SUPERFIELDS
European Research Council
Adv. Grant no. 226455

Perugia
June 25th, 2010

A Supergravity Dual for 4d SCFT's Universal Sector

Gianguido Dall'Agata

D. Cassani, G.D., A. Faedo, arXiv:1003.4283 + work in progress...

related work:

J. Liu, P. Szepietowski, Z. Zhao, arXiv:1003.5374

J. Gauntlett, O. Varela, arXiv:1003.5642

K. Skenderis, M. Taylor, D. Tsimpis, arXiv:1003.5657

The Messages

i) *Any SCFT with at least N=1 has a finite Universal spectrum of pure gauge operators* $\mathcal{O}$.

ii) *We constructed the dual 5d supergravity model that captures the full dynamics induced by* $\mathrm{Tr}\,\mathcal{O}$ *and/or* $\langle \mathrm{Tr}\,\mathcal{O} \rangle$

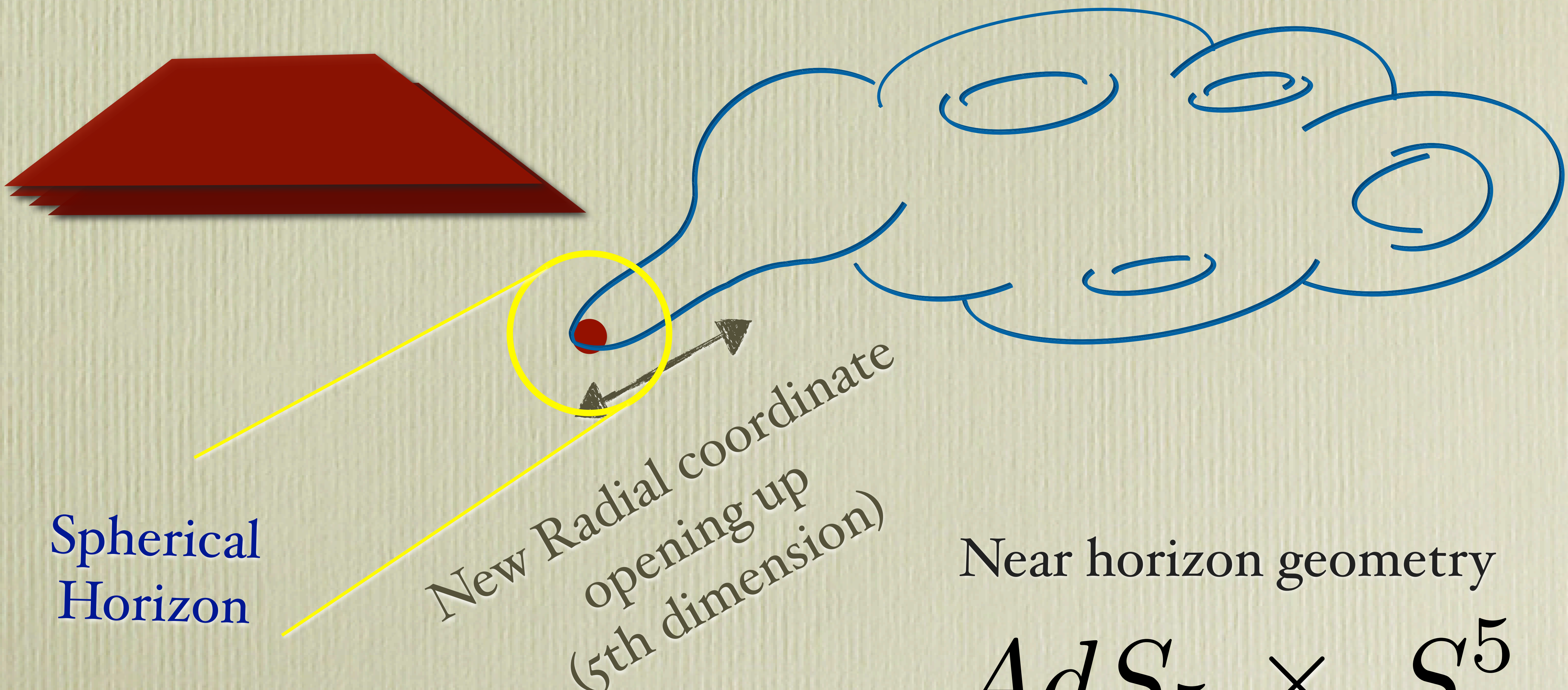
iii) *This model is a consistent truncation of Type IIB supergravity compactified on a "squashed" Sasaki–Einstein 5-manifold*

NON-SPHERICAL HORIZONS IN AdS/CFT

non-spherical horizons in AdS/CFT

$$M_4 \qquad \times \qquad CY$$

$$M_4 \quad \times \quad CY$$

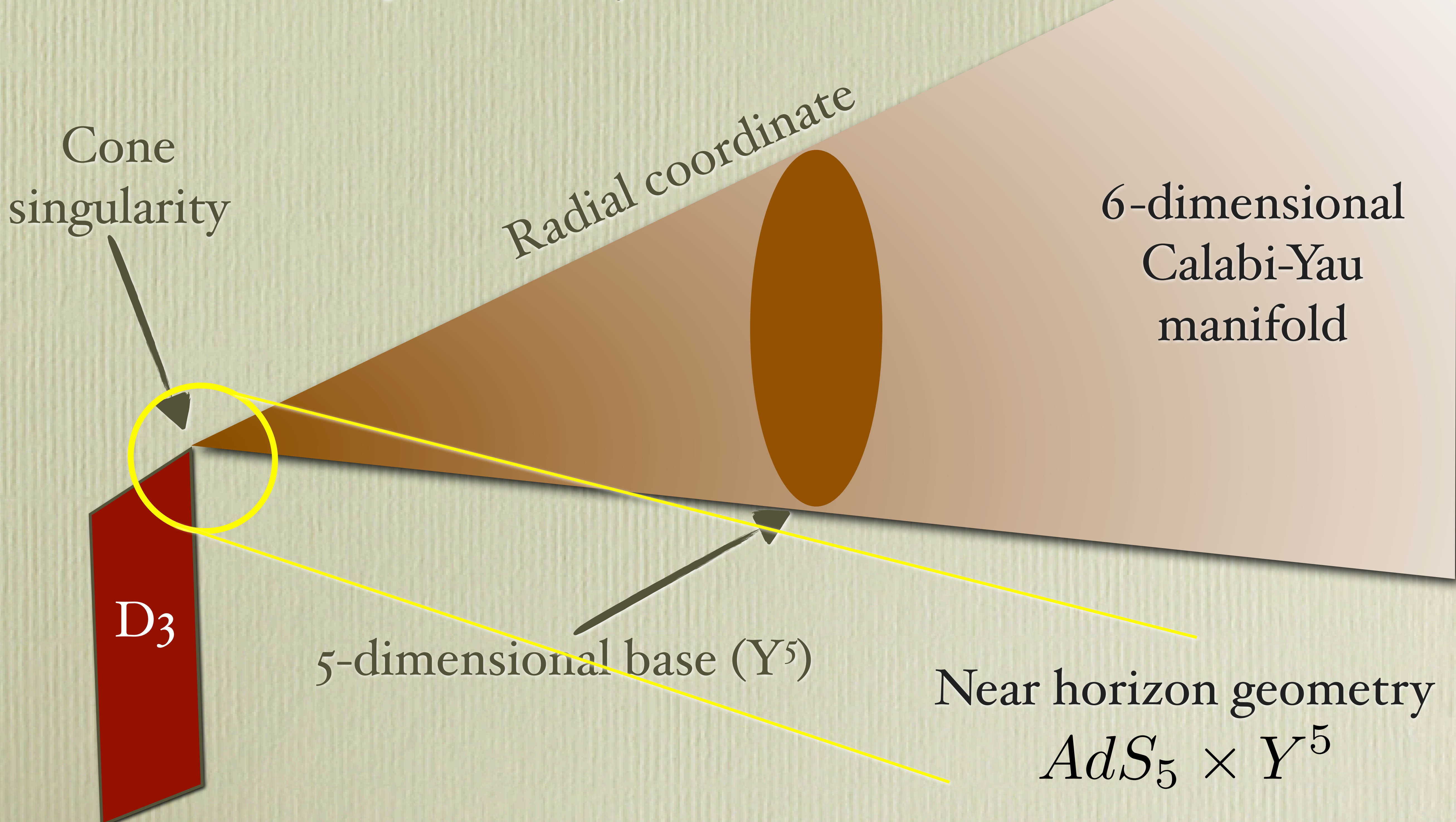


Near horizon geometry

$$AdS_5 \times S^5$$

NON-SPHERICAL HORIZONS IN AdS/CFT

When the branes are at a singular point the resulting horizon is not spherical anymore



D3-branes generate 5-form fluxes:

$$F_5^{\text{flux}} = (1 + *)2k\,\text{vol}(Y_5)$$

Conditions on the base manifold

SUPERSYMMETRY

SASAKI

$$(\nabla_M + k\Gamma_M)\,\epsilon = 0$$

EQUATIONS OF MOTION

EINSTEIN

$$R_{ab} = -2k^2 g_{ab}$$

- Definitions of Sasaki–Einstein manifold from:
  - globally defined *Killing spinor*;
  - existence of a constant-norm *Killing vector* (Reeb vector);
  - 5-dimensional spaces whose *cone* is a (non-compact) *Calabi–Yau* manifold;
- For our purposes, we use their **contact structure**:
  - Globally defined 1-form η and 2-forms J, Ω
    Reeb ⟷ U(1) R-symmetry
  - Locally: *U(1) fibration over a Kæhler–Einstein base*.

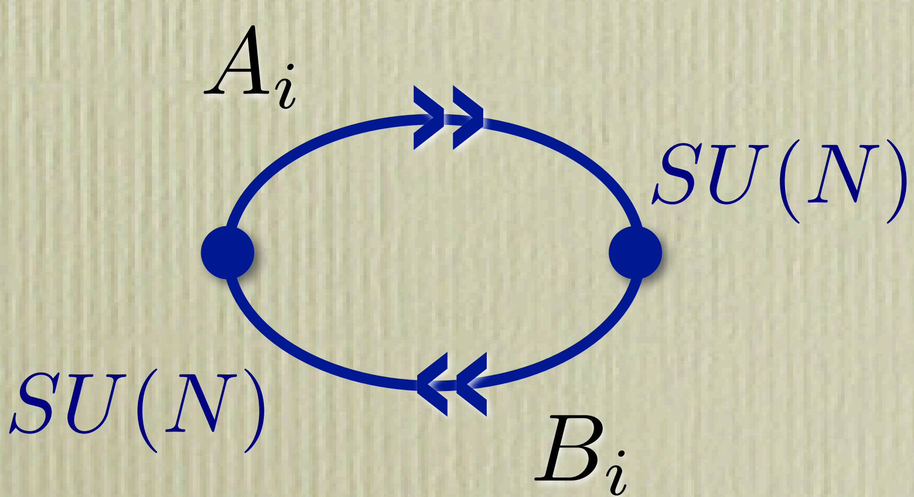
Examples Same topology : $S^2 \times S^3$

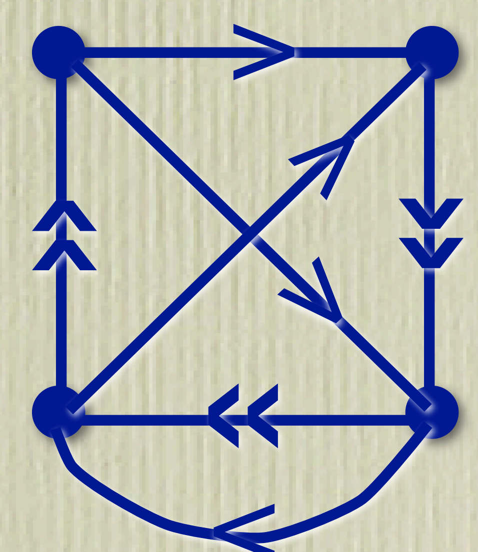
$$\frac{SU(2) \times SU(2)}{U(1)}$$

Different metrics:

$T^{1,1}$	$Y^{p,q}$	$L^{p,q,r}$
$SU(2) \times SU(2)$	$SU(2) \times U(1)$	$U(1) \times U(1)$

Isometries = Flavour + $U(1)_R$





$Y^{2,1}$

$\ldots$

- Which physics can we discuss?
- Predict anomalous dimensions of CFT spectrum
- Study deformations (*RG flows* and such...)
- Perturbative and non-perturbative effects on 4d effective theories
- *Holographic superconductors / CM applications*
- ....

- **Problem:** turning on operators and/or vevs

$$\mathcal{W}' = \mathcal{W} + \mathrm{tr}\,\mathcal{O}$$

forces *additional* operators and/or vevs.

- We need to solve 10d equations: 5d Kaluza–Klein modes talk to each other

- If the OPEs close

$$\mathcal{O}_i \mathcal{O}_j \sim \mathcal{O}_k$$

$$\mathcal{O}_i \mathcal{O}'_j \sim \mathcal{O}'_k \qquad \mathcal{O}'_i \mathcal{O}'_j \sim \mathcal{O}'_k$$

we can keep a **finite** number of states $\mathcal{O}_i$

CONSISTENT TRUNCATION

- Consistent truncations are **not** always effective theories

- Examples:

massless modes

$\mathcal{N}=8$ pure sugra from M-theory on $\mathbb{T}^7$

$\mathcal{N}=8$ *SO(8) gauged sugra* from M-theory on $\mathbb{S}^7$

Coset manifold reductions keeping all left-invariant modes

massive truncations

For a given $\dfrac{G}{H}$ one has a H-structure reduction

N=1 Universal Sector

4-dimensional N=1 SCFTs

Chiral singletons

Flavour indices

$$\mathcal{W}_\alpha \qquad\qquad S_i$$

Gauge superfields — Matter fields

Universal — **Y_5-dependent**

Generic operators = words using "letters" above

$$\mathrm{tr}\, W^2 + \ldots \qquad\qquad \mathrm{tr}\,(S_i S_j^\dagger) + \ldots$$

$$\mathrm{tr}\,(W_\alpha S_i S_j \ldots S_k) + \ldots$$

Ceresole, GD, D'Auria, Ferrara

The gauge superfield is fermionic. Hence there is only a finite number of primary states *(also using superspace identities)*

W_α	singleton	pure gauge
$tr\, W^2$	chiral	hypermultiplet
$tr\,(W_\alpha \overline{W}_{\dot\alpha})$	energy tensor	graviton
$tr\,(W_\alpha \overline{W}^2)$	semi-conserved	Ist KK tower (gravitino)
$tr\,(W^2 \overline{W}^2)$	long multiplet	2nd KK tower (breathing mode vector multiplet)

Gravity dual: ***use Sasakian contact structure*** (η, J, Ω)

Algebraic conditions

$$J \wedge \Omega = 0$$

$$\Omega \wedge \overline{\Omega} = 2\, J \wedge J$$

$$\eta \lrcorner J = \eta \lrcorner \Omega = 0$$

Differential conditions

$$d\eta = 2J$$

$$d\Omega = 3i\, \eta \wedge \Omega$$

N=1 Universal Sector

Gravity dual: ***use Sasakian contact structure*** (η, J, Ω)

Metric Ansatz:

$$ds^2 = e^{-\frac{2}{3}(4U+V)} ds^2(M) + e^{2U} ds^2(B_{\mathrm{KE}}) + e^{2V}(\eta + A) \otimes (\eta + A)$$

Forms reduction:

$$B = b_2 + b_1 \wedge (\eta + A) + b^J J + \mathrm{Re}(b^\Omega \Omega)$$

5d gauge symmetries inherited from form gauge $B \to B + d\Lambda$ and diffeos $\eta \to \eta + d\omega$

$$\delta b_2 = d\lambda_1 + \lambda_0\, dA, \qquad \delta b^\Omega = 3i\omega b^\Omega,$$

$$\delta b_1 = d\lambda_0, \qquad \delta b^J = 2\lambda_0,$$

N=1 Universal Sector

GD, Herrmann, Zagermann

- The resulting effective theory is a 5d N=4 gauged supergravity
- Content:
  - 1 graviton + 2 vector multiplets

$$(g_{\mu\nu}, 4\psi_\mu, 6A_\mu, 4\lambda, \Sigma) \qquad (A_\mu, 4\lambda, 5\phi)$$

- Scalar manifold

$$\mathcal{M}_{\rm scal} = \mathrm{SO}(1,1) \times \frac{\mathrm{SO}(5,2)}{\mathrm{SO}(5)\times \mathrm{SO}(2)}$$

- Gauge group $G = \mathrm{Heis}_3 \times \mathrm{U}(1)_R$

The Supergravity Model

- The N=4 structure follows from the existence of 2 globally defined spinors $\zeta^{1,2}(y)$ on the SE internal manifold

- Each IIB gravitino gives 2 5d pseudo-Majorana gravitinos

$$\Psi_\mu^\alpha(x,y) = \psi_\mu^{\alpha\,1}(x) \otimes \zeta^1(y) + \psi_\mu^{\alpha\,2}(x) \otimes \zeta^2(y)\,.$$

where $\quad \overline{\psi}_{\mu\,i} \equiv (\psi_\mu^i)^\dagger \gamma^0 = \Omega_{ij} (\psi_\mu^j)^T C\,,$

- Charged vectors become tensor fields (satisfying 1st order eoms)

$$dB^M \simeq M^M{}_N \star B^N$$

The Gauge group is $G = \mathrm{Heis}_3 \times \mathrm{U}(1)_R$ → $F_5 \neq 0$; → $d\eta \neq 0$, $d\Omega \neq 0$

Covariant curvatures include tensors

$$\mathcal{H}^{\mathcal{M}} = dA^{\mathcal{M}} + \frac{1}{2} X_{\mathcal{N}\mathcal{P}}{}^{\mathcal{M}} A^{\mathcal{N}} \wedge A^{\mathcal{P}} + Z^{\mathcal{M}\mathcal{N}} B_{\mathcal{N}}$$

4 vectors A^{Λ} are in the adjoint of G,

4 vectors A^{I} are in a non-trivial representation of G (dualized to tensors)

$$t_{\Lambda} = \begin{pmatrix} -f_{\Lambda\Sigma}{}^{\Gamma} & (t_{\Lambda})_{\Sigma}{}^{I} \\ 0 & (t_{\Lambda})_{J}{}^{I} \end{pmatrix}$$

Non-trivial mixing
First stringy realization

R-charged

The gauging induces a scalar potential

$$\mathcal{V} = -12\, e^{-\frac{14}{3}U-\frac{2}{3}V} + 2\, e^{-\frac{20}{3}U+\frac{4}{3}V} + \frac{9}{2}\, e^{-\frac{20}{3}U-\frac{8}{3}V-\phi}|b^\Omega|^2 + \frac{9}{2}\, e^{-\frac{20}{3}U-\frac{8}{3}V+\phi}|c^\Omega - C_0 b^\Omega|^2 + e^{-\frac{32}{3}U-\frac{8}{3}V}\left[3\,\mathrm{Im}\left(b^\Omega\,\overline{c^\Omega}\right) + k\right]^2$$

This potential has 2 vacua:

Round SE metric

N=2 G = U(1)

$$U = V = b^\Omega = c^\Omega = 0$$

N=0 G broken

$$e^{4U} = e^{-4V} = \frac{2}{3}\,, \quad b^\Omega = \frac{e^{i\theta+\phi/2}}{\sqrt{3}}$$

$$c^\Omega = b^\Omega \tau\,, \qquad \tau \equiv (C_0 + i\, e^{-\phi})$$

Squashed metric

The masses at the susy vacuum match the CFT spectrum

The N=0 vacuum scalars mix:

Mass Eigenstate	m^2	Δ
$\delta U + \delta V$	36	8
$\sqrt{3}\,(\mathrm{Im}\,\delta c^\Omega + \mathrm{Re}\,\delta b^\Omega) + 8\,\delta U$	36	8
$\sqrt{3}\,(\mathrm{Im}\,\delta c^\Omega - \mathrm{Re}\,\delta b^\Omega) + \delta\phi$	27	$2(1+\sqrt{7})$
$\sqrt{3}\,(\mathrm{Im}\,\delta b^\Omega + \mathrm{Re}\,\delta c^\Omega) - \delta C_0$	27	$2(1+\sqrt{7})$
$\sqrt{3}\,(\mathrm{Re}\,\delta b^\Omega + \mathrm{Im}\,\delta c^\Omega) - 4\,\delta U$	9	$2(1+\sqrt{3})$
$\sqrt{3}\,(\mathrm{Im}\,\delta c^\Omega - \mathrm{Re}\,\delta b^\Omega) - 2\,\delta\phi$	0	4
$\sqrt{3}\,\mathrm{Re}\,\delta c^\Omega + \delta C_0$	0	4
$\sqrt{3}\,\mathrm{Im}\,\delta b^\Omega + \delta C_0$	0	4

- Supersymmetric deformation: This truncation includes as a subcase the *Girardello–Petrini–Porrati–Zaffaroni* flow (turning on only the gaugino condensate operator $\mathrm{tr}\, W^2$)

- Non-susy deformations including both vevs and the operator may reach the IR point

- Holographic superconductors

  Gubser-Pufu-Rocha
  Gubser-Herzog-Pufu-Tesileanu

  U(1) symmetry gets broken from UV to IR

- String theory backgrounds with non-relativistic conformal symmetry

  Maldacena-Martelli-Tachikawa

  Massive vectors can be used to obtain *Schrödinger* or *Lifshitz* symmetry

In Progress...

- Additional flavour singlets come from the cycles of the internal manifold: **Betti Multiplets.**

  U(1) Baryonic current J_B Betti hyper $\mathrm{Tr}(W_1^2 - W_2^2)$

- This is Y-dependent. Hence we focused on the T^{11} manifold.

- There is a new vacuum! New Einstein metric for the T^{11}, not the standard one. (unfortunately, it's unstable)

- *New applications:*
  - gaugings corresponding to fluxes on the cycles
  - holographic RG-flows related to fractional branes...

Summary

i) *Any SCFT with at least N=1 has a finite Universal spectrum of pure gauge operators* $\mathcal{O}$.

ii) *We constructed the dual 5d supergravity model that captures the full dynamics induced by* $\mathrm{Tr}\,\mathcal{O}$ *and/or* $\langle \mathrm{Tr}\,\mathcal{O} \rangle$

iii) *This model is a consistent truncation of Type IIB supergravity compactified on a "squashed" Sasaki–Einstein 5-manifold*

iv) *Countless applications.... next time*