On SUSY-breaking in metastable vacua of perturbed $\mathcal{N} = 2$ theories

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Introduction

Long-lived metastable vacua which break supersymmetry are a generic feature in $\mathcal{N} = 1$ SQCD with massive fundamental matter (as shown by ISS)

Strong coupling makes the calculability of models of dynamical supersymmetry breaking challenging

In the ISS a weakly coupled dual description is used

Another setting where this is calculable is in perturbed $\mathcal{N} = 2$ theories

$\mathcal{N} = 2$ theories

Vector multiplet: $SU(N_c)$ gauge field A_μ, 1 complex scalar Φ in the adjoint representation, two Weyl fermion in the adjoint representation

Hypermultiplets: 2 comples scalars Q, $\tilde{Q}^\dagger$, two Weyl fermions ψ_q, $\psi^\dagger_{\tilde{q}}$ in the same representation (for example fundamental or adjoint)

$$W = Q_k \Phi \tilde{Q}_k + m_k Q_k \tilde{Q}_k$$

The VEV of Φ breaks $SU(N_c) \to U(1)^{N_c - 1}$: Coulomb Branch of the moduli space

Seiberg-Witten solution: $N_c = 2$

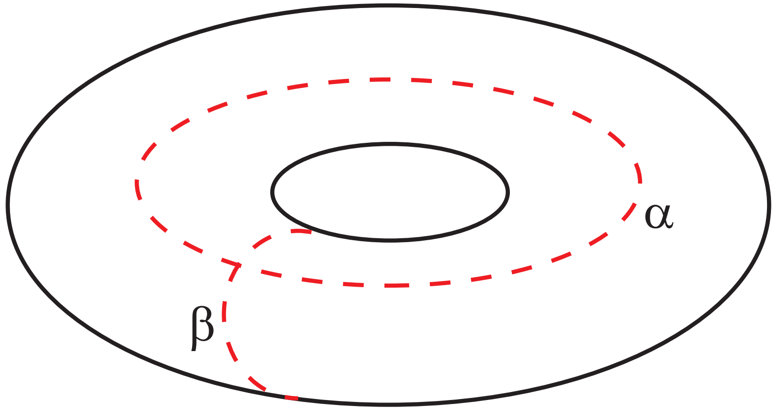
Moduli space coordinate $u = Tr\Phi^2$
Effective $U(1)$ theory:

$$\frac{1}{4\pi} Im \left[\int d^4\theta \frac{\partial \mathcal{F}}{\partial A} \bar{A} + \int d^2\theta \frac{1}{2} \frac{\partial^2 \mathcal{F}}{\partial A^2} W_\alpha W^\alpha \right]$$

$$A_D = \frac{\partial \mathcal{F}}{\partial A}.$$

$A_D(u), A(u)$ are function of the moduli space coordinate which can be be computed from the SW curve

The curve and the periods



$$A(u) = \oint_\alpha d\lambda_{SW} \,, \qquad A_D(u) = \oint_\beta d\lambda_{SW} \,,$$

$$ds^2 = g = (\mathrm{Im}\,\tau_e) A' \, \bar{A}' du d\bar{u} \,, \qquad \tau_e = \frac{dA_D}{dA}$$

Potential

Metastable vacua correspond to local maxima of

$$\frac{1}{V} = (\mathrm{Im}\,\tau_e)\left|\frac{a'}{W'}\right| .$$

This is a product of two factors which does not have local maxima (an harmonic function and the squared modulus of an holomorphic function)

Perturbing the theory

$$\delta W = \mu(u + \alpha u^2 + \beta u^3)\,.$$

Irrelevant operators α, β are needed

Theorem (Ooguri, Ookouchi, Park): for almost every point of the moduli space u_0, we can find (α, β) in such a way that a metastable vacuum is generated in u_0. Due to the fact that the sectional curvature of the moduli space metric is positive definite

$$\langle w_1, R(w_2, w_2) w_1 \rangle > 0\,.$$

Condition for metastability

$$\mathcal{W} = \tilde{\mu}\, W = \tilde{\mu}\left((u-u_0) + \kappa(u-u_0)^2 + \lambda(u-u_0)^3\right) \ .$$

$$V = |\tilde{\mu}|^2\, g^{-1}(u,\bar{u})\, W'(u)\, \bar{W}'(\bar{u})$$

$$\kappa = -\frac{1}{2}\, g\, \frac{\partial g^{-1}}{\partial u}$$

$$|\lambda - \lambda_0| < \frac{g}{6}\left(\frac{\partial^2 g^{-1}}{\partial u \partial \bar{u}} - g\left|\frac{\partial g^{-1}}{\partial u}\right|^2\right) = r_\lambda\,, \qquad \lambda_0 = \frac{g^2}{3}\left(\frac{\partial g^{-1}}{\partial u}\right)^2 - \frac{g}{6}\frac{\partial^2 g^{-1}}{\partial u^2}$$

Condition for metastability

In order to obtain a metastable vacuum in u_0:

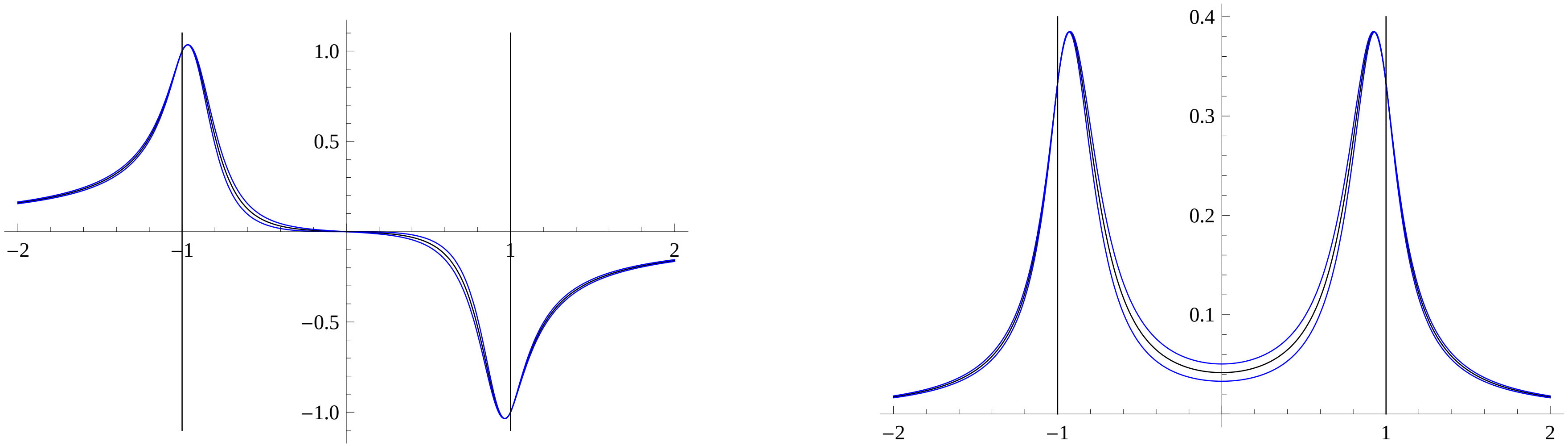
$$\alpha = \alpha_0 + \delta\alpha\,\epsilon$$

$$\beta = \beta_0 + \delta\beta\,\epsilon$$

ϵ is a complex number with $|\epsilon| < 1$

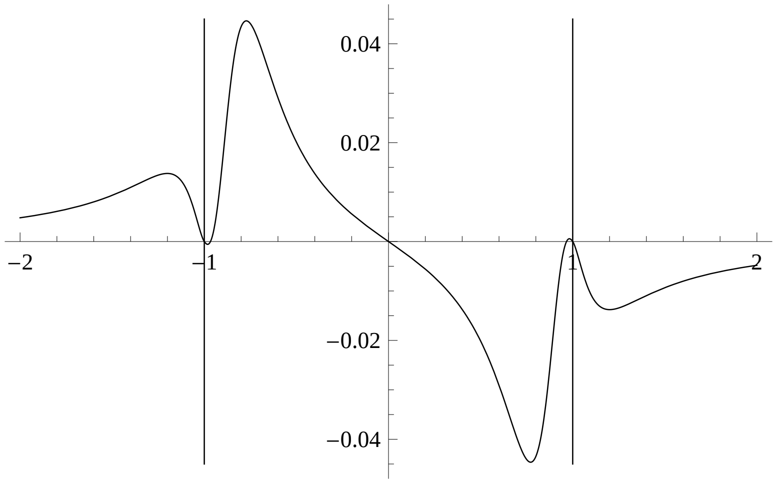
$(\alpha_0, \beta_0, \delta\alpha, \delta\beta)$ are functions of u_0 that can be computed from the moduli space metric (which can be computed from the SW curve)

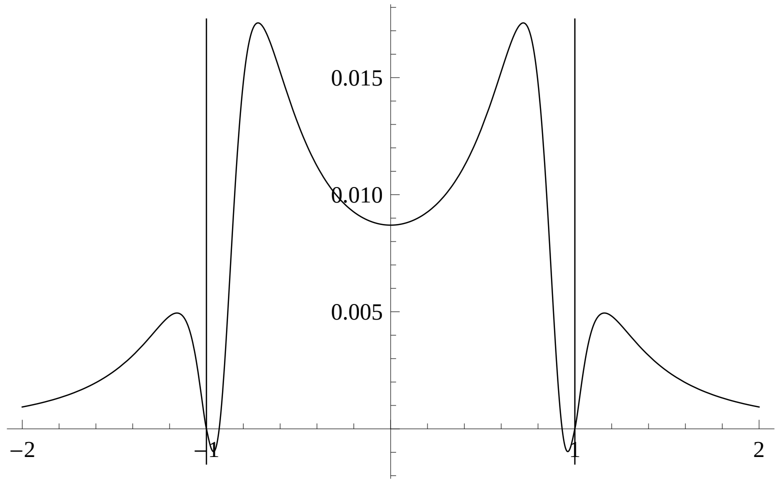
SYM, $N_f = 0$



Left: α_0 . Right: β_0.

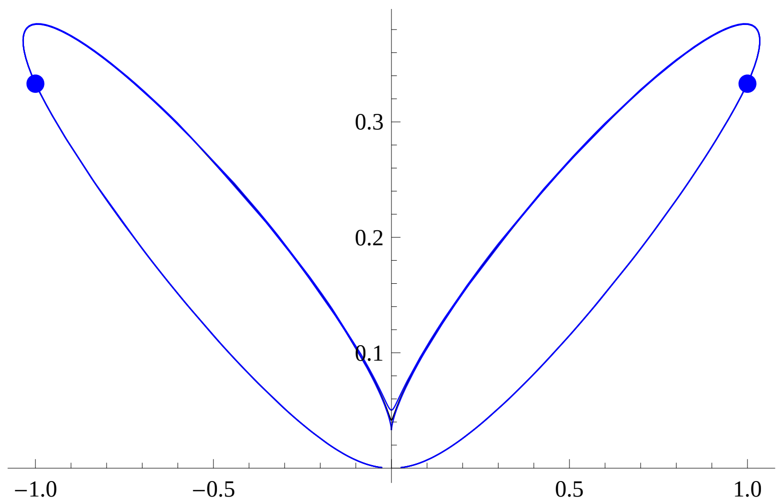
SYM, $N_f = 0$





Left: $\delta\alpha$. Right: $\delta\beta$.

SYM, $N_f = 0$



Region of parameters which achieve a metastable vacuum on the real axis of the moduli space u_0.

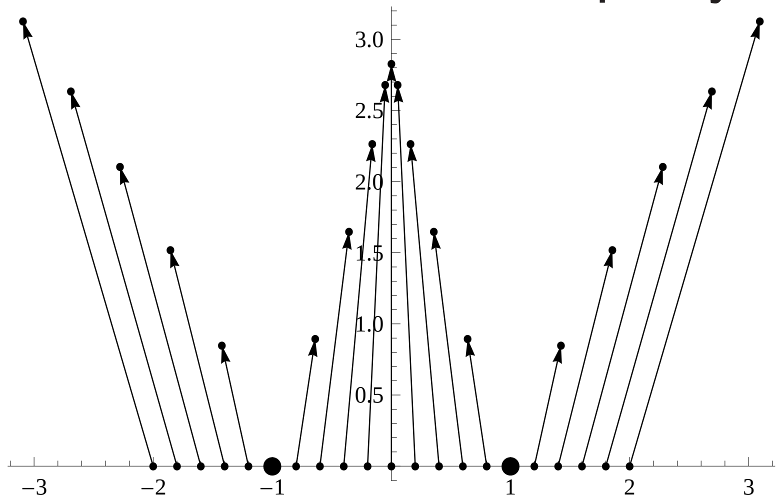
Is the vacuum long-lived ?

The tunneling rate proportional to e^{-S}, where

$$S \propto \frac{(\Delta u/\Lambda)^4}{\Delta V},$$

This can be achieved for very small μ, because $\Delta V \propto \mu^2\Lambda^2$, and Δu is independent of μ

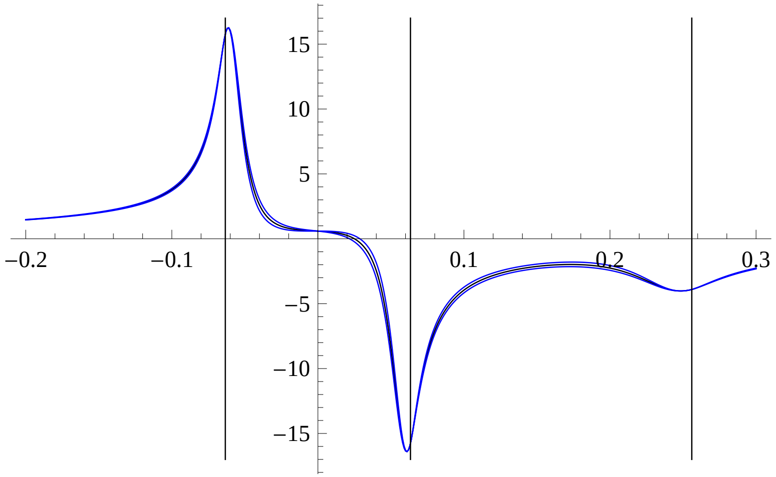
...this must be checked for all the supersymmetric vacua...

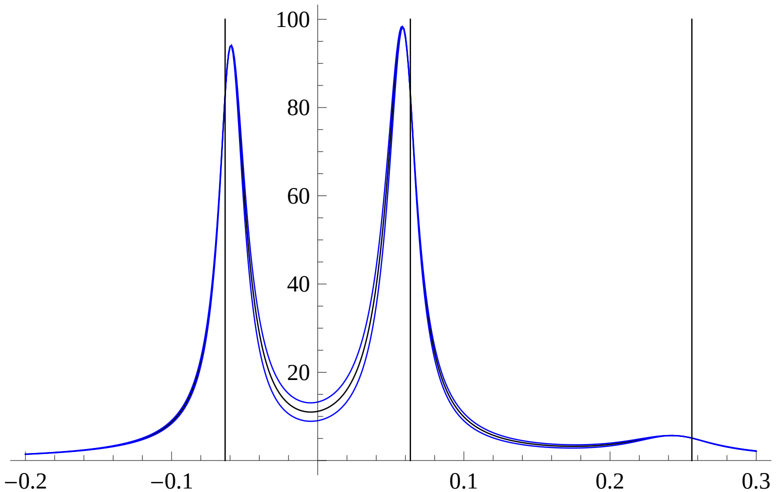


$N_f = 4$ **theory**

Can we do this for conformal theories, such as the $\mathcal{N} = 4$ or the theory with $N_f = 4$?

In order to do it we must add a mass for the hypermultiplets





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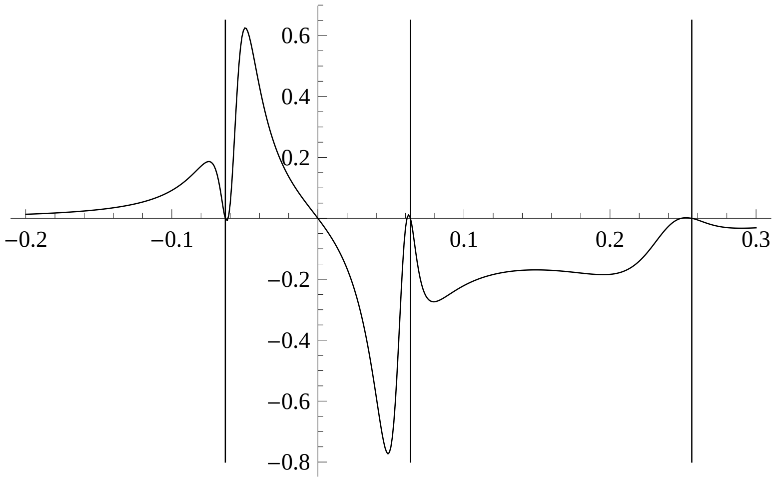
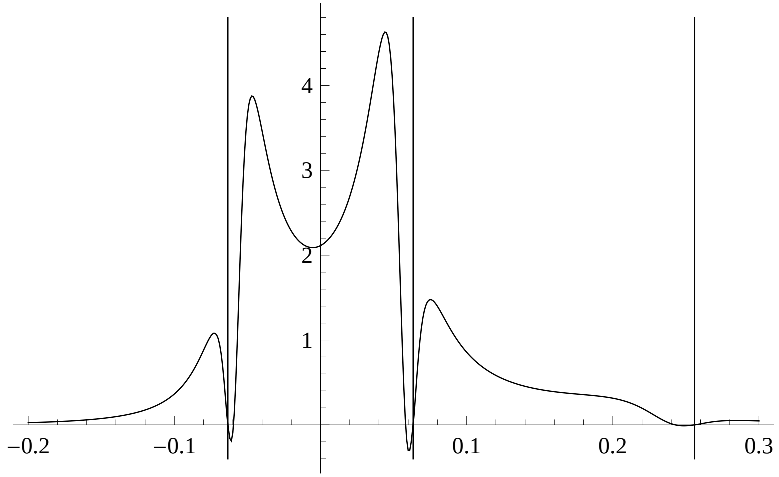
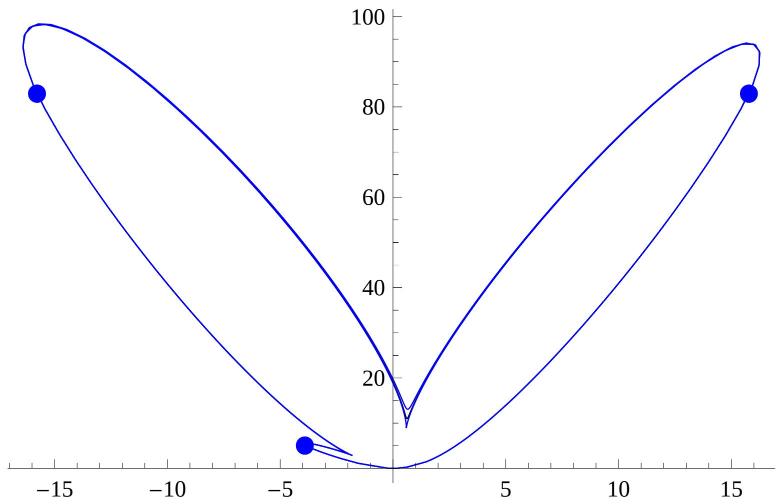
$N_f = 4$ theory





Left: $\delta\alpha$. Right: $\delta\beta$.

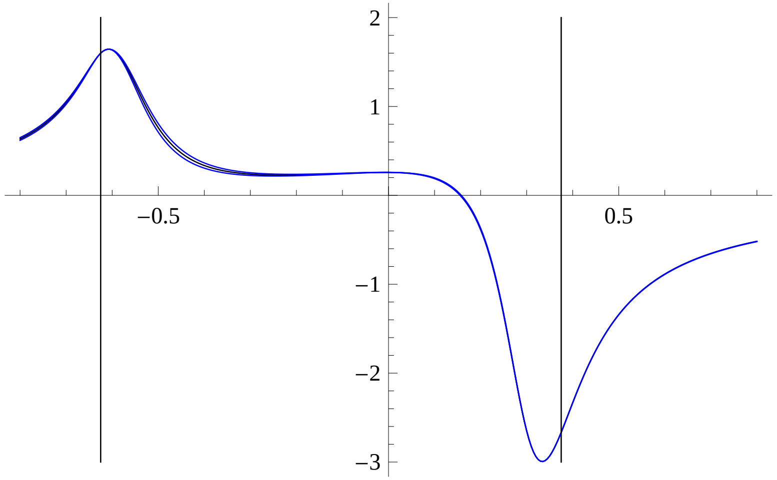
$N_f = 4$ theory

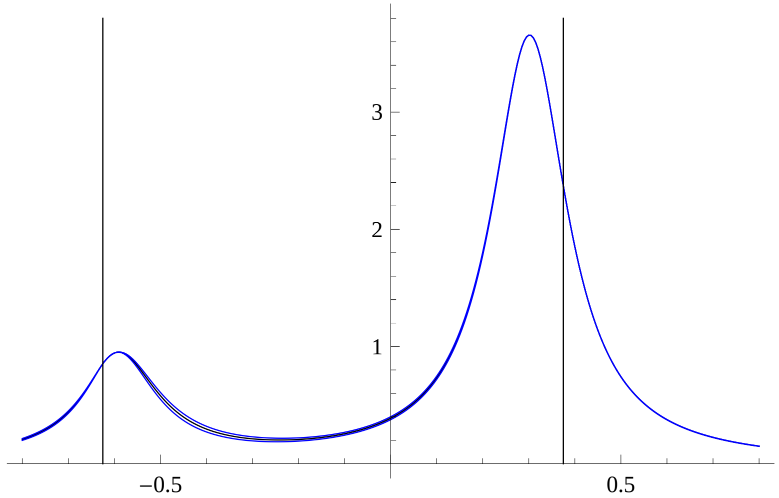


Region of parameters which achieve a metastable vacuum on the real axis of the moduli space u_0.

$N_f = 2$ theory ($m_1 = m_2 = m_c$)

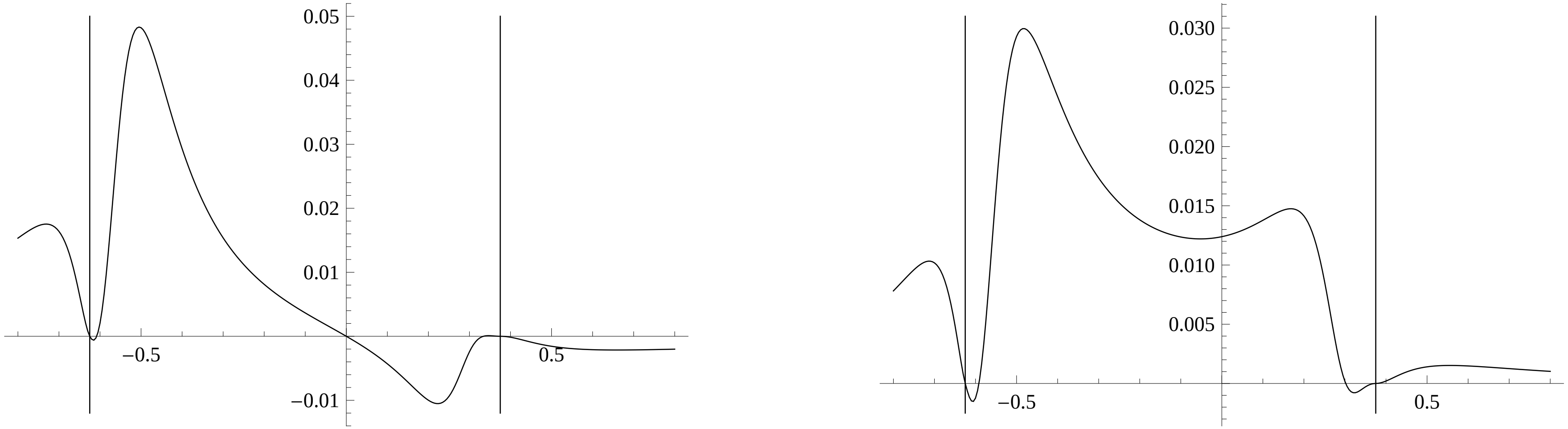
An example with a conformal Argyres-Douglas fixed point





Left: α_0 . Right: β_0.

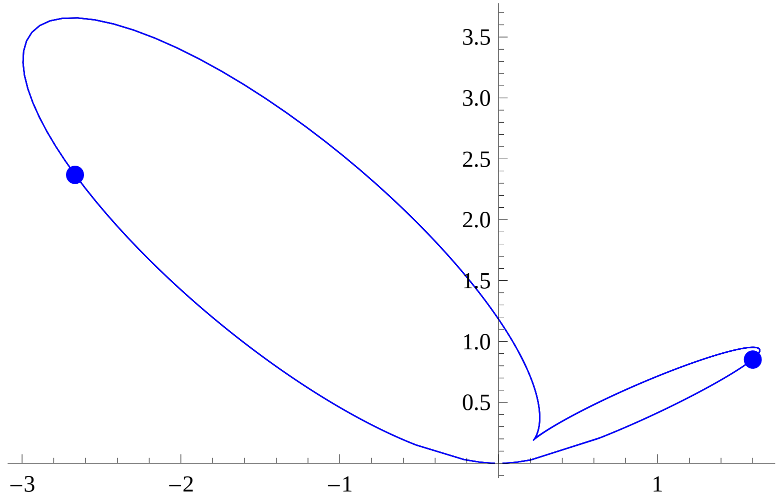
$N_f = 2$ **theory** $(m_1 = m_2 = m_c)$



Left: $\delta\alpha$. Right: $\delta\beta$.

Nearby a conformal point $\delta\alpha, \delta\beta \propto (u - u_{AD})^3$

$N_f = 2$ theory $(m_1 = m_2 = m_c)$



Region of parameters which achieve a metastable vacuum on the real axis of the moduli space u_0.

Direct gauge mediation

In these examples, a global flavor symmetry $U(1) \times SU(N_f)$ can be gauged and coupled to an external supersymmetric sector, in order to realize direct gauge mediation

$$m_\lambda = \frac{\alpha_r}{4\Pi}\Lambda_G\,, \qquad m^2_{\tilde{f}} = 2C_{\tilde{f}}\left(\frac{\alpha_r}{4\pi}\right)^2\Lambda_S^2\,,$$

$$\Lambda_G = F^a\left(\partial_a(\log\det\mathcal{M})\right)\,, \qquad \Lambda_S^2 = \frac{1}{2}|F^a|^2\frac{\partial^2}{\partial a\partial\bar{a}}\sum_i\left(\log|\mathcal{M}_i|^2\right)^2\,,$$

Direct gauge mediation

At weak coupling, the squarks can be identified with the messengers:

$$W = \sqrt{2} Q_k \left(a - \frac{m_k}{\sqrt{2}} \right) \tilde{Q}_k \, .$$

What about strong coupling ? The picture in principle is complex because in the spectrum there are monopoles or dyons carrying flavor quantum numbers

A formula for the gaugino masses

It is possible to gauge a global symmetry using another $\mathcal{N} = 2$ gauge theory; with this trick an exact expression at the leading order in SUSY-breaking can be found (arXiv:0806.4733, Ooguri, Ookouchi, Park, Song)

$$\mathcal{L} = \ldots + \frac{i}{16\pi}\mathcal{F}_{IJK}F^I\lambda^J\lambda^K\,,$$

In this expression the prepotential $\mathcal{F}$ is also function of the mass terms m^a , which correspond to the eigenvalues of the spectator gauge group.

An example

Let us gauge the $U(1)_F$ symmetry of the $N_f = N_c = 2$ theory, $m = 0$

$$(m_\lambda^{GGM})_{ab} = g_F^2 \frac{i}{8\pi} F^i \mathcal{F}_{iab}$$

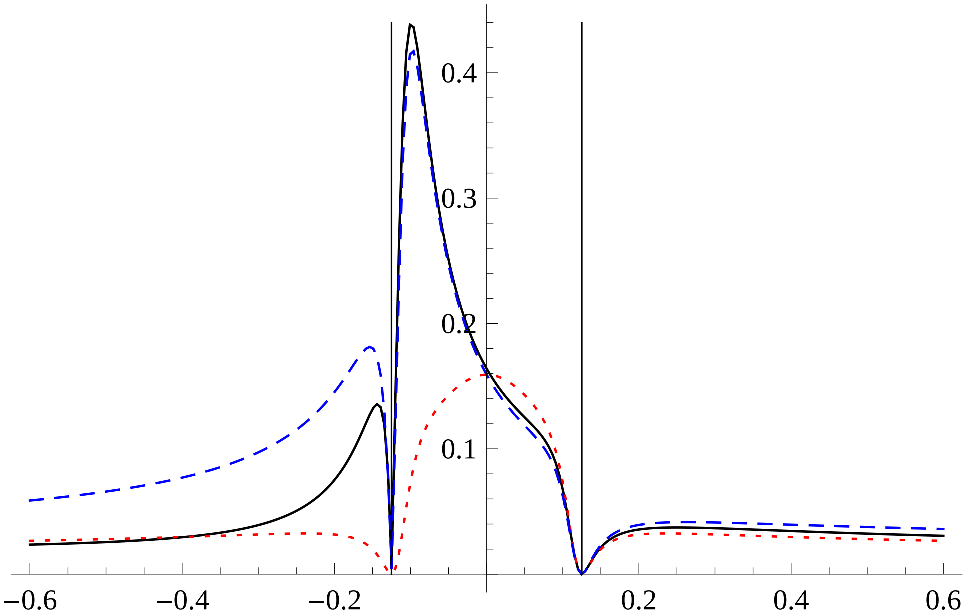
We can compare this formula with the contribution from the dyon which carry $U(1)_F$ and which is massless at one of the singularities

$$W = \sqrt{2}(a + a_D) D\tilde{D}$$

$$m_\lambda^{\mathrm{D}} = \frac{g_F^2}{8\pi^2} \left| F^a \frac{1+\tau_e}{a + a_D} \right| .$$

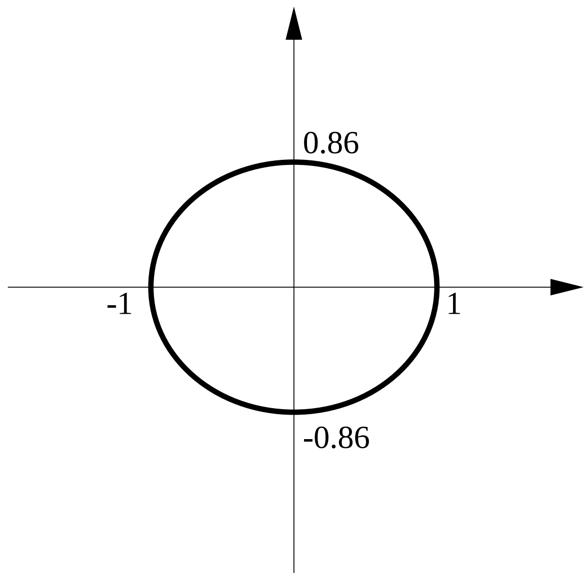
A comparison

Let us compare the contribution of the dyon (blue) with the exact formula (black)



There is a good agreement inside the marginal stability curve

Spectrum of BPS states

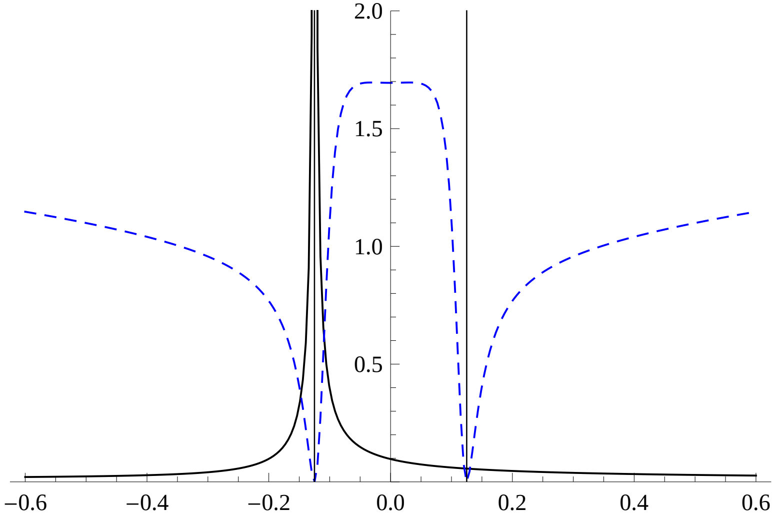


Inside the marginal stability curve the only BPS states are the monopole ($U(1)_F$ singlet) and the $(1,1)$ dyon (which is charged under $U(1_F)$)

Outside the curve the spectrum of BPS states is semiclassical

A cross-check

F-term (in blue) and $|\mathcal{A}|$ (in black)



$$m_\lambda^{\mathrm{GGM}} = g_F^2 \, |F^a \mathcal{A}| \; , \qquad \mathcal{A} = \frac{i}{8\pi} \left(\frac{\partial^2 a_D}{\partial m^2} - \left(\frac{\partial \tau_e}{\partial m} \right)^2 \frac{\partial a}{\partial \tau_e} \right)$$

Conclusions

Long-lived metastable vacua are possible in $\mathcal{N} = 2$ theories perturbed by a superpotential, even if they are not too generic (the allowed region of parameters (α, β) is rather small, especially in the weakly coupled region of the moduli space)

For conformal theories $(N_f = 2N_c, \mathcal{N} = 4)$ we need also to explicitly break conformal invariance by a mass for the hypermultiplets. Nearby a conformal point even more fine tuning is needed, $\delta\alpha, \delta\beta \propto (u - u_{AD})^3$

An explicit example of Direct Gauge Mediation where the messenger is a dyon was discussed