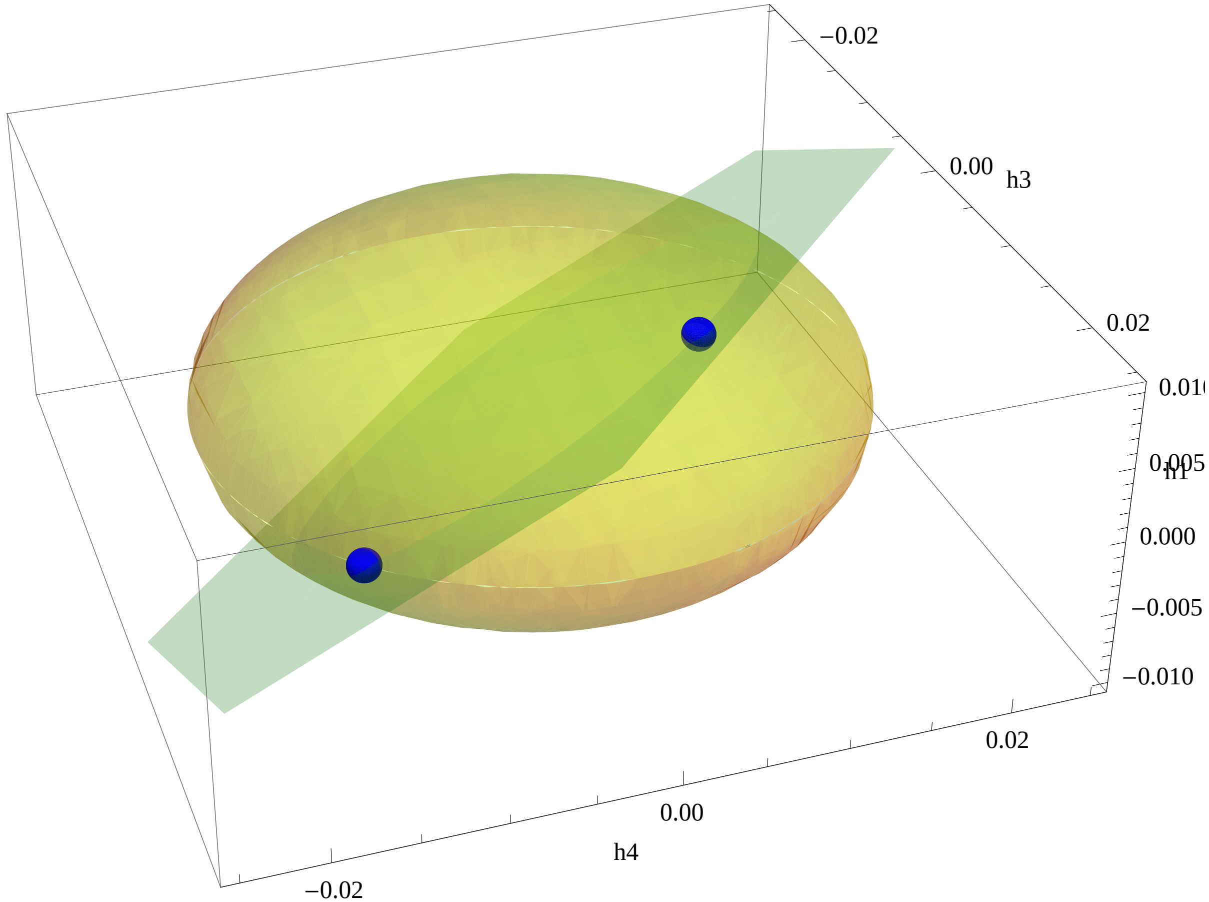
Fixed point with $y_1 = 0$ is a $\mathcal{N} = 2$ superconformal theory with $\mathrm{SU}(2)_A \times \mathrm{SU}(2)_B$ global symmetry

- $h_1 = h_2 \neq 0$



Fixed point $\quad h_1 = h_2 = \frac{1}{2}(h_3 + h_4),\ h_3 = \frac{1}{K_1},\ h_4 = \frac{1}{K_2}$

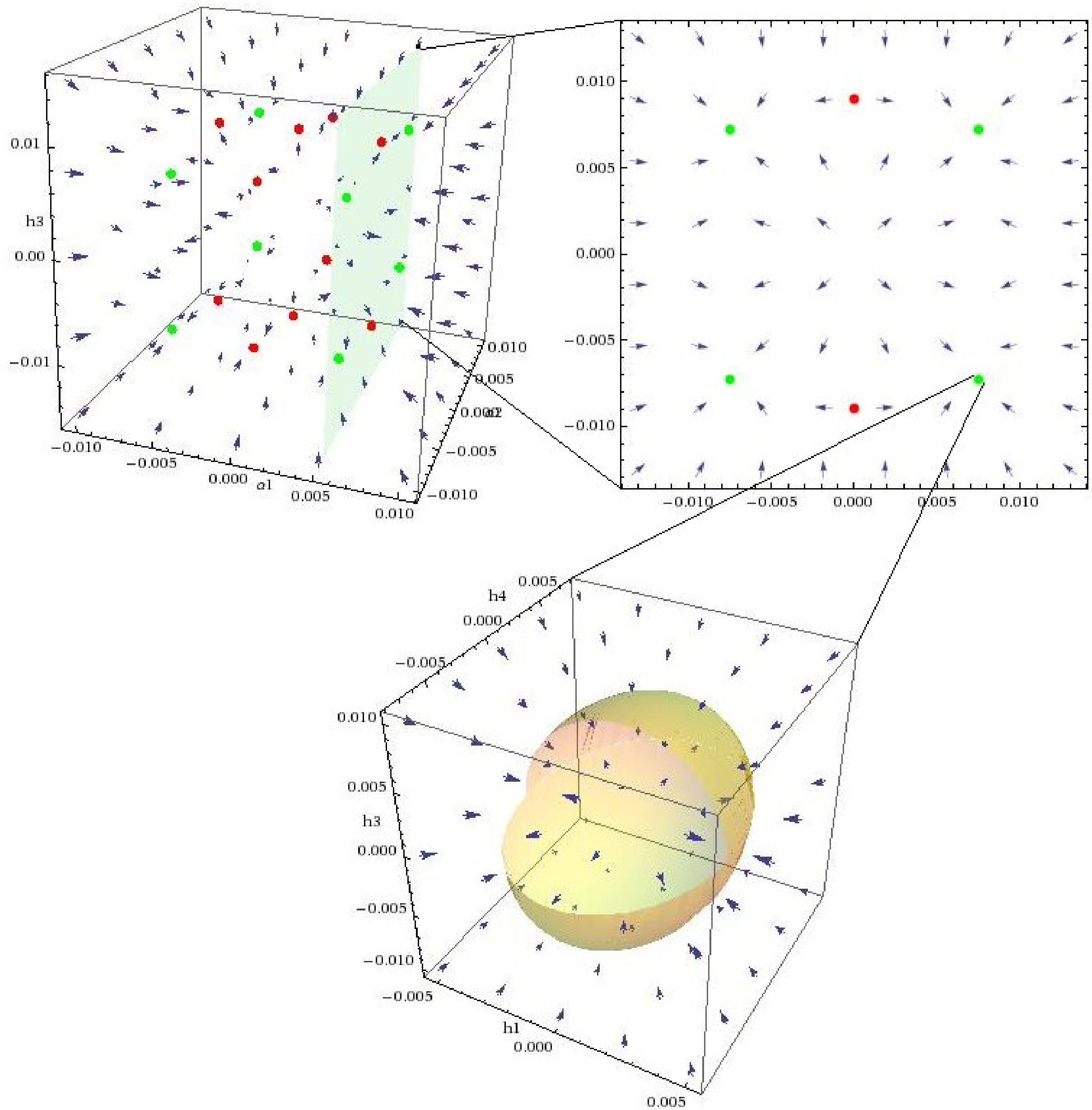
$\mathcal{N} = 3$ superconformal theory



RG flow which connects $\mathcal{N}=2$, $\mathrm{SU}(2)_A \times \mathrm{SU}(2)_B$ theory to $\mathcal{N}=3$ theory

ADDING FLAVORS with

$$\lambda_j = 0 \qquad \alpha_1 = \alpha_2 \qquad \alpha_3 = \alpha_4$$

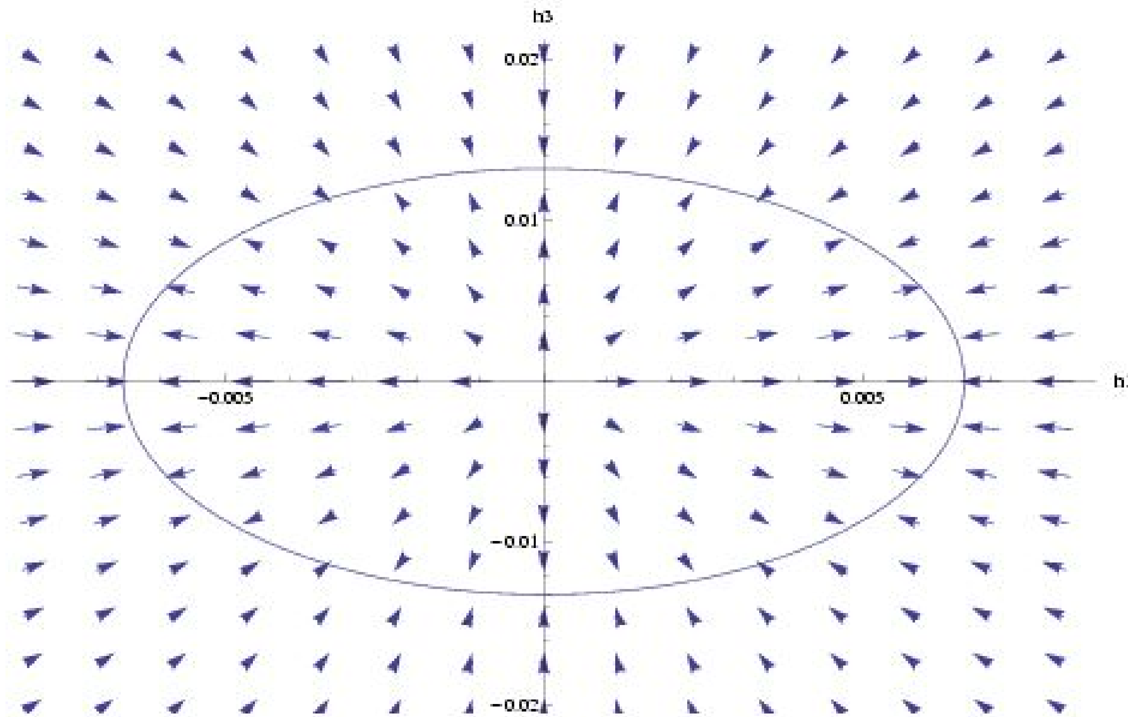


0.01
h3
0.00
-0.01
-0.010
-0.005
0.000
α1
0.005
0.010
0.010
0.005
0.000
α2
-0.005
-0.010
0.010
0.005
0.000
-0.005
-0.010
-0.010
-0.005
0.000
0.005
0.010
0.005
h4
0.000
-0.005
0.010
0.005
h3
0.000
-0.005
-0.010
-0.005
h1
0.000
0.005

Summary and conclusions

- We have determined the spectrum of IR fixed points for the ABJ–like class of theories and their generalizations
- Since the fixed points are at finite distance from the UV free theory, for a suitable choice of the parameters ($K_1, K_2 \gg N, M$) they can be studied perturbatively
- Our results are two–loop exact (no planar limit)
- We have confirmed the IR stability of fixed points already conjectured in the literature and found a larger class of IR stable fixed points
- Which are the dual supergravity descriptions of the fixed points we have found?

- We have discussed the role of global symmetries and clarified the meaning of IR stability. (D. Green, Z. Komargodski, N. Seiberg, Y. Tachikawa and B. Wecht, arXiv:1005.3546)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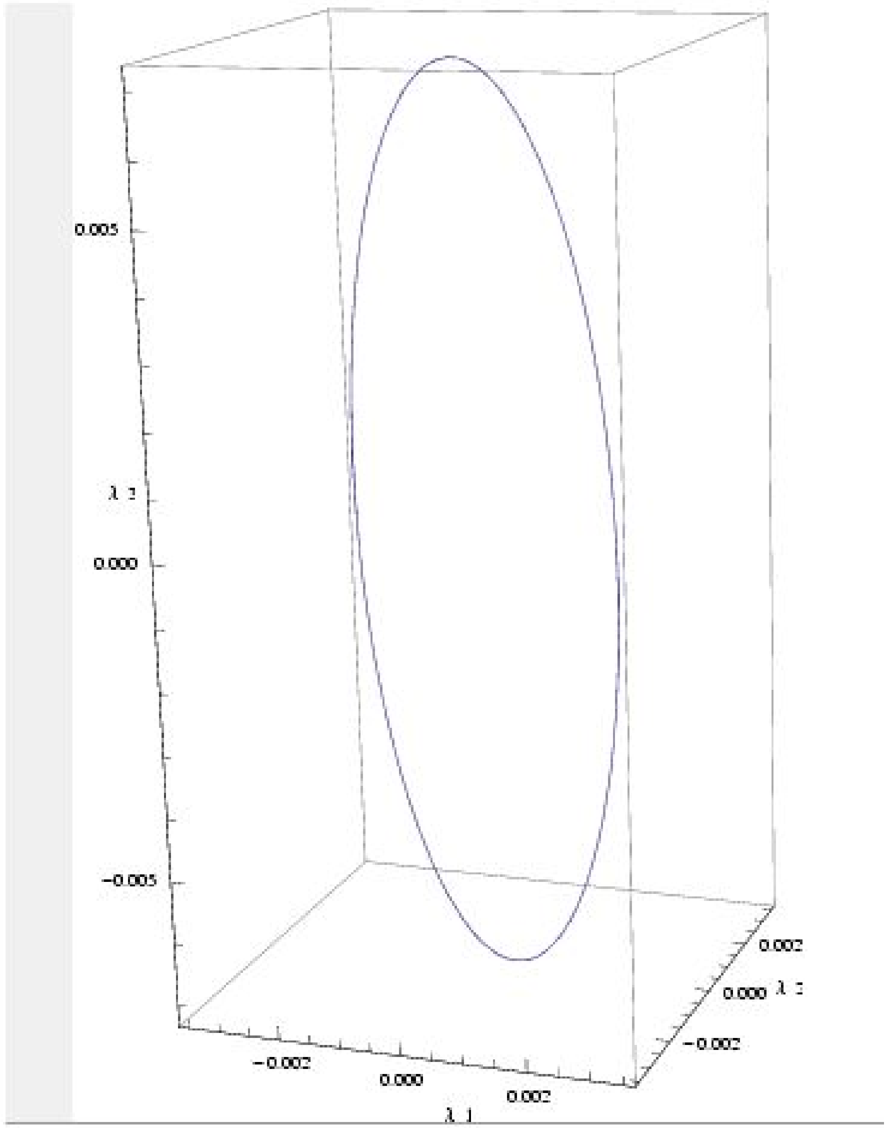
0.01
h1
−0.005
0.005
−0.01
−0.02



0.005
λ 3
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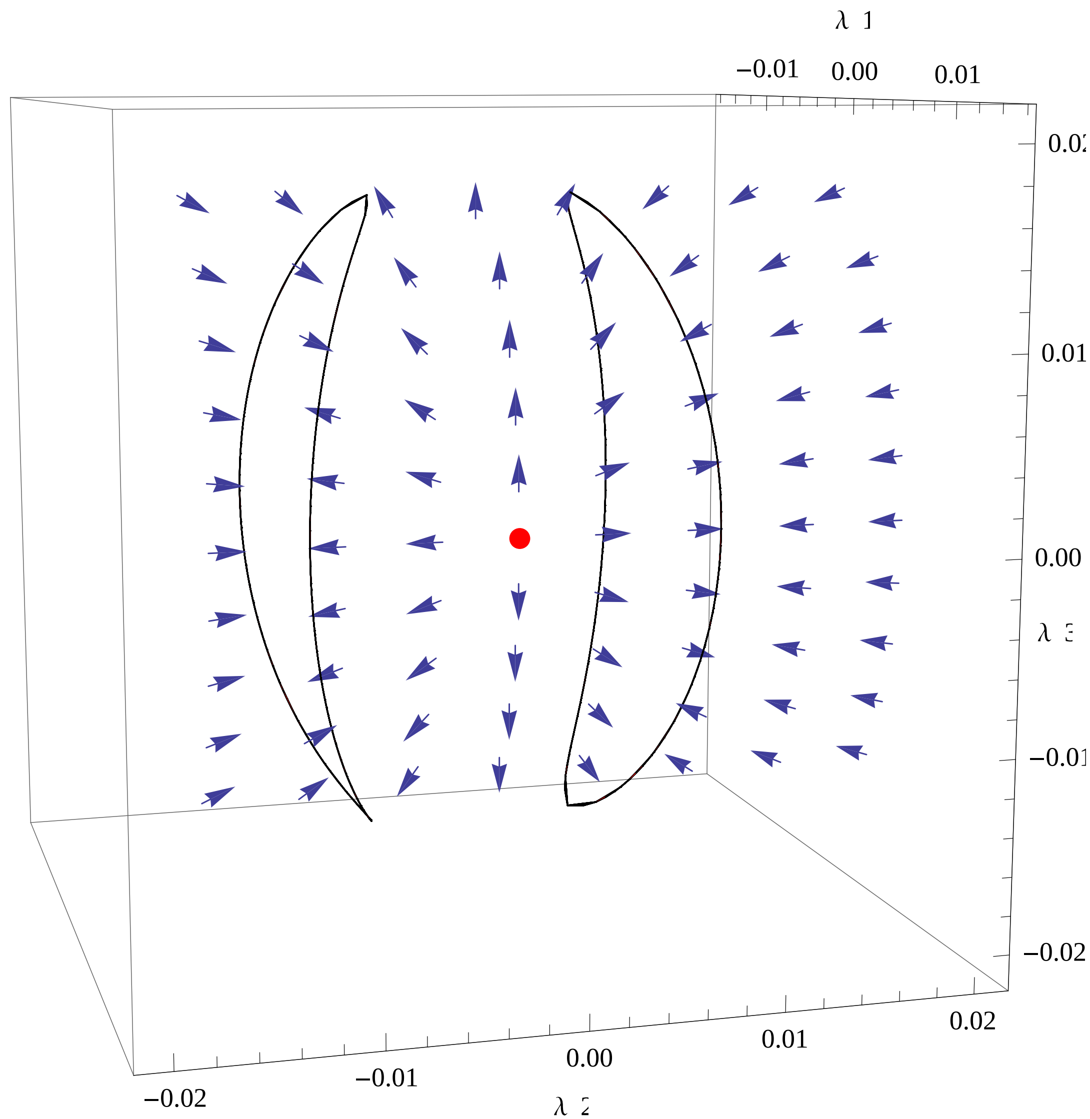
−0.002
0.000
0.002
λ 1
0.002
0.000
λ 2
−0.002


λ 1
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0.00
0.01
0.02
0.01
0.00
λ 3
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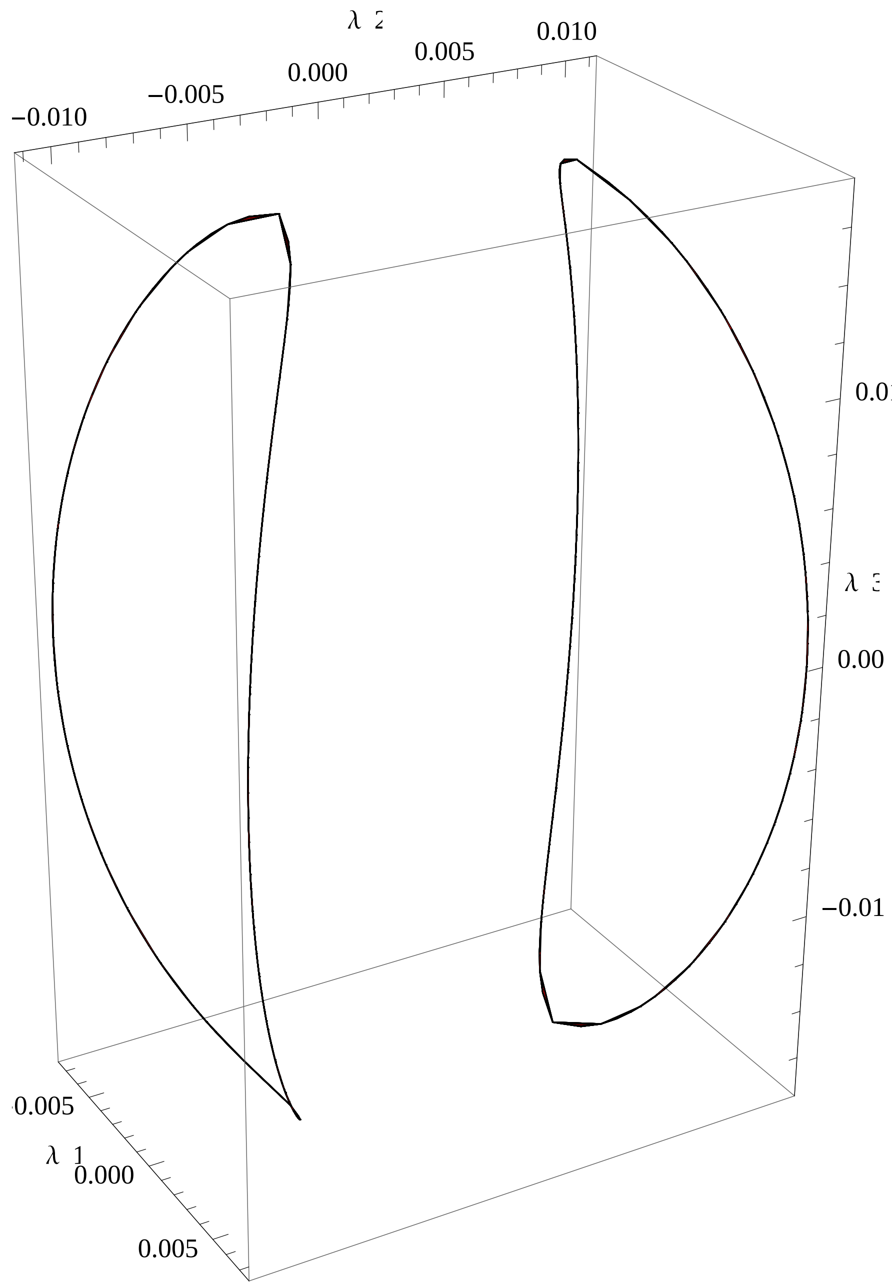
−0.02
−0.02
−0.01
0.00
0.01
0.02
λ 2



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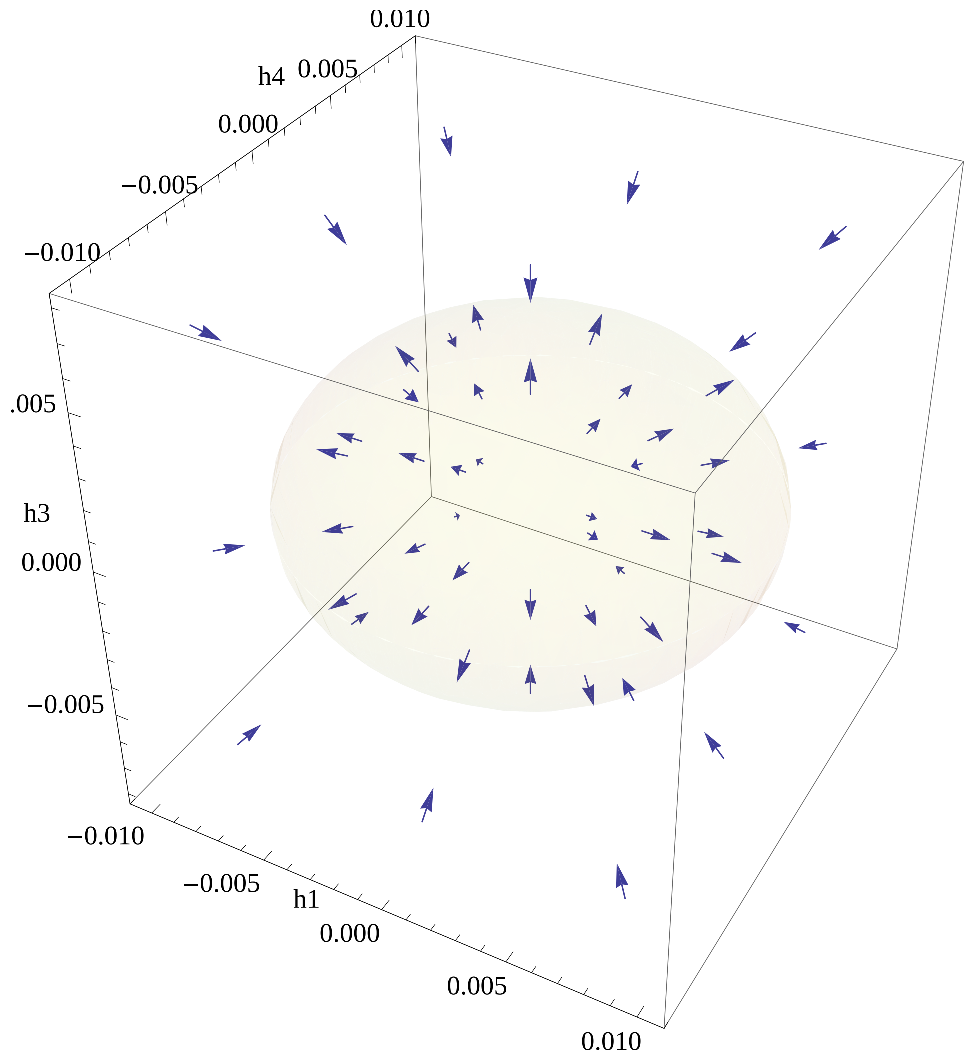
0.000
0.005
0.010
λ 3
0.01
0.00
−0.01
λ 1
0.005
0.000
0.005



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−0.010
h3
.005
0.000
−0.005
h1
−0.010
−0.005
0.000
0.005
0.010