A Decoupling Limit of AdS/CFT

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+ work in progress…

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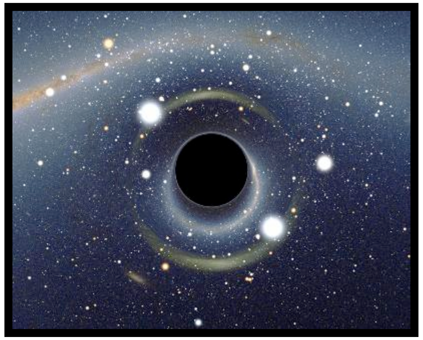
IDEA: obtain an understanding of the quantum mechanical nature of black holes

Important for:
- solving Hawking's information paradox
- understanding the origin and evolution of our Universe
- knowing the microstates of black holes

Black holes are classical solutions of Einstein's eq. of GR

→region of space from which nothing can escape



To obtain a quantum mechanical description of BH one needs a quantum theory of gravity

→String theory is the leading candidate for a quantum theory of gravity

GOAL: obtain an understanding of the quantum mechanical nature of BH in the context of string theory using the AdS/CFT correspondence

AdS/CFT correspondence: duality relating a CFT and a string theory

$\rightarrow$ Any physical phenomena in the CFT side can be described as a physical phenomena in the string theory side and viceversa

Why AdS/CFT useful:

- understand how BH emerge from gauge theories

  $\rightarrow$ Gauge theories are quantum mechanical ➔ we obtain a quantum mechanical description of BH

- study physics of BH using connection to the physics of gauge theory

  $\rightarrow$ Experiments on quark-gluon plasma at RHIC and LHC might represent a window into BH physics

➔ AdS/CFT could provide the first opportunity for an experimental foundation of a quantum mechanical description of BH

- understanding the physics of BH can be used to predict strong coupling behavior of gauge theories

The power of the AdS/CFT correspondence is that it gives a tool to describe the non perturbative physics of gauge theory and string theory

Understanding how BH can be described by QFT requires understanding how strings can be described by QFT

A lot of progress lately in the context of the

AdS_5/CFT_4 correspondence

$\rightarrow \mathcal{N}$= 4 SU(N) SYM on R $\times$ S^3 dual to type IIB string theory on $AdS_5 \times S^5$

Dictionary relating λ, N to g_s, l_s and R (the AdS_5, S^5 radius):

string tension $T_{\text{str}} = \frac{1}{2}\sqrt{\lambda}$ with $T_{\text{str}} = \frac{R^2}{4\pi l_s^2}$

string coupling $g_s = \frac{\lambda}{N}$ 't Hooft coupling $\lambda = g_{YM}^2 N$

This is in accordance with 't Hooft's expectations

- g_s is inversely proportional with N
- Large λ corresponds to semi-classical limit for world-sheet theory

AdS/CFT: $\Rightarrow$A strong/weak coupling duality

A quantitative matching of gauge and string theory needs to interpolate between weak and strong coupling

Many impressive results achieved in the planar limit of $\mathcal{N}$ = 4 SYM using the connection with certain **spin chains : Integrability**

Alday, Astolfi, Arutyunov, Bak, Beccaria, Beisert, Bianchi, Bombardelli, Cicuta, Dorey, Ferretti, Fiamberti, Fioravanti, Forcella, Forini, Freyhult, Frolov, Gaiotto, Giangreco Marotta Puletti, Giombi, Grignani, Gromov, Harmark, Hernandez, Janik, Kazakov, Klose, Korchemsky, Kristjansen, Kruczenski, Lopez, Lukowski, Maldacena, Mc Loughlin, Minahan, MO, Nishioka, Penati, Plefka, Rey, Ricci, Roiban, Sant'Ambrogio, Semenoff, Serban, Sieg, Staudacher, Stephanski, Takayanagi, Tateo, Tirziu, Torrielli, Tseytlin, Vieira, Zanon, Zarembo, Zoubos…….

Limitation: Works only for planar limit N = ∞

To go beyond the planar limit we should find another approach

Our approach:

We study both gauge theory and string theory in the regime:

$$E - J \ll \lambda \ll 1, \;\; J \gg 1 \qquad \text{with} \quad J = J_1 + J_2 \qquad \lambda = g_{YM}^2 N$$

E: Energy of a state (=dimension of operator)
J_i: The three R-charges for the SU(4) R-symmetry (gauge theory)
The three angular momenta on S^5 (string theory)

We claim that both string theory and gauge theory are reliable in this regime

If true $\Rightarrow$ Provides a new way to quantitatively match gauge and string theory

To implement this, we take the *decoupling limit*

$$\lambda \to 0, \;\; \frac{E - J}{\lambda} \text{ fixed}, \;\; J_i \text{ fixed}, \;\; N \text{ fixed}$$

We take the same limit on the gauge and string theory sides

The approach does not rely on taking the planar limit

$\Rightarrow$We hope to make quantitative matches beyond the correspondence between planar $\mathcal{N} = 4$ SYM and tree-level string theory on $AdS_5 \times S^5$

Plan of talk:

Decoupling limit on the gauge theory side

Decoupling limit on the string theory side

Conclusions & Future directions

Decoupling limit on the gauge theory side:

We consider $\mathcal{N} = 4$ super Yang-Mills (SYM) on R $\times$ S^3

State/operator correspondence:

State, CFT on R $\times$ S^3	$\Leftrightarrow$	Operator, CFT on R^4
Energy E		Scaling dimension D
Gauge singlet		Gauge invariant operator

Gauge singlets:
Because flux lines on S^3 cannot escape

We put R(S^3) = 1,
hence E=D

The set of gauge invariant operators is given by linear combinations of all possible multi-trace operators: $\mathrm{Tr}(\cdots)\mathrm{Tr}(\cdots)\cdots\mathrm{Tr}(\cdots)$

Multitrace operators are made from single-trace operators:

$$\mathrm{Tr}(A_1 A_2 \cdots A_L)\ ,\ \ A_i \in \mathcal{A}$$

$\mathcal{A}$: The set of letters of $\mathcal{N} = 4$ SYM $\leftarrow$

3 complex scalars
1 gauge boson
8 fermions
plus descendants using the
covariant derivative

We get the spectrum of operators by diagonalizing the Dilatation operator D

$$D = D_0 + \lambda D_2 + \lambda^{3/2} D_3 + \lambda^2 D_4 + \cdots$$

Here D_0 is the bare scaling dimension, D_2 is the one-loop dilatation operator, etc.

Take now the decoupling limit

$$\boxed{\lambda \to 0, \;\; H \equiv \frac{D-J}{\lambda} \text{ fixed}, \;\; J_i \text{ fixed}, \;\; N \text{ fixed}}$$

$J \equiv J_1 + J_2$

J_i: The three R-charges for the SU(4) R-symmetry
N: We are considering $\mathcal{N} = 4$ SYM with gauge group SU(N)

We see that
$$\frac{D-J}{\lambda} = \frac{D_0 - J}{\lambda} + D_2 + \mathcal{O}(\lambda)$$

Since D_0 - J is a integer or half-integer, we get that only states with

$$D_0 = J$$

can survive the $\lambda \to 0$ limit

What are the operators with $D_0 = J = J_1 + J_2$?

$\Rightarrow$All the operators made out of the two complex scalars Z, X

Z: $(J_1,J_2,J_3)=(1,0,0)$
X: $(J_1,J_2,J_3)=(0,1,0)$

E.g. a single-trace operator could be $\mathrm{Tr}(XZZXXXZZZX)$

This is known as the SU(2) sector of $\mathcal{N} = 4$ SYM

We see now that for operators in the SU(2) sector, we have $H = D_2$ for $\lambda \to 0$

Conclusion: In the limit $\lambda \to 0, \;\; H \equiv \dfrac{D-J}{\lambda}$ fixed, J_i fixed, N fixed

we decouple all states/operators except for the ones in the SU(2) sector, and the effective Hamiltonian for the surviving states are

$$H = D_2$$

i.e. just the one-loop dilatation operator.

Note that this is true for any value of N and J_1, J_2

Decoupling limit of planar $\mathcal{N}$ = 4 SYM:

Planar limit N = ∞

$\rightarrow$ we can focus on the single-trace sector

$$\mathrm{Tr}(A_1 A_2 \cdots A_L), \ A_i \in \{Z, X\}$$

$\rightarrow$like a spin chain $\quad Z : \uparrow, \ X : \downarrow$

$$\mathrm{Tr}(XZZX \cdots Z)$$
$$\Downarrow$$
$$|\downarrow\uparrow\uparrow\downarrow \cdots \uparrow\rangle$$

Which spin chain?

$$H = D_2 = \frac{1}{2} \sum_{i=1}^{J} (I_{i,i+1} - P_{i,i+1})$$

$J = J_1 + J_2$: Length of single-trace operator / spin chain

H: Hamiltonian of ferromagnetic $XXX_{1/2}$ Heisenberg spin chain (Minahan and Zarembo)

Conclusion: In the limit

$$\lambda \rightarrow 0, \ H \equiv \frac{D - J}{\lambda} \text{ fixed}, \ J_i \text{ fixed}$$

planar $\mathcal{N}$ = 4 SYM on R $\times$ S^3 reduces to the ferromagnetic $XXX_{1/2}$ Heisenberg spin chain

What is the low energy spectrum, i.e. the spectrum for H $\ll$ 1 ?

Spectrum: Vacua (D_2 = 0) plus excitations (magnons)

Vacua are given by: $D_2 = 0$

The vacua are precisely the chiral primaries of $\mathcal{N}$ = 4 SYM obeying D_0 = J
$\rightarrow$The low energy excitations are 'close' to BPS

The spectrum of the magnons is:

$$H = \frac{2\pi^2}{J^2} \sum_{n\neq 0} n^2 M_n \ , \quad \sum_{n\neq 0} n M_n = 0$$

A string-like spectrum
in weakly coupled gauge theory

This is the leading order spectrum for planar $\mathcal{N}$ = 4 SYM on R $\times$ S^3 in the regime

$$E - J \ll \lambda \ll 1, \ \ J \gg 1$$

Using Bethe ansatz + integrability of Heisenberg chain one gets the first correction:

$$H = \frac{2\pi^2}{J^2} \sum_{n\neq 0} \left(1 + \frac{2}{J}\right) n^2 M_n \ , \quad \sum_{n\neq 0} n M_n = 0$$

Low energy expansion of H $\Rightarrow$ Expansion in 1/J

Decoupling limit of string theory:

Consider now the dual string theory, i.e. type IIB string theory on $AdS_5 \times S^5$

We want to take the decoupling limit

$$\lambda \to 0, \quad \frac{E-J}{\lambda} \text{ fixed}, \quad J_i \text{ fixed}, \quad N \text{ fixed}$$

In string theory variables:

$$R \to 0, \quad \frac{E-J}{R^4} \text{ fixed}, \quad J_i \text{ fixed}, \quad \frac{g_s}{R^4} \text{ fixed}$$

$$\boxed{T_{\text{str}} = \frac{R^2}{4\pi l_s^2}}$$

R: Radius of AdS_5 and S^5

We see that $T_{str} \to 0$

$\Rightarrow$We are seemingly entering the quantum string regime

We argue that this is not the case for all string modes

We can write the metric for $AdS_5 \times S^5$ as:

$$ds^2 = R^2 \cos^2\xi[-dt^2+(d\Omega_3)^2] - R^2(\sinh^2\rho + \sin^2\xi)dt^2 + R^2 A_{ij} dx^i dx^j$$

Ω_3: The 3-sphere in S^5 related to J_1 and J_2

x^i: The 6 directions transverse to t and Ω_3

Consider first $\xi = \rho = 0$: $$ds^2 = R^2[-dt^2 + (d\Omega_3)^2]$$

$$(d\Omega_3)^2 = \frac{1}{4}(d\theta^2 + \cos^2\theta d\phi^2) + \left(d\alpha + \frac{1}{2}\sin\theta d\phi\right)^2$$

This gives a certain bosonic sigma-model Lagrangian, along with some constraints

Take now $\lambda \to 0$ limit

We show that it gives the reduced action:

$$I_{\text{red}} = \frac{J}{4\pi}\int d\tilde{t} \int_0^{2\pi} d\sigma \left[\sin\theta\dot{\phi} - \frac{1}{4}\left((\theta')^2 + \cos^2\theta(\phi')^2\right)\right]$$

with $(E-J)\frac{J^2}{\lambda} = i\partial_{\tilde{t}}$ and $S_z = \frac{J_1 - J_2}{2} = -i\partial_\phi$

$$\boxed{I_{\text{red}} = \frac{J}{4\pi}\int d\tilde{t}\int_0^{2\pi} d\sigma\left[\sin\theta\dot{\phi} - \frac{1}{4}\left((\theta')^2 + \cos^2\theta(\phi')^2\right)\right]}$$

We see now that if J $\gg$ 1, the action is valid semi-classically

What does this mean?

It means that for certain modes, lying in the regime

$$\boxed{E - J \ll \lambda \ll 1 \quad , \quad J \gg 1}$$

the full action is large and is well-approximated by I_{red}

Seems surprising since $\lambda \ll 1$.
However, on the gauge theory side we saw that the coupling is rather $\dfrac{\lambda}{E - J}$

$\Rightarrow$Going to small λ is compensated by having E-J very small

$\lambda = g_{YM}^2 N$

What about α' corrections to the sigma-model?

Sigma-model on $AdS_5 \times S^5$ is exact $\rightarrow$ No α' corrections

(Could have been very problematic since our limit goes into the quantum string regime)

What about the modes not coming from the bosonic sigma-model on $R \times S^3$?

$$ds^2 = R^2 \cos^2 \xi [-dt^2 + (d\Omega_3)^2] - R^2(\sinh^2 \rho + \sin^2 \xi)dt^2 + R^2 A_{ij} dx^i dx^j$$

I.e. what about fluctuations away from $\xi = \rho = 0$?

These modes have the potential $$\frac{\alpha' J^3}{2\lambda}(\sinh^2 \rho + \sin^2 \xi)$$

A confining potential

$\rightarrow$Forces the modes to have $\xi = \rho = 0$ as $\lambda \rightarrow 0$

$\Rightarrow$The modes decouple

(Similar story should work for the fermions)

$\Rightarrow$The extra modes can still give a contribution, even though they decouple

We need to consider also quantum effects

1) The extra modes can contribute when we go off-shell

To take this into account one should integrate out the extra modes

This can give corrections to the reduced action

$$I_{\mathrm{red}} = \frac{J}{4\pi}\int d\tilde{t}\int_0^{2\pi} d\sigma \left[\sin\theta\dot{\phi} - \frac{1}{4}\left((\theta')^2 + \cos^2\theta(\phi')^2\right)\right]$$

However, these corrections enter as $1/J^n$ contributions, with higher derivative terms

2) What about the zero-point energy?

Here we are saved by the fact that E = J corresponds to a half-BPS state
$\Rightarrow$There are no quantum corrections to the zero-point energy

On the gauge theory side we have:

$$H \equiv \frac{E-J}{\lambda} = D_2$$

for small λ, where D_2 in the planar limit is the Heisenberg spin chain Hamiltonian

For large J, it is well-known that the Heisenberg spin chain is equivalent to the following sigma-model

$$I_{\text{gauge}} = \frac{J}{4\pi}\int d\tilde{t}\int_0^{2\pi} d\sigma \left[\sin\theta\dot{\phi} - \frac{1}{4}(\vec{n}')^2 + \frac{\pi^2}{12J^2}(\vec{n}'')^2 + \cdots\right]$$

with $\vec{n} = (\cos\theta\cos\phi, \cos\theta\sin\phi, \sin\theta)$

Notice now:

$$\boxed{I_{\text{gauge}} = I_{\text{red}} + \text{higher derivative terms}}$$

Thus, we see that we have matched the actions for the leading order part for large J

Moreover, we expect that the quantum fluctuations for the extra modes precisely become responsible for the higher-derivative terms that one has on the gauge theory side

What about the spectrum?

We set $\sin\theta = \frac{p}{\sqrt{J}}, \;\; \phi = \frac{x}{\sqrt{J}}$

I.e. we are zooming in at a point on the equator of the two-sphere

We can then expand I_{red} in 1/J:

$$I_{\text{red}} = \frac{1}{4\pi}\int d\tilde{t}\int_0^{2\pi} d\sigma\,[p\dot{x} - H(x,p)]$$

$$H(x,p) = \frac{(x')^2+(p')^2}{4} + \frac{p^2}{4J}\Big((p')^2-(x')^2\Big) + \mathcal{O}(J^{-2})$$

This gives the following spectrum (up to order 1/J):

$$\boxed{H = \frac{2\pi^2}{J^2}\sum_{n\neq 0}\left(1+\frac{2}{J}\right)n^2 M_n \;, \quad \sum_{n\neq 0} n M_n = 0}$$

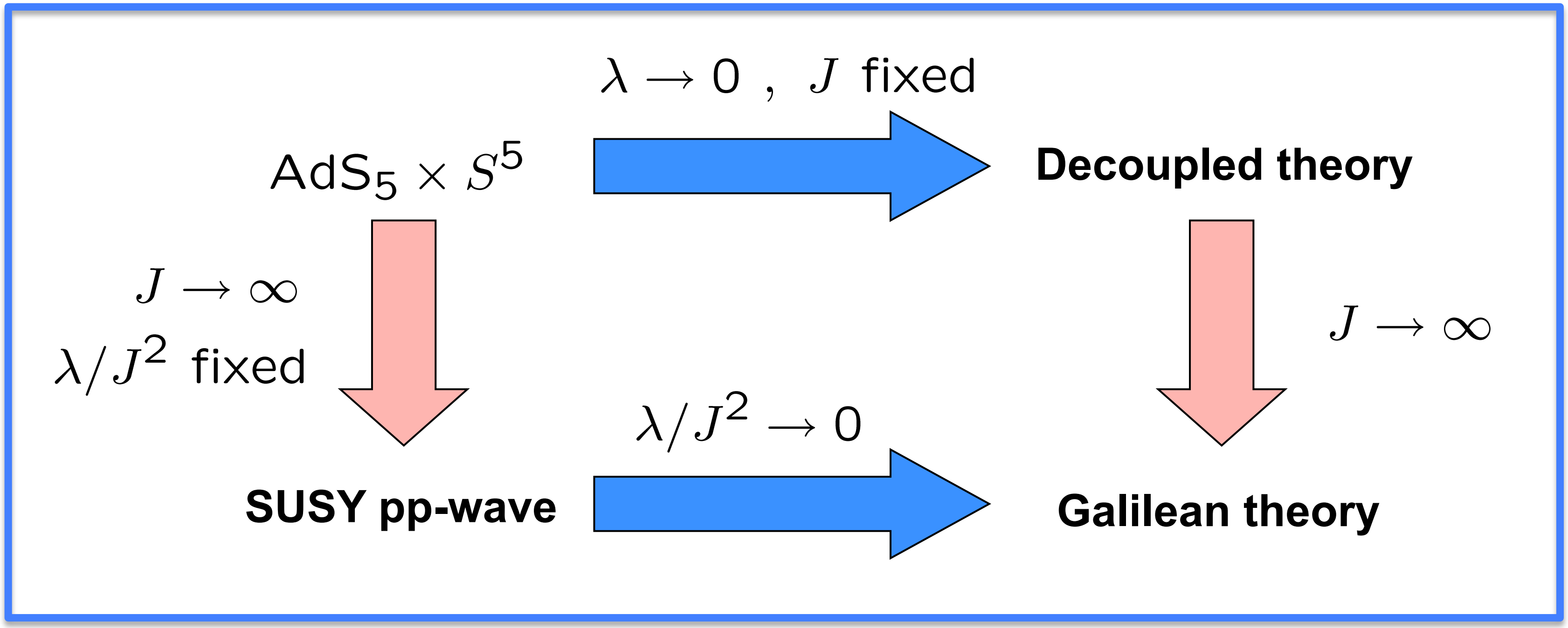
$\Rightarrow$This matches the result on the gauge theory side

We have matched the spectrum of gauge and string theory to leading order.

Further comments on the string side:

Decoupling limit of strings on $AdS_5 \times S^5$ is a non-relativistic limit where some of the modes decouple

J large $\Rightarrow I_{\rm red} \simeq \frac{1}{4\pi}\int d\tilde{t}\int_0^{2\pi} d\sigma \left[p\dot{x} - \frac{(x')^2+(p')^2}{4}\right]$ $\Rightarrow$A Galilean theory $H = \frac{2\pi^2}{J^2}\sum_{n\neq 0} n^2 M_n$



In this case the right pp-wave is:

$$ds^2 = -4dx^+dx^- - \mu^2 \sum_{I=3}^{8} x^I x^I (dx^+)^2 + \sum_{i=1}^{8} dx^i dx^i + 4\mu x^2 dx^1 dx^+$$

(Michelson. Bertolini, de Boer, Imeroni, Harmark and Obers)

See Grignani, Harmark, Marini and MO (2009) for a complete study of all possible pp-wave backgrounds which can be obtained as Penrose limits of the $AdS_5 \times S^5$ geometry.

The decoupling limit can also be expressed as a limit of the thermal partition function for $\mathcal{N} = 4$ SYM on $R \times S^3$ in the grand canonical ensemble

$$Z(\beta, \Omega_i) = \mathrm{Tr}(e^{-\beta E - \beta \Omega_i J_i})$$

$\beta = T^{-1}$: Inverse temperature

Ω_i : Chemical potentials corresponding to R-charges J_i of SU(4) R-symmetry

In terms of this the decoupling limit is

$$\boxed{T \to 0,\ \Omega \to 1,\ \lambda \to 0,\ \tilde{T} \equiv \frac{T}{1-\Omega},\ \tilde{\lambda} \equiv \frac{\lambda}{1-\Omega},\ N \text{ fixed}}$$

with $\Omega_1 = \Omega_2 = \Omega$ and $\Omega_3 = 0$.

The limit corresponds to going near the critical point T=0, Ω_1=Ω_2=1

We employ this limit to interpolate the Hagedorn temperature from weak to strong coupling

Hagedorn temperature :

Consider the partition function

$$\log Z(\tilde{\beta}) = \sum_{n=1}^{\infty} \sum_{J=1}^{\infty} \frac{1}{n} e^{-nJ\tilde{\beta}} \mathrm{Tr}_J \left(e^{-n\tilde{\beta}\tilde{\lambda} D_2} \right) \qquad \tilde{\beta} = \frac{1}{\tilde{T}}$$

Large $\tilde{\lambda}$/low temperatures:

Using the low-energy spectrum $H = \frac{2\pi^2}{J^2} \sum_{n \neq 0} n^2 M_n \ , \quad \sum_{n \neq 0} n M_n = 0$

we find

$$\boxed{\tilde{T}_H = (2\pi)^{1/3} \left[\zeta\left(\frac{3}{2}\right) \right]^{-2/3} \tilde{\lambda}^{1/3} \quad \text{for} \quad \tilde{\lambda} \gg 1}$$

Hagedorn temperature of weakly coupled planar $\mathcal{N} = 4$ SYM on R $\times$ S^3 in the limit

$$T \to 0, \ \Omega \to 1\ , \ \lambda \to 0\ , \ \tilde{T} \equiv \frac{T}{1-\Omega} \ \text{fixed}, \ \tilde{\lambda} \equiv \frac{\lambda}{1-\Omega} \ \text{fixed}$$

We have already shown that the leading part of the spectrum of planar $\mathcal{N} = 4$ SYM and tree-level string theory on $AdS_5 \times S^5$ both become

$$H = \frac{2\pi^2}{J^2} \sum_{n\neq 0} n^2 M_n \ , \qquad \sum_{n\neq 0} n M_n = 0$$

In the regime

$$E - J \ll \lambda \ll 1 \quad , \qquad J \gg 1$$

This regime corresponds to $\tilde{\lambda} \gg 1$ for the limit of the partition function

I.e. small energies H $\ll$ 1, $H \equiv \frac{E-J}{\lambda}$, corresponds to large effective coupling $\tilde{\lambda} \gg 1$

In conclusion, we can get the following result from our consideration on the string side

$$\boxed{\tilde{T}_H = (2\pi)^{1/3} \left[\zeta\left(\frac{3}{2}\right)\right]^{-2/3} \tilde{\lambda}^{1/3} \quad \text{for} \quad \tilde{\lambda} \gg 1}$$

which matches the gauge theory result, due to the matching of the spectra.

The Hagedorn temperature is the first example of a quantity not protected by susy that has been succesfully interpolated from the gauge theory to the string theory regime

(Harmark, MO)

Generalization to other decoupled sectors

The previous discussion can be applied to other decoupled sectors for which the gauge/string theory spectrum can be written as

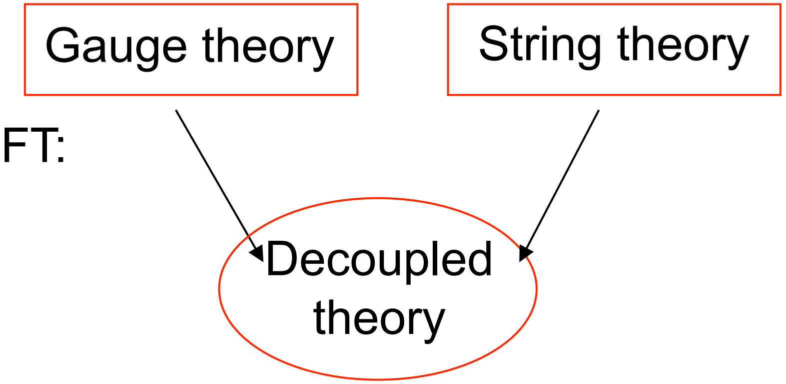
$$E = \frac{2\pi^2}{J^2}\sum_n n^2\left(\sum_{i=1}^{a} M_n^{(i)} + \sum_{j=1}^{b} N_n^{(j)} + \sum_{\alpha=1}^{c} F_n^{(\alpha)}\right)$$

SU(..)	(2)	$(1,1)_{bos}$	(1|1)	(1|2)	(2|3)	(1,1|1)	(1,1|2)	(1,2|2)	(1,2|3)
a	1	0	0	1	2	0	1	0	2
b	0	1	0	0	0	1	1	2	2
c	0	0	1	1	2	1	2	2	4

From the spectrum we find the Hagedorn temperature for large $\tilde{\lambda}$

$$\boxed{\tilde{T}_H = \left(\frac{2\sqrt{2\pi}}{(2a + 2b + (2-\sqrt{2})c)\zeta(3/2)}\right)^{2/3} \tilde{\lambda}^{1/3}}$$

Conclusions



- We have studied the decoupling limit of AdS/CFT:

$$\lambda \to 0,\ \frac{E-J}{\lambda}\ \text{fixed},\ J_i\ \text{fixed},\ N\ \text{fixed}$$

Decoupled theory for N = ∞/g_s = 0: Ferromagnetic $XXX_{1/2}$ Heisenberg spin chain

- Explicit matching of spectrum and Hagedorn temperature for large J

$$E = \frac{2\pi^2}{J^2}\sum_{n\neq 0} n^2 M_n \qquad \tilde{T}_H = (2\pi)^{1/3}\left[\zeta\left(\frac{3}{2}\right)\right]^{-2/3}\tilde{\lambda}^{1/3}$$

Other limits of AdS/CFT:

- We have found 14 limits giving non-trivial decoupled theories
- We derived a new pp-wave background where type IIB string theory can be quantized (*rotated pp-wave background*).

⇒Important for approaching non-perturbative physics of type IIB string theory in $AdS_5 \times S^5$, such as D-branes and black holes.

Ongoing research & Future directions:

1/N: Move away from planar limit/tree-level string theory

BH condense from strings when the coupling is increased

$\Rightarrow$ at low energies the gravitons condense and form BH

Our idea: establish a new connecting link between gauge and string theory when the string coupling is turned on

At low coupling, the linking theory should be a theory of splitting and joining of a gas of spin chains of various lengths

Increasing the coupling the spin chain gas condenses

The condensed phase of the spin chain gas should correspond to BH physics on the string side

Goal: find a quantitative description of the condensed phase

$\Rightarrow$ having a quantum mechanical description of BH

- Understand the Hagedorn phase transition using gauge theory
- 't Hooft: we can approach QCD (N=3) by first understanding the theory for $N=\infty$ and then making a perturbative series in 1/N