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Diffractive Survival Probabilities

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The following issues will be discussed:

1) The onset of s-channel unitarity at very high energies (LHC and above).

2) The roll of multi-Pomeron interactions (Pomeron enhancement).

3) Stability of survival probability calculations.

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I) Introduction 1

Soft survival probability is a consequence of s-channel unitarity in soft hadron scattering. b-space unitarity equation in a diagonal 1x1 representation is:

$$2\,\mathrm{Im}\, a_{el}(s,b) = |a_{el}(s,b)|^2 + G^{in}(s,b)$$

$$\sigma_{tot}(s,b) = \sigma_{el}(s,b) + \sigma_{in}(s,b)$$

A general solution can be written as:

$$a_{el}(s,b) = i\left(1 - e^{-\frac{1}{2}\Omega(s,b)}\right)$$

Ω is called opacity.

Generality is maintained as long as Ω is arbitrary. In this case $|a_{el}(s,b)| \leq 2$, and at the bound $G_{in} = 0$, i.e. $\sigma_{tot} = \sigma_{el}$.

In the eikonal Glauber approximation a_{el} is imaginary (Ω is real). In this case $|a_{el}(s,b)| \leq 1$, namely - the unitarity bound coincides with the black disc bound.

The second input of the eikonal model is that $\frac{\Omega}{2} = a_{el}^{input}$ which may violate unitarity.

The interpretation is that the unitarization of a^{in}_{el} is achieved through a sequence of repeated elastic rescatterings resulting in a^{out}_{el} which is unitarized.

An important observation is that

$$G^{in}(s,b) = 1 - e^{-\Omega(s,b)}$$

from which we deduce that $e^{-\Omega}$ is the probability that the two projectiles will reach the final interaction intact regardless of the intermediate rescatterings.

In a single channel model $\sigma_{diff} \ll \sigma_{el}$ and $\sigma_{el}(s,b) \leq \frac{1}{2}\sigma_{tot}(s,b)$. Regardless, the unitarization of a hard LRG process such as $pp \to p + LRG + \underbrace{2 \text{ hard dijets}}_{JJ \text{ (or Higgs)}} + LRG + p$ leads to the definition of the soft survival probability

$$S^2_{LRG} = \frac{\sigma^{out}_{LRG}}{\sigma^{in}_{LRG}} = \frac{\int d^2b\, |M_{LRG}|^2 e^{-\Omega(s,b)}}{\int d^2b\, |M_{LRG}|^2}$$

In reality, σ^{soft}_{diff} can not be neglected in the rescattering chain. Technically, this leads to a multichannel description at which elastic and diffractive rescatterings take pla

A LRG is an experimental signature that no color was exchanged between the two interacting hadrons (or partons).

Actually, lacking a satisfactory defenition of diffraction in hadronic interactions, BJ suggested, years ago, to define diffraction by its signature.

A LRG survival probability is that an intrinsically produced LRG will survive all the way to the detected final state.

S^2 has, in general, a few factorizable components:

1) Rescattering of the spectator partons. This is initial state interaction denoted S_s^2.

2) Bremsstrahlung of soft gluons radiated in the intrinsic partonic reaction. This is logarithmically suppressed by the Sudacov factor wich is included in the calculation of M_{LRG}.

3) Final state corrections are generally ignored. I shall return to this in the continuation.

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II Introduction 2

Most (but not all) calculations of the soft survival factor for exclusive central diffractive Higgs production are calculated within multi channel eikonal models. An important signature of such models is that $\sigma_{el}(s,b) + \sigma^{GW}_{diff}(s,b) \leq \frac{1}{2}\sigma_{tot}(s,b)$ (The Pumplin bound). Frequently quoted results were produced by GLM (Tel-Aviv) and KMR (Durham), FS (Frankfurt, Strikman et al.). In the above models the soft survival probability for an exclusive LRG final state (hard!) such as exclusive Higgs central production is actually described as a sumation of

$$\underbrace{pp \to (i+k)}_{\text{soft}} \underbrace{\longrightarrow p + LRG + H + LRG + p}_{\text{hard}}$$

Where, i,k are the intermediate elastic + diffractive states which are defined and counted diffently by GLM and KMR. FS is a single channel model.

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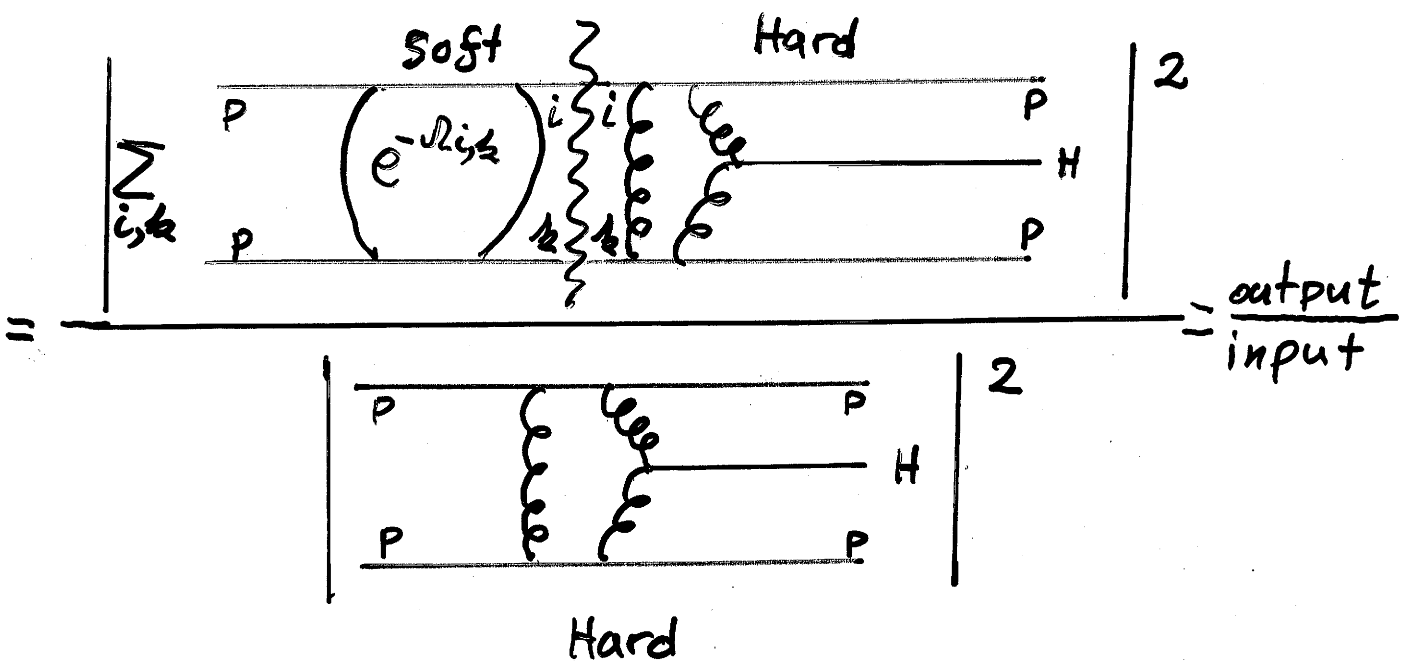
The resulting survival probability is:

$$S_H^2 = \frac{\int d^2b \left(\sum_{i,k} \left(M^S_{pp \to i,k}\, e^{-\frac{1}{2}\Omega^S_{i,k}} \right) M^H_{i,k \to pHp} \right)^2}{\int d^2b \left| M^H_{pp \to pHp} \right|^2} =$$



$\Omega^S_{i,k}$ are the soft opacities of the process $p+p \to i+k$. $\Rightarrow$ fit to soft scattering data.

The hard process of the nominator is $i+k \to p+H+p$. $\Rightarrow$

The hard process of the denominator is $p+p \to p+H+p$. $\Rightarrow$

$\Rightarrow$ Since the s-dependent terms in the $\frac{\text{nominator}}{\text{denominator}}$ cancell, we need only the b-profiles of the $i+k$ and $p+p$ reactions. These are obtained from HERA

The above defines the ingredients of the S^2_{Higgs} calculation:

1) Determination of $\Omega^{S}_{i,k}$. To this end you have to define a suitable data base.

2) Parametrization of the HERA t-dependence J/ψ data so as to obtain the profiles $\Gamma^{H}_{i,k}(s,b)$. The differences between models are not decisive!

As we shall see there are severe differences between the two channel GLM and KMR calculations. FS adopted a different philosophy which I shall not elaborate upon.

Over the last few years GLM, KMR, FS produced essentially identical numbers for S^2_{Higgs}, even though conceptually different. This pleasant industrial co-existence was shattered in 2007 with GLM and FS predicting $S^2_{Higgs} = 0.5-0.8\%$ and KMR predicting $S^2_{Higgs} = 1.2 - 3.2\%$.

This difference initiated a heated debate!

III) The Soft Sector

a) Basic formulation: GLM is a two channel eikonal model where the basic input approximation, compatible with the Good-Walker clasical paper, is that all diffractive final states at a vertex are presented by a single diffractive state $|D\rangle$. We have, thus, two orthonormal vertex wave functions $\langle \Psi_h | \Psi_D \rangle = 0$.
Corresponding is a 2×2 interaction matrix **T**.
Define its eigen functions ψ_1, ψ_2, then:

$$\Psi_h = \alpha \psi_1 + \beta \psi_2 \qquad \alpha^2 + \beta^2 = 1$$
$$\Psi_D = -\beta \psi_1 + \alpha \psi_2$$

Corresponding are 4 amplitudes $A_{i,k}$. Eeach satisfies a diagonal unitarity equation

$$2\, Im\, A_{i,k}(s,b) = |A_{i,k}(s,b)|^2 + G^{in}_{i,k}$$

and a differential survival probability factor $P_{i,k} = e^{-\Omega_{i,k}}$. The corresponding elastic and diffractive amplitudes in (s,b) space are:

$$a_{el} = i[\alpha^4 A_{1,1} + 2\alpha^2\beta^2 A_{1,2} + \beta^4 A_{2,2}]$$
$$a_{sd} = i\alpha\beta[\alpha^2 A_{1,1} + (\alpha^2 - \beta^2) A_{1,2} + \beta^2 A_{2,2}]$$
$$a_{dd} = i\alpha^2\beta^2[A_{1,1} - 2A_{1,2} + A_{2,2}]$$

} GW

We define, in complete analogy to the single channel eikonal model:

$$A^s_{i,k} = i\left(1 - e^{-\frac{1}{2}\Omega^s_{i,k}}\right)$$

$$\Omega^s_{i,k} = \nu^s_{i,k}(s)\, \Gamma^s_{i,k}(s,b)$$

$$\nu^s_{i,k} = \sigma^{s,0}_{i,k}\left(\frac{s}{s_0}\right)^{\Delta_k}$$

$$G^{in,s}_{i,k} = 1 - e^{-\Omega^s_{i,k}}$$

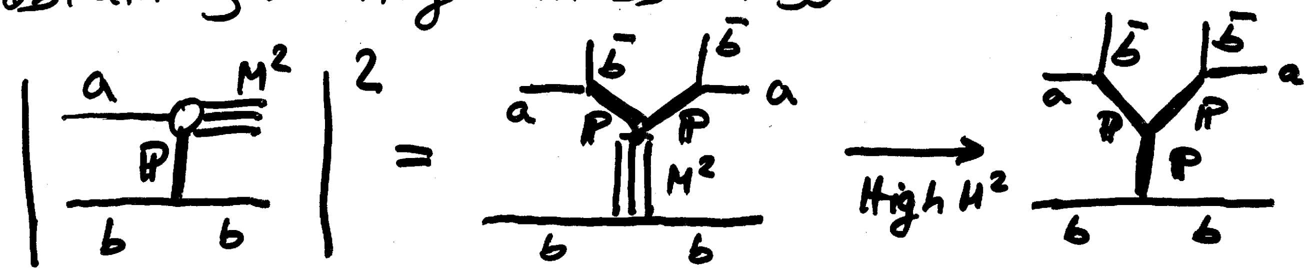
$\Rightarrow$ $e^{-\Omega^s_{i,k}(s,b)}$ is the differential i,k survival probability.

As of this point GLM and KMR diverge.

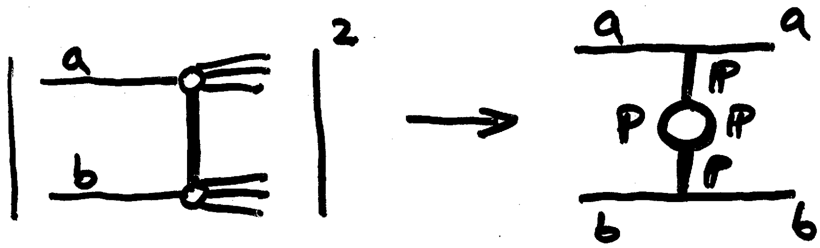
b) <u>Define diffraction:</u> Historically, diffraction was defined by Good and Walker very much in the way we wrote $A^s_{i,k}$ as the elements building a_{el}, a_{sd} and a_{dd}. In this interpretation we consider $A^s_{i,k}$ to be the components of the proton wave function from which we can construct the combinations corresponding to the elastic and diffractive channels. This is the basic philosophy of the GLM model. As such we neglect the dependence on the diffracted mass M^2.

Assume that the soft $I\!P$ is a simple (Regge like) pole in the J plane. Following AC Mueller papers (30 years ago!) we obtain for high mass diffraction:

SD:



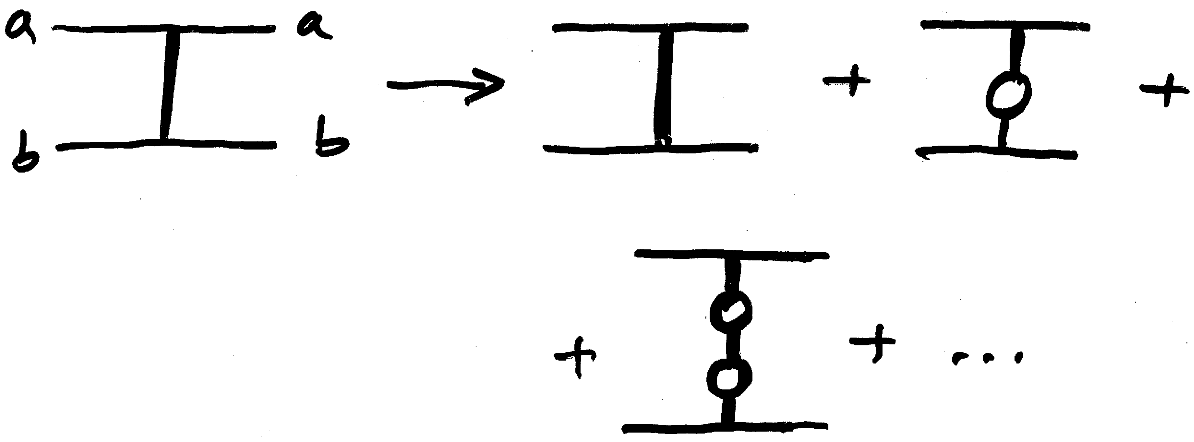
From which we obtain



One of the beutiful aspects of this formalism is that $\frac{d\sigma}{dt\,dM^2}$ acquires a Regge like dependence on $\alpha_{I\!P}(t) = 1+\Delta_{I\!P}+\alpha'_{I\!P}t$.

For this formalism to be significant we need to have $g_{3I\!P}$ which is not too small.

If $g_{3I\!P}$ is not very small, we have a large family of new diagrams containing $3I\!P$ vertices which have to be included not only in the diffractive calculation but also in the elastic scattering:



The net result is that $\Delta_{in} \to \Delta_{eff(out)} \xrightarrow{s} 0$.

As we shall see, IP enhancement results in significant reduction of Δ_{eff} which is expected to be significant enough to be observed in LHC and more so in Auger.

As it stands we do not have a decisive experimental support for the crucial importance of this mechanism.

The primary difference between GLM and KMR modeling is that GLM assumes diffraction to be mostly a GW type, i.e. that either the soft IP is not a simple J pole or that even if it is g_{3P} is too small to be dynamically important. KMR assumes that the soft IP is a simple J pole and that ... is significant.

As such KMR modelling reflects this basic assumption.

Both groups have suggested similar experimental procedures to extract $g_{3I\!P}$ including its reduction due to unitarity corrections.

c) <u>Parametrizations:</u> The consequent parametrizations of GLM and KMR are radically different

GLM	KMR
$\Gamma^S_{i,k} =$ exponential in t ⟹ Gaussian in b	$\Gamma^S_{i,k} =$ modified dipole in t ⟹ sum of Bessel functions in b
* $R^2_{i,k} = R^2_{i,k}(0) + 4\alpha'_{I\!P} \ln s$	$R^2_{i,k} = R^2_{i,k}(0)$ *
* fitted $\alpha'_{I\!P} = 0.17 - 0.18$	assumed $\alpha'_{I\!P} \equiv 0$ *
⇓ fitted $\Delta_{in} = 0.15$ ⟹ $\Delta_{out} \cong 0.08$; low Δ_{in} possible since $\alpha'_{I\!P} \neq 0$	fitted $\Delta_{in} = 0.55$ ⇓ ⟹ Δ_{out} reduces rapidly with $\sqrt{s}$; high Δ_{in} needed to compensate for $\alpha'_{I\!P} \equiv 0$

The different modeling reflects also in GLM and KMR chosing different data bases in their fitting procedures. The aim is to adjust $\Omega^S_{i,k}$ corresponding to soft $I\!P$ exchange. Best would be a simultaneous [illegible] fit!

The problem: data in the UA(4) – Tevatron range is not sufficient to constrain the $I\!P$ parameters.

GLM: fitted the ISR – Tevatron data in a $I\!P + R$ model from which we extract the $I\!P$ parameters. Our choice of Gaussian profiles limits our reproduction of $\frac{d\sigma_{el}}{dt}$ to the very forward t cone with $t_{max} = 0.10 - 0.15\ GeV^2$. This covers more than 85% of the elastic data. Is this enough? Shall discuss it.

Our data base contains: σ_{tot}; integrated σ_{el}, σ_{sd}, σ_{dd} and B_{el}. B_{sd}, B_{dd} and $\rho = \frac{Re}{Im} a(s,t)$ are calculated from the fitted parameters.

The GLM fit to a factorizable Regge like $I\!P$, i.e. assuming the $I\!P$ to be a simple J pole, results in a non satisfactory $\frac{\chi^2}{d.o.f.} = 2.30$ for 55 data points (with 12 parameters). Changing $\Gamma^{S}_{i,k}$ did not improve the fit.

⇒ options: i) give up coupling factorization

Bartels + GLM — i.e. the soft $I\!P$ is NOT a simple J pole.

Result: $\frac{\chi^2}{d.o.f.} = 1.25$ which is very good considering [illegible]

ii) Maintain input factorization but effectively break it by introducing $\mathbb{P}$ enhancement. This is GLM work in progress and I shall report some of our output results.

KMR are fitting just $\frac{d\sigma_{el}}{dt}$ reflecting the fact that their profiles Γ^S_{ijk} are more realistic (and more complicated!) than ours. They present diffractive output only at $\sqrt{s} = 1800$ GeV, but not the energy dependence even within their 540-1800 GeV fitting range. Is their fit satisfatory?!

Clearly GLM and KMR present, at best, their approximation to Ω^S_{ijk}. I shall try to asses how effective are these approximati[ons] so as to obtain reliable estimates of

i) Survival probabilities (wich depend also on the estimates of the hard sector).

ii) The rate at which the scattering amplitudes progress with energy toward unitarity saturation.

In the following I shall also quote some results from a very recent (to be published) GLM calculation of a factorizable model which includes $\mathbb{P}$ enhanced contributions.

IV) Unitarity Saturation

As we saw, for GW diffraction we have

$$a_{el} = i\left[\alpha^4 A_{1,1} + 2\alpha^2\beta^2 A_{1,2} + \beta^4 A_{2,2}\right]$$

$$a_{sd} = i\alpha\beta\left[-\alpha^2 A_{1,1} + (\alpha^2-\beta^2) A_{1,2} + \beta^2 A_{2,2}\right]$$

$$a_{dd} = i\alpha^2\beta^2\left[A_{1,1} - 2A_{1,2} + A_{2,2}\right]$$

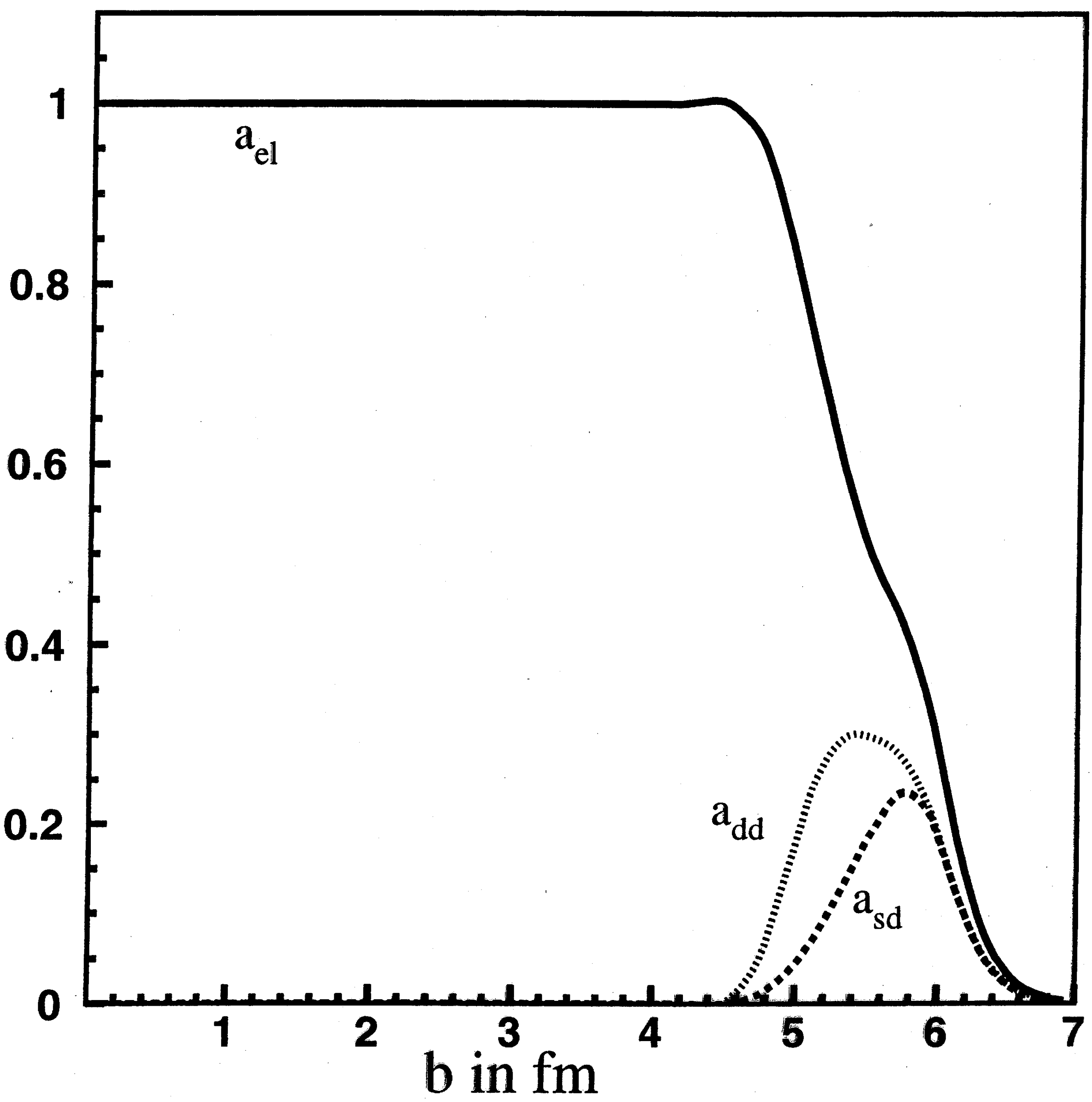
$$\alpha^2 + \beta^2 = 1$$

It is easy to prove that, independently of β, the s-unitarity black bound is reached when and only when $A_{1,1} = A_{1,2} = A_{2,2} \equiv 1$.
At this limit $a_{el} = 1$ and $a_{sd} = a_{dd} \equiv 0$.
See Fig 1.

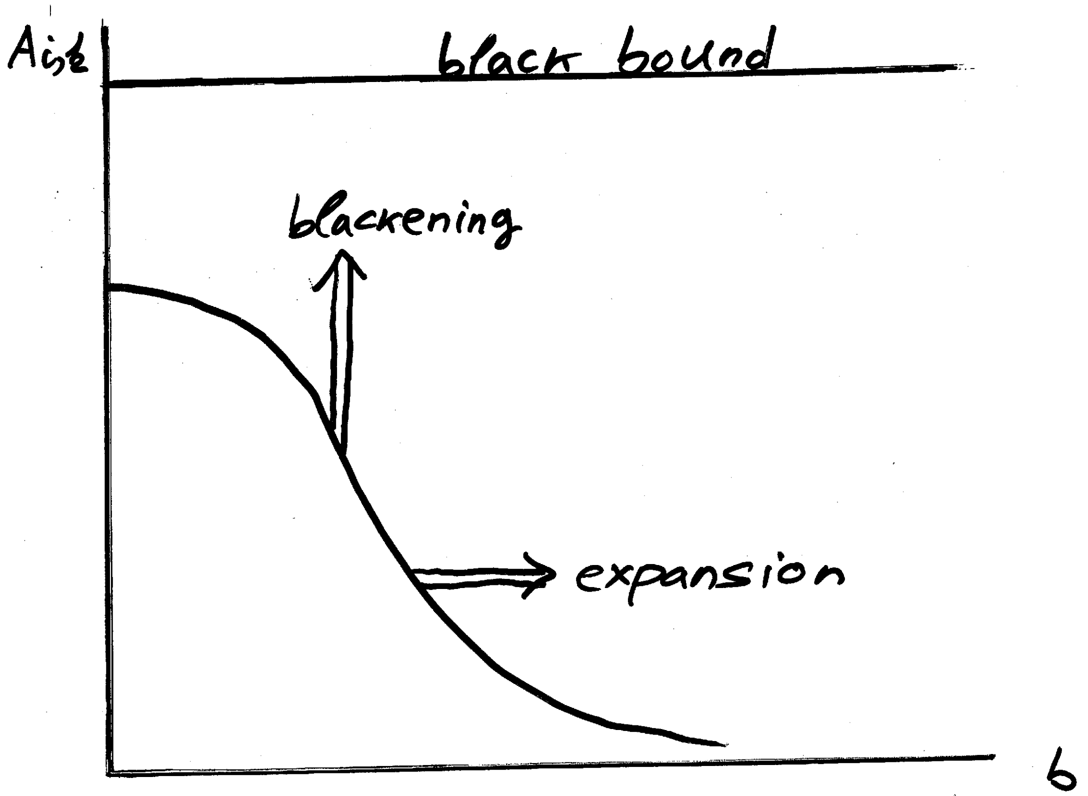
Actually, a basic feature of the GLM model is that $A_{2,2}$ reaches the black bound at low energies ($\sqrt{s} \simeq 20$ GeV). $A_{1,2}$ reaches the black bound quite fast in a factorizable model and more slowly in our non fatorizable version. It is the smallness of $A_{1,1}$ which determins the rate at which the black bound is reached.

KMR did not present the b-dependence of their $A_{i,k}$ components, but judging from their $a_{el}(s,b)$ figures it is very probable that they follow the same patern.

Fig 1



The $A_{i,k}$ amplitudes are growing due to 2 correlated processes: blackening and expansion.



In the GLM models $A_{i,k}$ can expand as $R_{i,k}^2$ has a term proportional to ln s. The cross sections oblige the Pumplin bound

$$\sigma_{el} + \sigma_{diff} \le \frac{1}{2}\sigma_{tot}.$$

In the KMR model $R_{i,k}^2 = R_{i,k}^2(0) = $ const. and consequently blackaning is the only option. Once $a_{el} = 1$ for all $b^2 \le R_{i,k}^2$, the GW amplitude can not increase any more. In this limit $\sigma_{el}^{GW} = \frac{1}{2}\sigma_{tot}^{GW}$ which is a constant is energy. Accordingly, $\sigma_{sd}^{GW} = \sigma_{dd}^{GW} \equiv 0$.

However, in the KMR model the diffractive channels are increasingly being fed by the Pomeron enhanced contributions. As a result the Pumplin bound is not valid. Note, also, that due to the $\mathbb{P}$ enhancement

$$\Delta_{eff}^{KMR} \to 0, \text{ so } \sigma_{tot} \to \text{const.}$$

A consequence of the above is that in the KMR model $a_{el}(s, b=0)$ approaches the black bound quite fast — just above LHC, while in the GLM models this growth is much slower. In the non factorizable model $a_{el}(b=0)$ reaches the black bound above the GZK knee at $\sqrt{s} \simeq 10^7 - 10^8$ GeV. As mentioned earlier, GLM have just completed a model calculation GLM2, based on factorization + $\mathbb{P}$ enhancement.

There are two differences relative to KMR

1) We sum the $\mathbb{P}$ diagrams differently.
2) Both $\Delta_{\mathbb{P}}$ and $\alpha'_{\mathbb{P}}$ are fitted rather than assumed in KMR. We get $\Delta_{\mathbb{P}} = 0.335$ and $\alpha'_{\mathbb{P}} = 0.01$, $\beta = 0.34$. At low energy our $\alpha'_{\mathbb{P}}$ is too small to produce significant expansion. However, this is changed once $4\mathbb{P}$s becomes significant. IF gets similar results to KMR assuming that ...

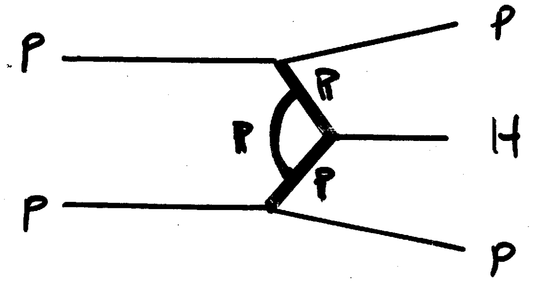
V) Survival Probabilities

Why are the calculated values of S^2 obtained by GLM and KMR so different?

1) 15 – 20% of the difference is traced to different values of $B(J/\psi)$ used in the calculations. GLM took its values from ZEUS using both $B_{ee}(J/\psi)$ and $B_{in}(J/\psi)$. KMR used $B_{ee}(J/\psi)$ in all 3 amplitudes taking their data from H1.

2) The main difference is traced to different profiles obtained in the two models. Fig 2, Fig 3.

The KMR S^2 values are, in our opinion, over estimated due to a final stat $I\!P$ enhanced correction missed in their calculation.



If $g_{3I\!P}$ is not too small there is a class of $I\!P$ enhanced diagrams which can not be neglected.

See Table I. 2 important conclusions emerg:

1) Mesurments of soft cross sections in LHC and Auger are cruicial to determine the roll of $3I\!P$.

2) GLM and KMR S^2

Fig 2

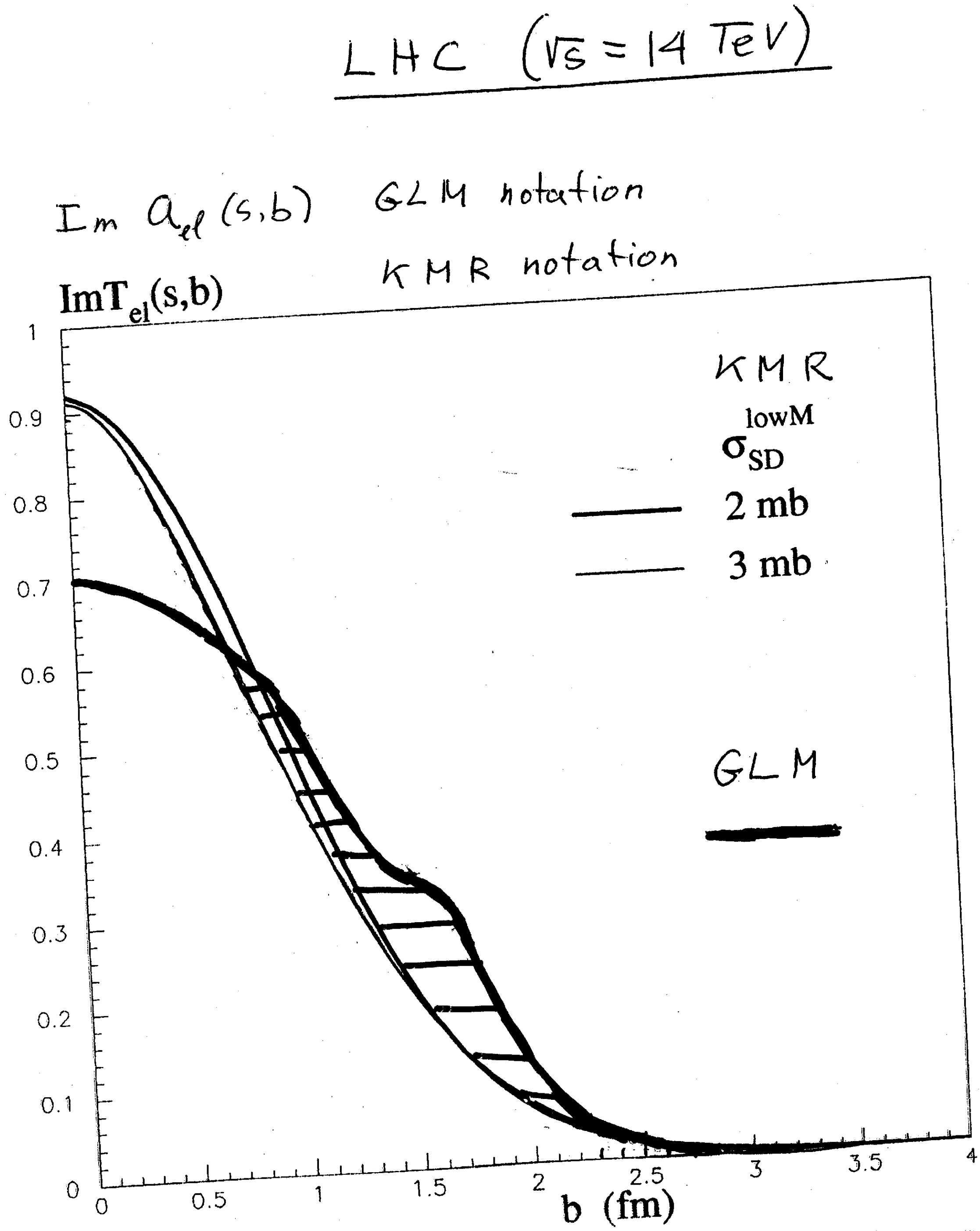

LHC ($\sqrt{s}$ = 14 TeV)
Im a_{el}(s,b) GLM notation
KMR notation
ImT$_{el}$(s,b)
KMR
σ_{SD}^{lowM}
2 mb
3 mb
GLM
b (fm)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0.5
1.5
2
2.5
3
3.5
4

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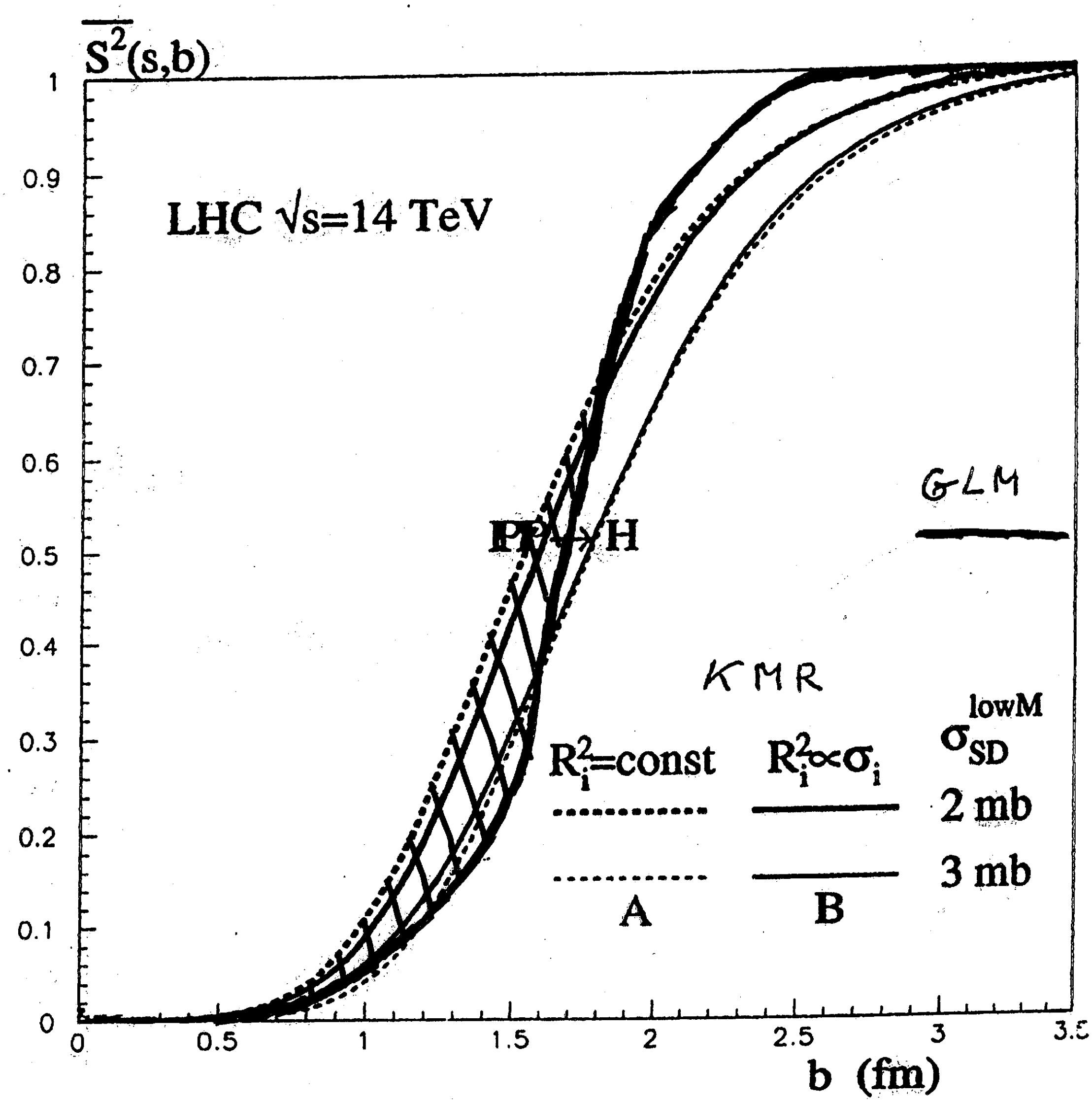
Fig 3



	$\sqrt{s} = 1800$ GeV			$\sqrt{s} = 14 \times 10^3$ GeV			$\sqrt{s} = 10^5$ GeV		
	GLM1	GLM2	KMR	GLM1	GLM2	KMR	GLM1	GLM2	KMR
mb σ_{tot}	78.0	73.3	74.0	110.5	92.1	88.0	150.0	108.0	98.0
mb σ_{el}	16.3	16.3	16.3	25.3	20.9	20.1	37.0	24.0	22.9
mb σ_{sd}	9.6	9.8	10.9	11.6	11.8	13.3	12.5	14.4	15.7
$\sigma_{sd}(LM)$		8.6	4.3		10.5	5.1		12.2	5.7
$\sigma_{sd}(HM)$		1.2	6.5		1.3	8.2		2.2	10.0
mb σ_{dd}	3.8	5.4	7.2	4.9	6.1	13.4	5.5	6.3	17.3
S^2_H %	2.7	3.2	2.7-4.8	0.7	2.35	1.2-3.2	~0	2	0.9-3.5
S^2_{enh} %		28.5			6.3			3.3	
S^2_{Higgs} %	2.7	1.2	2.7-4.8	0.7	0.2	1.2-3.2	~0	0.066	0.9-3.5
including S^2_{enh}			⇓ 0.77-1.37			⇓ 0.076-0.2			⇓ 0.03-0.083