Metastable Vacua from SCFTs

Based on arXiv:1003.0523
with A. Amariti, A. Mariotti and M.Siani

Luciano Girardello - Università di Milano - Bicocca

- $\mathcal{N}=1$ SUSY $SU(N_c)$ gauge theory, N_f Q and $\tilde{Q}$ with $SU(N_f)^2$ sym.

$$\mathcal{L}_{SQCD} = \int d^4\theta K_{ij} \left(Q_i e^{-V} Q_j^\dagger + \tilde{Q}_i e^V \tilde{Q}_j^\dagger \right) + \frac{1}{2g^2} \int d^2\theta \mathcal{W}_\alpha \mathcal{W}^\alpha + \int d^4\theta W(Q, \tilde{Q}) + \text{h.c.}$$

If $W(Q,\tilde{Q}) \neq 0$
$SU(N_f)^2$ usually broken
to $SU(N_f)_D$

$\longrightarrow$
Matter Fields

	N_c	$\bar{N}_f$	$U(1)_B$	$U(1)_R$
Q	$\bar{N}_c$	N_f	1	$1-\frac{N_f}{2N_c}$
$\tilde{Q}$	N_c	N_f	1	$1-\frac{N_f}{2N_c}$

$\boxed{\text{Gauge Coupling constant}}$

$$\frac{8\pi^2}{bg^2} = \log\frac{E}{\Lambda_h} \qquad \longrightarrow \qquad \Lambda_h = E e^{-\frac{8\pi^2}{bg^2}}$$

$\Lambda_h =$ holomorphic scale, $E =$ ren. scale

- Seiberg duality

If $N_f > N_c + 1$ the *electric* SQCD theory in the IR is equivalently described by a *magnetic* theory

	Electric theory	Magnetic Theory
Gauge	$SU(N_c)$	$SU(N_f - N_c)$
Flavor	$SU(N_f)$	$SU(N_f)$
Fields	$Q, \tilde{Q}$	$q, \tilde{q}$ and $M = \frac{Q\tilde{Q}}{\Lambda}$
Superpotential	$W = 0$	$W = Mq\tilde{q}$
Mass terms	$\Delta W = mQ\tilde{Q}$	$\Delta W = \mu^2 M$
Holomorphic scale	$\Lambda_h = Ee^{\frac{-8\pi^2}{bg^2}}$	$\tilde{\Lambda}_h = Ee^{\frac{-8\pi^2}{\tilde{b}\tilde{g}^2}}$

Scale matching (mapping of the SUSY vacua) $\Lambda_h^b \tilde{\Lambda}_h^{\tilde{b}} = (-1)^{2N_f} \hat{\Lambda}^{2N_f}$

with $b = 3N_c - N_f$, $\tilde{b} = 2N_f - 3N_c$ and $\hat{\Lambda}$ is an intermediate scale

- UV/IR duality maps a strongly coupled theory to a weakly coupled one
- $\boxed{N_c + 1 < N_f < \frac{3}{2} N_c}$ $\qquad b > 0$ and $\tilde{b} < 0$

  UV free electric window - IR free magnetic window
- $\boxed{\frac{3}{2} N_c < N_f < 3N_c}$ $\qquad$ Conformal Window

  IR interacting fixed point in both sides of duality $\qquad$ If $N_f < 2N_c \rightarrow \tilde{g} < g$; If $N_f > 2N_c \rightarrow \tilde{g} > g$
- $\boxed{N_f > 3N_c}$ $\qquad b < 0$ and $\tilde{b} > 0$

  IR free electric window - UV free magnetic window

- SUSY breaking in the IR free magnetic window
- $W_{ele} = mQ\tilde{Q}, \qquad N_c + 1 < N_f < \frac{3}{2}N_c$
- Witten Index: N_c SUSY vacua
- Strongly coupled IR physics $\rightarrow$ not accessible
- UV/IR duality, magnetic descriprion

$$W = hMq\tilde{q} - \mu^2 M$$

- where

$$\mu \sim -m\hat{\Lambda}, \quad h \sim \frac{\Lambda}{\hat{\Lambda}}, \quad M \sim \frac{Q\tilde{Q}}{\Lambda}$$

Kahler canonical, correction suppressed

Tree level SUSY breaking

- Solving equation of motion: $F_M = hq^i_\alpha \tilde{q}^\alpha_j - h\mu^2\delta^i_j$

  with $\alpha = 1,\dots,N_f - N_c$, $i,j = 1,\dots,N_f$

- Rank Condition: $F_M = h\mu^2\delta^{i'}_{j'}$ with $i',j' = N_f - N_c + 1,\dots,N_f$
- At tree level SUSY is broken, and $V_{tree} = N_c|h\mu^2|^2$
- (Pseudo)-moduli space

$$q = \begin{pmatrix} \mu e^{\theta} \\ 0 \end{pmatrix}, \quad \tilde{q}^T = \begin{pmatrix} \mu e^{-\theta} \\ 0 \end{pmatrix}, \quad M = \begin{pmatrix} 0 & 0 \\ 0 & X \end{pmatrix}$$

- Tree level analysis inconclusive, one loop analysis necessary
- Ignoring corrections from the Kahler ($\gamma_i \sim \frac{h^2}{32\pi^2}$)
- Effective potential: CW

$$V_{eff}^{(1)} = \frac{1}{64\pi^2} STr \mathcal{M}^4 \log \frac{\mathcal{M}^2}{\Lambda^2} = \frac{1}{64\pi^2} STr \left(Tr m_B^4 \log \frac{m_B^2}{\Lambda^2} - Tr m_F^4 \log \frac{m_F^2}{\Lambda^2} \right)$$

- Fluctuations around the vacuum

$$q = \begin{pmatrix} \mu e^{\theta} + \sigma_1 \\ \phi_1 \end{pmatrix}, \quad \tilde{q}^T = \begin{pmatrix} \mu e^{-\theta} + \sigma_2 \\ \phi_2 \end{pmatrix}, \quad M = \begin{pmatrix} \sigma_3 & \phi_3 \\ \phi_4 & X \end{pmatrix}$$

- Relevant superpotential

$$W = X\phi_1\phi_2 - h\mu^2 X + h\mu e^{-\theta}\phi_1\phi_4 + h\mu e^{\theta}\phi_2\phi_3$$

- Mass matrices

$$m_B^2 = \begin{pmatrix} W^{\dagger ac} W_{cb} & W^{\dagger abc} W_c \\ W_{abc} W^{\dagger c} & W_{ac} W^{\dagger cb} \end{pmatrix} \quad m_f^2 = \begin{pmatrix} W^{\dagger ac} W_{cb} & 0 \\ 0 & W_{ac} W^{\dagger cb} \end{pmatrix}$$

- Classical spectrum with $m_f^2 = m_f^2(X)$ and $m_b^2 = m_b^2(X)$
- Minimum of the CW effective potential at

$$\langle X \rangle = 0 \quad \& \quad \langle \theta + \theta^* \rangle = 0$$

$$m_X^2 = N_c \frac{h^2 \mu^2}{8\pi^2} (\log 4 - 1)$$

$$m_{\theta+\theta^*}^2 = (N_f - N_c) \frac{h^2 \mu^2}{8\pi^2} (\log 4 - 1)$$

- Electric theory: N_c SUSY vacua
- Duality $\rightarrow$ N_c SUSY vacua also in the magnetic model
- Indeed at large $\langle M \rangle$ the q and $\tilde{q}$ quarks are integrated out
- Scale matching $\Lambda_h^{\tilde{b}} = \Lambda_L^{\tilde{b}-N_f} \det hM$
- The effective theory at large M is pure SYM with singlets

$$W = W_{dyn} + W_{tree} = (N_f - N_c)\Lambda_L^3 - h\mu^2 M$$

with

$$W_{dyn} = (N_f - N_c)\left(\tilde{\Lambda}_h^{\tilde{b}} \det(hM)\right)^{\frac{1}{N_c}}$$

- $\langle hM \rangle_{SUSY} = \mu \left(\frac{\mu}{\tilde{\Lambda}_h}\right)^{\frac{\tilde{b}}{N_c}} \gg \mu \qquad if \qquad \tilde{\Lambda}_h \gg \mu$

- Lifetime: $\Gamma \sim e^{-S_B}$
- For a scalar field the classical bounce action is

$$S_B = \int d^4x\, \partial_\mu \phi\, \partial^\mu \phi + V(\phi)$$

- If $V(\phi)$ is a triangular barrier $S_B \sim \frac{(\Delta\phi)^4}{\Delta V}$
- Here $S_B \sim \left(\frac{\mu}{\tilde{\Lambda}_h}\right)^{\frac{4\tilde{b}}{N_c}} \gg 1$ with $\tilde{b} < 0$ and $\tilde{\Lambda}_h \gg \mu$
- $\rightarrow$ Parametrically large lifetime
(the more the theory is weakly coupled the more the lifetime is large)

- Massive SQCD with $\frac{3}{2}N_c < N_f < 3N_c$
- Electric and magnetic theories have are interacting SCFTs
- $\tilde{g} < g$ if $N_f < 2N_c$
- Classically supersymmetry is broken as in the IR free window
- But naively $S_B = \left(\frac{\mu}{\tilde{\Lambda}_h}\right)^{\frac{\tilde{b}}{N_c}}$ is not large

$$\tilde{b} = 3(N_f - N_c) - N_f = 2N_f - 3N_c > 0$$

- Equivalently: W_{dyn} is not negligible at small M

- Deeper analysis
- There are two important differences with the IR free case
  - The $\gamma_i \neq 0 \to Z_i \neq 1$: RG effects not negligible
  - RG flow from Λ_{UV} to $\Lambda_{IR} \equiv \Lambda_c$, CFT exit scale
- When $\gamma_i \sim cost. \neq 0$: holomorphic $\to$ canonical basis

Holomorphic basis	Canonical basis
ϕ_i	$\phi'_i = \sqrt{Z}_i \phi_i$
$K = Z \phi_i \phi_i^\dagger$	$K = \phi'_i \phi_i'^\dagger$
$W = W(\lambda_{bare}, \phi_i)$	$W = W(\lambda_{phys.}, \phi'_i)$

- If $Z_{UV} = 1$ in the UV, then $Z_{IR} \neq 1$ after RG flow.
- Canonical basis: absorb the WFR in the fields at every scale.
- RG effects on the couplings in W: from

$$\Delta W = \lambda_i(E)\phi_i$$

  with $\lambda(E) = \Lambda(E')Z(E,E')^{-\frac{\gamma_i}{2}}$
- Running of the holomorphic scale

$$\Lambda_h(E') = \Lambda_h(E) \prod z_{\phi_i}^{T_{F_i}/2}$$

  In the canonical basis $\Lambda_h(E') = \Lambda_h(E)\frac{E'}{E}$
- Renormalization of S_B: $\quad S_B(E') = S_B(E)Z_M^3$

- CFT exit scale: two possible sources, $\langle M\rangle_{susy}$ or μ
- From F_M $\quad M = \mu(\Lambda_{IR}) \left(\frac{\tilde{\Lambda}_h(\Lambda_{IR})}{\mu(\Lambda_{IR})}\right)^{\frac{\tilde{b}}{N_c}}$
- If $M = \Lambda_c$, $\tilde{\Lambda}_h = \Lambda_c e^{-\frac{8\pi^2}{g^2\tilde{b}}}$ and

$$\mu(\Lambda_c) = e^{-\frac{4\pi^2}{g*^2(N_f-N_c)}}\Lambda_c \ll \Lambda_c$$

(If $\mu(\Lambda_C) = \Lambda_c$ then $M \gg \Lambda_c$)

- $$S_B = \left(\frac{\mu_{IR}}{\tilde{\Lambda}_{h,IR}}\right)^{4\frac{2N_f-3N_c}{N_f-N_c}} = e^{\frac{16\pi^2}{g_*^2(N_f-N_c)}}$$

The lifetime cannot be parametrically large in the ISS model in the CFT window. Metastable vacua cannot be long lifetime.

- Is it possible to overcome this problem?
- Yes, but other relevant deformations must be added
- Indeed in ISS there are only two scales
  - μ the SUSY breaking scale
  - M the SUSY restoration scale

  and they are proportional.
- S_B is their ratio, and cannot be made parametrically large
- With other deformations S_B can be parametrically large

  OR equivalently

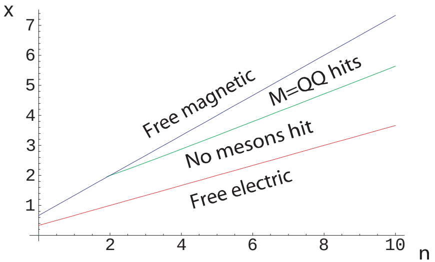
  The non perturbative potential can be negligible at small vevs.

Example: the SSQCD

- $\mathcal{N} = 1$ $SU(N_c)$ gauge theory with $SU(N_1) \times SU(N_2)$ flavour symmetry, N_1 massive and N_2 massless fundamentals, and a bifundamental of $SU(N_2)$
- $$\begin{array}{cccc} & SU(N_c) & SU(N_1) & SU(N_2) \\ Q \oplus \tilde{Q} & N_c \oplus \bar{N}_c & \bar{N}_1 \oplus N_1 & 1 \\ P \oplus \tilde{P} & N_c \oplus \bar{N}_c & 1 & \bar{N}_2 \oplus N_2 \\ S & 1 & 1 & N_2 \otimes \bar{N}_2 \end{array}$$
- Superpotential: $W = mQ\tilde{Q} + SP\tilde{P} + k^2 S$

- Seiberg duality: if $N_1 + N_2 > N_c + 1$ there is a dual $SU\left(\widetilde{N} = N_1 + N_2 - N_c\right)$ dual gauge theory
	$SU(\tilde{N})$	$SU(N_1)$	$SU(N_2)$
$q \oplus \widetilde{q}$	$\widetilde{N} \oplus \overline{\widetilde{N}}$	$\overline{N}_1 \oplus N_1$	1
$p \oplus \widetilde{p}$	$\widetilde{N} \oplus \overline{\widetilde{N}}$	1	$\overline{N}_2 \oplus N_2$
M	1	$N_1 \otimes \overline{N}_1$	1
$L \oplus K$	1	$N_1 \oplus \overline{N}_1$	$\overline{N}_2 \oplus N_2$
- $W = Mq\tilde{q} + Lq\tilde{p} + Kp\tilde{q} - \mu^2 M + \rho p\tilde{p}$

- Conformal window: $3/2N_c < N_1 + N_2 < 3N_c$
- Another constraint imposed by the rank condition:
  Tree level SUSY breaking if $N_1 > \widetilde{N} \rightarrow N_2 < N_c$
- Defining $n_{ele} = \frac{N_2}{N_1}$ and $x_{ele} = \frac{N_c}{N_1}$ there is no meson hitting the unitary bound $(R(\mathcal{O}) < \frac{2}{3})$ if $n_{ele} < 1$ and $x_{ele} < 1$



The SUSY breaking vacuum

- Rank condition

$$F_M = q_i^\alpha \tilde{q}_\alpha^j - \mu^2 \delta_i^j \qquad \alpha = 1, \ldots, \widetilde{N}, \quad i,j = 1, \ldots, N_1$$

Since $N_1 > \widetilde{N} \rightarrow N_1 - \widetilde{N}$ SUSY breaking equations

- $q = \tilde{q}^T = \begin{pmatrix} \mu \mathbf{1}_{\tilde{N}} \\ 0 \end{pmatrix} \quad p = \tilde{p} = 0 \quad M = \begin{pmatrix} 0 & 0 \\ 0 & X \mathbf{1}_{N_1 - \tilde{N}} \end{pmatrix}$

$L = \begin{pmatrix} 0 \\ Y \mathbf{1}_{N_1 - \tilde{N}} \end{pmatrix} \quad K^T = \begin{pmatrix} 0 \\ \widetilde{Y} \mathbf{1}_{N_1 - \tilde{N}} \end{pmatrix}$

- One loop CW for the pseudomoduli from W_{IR}:

$$W_{IR} = X\phi_1\phi_2 - \mu^2 X + \mu(\phi_1\phi_4 + \phi_2\phi_3) + Y\phi_1\phi_5 + Y\phi_2\phi_6 + \rho\phi_5\phi_6$$

- Assignment of R charges 0 or 2 for all the fields
- X, Y and $\widetilde{Y}$ stable at the origin
- $\boxed{\text{Possible sources of destabilizations}}$
  - Corrections from the kahler potential
  - Non perturbative dynamics

Non perturbative dynamics and SUSY vacua

- At large M the fields q and $\tilde{q}$ get a dynamical mass
- The fields p and $\tilde{p}$ are massive at tree level
- Integrating out the massive fields there is a scale matching

  $$\Lambda_L^{3\tilde{N}} = \tilde{\Lambda}_h^{3\tilde{N}-N_1-N_2} \det \rho \det M$$
- Non perturbative superpotential

$$W_{np} = \tilde{N}\Lambda_L^3 = N\left(\tilde{\Lambda}_h^{3\tilde{N}-N_1-N_2} \det \rho \det M\right)^{\frac{1}{N_c}}$$

- SUSY vacua: all the fields at the origin except M

$$\langle M \rangle_{SUSY} = \mu \left(\frac{\mu}{\tilde{\Lambda}_h}\right)^{\frac{3\tilde{N}-N_1}{\tilde{N}}} \left(\frac{\tilde{\Lambda}_h}{\rho}\right)^{\frac{N_2}{\tilde{N}}}$$

The bounce action

- $S_B \sim \frac{M^4}{\mu^4} = \left(\frac{\mu}{\tilde{\Lambda}_h}\right)^{4\frac{3\tilde{N}-N_1}{\tilde{N}}} \left(\frac{\tilde{\Lambda}_h}{\rho}\right)^{\frac{4N_2}{\tilde{N}}}$

  S_B renormalized action
  In the canonical basis
  renormalization effects
  on the couplings

- Large Lifetime
  OR
  Small W_{np} at the origin

  if $\frac{\rho}{\tilde{\Lambda}_h} \ll \left(\frac{\mu}{\Lambda}\right)^{\frac{3\tilde{N}-N_1}{N_2}}$

- Relations at the CFT exit scale Λ_c
- $\Lambda_c = \langle M \rangle_{SUSY} \gg \mu_{IR}, \rho_{IR}$
- Indeed
  $\epsilon_{IR} \equiv \frac{\rho_{IR}}{\mu_{IR}} \ll 1$ $\longrightarrow$

$$\boxed{\begin{aligned} \mu_{IR} &= \Lambda_c^{-\frac{8\pi^2}{g_*^2(2\tilde{N}-N_2)}} \epsilon_{IR}^{\frac{N_2}{(2\tilde{N}-N_2)}} \ll \Lambda_c \\ \rho_{IR} &= \Lambda_c^{-\frac{8\pi^2}{g_*^2(2\tilde{N}-N_2)}} \epsilon_{IR}^{\frac{2\tilde{N}}{(2\tilde{N}-N_2)}} \ll \Lambda_c \end{aligned}}$$

- The renormalized bounce action at Λ_C is

$$S_B = \frac{e^{\frac{32\pi^2}{g_*^2(2\tilde{N}-N_1))}}}{\epsilon^{\frac{4N_2}{\tilde{N}-N_2}}} \qquad \text{(As ISS if } N_2 \to 0\text{)}$$

- Parametric dependence on the small relevant deformations
- What constraints have to be imposed in the UV?
  - ρ_{UV} and μ_{UV} small s.t. the model is approximately a SCFT
  - Bound on ϵ_{UV} s.t. it runs to an allowed ϵ_{IR}
- In the canonical basis analysis based on the running of the couplings in W
- $$\begin{aligned} \mu_{IR} &= \mu_{UV} Z_M^{-1} \\ \rho_{IR} &= \rho_{UV} Z_p Z_{\tilde{p}} \end{aligned} \quad \text{where} \quad \boxed{\begin{array}{c} Z_{\phi_i} \quad \text{given by solving} \\ \frac{\partial \log Z_i}{\partial \log E} = -\gamma_i \\ \text{with } \gamma_i \sim cost. \end{array}}$$
- $$Z_{\phi_i} = \left(\frac{\Lambda_{IR}}{\Lambda_{UV}}\right)^{-\gamma_i} \qquad \text{where here } \Lambda_{IR} = \Lambda_c$$

Anomalous dimensions

- Δ_i = Scaling dimension of $\phi_i \rightarrow \Delta_i = 1 + \gamma_i/2$
- In a SCFT $\Delta_i = \frac{3}{2}R_i$, whith R_i the exact R-charge
- R-exact at the fixed point is computed by a-maximization
-
$$\boxed{a_{trial}(R) = \frac{3}{32\frac{1}{1}}\left(3TrR^3 - TrR\right)}$$
TrR^3 Coeff.of the gauge anomaly
TrR Coeff. of the grav. anomaly

$R = R_0 + \sum F_i$ non anomalous combinations of the $U(1)$s
- Constraints from anomalies and from W impose
- $\tilde{N} + N_1(R[q]-1) + N_2(R[p]-1) = 0, \quad 2R[q] + R[N] = 2, \quad R[K] + R[p] + R[q] = 2$

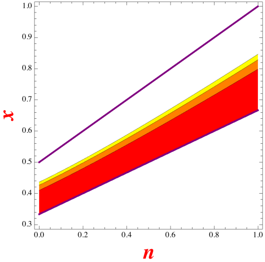
$R[q] = R[\tilde{q}]$, $R[p] = R[\tilde{p}]$ and $R[K] = R[L]$ from the symmetries

- Define $n = \frac{N_1}{N_2}$ and $x = \frac{\tilde{N}}{N_2}$
- The relevant R-charges are $\begin{array}{l} R[p] = \frac{n-x+y_{max}}{n} \\ R[M] = 2y_{max} \end{array}$
- By using $\gamma_i = 3R_i - 2$

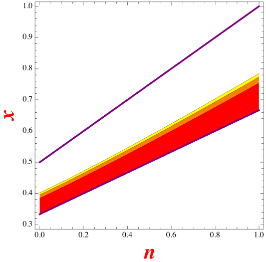
$$\epsilon_{IR} = \epsilon_{UV} \left(\frac{\Lambda_c}{E_{UV}} \right)^{\frac{3}{2n}(n-2x+y_{max}(2-n))} \ll 1$$

- Is this inequality satisfied in the whole CFT region ?
- Does E_{UV}/Λ_c have to be large?

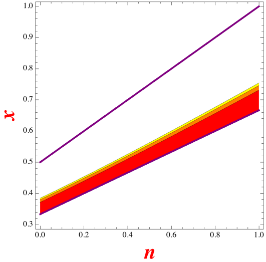
Allowed region for the existence of metastable vacua in the weakly coupled region of the conformal window



$\Lambda_c/E_{UV} = 10^{-4}$



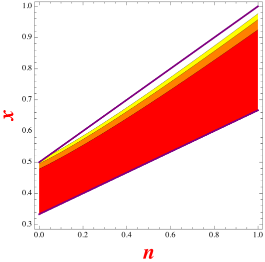
$\Lambda_c/E_{UV} = 10^{-6}$



$\Lambda_c/E_{UV} = 10^{-8}$

$$\epsilon_{UV} = 10^{-2}$$

Allowed region for the existence of metastable vacua in the weakly coupled region of the conformal window



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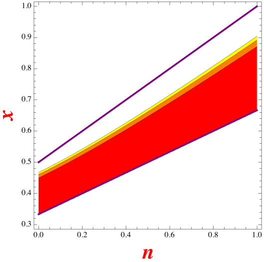

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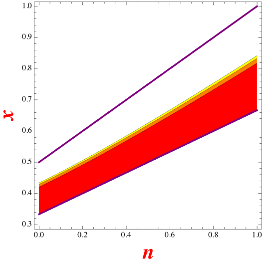
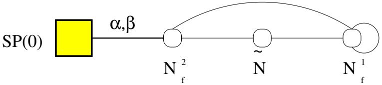


$\Lambda_c/E_{UV} = 10^{-8}$

$$\epsilon_{UV} = 10^{-4}$$

UV hyerarchies

- Metastable vacua in SSQCD exist only if $\rho_{UV} \ll \mu_{UV}$
- This constraint can be imposed by hand
- Alternatively a dynamical mechanism is necessary
- If $N_2 = 1$ mass term from an $SP(0)$ stringy instanton



- $S_{inst} = \alpha p \tilde{p} \beta \rightarrow W_{inst} = \int d\alpha d\beta e^{S_{inst}} = \Lambda e^{-A} p \tilde{p}$

Extension to other $\mathcal{N} = 1$ SCFTs

- Other SCFTs can possess metastable vacua in the IR
- Along the lines of the SSQCD case it is possible to give a general recipe for building these models
- Take a $\mathcal{N} = 1$ gauge theory with metastable vacua in the IR free magnetic window
- Consider the dual theory and add some massive quarks, s.t. the theory is in a conformal window
- In most of the cases other fields and interactions must be added, s.t. no mesons hit the unitary bound
- S_B has a parametrical dependence on the relevant couplings
- With the RG running look for the UV regimes of scales and ranks consistent with the requirement of large S_B

- Genericity in $\mathcal{N} = 1$ SCFTs
- R-symmetry breaking by relevant, marginal or irrelevant deformations
- Gauge mediation and $SU(5)$ GUT
- Suppressing flavour anarchy in single sector of SUSY breaking
  - Small anomalous dimensions (near the IR free regime)
  - Long running in the CFT $\rightarrow$ Landau poles tipically below the unification scale
  - a-maximization with more groups not straightforward