Worldline approach to higher spin fields

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Outline

Worldline approaches

- Generalities
- $N = 2$ spinning particle

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- $N = 2$ spinning particle

$O(N)$ particles and HS fields

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- ... and HS fields

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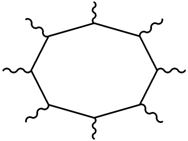
- An example: scalar contribution to scalar QED at 1-loop

$$S[\phi,\phi^*,A] = \int d^D x \left(|(\partial_\mu + iqA_\mu)\phi|^2 + m^2|\phi|^2 \right)$$

$$\mathrm{e}^{-\Gamma[A]} = \int D\phi D\phi^* \, \mathrm{e}^{-S[\phi,\phi^*,A]} = \mathrm{Det}^{-1}(-\nabla_A^2 + m^2)$$

$$\Gamma[A] = \mathrm{Tr}\log(-\nabla_A^2 + m^2) = -\int_0^\infty \frac{dT}{T} \; \mathrm{Tr}\,\mathrm{e}^{-(-\nabla_A^2+m^2)T}$$

$$= -\int_0^\infty \frac{dT}{T} \int_{PBC} Dx \; \mathrm{e}^{-\int_0^T d\tau \left(\frac{1}{4}\dot{x}^2 + iqA_\mu(x)\dot{x}^\mu + m^2\right)} =$$



- Notice the (gauge fixed) action of the scalar particle running in the loop

- Use $A_\mu(x) = \sum_{i=1}^N \varepsilon_\mu^{(i)}\, e^{ip_i\cdot x}$ and expand interaction to get averages of "photon vertex operators"

$$\left\langle \varepsilon_{\mu_1}^{(1)}\, \dot{x}^{\mu_1}(\tau_1) e^{ip_1\cdot x(\tau_1)} \;\cdots\; \varepsilon_{\mu_N}^{(N)}\, \dot{x}^{\mu_N}(\tau_N) e^{ip_N\cdot x(\tau_N)} \right\rangle$$

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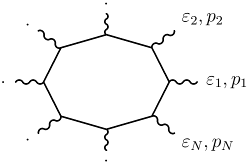
- Obtain "Bern-Kosower master formula"

$$\Gamma[p_1,\varepsilon_1;..;p_N,\varepsilon_N] = (2\pi)^D\delta^D\Big(\sum_{i=1}^N p_i\Big)(-iq)^N\int_0^\infty \frac{dT}{T}\,\frac{e^{-m^2T}}{(4\pi T)^{\frac{D}{2}}}$$

$$\Big(\prod_{i=1}^N\int_0^T d\tau_i\Big)\exp\sum_{i,j=1}^N\Big[\frac{1}{2}\Delta_{ij}\,p_i\cdot p_j - i\,{}^\bullet\!\Delta_{ij}\,\varepsilon_i\cdot p_j + \frac{1}{2}\,{}^{\bullet\bullet}\!\Delta_{ij}\,\varepsilon_i\cdot\varepsilon_j\Big]\Big|_{\mathrm{lin}\,\varepsilon_i}$$

$=$ 

- action of scalar particle
  $S[x] = \int d\tau \left(\frac{1}{2} e^{-1} \dot{x}^2 + \frac{1}{2} e m^2 + iqA(x) \cdot \dot{x} \right)$

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  QM path integrals in curved space subtle but understood, they require regularization with corresponding (finite) counterterms

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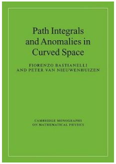
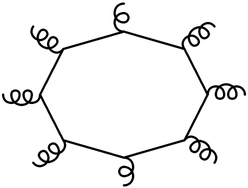
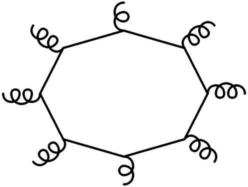


F. Bastianelli and P. van Nieuwenhuizen
"Path Integrals and Anomalies in Curved Space"
(CUP 2006)



- N=0: scalar (with Zirotti)



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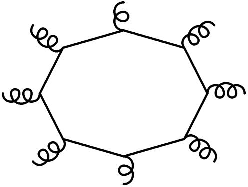

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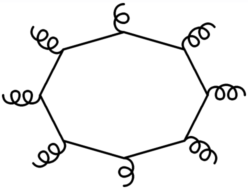
- N=2: vector and differential forms (Benincasa, Giombi)



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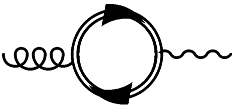

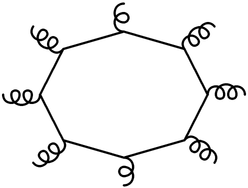

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- Applications on photon-graviton mixing (Schubert)
- Higher N: can study **higher spin fields** (Corradini, Latini; Bonezzi)

Outline

$N=2$ spinning particle

Dynamical variables $(\underbrace{x^\mu, p_\mu, \psi^\mu, \bar\psi_\mu}_{X}; \underbrace{e, \chi, \bar\chi, a}_{G})$

$$S = \int dt \left[p_\mu \dot{x}^\mu + i\bar\psi_\mu \dot\psi^\mu - eH - i\bar\chi Q - i\chi \bar{Q} - aJ \right]$$

where

$$H = \frac{1}{2} p_\mu p^\mu, \; Q = p_\mu \psi^\mu, \; \bar{Q} = p_\mu \bar\psi^\mu, \; J = \bar\psi^\mu \psi_\mu$$

are first class constraints

$$\{Q, \bar{Q}\} = -2iH, \quad \{J, Q\} = iQ, \quad \{J, \bar{Q}\} = -i\bar{Q}$$

Canonical quantization $[\hat{x}^\mu, \hat{p}_\nu] = i\delta^\mu_\nu \ , \quad \{\hat{\psi}^\mu, \hat{\psi}^\dagger_\nu\} = \delta^\mu_\nu$

Wave functions

$$\phi(x,\psi) \ = \ F(x) + F_\mu(x)\psi^\mu + \frac{1}{2}F_{\mu_1\mu_2}(x)\psi^{\mu_1}\psi^{\mu_2} + \dots$$

Constraints

$$\hat{H} = -\frac{1}{2}\partial_\mu\partial^\mu, \ \hat{Q} = -i\psi^\mu\partial_\mu, \ \hat{\bar{Q}} = -i\partial_\mu\frac{\partial}{\partial\psi_\mu}, \ \hat{J} = -\psi^\mu\frac{\partial}{\partial\psi^\mu}+p+1$$

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$$\hat{J}\phi_{phys} = 0 \;\Rightarrow\; \phi_{phys} \sim F_{\mu_1\ldots\mu_{p+1}}(x)\psi^{\mu_1}\ldots\psi^{\mu_{p+1}}$$

$$\hat{Q}\phi_{phys} = 0 \;\Rightarrow\; dF_{p+1} = 0, \qquad \hat{\bar{Q}}\phi_{phys} = 0 \;\Rightarrow\; d^\dagger F_{p+1} = 0$$

$\Rightarrow$ **Maxwell equations** for a p-forms gauge field A_p

Using the N=2 spinning particle one can obtain a nice representation on the one-loop effective action of massless (and massive) p-forms in a gravitational background

$$
\begin{aligned}
\Gamma_p^{QFT}[g] &= \int_{S^1} \frac{\mathcal{D}G\mathcal{D}X}{Vol(\text{gauge})}\, e^{-S[G,X;g]} \\
&= \underbrace{\int_0^\infty \frac{d\beta}{\beta}}_{\text{proper}-\text{time}} \underbrace{\int_0^{2\pi} \frac{d\phi}{2\pi}}_{U(1)\ \text{mod.}} \underbrace{\left(2\cos\frac{\phi}{2}\right)^{-2}}_{FP\ \text{det.}} \underbrace{\int_{S^1} \mathcal{D}X\, e^{-S[X,\hat{G};g_{\mu\nu}]}}_{\text{path integral for } N=2 \text{ nl}\sigma\text{m}}
\end{aligned}
$$

where we used PBC for (e, x), ABC for $(\psi, \bar{\psi}, \chi, \bar{\chi})$, and chose the gauge $G = \hat{G} = (\hat{e}, \hat{\chi}, \hat{\bar{\chi}}, \hat{a}) = (\beta, 0, 0, \phi)$.

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It allows to compute amplitudes and heat kernel coefficients, and derive and check duality relations.

F.B., P. Benincasa, S. Giombi, JHEP **0504** (2005) 010 [hep-th/0503155]; JHEP **0510** (2005) 114 [hep-th/0510010]

Outline

$O(N)$ spinning particle

- Consider worldline action ($i = 1, .., N$)

$$S = \int dt \Big[p\dot{x} + \frac{i}{2}\psi_i\dot{\psi}_i - \frac{1}{2}p^2 \Big]$$

- Rigid $SO(N)$ extended supersymmetry

$$H = \frac{1}{2}p^2, \qquad Q_i = p\psi_i \ , \qquad J_{ij} = i\psi_i\psi_j$$

$$\begin{aligned} \{Q_i, Q_j\}_{PB} &= -2i\delta_{ij}H \\ \{J_{ij}, Q_k\}_{PB} &= \delta_{jk}Q_i - \delta_{ik}Q_j \\ \{J_{ij}, J_{kl}\}_{PB} &= \delta_{jk}J_{il} - \delta_{ik}J_{jl} - \delta_{jl}J_{ik} + \delta_{il}J_{jk} \end{aligned}$$

$O(N)$ spinning particle action

- Algebra is first class $\rightarrow$ can be gauged

$$S = \int dt \Big[p\dot{x} + \frac{i}{2}\psi_i\dot{\psi}_i - eH - i\chi_i Q_i - \frac{1}{2}a_{ij}J_{ij} \Big]$$

- Eliminating momenta p

$$S = \int dt \Big[\frac{1}{2}e^{-1}(\dot{x} - i\chi_i\psi_i)^2 + \frac{i}{2}\psi_i(\delta_{ij}\partial_\tau - a_{ij})\psi_j \Big]$$

(model originally worked out by Gershun and Tkach)

Bargmann-Wigner equations

- Canonical analysis:

  $D = 4 \;\rightarrow\;$ spin $\frac{N}{2}$ particle (Bargmann-Wigner eq.)

  - wave function $|\Psi\rangle$ is a multispinor $\Psi_{\alpha_1\cdots\alpha_N}(x)$
  - constraints $\hat{J}_{ij}|\Psi\rangle = 0 \;\rightarrow\;$ reduce the number of independent components of the multispinor
  - constraints $\hat{Q}_i|\Psi\rangle = 0 \;\rightarrow\; \partial\!\!\!/_{\alpha_i}{}^{\tilde{\alpha}_i}\Psi_{\alpha_1..\tilde{\alpha}_i..\alpha_N}(x) = 0$

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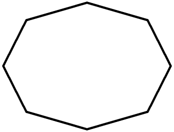
- Study 1-loop quantization $\;\Rightarrow\;$ path integral on the circle



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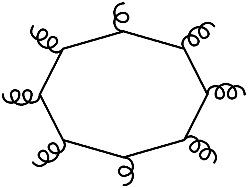

Outline

Bargmann Wigner eq. in different base

- Consider $N = 2s$ (integer spin)
  Use complex combinations ψ_I^μ and $\bar{\psi}_I^\mu \equiv \bar{\psi}^{\mu I}$, $I = 1, .., s$.
  Wave function:

$$\Psi(x, \psi) \sim \sum R_{\mu_1 \mu_2 ., ..., \nu_1 \nu_2 ..}(x)\, \psi_1^{\mu_1} \psi_1^{\mu_2} ., ..., \psi_s^{\nu_1} \psi_s^{\nu_2} ..$$

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- $\underline{J \text{ constraints}} \;\rightarrow\;$ tensor R:

  rectangular Young tableau of SO(D)

$$R \quad \sim \quad n = \frac{D}{2} \left\{ \underbrace{\begin{array}{|c|c|c|c|} \hline & & & \\ \hline & & & \\ \hline & & & \\ \hline \end{array}}_{s = \frac{N}{2}} \right.$$

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- $\underline{J \text{ constraints}} \ \rightarrow$ tensor R:
  - has s blocks of $n = \frac{D}{2}$ indices each: $R_{[\mu_1^1..\mu_n^1],\dots,[\mu_1^s..\mu_n^s]}$
  - antisymmetric inside each block
  - satisfies algebraic Bianchi identities
  - symmetric under exchanges of the s blocks of indices
  - traceless

Bargmann Wigner eq. in different base

- Half of the $\underline{Q \text{ constraints}}$: differential Bianchi identities

$$\partial_{[\mu} R_{\mu_1^1..\mu_n^1],...,\mu_1^s..\mu_n^s} = 0$$

Other half of the $\underline{Q \text{ constraints}}$: Maxwell equations

$$\partial^\mu R_{\mu\mu_2^1..\mu_n^1,...,\mu_1^s..\mu_n^s} = 0$$

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- Gauge invariant description of higher spin fields

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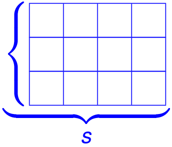
- Gauge invariant description of higher spin fields
- Integrate Bianchi identities $\rightarrow$ HS gauge fields

HS gauge fields

Manifest $U(s) \subset O(2s)$ $\quad J_{ij} = (J_{I\bar{J}}, J_{IJ}, J_{\bar{I}\bar{J}}) \equiv (J_I{}^J, K_{IJ}, \bar{K}^{IJ})$

$$J_I{}^J|R\rangle = 0 \quad \Rightarrow \quad R \sim n$$



$$\bar{K}^{IJ}|R\rangle = 0 \quad \Rightarrow \quad R \text{ traceless}$$

$$K_{IJ}|R\rangle = 0 \quad \Rightarrow \quad \text{already satisfied } (R \text{ traceless})$$

$$Q_I|R\rangle = 0 \quad \Rightarrow \quad R \text{ closed (Bianchi identities)}$$

$$\bar{Q}^I|R\rangle = 0 \quad \Rightarrow \quad R \text{ co} - \text{closed (Maxwell equations)}$$

HS gauge fields

- Solve Q_I by

$$|R\rangle = Q_1 Q_2 .. Q_s |\varphi\rangle$$

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$$J_I{}^J |\varphi\rangle = -i\delta_I{}^J |\varphi\rangle \qquad \varphi \sim (n-1)\left\{ \underbrace{\begin{array}{|c|c|c|c|}\hline & & & \\ \hline & & & \\ \hline \end{array}}_{s} \right.$$

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- To solve last nontrivial constraints $\bar{K}^{IJ}$ compute

$$\bar{K}^{12}Q_1..Q_s|\varphi\rangle = Q_3..Q_s i \underbrace{\left[-2H + Q_I\bar{Q}^I + \frac{i}{2}Q_IQ_J\bar{K}^{JI}\right]}_{\text{Fronsdal–Labastida operator } G}|\varphi\rangle = 0$$

HS gauge fields

- Solution described by the equation

$$G|\varphi\rangle = Q_I Q_J Q_K |\rho^{KJI}\rangle$$

where the compensators can be presented as

$$|\rho^{KJI}\rangle = \bar{W}^K \bar{W}^J \bar{W}^I |\rho\rangle$$

(W^μ arbitrary vector field, $\bar{W}^I \equiv W^\mu \bar{\psi}^I_\mu$, $|\rho\rangle$ with same YT as $|\varphi\rangle$)

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- Gauge symmetries ($|\xi^I\rangle = \bar{V}^I|\xi\rangle$)

$$\begin{aligned} \delta|\varphi\rangle &= Q_K|\xi^K\rangle \\ \delta(|\rho^{KJI}\rangle) &= \frac{i}{2}\bar{J}^{[KJ}|\xi^{I]}\rangle \end{aligned}$$

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- Can gauge fix compensator to get Fronsdal-Labastida eq.

$$G|\varphi\rangle = 0$$

Fronsdal-Labastida equation

$$\left[-2H + Q_I\bar{Q}^I + \frac{i}{2}Q_IQ_J\bar{K}^{JI}\right]|\varphi\rangle = 0$$

$|\varphi\rangle$ is double traceless (for consistency with gauge fixing)

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- For $D = 4$ and $N = 2s$ it gives Fronsdal eq.

$$R \sim \underbrace{\begin{array}{|c|c|c|c|c|}\hline & & & & \\ \hline & & & & \\ \hline \end{array}}_{s} , \qquad \varphi \sim \underbrace{\begin{array}{|c|c|c|c|c|}\hline & & & & \\ \hline \end{array}}_{s}$$

$$-2H \sim \Box, \;\; Q_I \sim \psi_I^\mu \partial_\mu, \;\; \bar{Q}^I \sim \frac{\partial}{\partial \psi_{\mu I}} \partial_\mu, \;\; \bar{K}^{IJ} \sim \frac{\partial}{\partial \psi_{\mu I}} \eta^{\mu\nu} \frac{\partial}{\partial \psi_{\nu J}}$$

$$\Box \varphi_{\mu_1 \dots \mu_s} - (\partial_{\mu_1} \partial^\alpha \varphi_{\alpha \mu_2 \dots \mu_s} + \dots) + (\partial_{\mu_1} \partial_{\mu_2} \varphi^\alpha{}_{\alpha \mu_3 \dots \mu_s} + \dots) = 0$$

$$\varphi^\alpha{}_\alpha{}^\beta{}_{\beta \mu_5 \dots \mu_s} = 0 \; ; \qquad \delta \varphi_{\mu_1 \dots \mu_s} = \partial_{(\mu_1} \xi_{\mu_2 \dots \mu_s)} \; , \qquad \xi^\alpha{}_{\alpha \mu_3 \dots \mu_{s-1}} = 0$$

Outline

$U(N)$ spinning particle

We now consider a similar model of a spinning particle, constructed by Marcus from the particle limit of $N=2$ strings. It is defined on a (flat) complex space.

- Consider a worldline action ($i=1,..,N$)

$$S=\int dt\Big[p_\mu\dot{x}^\mu+\bar{p}_{\bar{\mu}}\dot{\bar{x}}^{\bar{\mu}}+i\bar{\psi}^i_\mu\dot{\psi}^\mu_i-p_\mu\bar{p}^\mu\Big]$$

- Rigid $U(N)$ extended supersymmetry

$$H=p_\mu\bar{p}^\mu,\quad Q_i=p_\mu\psi^\mu_i\,,\quad \bar{Q}^i=\bar{p}_{\bar{\mu}}\bar{\psi}^{\bar{\mu}i}\,,\quad J^j_i=\psi^\mu_i\bar{\psi}^j_\mu$$

$$\{Q_i,\bar{Q}^j\}_{PB}=-i\delta^j_i H$$

$$\{J^j_i,Q_k\}_{PB}=-i\delta^j_k Q_i\,,\qquad \{J^j_i,\bar{Q}^k\}_{PB}=i\delta^k_i\bar{Q}^j$$

$$\{J^j_i,J^l_k\}_{PB}=i\delta^l_i J^j_k-i\delta^j_k J^l_i$$

$U(N)$ spinning particle action

- Algebra is first class $\rightarrow$ can be gauged

$$S = \int_0^1 d\tau \left[p_\mu \dot{x}^\mu + \bar{p}_{\bar{\mu}} \dot{\bar{x}}^{\bar{\mu}} + i \bar{\psi}^i_\mu \dot{\psi}^\mu_i - eH - i\bar{\chi}^i Q_i - i\chi_i \bar{Q}^i - a^i_j J^j_i \right]$$

- Eliminating momenta p and $\bar{p}$

$$S = \int_0^1 d\tau \left[e^{-1} \left(\dot{x}^\mu - i\bar{\chi}^i \psi^\mu_i \right) \left(\dot{\bar{x}}_\mu - i\chi_j \bar{\psi}^j_\mu \right) + i\bar{\psi}^i_\mu \left(\delta^j_i \partial_\tau - i a^j_i \right) \psi^\mu_j \right]$$

Outline

Canonical analysis

- Choose coordinates $(x^\mu, \bar{x}^{\bar{\mu}}, \psi_i^\mu)$ and momenta $(p_\mu, \bar{p}_{\bar{\mu}}, \bar{\psi}^i_\mu)$
  Wave functions:

$$\Psi(x, \bar{x}, \psi) \sim \sum F_{\mu_1\mu_2.,\ldots,\nu_1\nu_2..}(x, \bar{x})\, \psi_1^{\mu_1} \psi_1^{\mu_2}.,\ldots, \psi_N^{\nu_1} \psi_N^{\nu_2}..$$

- <u>J constraint</u> $\rightarrow$ tensor F:
  - has N blocks of m indices each: $F_{[\mu_1^1..\mu_m^1],\ldots,[\mu_1^N..\mu_m^N]}$
  - antisymmetric inside each block
  - satisfies algebraic Bianchi identities
  - symmetric under exchanges of the s blocks of indices

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  - symmetric under exchanges of the s blocks of indices
  - i.e. Young tableau of $GL(d)$, where d are the complex dimensions of the manifold

Canonical analysis

- $\underline{Q \text{ constraints}}$: "Bianchi" identities

$$\partial_{[\mu} F_{\mu^1_1..\mu^1_m],\dots,\mu^N_1..\mu^N_m} = 0$$

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- This formulation gives a gauge invariant description for a class of higher spin fields on complex manifolds
- Can integrate Bianchi identities $\rightarrow$ HS gauge fields

Complex HS gauge fields

$$J_i^{\,j}|F\rangle = 0 \quad \Rightarrow \quad F \sim m\left\{\underbrace{\begin{array}{|c|c|c|c|}\hline & & & \\ \hline & & & \\ \hline & & & \\ \hline \end{array}}_{N}\right.$$

$$Q_i|F\rangle = 0 \quad \Rightarrow \quad F \text{ closed (Bianchi identities)}$$

$$\bar{Q}^i|F\rangle = 0 \quad \Rightarrow \quad F \text{ coclosed (Maxwell equations)}$$

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- Impose last constraints ($\bar{Q}^i$) and compute

$$\bar{Q}^1 \, Q_1 .. Q_N |\phi\rangle = -Q_2 .. Q_N \underbrace{\Big[-H + Q_i \bar{Q}^i \Big]}_{\text{operator } G} |\varphi\rangle = 0$$

Complex HS gauge fields

- Solution described by

$$\big(-H + Q_i \bar{Q}^i\big)|\phi\rangle = Q_i Q_j |\rho^{ij}\rangle$$

where the compensators can be represented by

$$|\rho^{ij}\rangle = \bar{V}^i \bar{V}^j |\rho\rangle$$

(V^μ arbitrary vector field, $\bar{V}^i \equiv V^\mu \bar{\psi}^i_\mu$, $|\rho\rangle$ with same YT as $|\phi\rangle$)

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- Gauge symmetries

$$\begin{aligned}\delta|\phi\rangle &= Q_i|\xi^i\rangle\\ \delta|\rho^{ij}\rangle &= -\bar{Q}^{[i}|\xi^{j]}\rangle\end{aligned}$$

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Complex HS equations

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$$\left(-H + Q_i\bar{Q}^i\right)|\phi\rangle = 0$$

- For consistency with partial gauge fixing, $|\phi\rangle$ is double divergence-less

$$\bar{Q}^i\bar{Q}^j|\phi\rangle = 0$$

  and gauge symmetry ($\delta|\phi\rangle = Q_i|\xi^i\rangle$) has constrained gauge parameters ($i \neq j$)

$$\bar{Q}^i|\xi^j\rangle = 0$$

Explicit examples

Case with $p = 1$ and arbitrary N without compensators

$$\varphi \sim \underbrace{\boxed{}\boxed{}\boxed{}\boxed{}\boxed{}}_{N} \quad \rightarrow \quad \partial_\alpha \bar{\partial}^\alpha \varphi_{\mu_1 \ldots \mu_N} - \sum_{i=1}^{N} \partial_{\mu_i} \bar{\partial}^\alpha \varphi_{\mu_1 .. \alpha .. \mu_N} = 0$$

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Very much reminiscent of Fronsdal's equations: no invariant concept of taking traces on holomorphic indices, usual trace constraints naturally substituted by differential constraints.

Case with $p = 1$ and $N = 2$ with compensator

$$\varphi \sim \begin{array}{|c|c|}\hline & \\ \hline \end{array}$$

$$\partial_\alpha \bar{\partial}^\alpha \varphi_{\mu\nu} - \partial_\mu \bar{\partial}^\alpha \varphi_{\alpha\nu} - \partial_\nu \bar{\partial}^\alpha \varphi_{\mu\alpha} = \partial_\mu \partial_\nu \rho$$

gauge symmetry

$$\delta\varphi_{\mu\nu} = \partial_\mu \lambda_\nu + \partial_\nu \lambda_\mu \ , \qquad \delta\rho = -2\bar{\partial}^\alpha \lambda_\alpha$$

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Reminiscent of the Francia-Sagnotti construction for relaxing the trace constraints on standard higher spin gauge theories using compensator fields.

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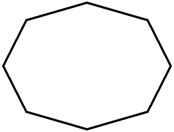

Worldline formalism

- These field equations can be quantized and described using the worldline formalism
- At free field level can just compute the number of degrees of freedom
- Coupling to curved manifolds:
  for $N > 2$ it is possible to extend these equations on manifold with constant holomorphic curvature, and more generally to manifolds with vanishing Bochner tensor (a Kähler analogue of the Weyl tensor)

Worldline formalism

- $O(N)$ particles on maximally symmetric spaces $\Rightarrow$ zero mode of the nonlinear Bershadsky–Knizhnik $O(N)$ superconformal algebras in 2D (quantum version of couplings to (A)dS by Kuzenko and Yarevskaya)

  (works with Olindo Corradini and Emanuele Latini,
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  (work with Roberto Bonezzi,
  JHEP 0903:063,2009 [arXiv:0901.2311],
  JHEP 1005:020,2010 [arXiv:1003.1046], ...)

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- and to conclude ....