How does the string in flat space-time know about curved space-time?

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Foreword

- This talk is based on the work done together with

  Giuseppe D'Appollonio, Rodolfo Russo and Gabriele Veneziano, to appear.

Plan of the talk

Introduction

- String theory, as originally formulated, is a theory in flat Minkowski space-time.
- It contains in its spectrum a massless spin 2 particle that has all the properties of a graviton.
- It turns out that the low-energy string effective action that one derives from scattering of strings is a (super)gravity theory with string corrections.
- Curved space-time is not put by hand, as in GR, but emerges from string scattering amplitudes.
- At the perturbative level ($g_s \sim 0$) string theory is only a theory of strings.
- If we take into account non-perturbative effects string theory contains additional p-dimensional states called D(irichlet)p branes.

- On the one hand, they are classical solutions of the low-energy 10-dim string effective action:

$$S = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g} \left[R - \frac{1}{2} (\nabla\phi)^2 - \frac{1}{2(p+2)!} \mathrm{e}^{-a\phi} \left(F_{p+2}\right)^2 \right]$$

coupled to graviton, dilaton and RR $(p+1)$-form potential given by:

$$ds^2 = [H(r)]^{2A} \left(\eta_{\alpha\beta} dx^\alpha dx^\beta \right) + [H(r)]^{2B} \left(\delta_{ij} dx^i dx^j \right)$$

with $r^2 \equiv \delta_{ij} x^i x^j$ and

$$\mathrm{e}^{-\phi(x)} = [H(r)]^{\frac{p-3}{8}} \quad , \quad \mathcal{C}_{01\ldots p}(x) = \left([H(r)]^{-1} - 1 \right)$$

- A, B and are equal to

$$A = -\frac{7-p}{16} \quad , \quad B = \frac{p+1}{16}$$

- $H(r)$ is an harmonic function given by

$$H(r) = 1 + \left(\frac{R}{r}\right)^{7-p} \;\; ; \;\; R_p^{7-p} = \frac{2\kappa_{10} T_p N}{(7-p)\Omega_{8-p}} = \frac{g_s N (2\pi\sqrt{\alpha'})^{7-p}}{(7-p)\Omega_{8-p}}$$

$$\kappa_{10} = \frac{(2\pi)^{7/2}}{\sqrt{2}} g_s (\alpha')^2 \;\; ; \;\; T_p = \frac{\sqrt{\pi}}{(2\pi\sqrt{\alpha'})^{p-3}} \;\; ; \;\; \Omega_q = \frac{2\pi^{\frac{q+1}{2}}}{\Gamma(\frac{q+1}{2})}$$

- Mass per unit volume and RR charge

$$M_p = \frac{T_p}{\kappa_{10}} N = \frac{(2\pi\sqrt{\alpha'})^{1-p}}{2\pi\alpha' g_s} N \;\; ; \;\; \mu_p = \sqrt{2} T_p N$$

Non-perturbative in g_s.

- The low-energy string effective action is an action in curved space-time and therefore the classical solution will inherit all the properties of a theory of (super)gravity.

- On the other hand, in string theory the Dp branes are characterized by having open strings attached to their $(p+1)$-dim world-volume.
- Therefore, the open strings satisfy Dirichlet boundary conditions along the directions transverse to the world-volume of the Dp brane.
- In string theory a Dp brane is described by a closed string state, called the boundary state.
- Dp branes interact by exchanging open strings and therefore the lowest order interaction is given by the annulus diagram. [Polchinski, 1995]
- By open/closed string duality the annulus diagram satisfies the following identity:

$$-\mathrm{Tr}\log(L_0 - a) = \int_0^\infty \frac{d\tau}{\tau} \mathrm{Tr}\left(\mathrm{e}^{-2\pi\tau(L_0-a)}\right) = \langle B|D|B\rangle$$

that determines the boundary state

$$|B\rangle \equiv \frac{T_p}{2} N|B_X\rangle|B_\psi\rangle \quad ; \quad D = \frac{\alpha'}{4\pi}\int_{|z|<1} \frac{d^2 z}{|z|^2} z^{L_0-a}\bar{z}^{\tilde{L}_0-a}$$

- The bosonic part of the boundary state is equal to

$$|B_X\rangle = \delta^{d-p-1}(\hat{q}^i - y^i)\left(\prod_{n=1}^{\infty} e^{-\frac{1}{n}\alpha_{-n}S\cdot\widetilde{\alpha}_{-n}}\right)|0\rangle_\alpha|0\rangle_{\widetilde{\alpha}}|p=0\rangle$$

$S \equiv (\eta_{\alpha\beta}; -\delta_{ij})$

- Using the boundary state and the vertex operators for open and closed strings one can compute any amplitude involving scattering of strings on the D branes.
- In particular, these amplitudes determine the structure of the Born-Infeld action.
- All calculations are done in flat space where we are able to quantize the string.
- On the other hand, the D branes are classical solutions of the low-energy string effective action in curved space.
- How are we going to recover the curved space properties of the Dp branes computing scattering of strings in flat space?
- This is what I am going to show in some example in this seminar.

The large distance behaviour of the classical solution [M. Frau, A. Lerda, I. Pesando, R. Russo, S. Sciuto and PDV, 1997]

- Given the boundary state one can compute the large distance behaviour of the various fields of the classical solution as follows:

$$\langle\psi|D|B\rangle \quad ; \quad D = \frac{\alpha'}{4\pi}\int \frac{d^2z}{|z|^2} z^{L_0 - a}\bar{z}^{\tilde{L}_0 - a}$$

$\langle\psi|$ is the string state corresponding to the field of the classical solution.

- Let us start by computing the expression for the generic NS-NS massless field which is given by

$$J^{\mu\nu} \equiv {}_{-1}\langle\widetilde{0}, \frac{k_\perp}{2}|_{-1}\langle 0, \frac{k_\perp}{2}|\psi^\nu_{1/2}\,\widetilde{\psi}^\mu_{1/2}|D|B\rangle_{NS} = -\frac{T_p}{2k_\perp^2}V_{p+1}S^{\nu\mu}$$

- Specifying the different polarizations corresponding to the various fields we get for the dilaton

$$\delta\phi = \frac{1}{\sqrt{8}}\left(\eta^{\mu\nu} - k^\mu\ell^\nu - k^\nu\ell^\mu\right)J_{\mu\nu} = \frac{3-p}{4\sqrt{2}}\,T_p\,\frac{V_{p+1}}{k_\perp^2}$$

- For the graviton we get:

$$\delta h_{\mu\nu}(k) = \frac{1}{2}\Big(J_{\mu\nu} + J_{\nu\mu}\Big) - \frac{\delta\phi}{\sqrt{8}}\,\eta_{\mu\nu}$$

$$= T_p \frac{V_{p+1}}{k_\perp^2}\,\mathrm{diag}\,(-A, A \ldots A, B \ldots B) \quad ,$$

where $A = -\frac{7-p}{16}$ and $B = \frac{p+1}{16}$

- For the Kalb-Ramond field we get:

$$\delta B_{\mu\nu}(k) = \frac{1}{\sqrt{2}}\Big(J_{\mu\nu} - J_{\nu\mu}\Big) = 0$$

- In the R-R sector we get instead

$$\delta C_{01\ldots p}(k) \equiv \langle P^{(C)}_{01\cdots p}|\, D\, |B\rangle_{\mathrm{R}} = \mp\, \mu_p \frac{V_{p+1}}{k_\perp^2} \quad ; \quad T_p = \sqrt{2}\mu_p$$

- We can express the previous fields in configuration space using the following Fourier transform valid for $p < 7$

$$\int d^{(p+1)}x \, d^{(9-p)}x \frac{e^{ik_\perp \cdot x_\perp}}{(7-p)\, r^{7-p}\, \Omega_{8-p}} = \frac{V_{p+1}}{k_\perp^2} ,$$

- We must rescale the various fields according to

$$\varphi = \sqrt{2}\kappa_{10}\phi \quad , \quad \tilde{h}_{\mu\nu} = 2\kappa_{10} h_{\mu\nu} \quad , \quad \mathcal{C}_{01\ldots p} = \sqrt{2}\kappa_{10} C_{01\ldots p}$$

- We get the following large distance behaviour for the dilaton

$$\delta\varphi(r) = \frac{3-p}{4} \left(\frac{R_p}{r}\right)^{7-p}$$

- For the graviton we get

$$\delta\tilde{h}_{\mu\nu}(r) = 2\left(\frac{R_p}{r}\right)^{7-p} \mathrm{diag}\left(-A, \ldots A, B \ldots B\right) \quad ,$$

- For the RR field we get

$$\delta\mathcal{C}_{01\ldots p} = \left(\frac{R_p}{r}\right)^{7-p}$$

- The previous equations reproduce exactly the behavior for $r \to \infty$ of the metric, the dilaton and the R-R potential of the solution.
- The next to the leading behaviour is expected to come from the one-point function with two boundary states:

$$\mathcal{N}\sum_{\alpha,\beta}\langle B|\int d^2 z_1 \;\, W(z_1, \bar{z}_1) \;\, D \;|B\rangle_{\alpha,\beta}$$

W is the vertex operator corresponding to the massless closed string.

- The explicit calculation gave zero after the sum over the spin structures [R. Marotta, I. Pesando, PDV (1998), unpublished].

- Two alternative ways of getting the classical solution.
- One is by solving the classical supergravity equations of motion with the Dp brane as a source.
- The other is by computing the one-point function for the closed string fields in an action that contains their interaction in the bulk and their interaction with the D brane:

$$\langle \Phi(x)\ \mathrm{e}^{iS_{bulk}+iS_{boundary}} \rangle$$

where

$$S_{boundary} = \int d^{p+1}x\ T_p \left[-\eta_{\alpha\beta} h^{\alpha\beta} + \frac{3-p}{2\sqrt{2}} \phi + \sqrt{2} C_{01\ldots p} \right]$$

- By explicit calculation one can reproduce the leading and the next to the leading behaviour of the classical solution.
- It is still not clear why the stringy calculation does not work as in field theory. One may need an off-shell extrapolation.

The approach of Amati, Ciafaloni and Veneziano(ACV) [ACV, 1987; Sundborg, 1987]

- The starting point of the ACV approach is the four graviton scattering amplitude on the sphere in the Regge limit ($s = 4E^2 \to \infty$ and small t):

$$\frac{a_{Tree}}{s} = \frac{32\pi G_N}{\alpha' s} \frac{\Gamma(-\frac{\alpha'}{4}t)}{\Gamma(1+\frac{\alpha'}{4}t)} \left(\frac{\alpha'}{4}s\right)^{2+\frac{\alpha'}{2}t} \mathrm{e}^{-i\pi\frac{\alpha'}{4}t} \Longrightarrow 8\pi G_N \frac{s}{(-t)}$$

- At high energy its Fourier transform in the space of impact parameter exponentiates and one can compute the classical deflection angle for large impact parameter:

$$\Theta = \frac{\sqrt{\pi}\ \Gamma(\frac{D}{2})}{\Gamma(\frac{D-1}{2})} \left(\frac{r_s}{b}\right)^{D-3} \quad ; \quad r_s^{D-3} = \frac{16\pi G_N \sqrt{s}}{(D-2)\Omega_{D-2}}$$

  D is the number of non-compact directions and r_s is the Schwarzschild radius.

- Then there are classical next to the leading corrections in the large impact parameter and string corrections.

- It corresponds to the deflection angle of a particle moving in the Aichelburg-Sexl metric that can be seen as the boosted Schwarzschild metric.
- It is the metric created by a fast moving particle as seen from the other particle.
- In the case of a scattering on a Dp brane, there is a background metric, namely the metric created by the Dp brane.
- How can we see the effect of this metric in the scattering of a massless closed string on a Dp brane?

The classical deflection angle in brane background

- At the classical level we can compute the deflection angle of a massless probe moving in the metric created by a Dp brane.
- Consider a general metric of the kind:

$$ds^2 \equiv g_{\mu\nu}(x)dx^\mu dx^\nu = -\alpha(r)dt^2 + \beta(r)(dr^2 + r^2 d\theta^2)$$

  where we have neglected coordinates that are not involved for a geodesic in which only t, r and θ vary.
- The geodesic equations can be best derived from the action of a massless point-particle in this metric:

$$S = \frac{1}{2}\int \frac{d\tau}{e}\dot{x}^\mu \dot{x}^\nu g_{\mu\nu}(x) = \frac{1}{2}\int \frac{d\tau}{e}\left(-\dot{t}^2\alpha(r) + \beta(r)\left(\dot{r}^2 + r^2\dot{\theta}^2\right)\right)$$

  where e is the einbein to take care of the reparametrization invariance of the world line coordinate τ.
- The conjugate momenta are given by:

$$p_t \equiv \frac{\partial L}{\partial \dot{t}} = -\frac{\dot{t}\alpha}{e} \;\; ; \;\; p_r \equiv \frac{\partial L}{\partial \dot{r}} = \frac{\beta(\rho)\dot{r}}{e} \;\; ; \;\; p_\theta = \frac{\partial L}{\partial \dot{\theta}} = \frac{\dot{\theta} r^2 \beta(r)}{e}$$

- The Eq. of motion for e gives:

$$\beta(r)\dot{r}^2 + \beta(r)r^2\dot{\theta}^2 = \alpha(r)\dot{t}^2$$

- Since the Lagrangian does not depend explicitly on either t or θ there are two conserved quantities: the energy and the angular momentum

$$E = -\alpha(r)\dot{t} \quad ; \quad J = \beta(r)r^2\dot{\theta}$$

where a dot denotes derivative with respect to τ and we have taken $e = 1$.

- Combining the three previous equations we get

$$\frac{\dot{\theta}}{\dot{r}} = \frac{J}{\beta r^2}\frac{1}{\sqrt{\frac{E^2}{\alpha\beta} - \frac{J^2}{\beta^2 r^2}}} = \frac{b}{r^2}\frac{1}{\sqrt{\frac{\beta}{\alpha} - \frac{b^2}{r^2}}}$$

where $b \equiv J/E$ is the impact parameter.

- The deflection angle is therefore given by:

$$\Theta_p = 2\int_{r*}^{\infty} \frac{dr}{r^2} \frac{b}{\sqrt{\frac{\beta}{\alpha} - \frac{b^2}{r^2}}} - \pi$$

  $r*$ is the turning point i.e. the largest root of the equation $\frac{\beta}{\alpha} - \frac{b^2}{r^2} = 0$.
- The result depends only on α/β.
- It is therefore invariant under a r-dependent rescaling of the whole metric.
- Therefore, we can work alternatively in either the string or the Einstein frame.
- In our case we find, for a Dp brane:

$$\frac{\beta}{\alpha} = 1 + \left(\frac{R_p}{r}\right)^{7-p}$$

- Changing variable to $u = \frac{b}{r}$ one gets:

$$\Theta_p = 2 \int_0^{u^*} \frac{du}{\sqrt{1 - u^2 + \left(\frac{R_p}{b}\right)^{7-p} u^{7-p}}} - \pi$$

where u^* is the smallest root of the equation:

$$1 - (u^*)^2 + \left(\frac{R_p}{b}\right)^{7-p} (u^*)^{7-p} = 0$$

- The integral can be done exactly for the cases $p = 5, 6$:

$$\tan \frac{\Theta_6}{2} = \frac{R_6}{2b} \quad ; \quad \Theta_5 = \frac{\pi}{\sqrt{1 - \left(\frac{R_5}{b}\right)^2}} - \pi$$

- For the case $p = 3$ we get instead:

$$\Theta_3 = 2\sqrt{1 + k^2} K(k) - \pi \;\; ; \;\; K(k) = \int_0^1 \frac{dv}{\sqrt{(1 - v^2)(1 - k^2 v^2)}}$$

K is the complete elliptic integral of first kind.

- For general p we have not yet been able to write the deflection angle in closed form.
- We have computed the leading and the next to the leading behaviour for large impact parameter:

$$\Theta_p = \sqrt{\pi}\left[\frac{\Gamma(\frac{8-p}{2})}{\Gamma(\frac{7-p}{2})}\left(\frac{R_p}{b}\right)^{7-p} + \frac{1}{2}\frac{\Gamma(\frac{15-2p}{2})}{\Gamma(6-p)}\left(\frac{R_p}{b}\right)^{2(7-p)} + \dots\right]$$

Scattering of a closed string on a Dp brane: disk

- We consider the scattering of a massless closed string of the NS-NS sector on a Dp brane.
- The two closed strings have respectively momentum p_1 and p_2.
- Along the directions of the world-volume of a Dp brane there is conservation of energy and momentum:

$$(p_1 + p_2)_{\|} = 0 \quad ; \quad p_1^2 = p_2^2 = 0$$

- The scattering is described by two Mandelstam variables:

$$t = -(p_{1\perp} + p_{2\perp})^2 = -4E^2 \cos^2 \frac{\theta}{2} \;\; ; \quad s = E^2 = |p_{1\perp}|^2 = |p_{2\perp}|^2$$

$\theta =$ the angle between the d-dim ($d \equiv 9 - p$) vectors $p_{1\perp}$ and $p_{2\perp}$.

- At high energy we consider the following kinematical configuration:

$$p_1 = (E, \underbrace{0 \ldots 0}; E, \vec{p}_1) \quad ; \quad p_2 = (-E, \underbrace{0 \ldots 0}; -E, \vec{p}_2)$$

$\vec{p}_1, \vec{p}_2$ are $(d-1)$-dim vectors orthogonal to the $(p+1)$ direction.

- At high energy ($s \to \infty$) only the coefficient of the term $\mathrm{Tr}(\epsilon_1 \epsilon_2^T)$ survives:

$$A_1 = -\left(\frac{\kappa_{10} T_p N}{2}\right)(\alpha' E^2)^2 \frac{\Gamma(-\alpha' E^2)\Gamma(-\frac{\alpha'}{4}t)}{\Gamma(1-\alpha' E^2 - \frac{\alpha'}{4}t)}$$

where N is the number of D branes,

$$\kappa_{10} = \frac{(2\pi)^{7/2}}{\sqrt{2}} g_s (\alpha')^2 \; ; \; T_p = \frac{\sqrt{\pi}}{(2\pi\sqrt{\alpha'})^{p-3}} \; ; \; \frac{\kappa_{10} T_p N}{2} = \frac{R_p^{7-p} \pi^{\frac{9-p}{2}}}{\Gamma(\frac{7-p}{2})}$$

[Ademollo et al, 1974, Klebanov and Thorlacius, 1995; Klebanov and Hashimoto, 1996, Garousi and Myers, 1996]

- The poles in the t-channel correspond to exchanges of closed strings, while those in the s-channel correspond to exchanges of open strings:

$$2 + \frac{\alpha'}{2} t = 2m \;\; ; \;\; m = 2, 4, \ldots \quad ; \quad 1 + \alpha' E^2 = n \;\; ; \;\; n = 1, 2 \ldots$$

- Regge behaviour at high energy:

$$A_1 = \left(\frac{\kappa_{10} T_p N}{2} \right) \mathrm{e}^{-i\frac{\alpha'}{4}t} \, (\sqrt{\alpha'} E)^{2+\frac{\alpha'}{2}t} \Gamma(-\frac{\alpha'}{4}t)$$

- A_1 diverges when $E \to \infty$ and this creates problems with the unitarity of the S matrix.
- This problem is cured by higher orders in the perturbative expansion.
- They contribute with higher power of the energy in such a way that they can be summed to get an imaginary exponential: eikonal approximation $\Longrightarrow$ no problems with unitarity.
- The properly normalized S matrix is:

$$S = 1 + iT = 1 + i\frac{A}{\sqrt{2E_1}\sqrt{2E_2}} = 1 + i\frac{A}{2E} \;\; ; \;\; E_1 = E_2 \equiv E$$

- The quantity that exponentiates at high energy and small t is:

$$iT_1 \equiv i\frac{A_1}{2E} \Longrightarrow i\left(\frac{\kappa_{10}T_pN}{2}\right)\mathrm{e}^{-i\frac{\alpha'}{4}t}\ \frac{(\sqrt{\alpha'}E)^{2+\frac{\alpha'}{2}t}}{2E}\Gamma(-\frac{\alpha'}{4}t)$$

- Assume that the amplitude is dominated by the graviton massless pole at $t \sim 0$ ($\alpha' \to 0$):

$$iT_1(t,E) = i\left(\frac{\kappa_{10}T_pN}{2}\right)\left(\frac{2E}{(-t)} + i\pi\frac{\alpha' E}{2}\left(\sqrt{\alpha'}E\right)^{\frac{\alpha'}{2}t}\right)$$

- The real part describes the scattering of the closed string on the Dp brane, while the imaginary part describes the absorption of the closed string by the Dp brane.
- Go to impact parameter space:

$$T_1^R(b,E) + iT_1^I(b,E) = \int \frac{d^{d-1}q_t}{(2\pi)^{d-1}}\mathrm{e}^{-ib\cdot q_t}\,T_1(t=-q_t^2,E)$$

Deflection angle from string theory

- For the real part one gets:

$$iT_1^R(b,E) \equiv 2i\delta(E,b) = i\frac{E\,\sqrt{\pi}\,R_p^{d-2}}{(d-3)b^{d-3}}\cdot\frac{\Gamma(\frac{d-1}{2})}{\Gamma(\frac{d-2}{2})}\ \ ;\ \ d \equiv 9-p$$

- Assuming that it exponentiates, we get:

$$S(E,b) \equiv \mathrm{e}^{2i\delta(E,b)} = \mathrm{e}^{i\frac{E\,\sqrt{\pi}\,R_p^{d-2}}{(d-3)b^{d-3}}\cdot\frac{\Gamma(\frac{d-1}{2})}{\Gamma(\frac{d-2}{2})}}$$

- Going back to momentum space, we get:

$$\int d^{d-1}b\ \mathrm{e}^{i(b\cdot q_t+2\delta(E,b))}$$

- For large impact parameter we have the saddle point equation:

$$\vec{q}_t - \vec{b}\ \frac{E\,\sqrt{\pi}\,R_p^{d-2}}{b^{d-1}}\cdot\frac{\Gamma(\frac{d-1}{2})}{\Gamma(\frac{d-2}{2})} = 0$$

- From which we compute the deflection angle:

$$\Theta_p = \frac{|q_t|}{E} = \sqrt{\pi}\left(\frac{R_p}{b}\right)^{7-p} \cdot \frac{\Gamma(\frac{8-p}{2})}{\Gamma(\frac{7-p}{2})}$$

- It agrees with the classical calculation for large impact parameter!!

- Assuming that also the imaginary part exponentiates, we get the absorption amplitude:

$$S^{abs}(E,b) \;=\; \mathrm{e}^{-g_s\sqrt{\alpha'}E\frac{(2\pi)^{\frac{d-1}{2}}}{16}\left(\log(\sqrt{\alpha'}E)\right)^{\frac{1-d}{2}}\mathrm{e}^{-\frac{b^2}{2\alpha'\log(\sqrt{\alpha'}E)}}}$$

  that is a purely stringy effect.

- To check the exponentiation and to compute the next to the leading behaviour in the expansion for large impact parameter we need to compute the annulus diagram.

The annulus diagram

- The annulus diagram is given by:

$$A_2 = \mathcal{N} \int d^2 z_a d^2 z_b \sum_{\alpha,\beta} \langle B | W_a(z_a, \bar{z}_a) W_b(z_b, \bar{z}_b) D | B \rangle_{\alpha,\beta}$$

  $W_{a,b}(z_{a,b}, \bar{z}_{a,b})$ are the closed string vertices, $\mathcal{N}$ is a normalization factor and $\sum_{\alpha,\beta}$ is the sum over the spin structures.
- The sum over the spin structures can be explicitly performed obtaining in practice only the contribution of the bosonic degrees of freedom without the bosonic partition function.
- The final result is rather explicit.
  [Pasquinucci, 1997 and Lee and Rey, 1997]

- In the closed string channel the coefficient of the term with $\mathrm{Tr}(\epsilon_1 \epsilon_2^T)$ (relevant at high energy) of annulus diagram is equal to:

$$
\begin{aligned}
A_2 &= \left(\frac{\kappa_{10}}{\pi}\right)^2 (8\pi^2\alpha')^{-\frac{p+1}{2}} \frac{(2\pi)^4}{2} N^2 (\alpha' s)^2 \\
&\times \int_0^\infty \frac{d\lambda}{\lambda} \lambda^{\frac{p+1}{2}} \int_0^{\frac{1}{2}} d\rho_a \int_0^{\frac{1}{2}} d\rho_b \int_0^{\frac{1}{\lambda}} d\omega_a \int_0^{\frac{1}{\lambda}} d\omega_b \\
&\times \mathrm{e}^{-\alpha' s V_s - \frac{\alpha'}{4} t V_t} \quad ; \quad z_{a,b} \equiv \mathrm{e}^{2\pi i(\rho_{a,b} + i\omega_{a,b})}
\end{aligned}
$$

- where

$$
V_s = -2\pi\lambda\rho_{ab}^2 + \log \frac{\Theta_1(i\lambda(\zeta + \rho_{ab})|i\lambda)\Theta_1(i\lambda(\zeta - \rho_{ab}|)i\lambda)}{\Theta_1(i\lambda(\zeta + i\omega_{ab})|i\lambda)\Theta_1(i\lambda(\zeta - i\omega_{ab})|i\lambda)}
$$

and

$$
V_t = 8\pi\lambda\rho_a\rho_b + \log \frac{\Theta_1(i\lambda(\rho_{ab} + i\omega_{ab})|i\lambda)\Theta_1(i\lambda(\rho_{ab} - i\omega_{ab})|i\lambda)}{\Theta_1(i\lambda(\zeta + i\omega_{ab})|i\lambda)\Theta_1(i\lambda(\zeta - i\omega_{ab})|i\lambda)}
$$

$$
\rho_{ab} \equiv \rho_a - \rho_b \quad ; \quad \zeta = \rho_a + \rho_b \quad ; \quad \omega_{ab} \equiv \omega_a - \omega_b
$$

- The high energy behaviour ($E \to \infty$) of the annulus diagram can be studied, by the saddle point technique, looking for points where V_s vanishes.
- This happens for $\lambda \to \infty$ and $\rho_{ab} \to 0$.
- Performing the calculation one gets the leading term for $E \to \infty$:

$$A_2(E, t) \to \frac{i}{4E} \int \frac{d^{d-1}k}{(2\pi)^{d-1}} A_1(E, t_1) A_1(E, t_2) V(t_1, t_2, t)$$

where

$$t_1 \equiv -(\frac{q}{2} + k)^2 \;\; ; \quad t_2 \equiv -(\frac{q}{2} - k)^2 \;\; ; \quad t = -q^2$$

and

$$V(t_1, t_2, t) = \frac{\Gamma(1 + \frac{\alpha'}{2}(t_1 + t_2 - t))}{\Gamma^2(1 + \frac{\alpha'}{4}(t_1 + t_2 - t))} \Longrightarrow 1$$

in the field theory limit ($\alpha' \to 0$).

- Going to impact parameter space:

$$A_2(E, b) = \int \frac{d^{d-1}q}{(2\pi)^{d-1}} \mathrm{e}^{-ib\cdot q} A_2(E, -q^2) = \frac{i}{4E} (A_1(E, b))^2$$

- In terms of the matrix $T \equiv \frac{A}{2E}$:

$$T_2(E, b) = \frac{i}{2} (T_1(E, b))^2$$

- This implies that:

$$S(E, b) = 1 + iT_1 + iT_2 + \cdots = 1 + iT_1 - \frac{1}{2} (T_1)^2 + \cdots = \mathrm{e}^{iT_1}$$

- At high energy the amplitude exponentiates: no problems with unitarity.
- We have extracted the leading behaviour at high energy:

$$T_1 \sim E \quad ; \quad T_2 \sim E^2$$

But there is also a next to the leading contribution to $T_2^{nl} \sim E$.

- It can be extracted from the annulus amplitude.
- It must also exponentiate.
- It gives the next to the leading correction to the deflection angle that agrees with the one obtained from the classical calculation.
- We have seen that both the leading and the next to the leading terms in the expansion for large impact parameter are reproduced by the string scattering amplitude.
- Being brave, one can assume that all the terms are reproduced.
- This implies an equation for the phase shift:

$$-\frac{2}{E}\frac{\partial\delta(E,b)}{\partial b} = \Theta + \pi$$

- Integrating the previous equation one gets, for the simple case $p = 5$:

$$\delta_5(E,b) = \frac{\pi E b}{2}\left[1 - \sqrt{1 - \left(\frac{R_5}{b}\right)^2}\right]$$

- The phase shift has an imaginary part for $b \leq R_5$.
- For $b \leq R_5$ the probe particle is absorbed by the brane with an absorption cross section that is proportional to:

$$|S| \sim \mathrm{e}^{-2\pi E R_5}$$

  and is independent from b.
- Similar results for any p.

Conclusion and outlook

- We have seen how from string scattering in flat space-time we can recover properties of curved space-time.
- In particular, from the scattering of a massless closed string on a Dp brane at high energy and low transfer momentum we have computed the deflection angle of a probe particle moving in the metric of the Dp brane.
- The result reproduces the leading and the next to the leading contributions for large impact parameter computed from classical gravity in the metric of a Dp brane.
- String corrections to the field theory results can also be computed.

- We have not seen any effect from the dilaton and the RR field on the deflection angle.
- Work directly with the angular momentum J instead of the impact parameter b ($b \sim \frac{J}{E}$).

The absorption cross section from a D3 brane
[Klebanov, 1997]

- The low energy absorption cross section of a dilaton by a D3 brane can be calculated and one gets:

$$\sigma_{abs.D3} = \frac{\pi^4}{8}\omega^3 R^8 \quad ; \quad H(r) = 1 + \frac{R^4}{r^4}$$

  ω is the dilaton energy.
- This calculation is done using the curved space formalism in the metric of a D3 brane.
- On the other hand, the same quantity can be computed from the the scattering of a closed string on a D3 brane that generates two open strings (gluons).
- At low energy the coupling of the DBI action that is relevant is the one involving a dilaton and two gauge fields that gives the following amplitude:

$$\mathcal{A} = -\frac{\kappa_{10}}{\sqrt{2}} \cdot 2 \cdot \frac{p_1 \cdot p_2}{\sqrt{2\omega}\ \omega} = -\frac{\kappa_{10}\sqrt{\omega}}{\sqrt{2}} \quad ; \quad E_1 = E_2 = \frac{\omega}{2}$$

- The absorption cross section is equal to:

$$\frac{1}{2}\int \frac{d^3 p_1}{(2\pi)^3}\int \frac{d^3 p_2}{(2\pi)^3}(2\pi)^4 \delta(E_1 + E_2 - \omega)\delta^3(\vec{p}_1 + \vec{p}_2)\mathcal{A}^2$$

  Factor $\frac{1}{2}$ because the two particles in the final state are identical.
- One gets:

$$\sigma_{abs.D3} = \frac{\kappa_{10}^2 N^2 \omega^3}{32\pi} = \frac{\pi^4}{8}\omega^3 R^8 \quad ; \quad R^4 = \frac{\kappa_{10} N}{2\pi^{\frac{5}{2}}}$$

- The same result from the coupling of a dilaton with two gauge fields that has apriori nothing to do with curved space-time.
- This calculation is at the origin of the Maldacena conjecture.