On the Effects of NN Correlations on Medium- and High-Energy Processes off Nuclei

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1. Exclusive processes off few-body nuclei: ^3He

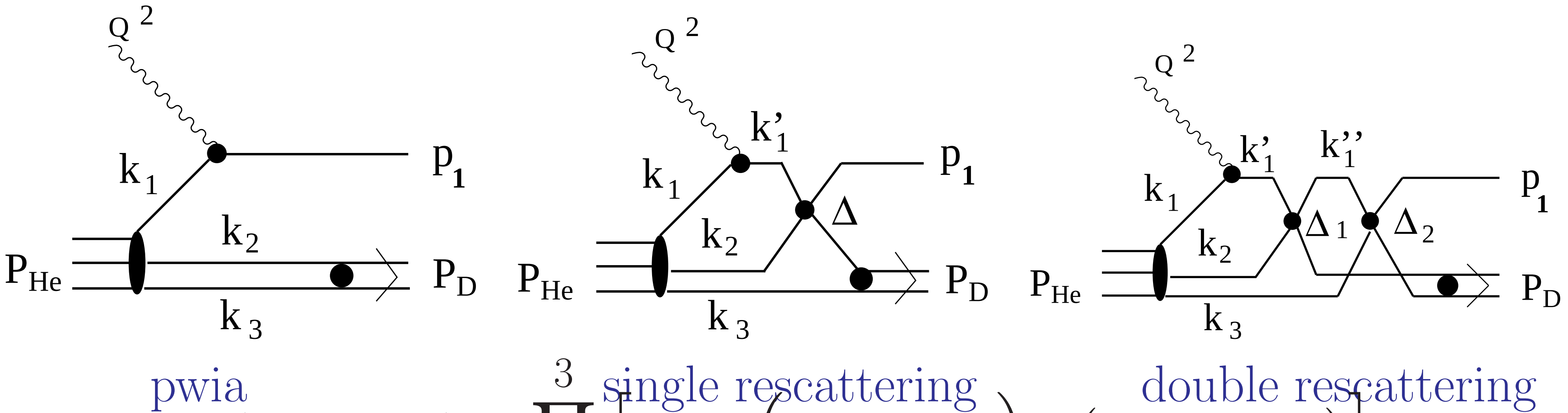
- NN correlations taken into account by realistic, state-of-the-art few-body wave functions obtained with $AV18$ interaction
  $\rightarrow$ *initial state*: $\Psi_{^3He} = \hat{\mathcal{A}}\, e^{i\boldsymbol{P}\cdot\boldsymbol{R}}\, \psi_3(\boldsymbol{\rho}, \boldsymbol{r})$
  $\rightarrow$ *final state*: $\Phi_f^*(\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{r}_3) = \hat{\mathcal{A}} S(\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{r}_3) e^{-i\boldsymbol{p}_1\boldsymbol{r}_1} e^{-i\mathbf{P}_D\mathbf{R}_D} \Psi^*_{D/pn}(\boldsymbol{r})$
- Factorization approximation and non-relativistic reduction removed by performing calculations in momentum space; current operator chosen according to the $CC1$ prescription:

$$\langle\, s_f | J_\mu(\boldsymbol{k}_1, \boldsymbol{q}) | \lambda \,\rangle =$$
$$= \bar{u}(\boldsymbol{k}_1 + \boldsymbol{q}, s_f) \left[\gamma_\mu \left(F_1(Q^2) + F_2(Q^2) \right) - (2k_1 + q)_\mu F_2(Q^2) \right] u(\boldsymbol{k}_1, \lambda)$$

- FSI in $S(\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{r}_3)$, calculated within generalized eikonal approximation

$^3He(e,e'p)^2H$: diagrammatic approach to FSI

Δ: momentum transfer in the NN rescattering

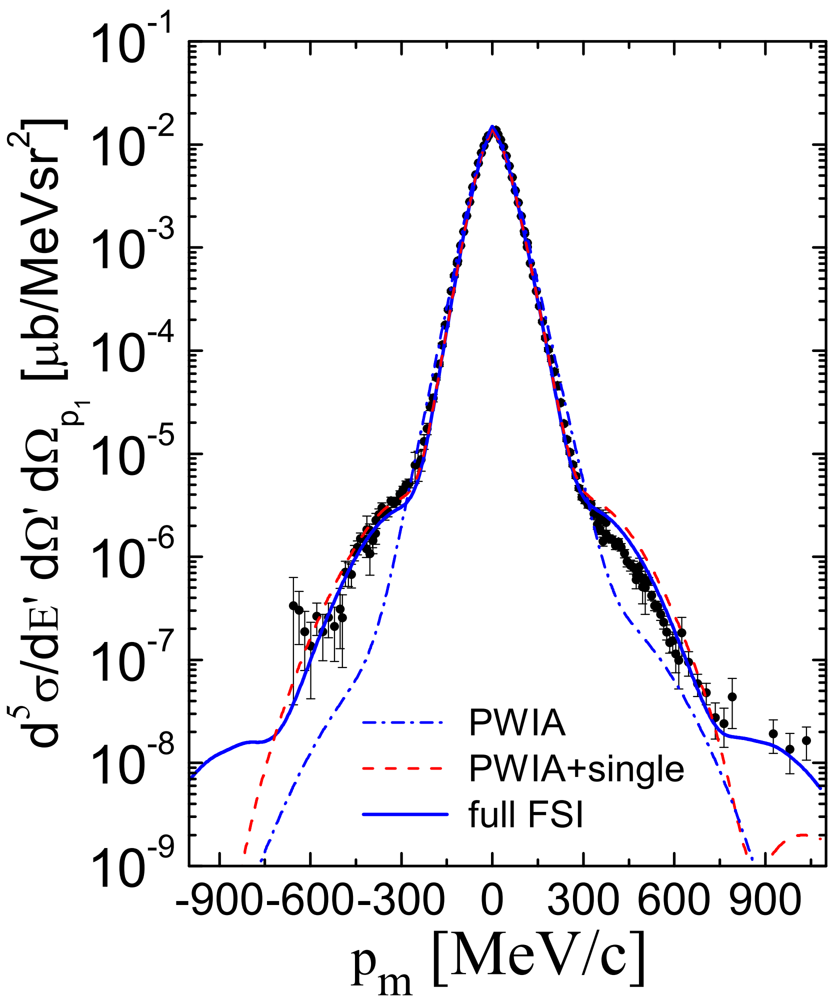


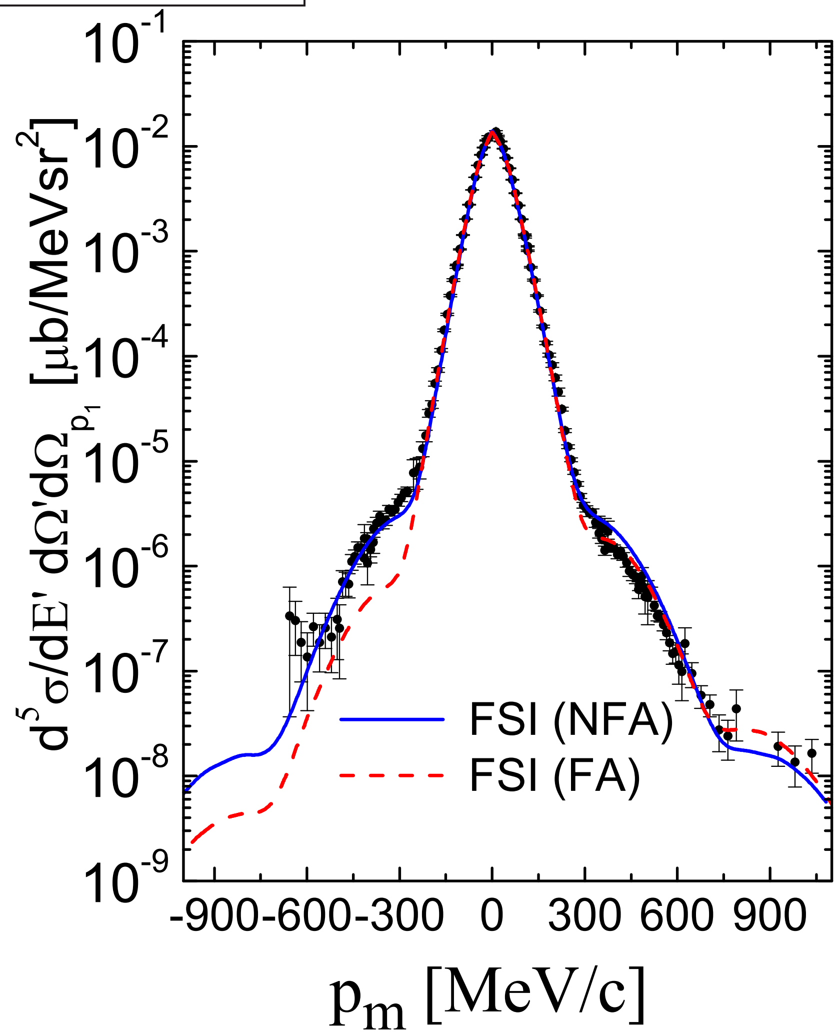
$$S(\boldsymbol{r}_1,\boldsymbol{r}_2,\boldsymbol{r}_3) = \prod_{j=2}^{3}\left[1-\theta\left(\boldsymbol{r}_{j||}-\boldsymbol{r}_{1||}\right)\Gamma\left(\boldsymbol{r}_{j\perp}-\boldsymbol{r}_{1\perp}\right)\right]$$

$$\Gamma(\boldsymbol{r}_\perp) = \frac{1}{2\pi i K^2}\int d^2\kappa_\perp f_{NN}(\kappa_\perp)e^{-i\kappa_\perp \boldsymbol{r}_\perp}$$

$$f_{NN}(\boldsymbol{\Delta}_\perp) = K\frac{\sigma^{tot}(i+\alpha)}{4\pi}e^{-b^2\boldsymbol{\Delta}_\perp^2/2}$$

$^{3}He(e,e'p)^{2}H$: results

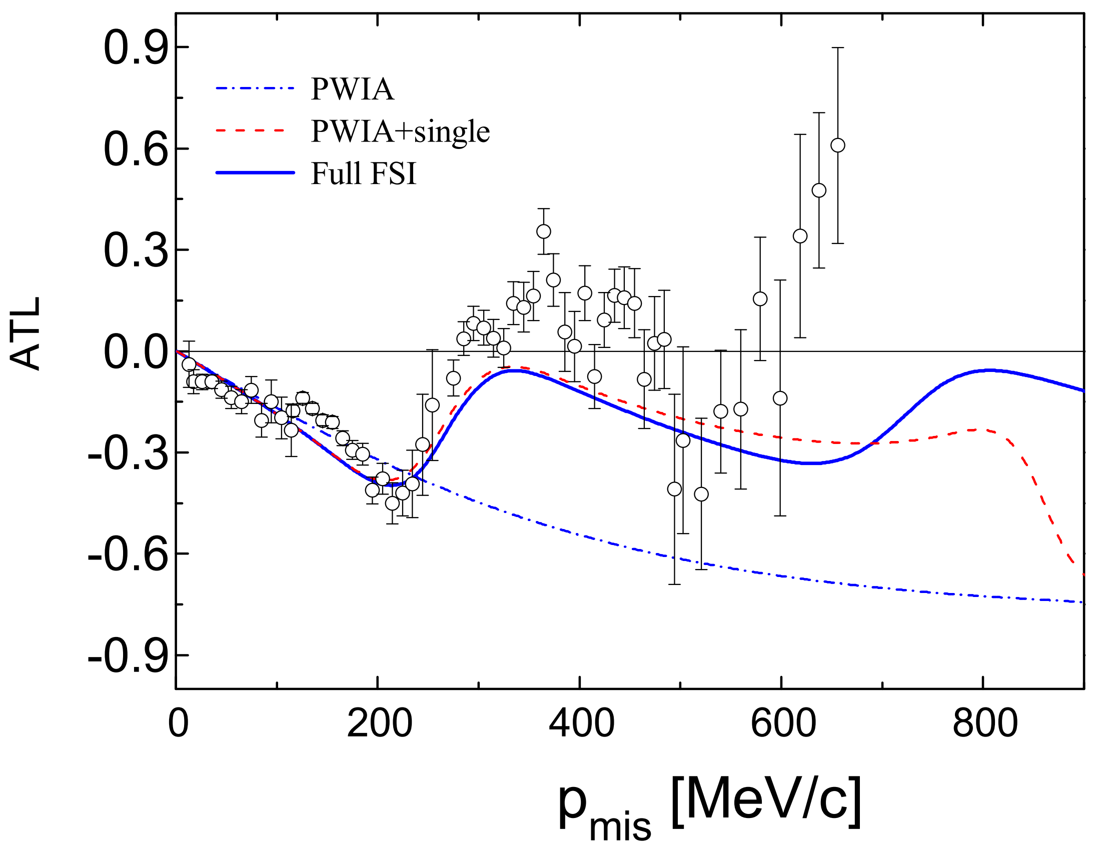




Data: *M.M. Rvachev et al., PRL**94** (2005) 192302*

Calculations: *C.Ciofi degli Atti, L.P. Kaptari, PRL**100** (2008) 122301*

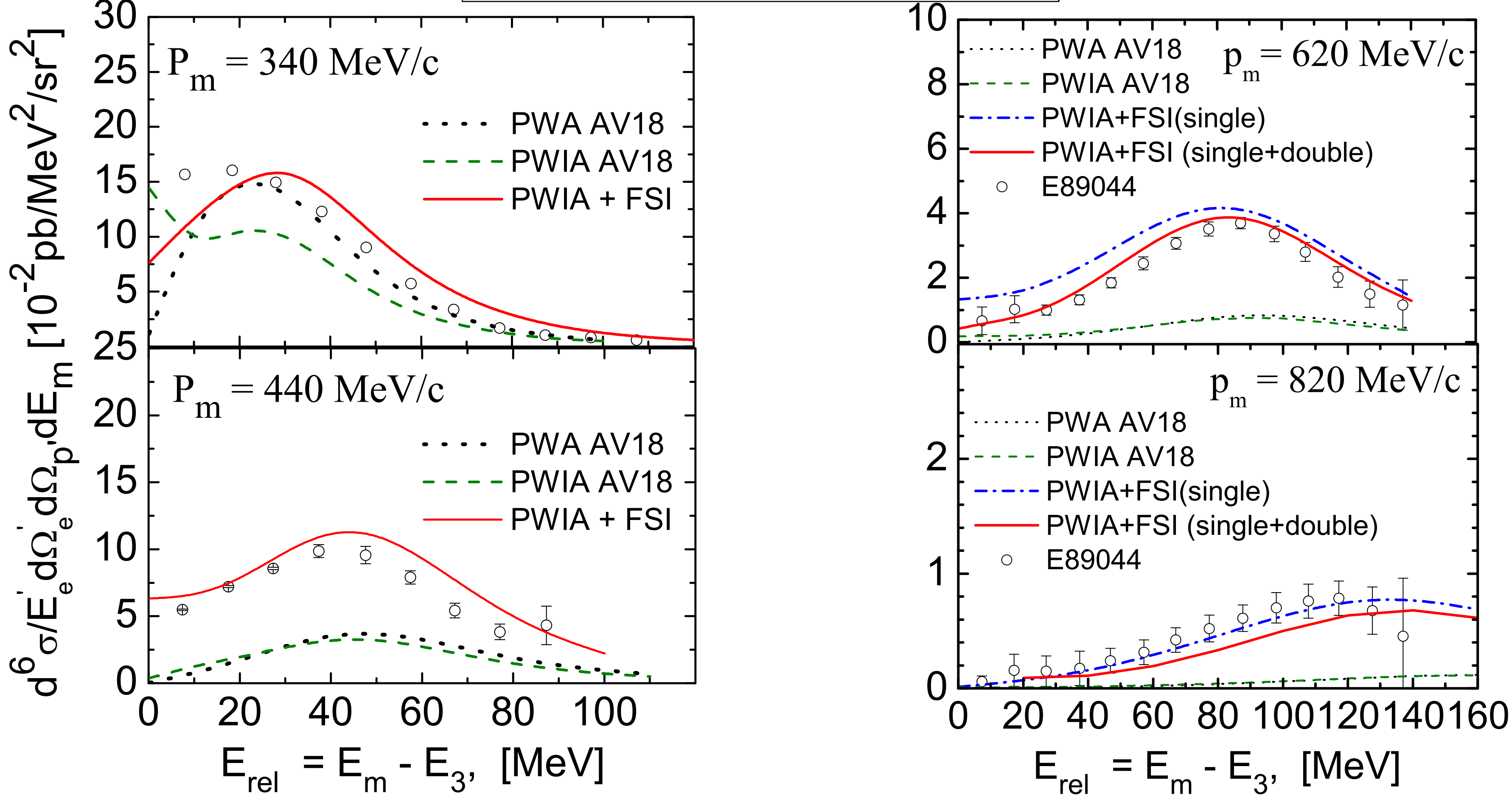
$^3He(e,e'p)^2H$: **results**



Data: *M.M. Rvachev et al., PRL**94** (2005) 192302*

Calculations: *C.Ciofi degli Atti, L.P. Kaptari, PRL**100** (2008) 122301*

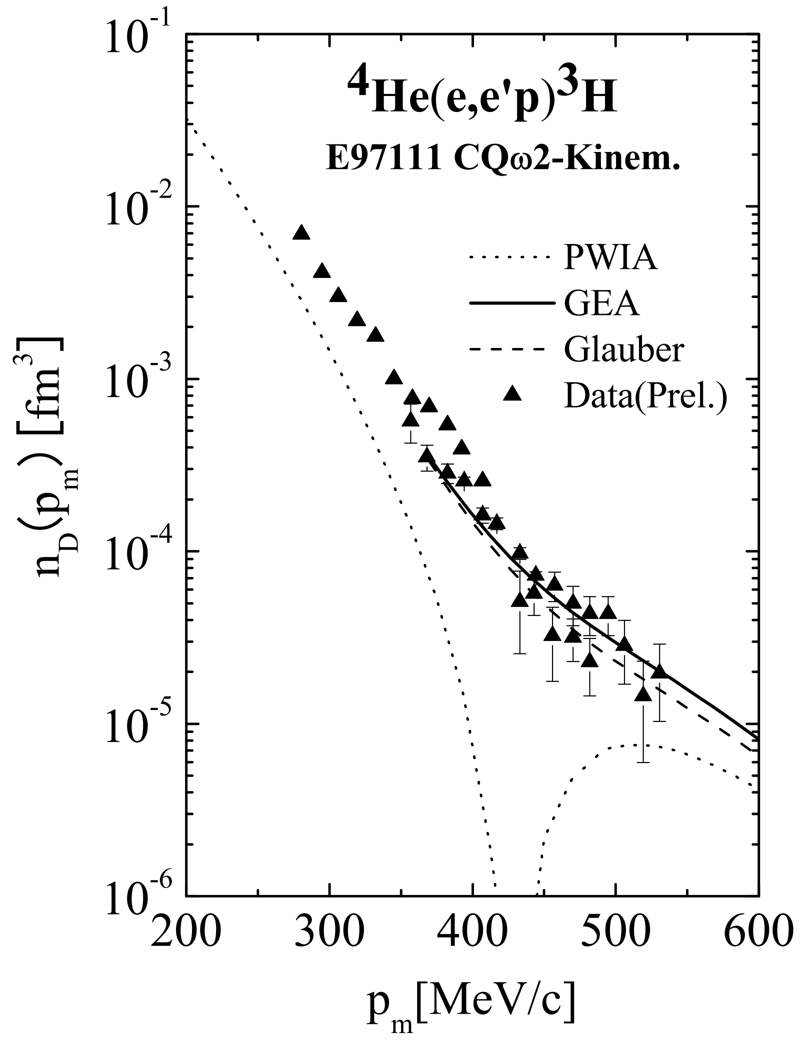
$^3He(e,e'p)(pn)$: **results**



FSI in (n-p) pair- irrelevant; FSI in p-(n-p) system- very large

(preliminary un-factorized results on PWIA show no sizeable effect)

$^4He(e,e'p)^3He$: results within factorized approximation



$$n_D(\mathbf{p}_m) = \frac{d^5\sigma}{d\Omega' dE'\, d\Omega_p}(\mathcal{K}\sigma_{ep})^{-1}$$

Data: *B. Reitz et al.*, EPJ A **S19** (2004) 165

Calculations: *C.Ciofi degli Atti*, L.P. Kaptari, H. Morita in "Few Body Systems", *in press* `arXiv:0803.1162`

A corresponding *un-factorized* calculation is in progress

2. Spectral Function for Complex Nuclei: TNC Model

$$P_A(|\mathbf{k}|, E) = P_0^A(|\mathbf{k}|, E) + P_1^A(|\mathbf{k}|, E)$$

$$P_0^A(|\mathbf{k}|, E) = \sum_{\alpha<\alpha_F} \tilde{n}_\alpha(|\mathbf{k}|)\delta(E-|\epsilon_\alpha|)\,; \int \tilde{n}_\alpha d\boldsymbol{k} < 1 \; (\textit{renormalized MF})$$

$$P_1^A(|\mathbf{k}|, E) = \sum_{f\neq\alpha} \left| \int d\mathbf{r}\, e^{i\mathbf{k}\cdot\mathbf{r}}\, G_{f0}(\vec{r}) \right|^2 \delta[E-(E_{A-1}^f - E_A)] \; (\textit{correlations})$$

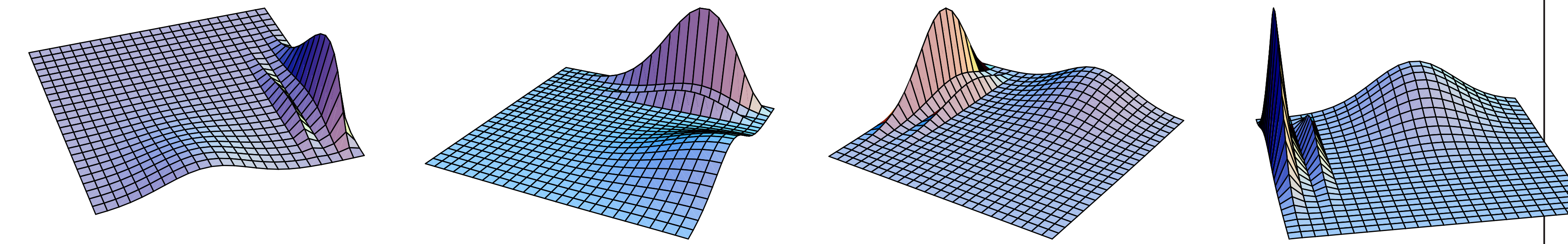
Two-Nucleon Correlation (TNC) Model

(*Frankfurt & Strikman; Ciofi & Simula*)

$$P_1^A(|\mathbf{k}|, E) = \int d\mathbf{P}_{cm}\, n_{rel}^A(|\mathbf{k} - \mathbf{P}_{cm}/2|)\, n_{cm}^A(|\mathbf{P}_{cm}|) \cdot$$
$$\cdot\, \delta\left[E - E_{thr}^{(2)} - \frac{(A-2)}{2M(A-1)} \cdot \left(\mathbf{k} - \frac{(A-1)\mathbf{P}_{cm}}{(A-2)} \right)^2 \right]$$

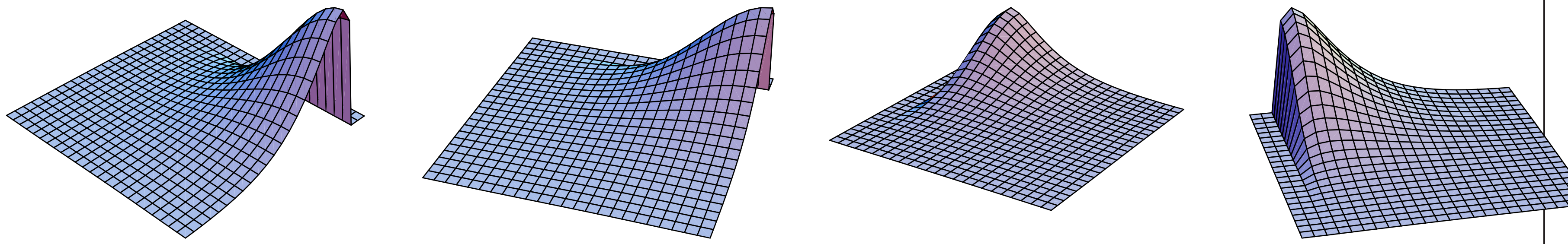
TNC Spectral Function - ^{12}C

$P_0^p(k,E)$:



$$\int dE\, dk\, P_0^p(k,E) = 0.8\, Z_p \qquad \longrightarrow \qquad \textbf{\textit{Shell Model: 80\%}}$$

$P_1(k,E)$:



$$\int dE\, dk\, P_1^p(k,E) = 0.2\, Z_p \qquad \longrightarrow \qquad \textbf{\textit{Correlations: 80\%}}$$

C. Ciofi degli Atti, S. Simula - PRC 53 (1996)

Many-Body Calculations: Cluster Expansion

- The nuclear many-body problem:

$$\hat{\mathbf{H}}\,\Psi_n = E_n\,\Psi_n\,, \qquad \hat{\mathbf{H}} = -\frac{\hbar^2}{2\,m}\sum_i \hat{\nabla}_i^2 + \frac{1}{2}\sum_{i<j}\hat{v}_{ij}$$

- The ground state energy E_0 is given by:

$$E_o = -\frac{\hbar^2}{2\,m}\int d\boldsymbol{r}\left[\hat{\nabla}^2\,\rho^{(1)}(\boldsymbol{r},\boldsymbol{r}')\right]_{\boldsymbol{r}=\boldsymbol{r}'} + \sum_n \int d\boldsymbol{r}_1 d\boldsymbol{r}_2\,\hat{v}^{(n)}\rho^{(2)}_{(n)}(\boldsymbol{r}_1,\boldsymbol{r}_2)$$

$$\longrightarrow\ \rho^{(1)}(\boldsymbol{r},\boldsymbol{r}') = A\int\prod_{j=2}^{A} d\boldsymbol{r}_j\,\Psi_o^\dagger(\boldsymbol{r},\boldsymbol{r}_2\ldots,\boldsymbol{r}_A)\,\Psi_o(\boldsymbol{r}',\boldsymbol{r}_2\ldots,\boldsymbol{r}_A)$$

$$\longrightarrow\ \rho^{(2)}_{(n)}(\boldsymbol{r}_1,\boldsymbol{r}_2) = \frac{A(A-1)}{2}\int\prod_{j=3}^{A} d\boldsymbol{r}_j\,\Psi_o^\dagger(\boldsymbol{r}_1\ldots,\boldsymbol{r}_A)\,\hat{O}_{12}^{(n)}\,\Psi_o(\boldsymbol{r}_1\ldots,\boldsymbol{r}_A)$$

- $\rho^{(1)}(\boldsymbol{r}, \boldsymbol{r}')$ and $\rho^{(2)}_{(n)}(\boldsymbol{r}_1, \boldsymbol{r}_2)$ are *cluster expanded*, with:

$$\Psi_o = \hat{\mathbf{F}}\,\phi_o$$

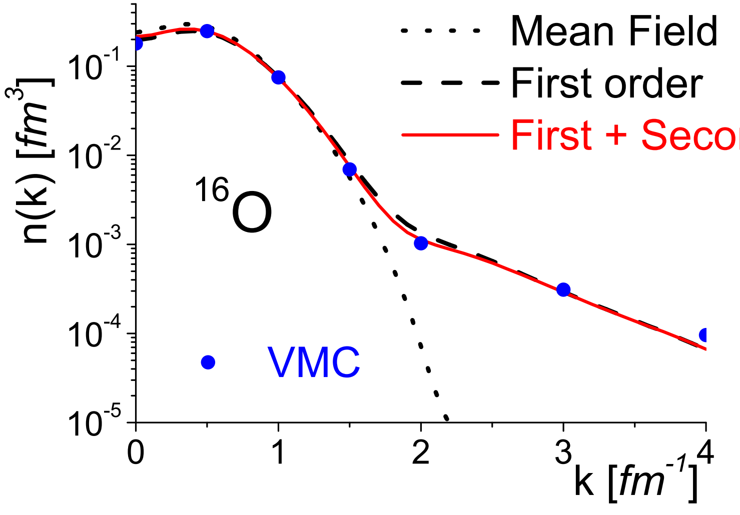
  where ϕ_o is the *mean-field* wave function and

$$\hat{\mathbf{F}} = \hat{S} \prod_{i<j} \hat{f}_{ij} = \hat{S} \prod_{i<j} \sum_n f^{(n)}(r_{ij})\, \hat{\mathcal{O}}^{(n)}_{ij}$$

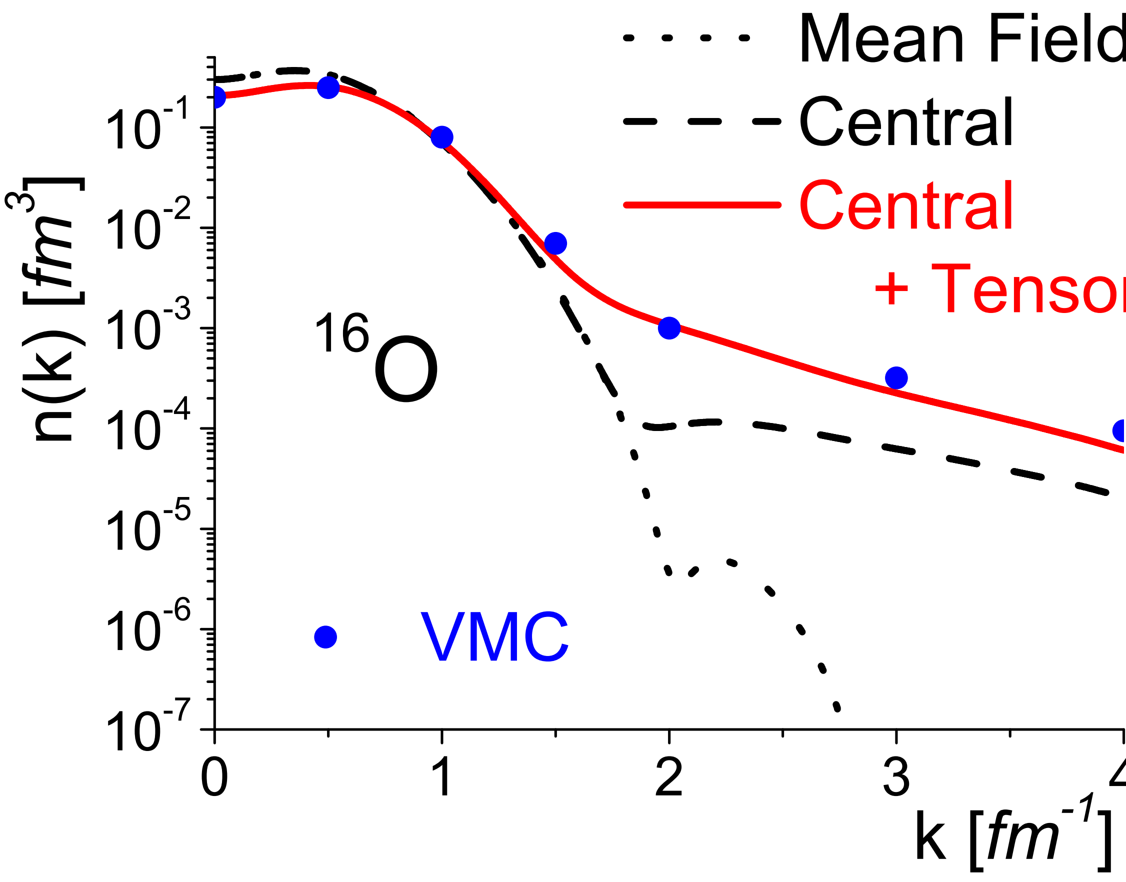
  is a *correlation* operator.
- cluster expansion truncated at *2nd order* in $\hat{f}_{ij}$;
- the wave function and correlation functions which minimize the ground-state energy are used to calculate the *expectation value of any operator*
- we can calculate *diagonal* as well as *non-diagonal*, one- and two-body **realistically correlated** densities and *Spectral Functions*

  (*M. Alvioli, C.Ciofi degli Atti, H.Morita, PRC* ***72*** *(2005)*)

Momentum Distributions and Tensor Forces

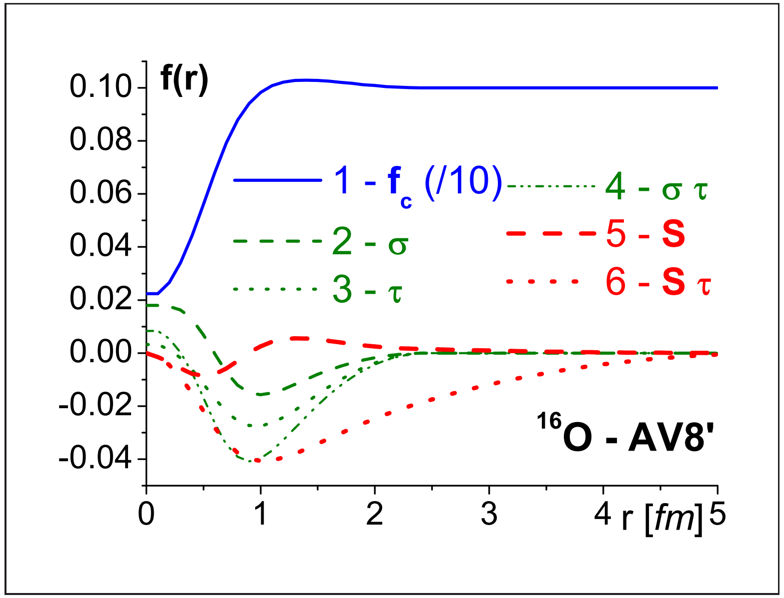


⟵ cluster expansion convergence



Tensor $(S + S \otimes \tau)$ ***correlations!***

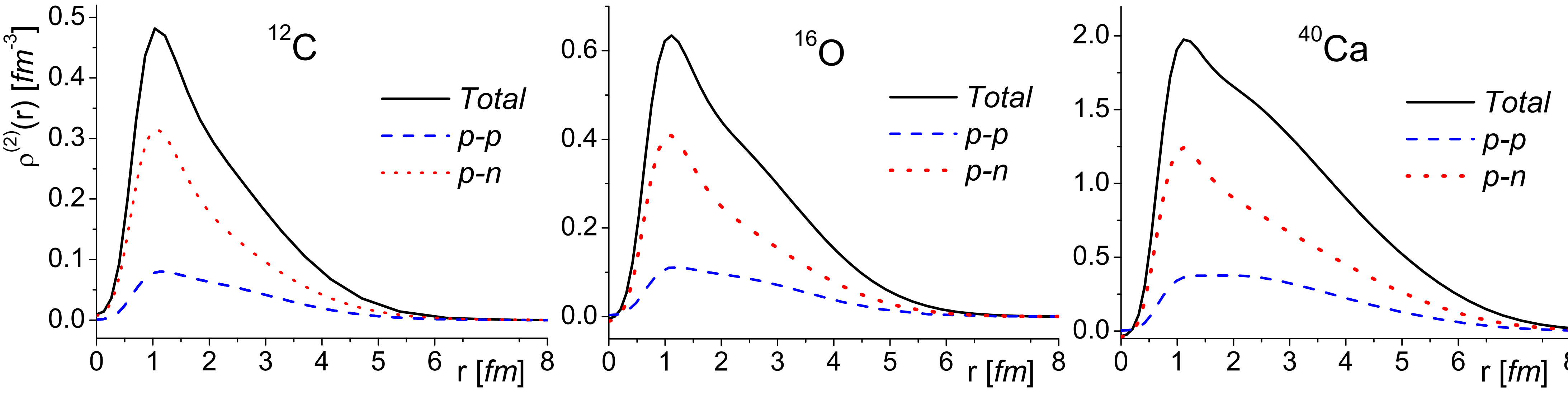
$\mathbf{k > 1.5\ fm^{-1}}$ ⟶



⟵ correlation functions

Two-Body Densities: isospin separation

$$\rho^{(2)}(r) = \int d\boldsymbol{R}\, \rho^{(2)}\left(\boldsymbol{R}+\frac{1}{2}\boldsymbol{r}\,,\,\boldsymbol{R}-\frac{1}{2}\boldsymbol{r}\,;\,\boldsymbol{R}+\frac{1}{2}\boldsymbol{r}\,,\,\boldsymbol{R}-\frac{1}{2}\boldsymbol{r}\right)$$



- normalization (**number of pairs**) conserved by the expansion
- **isospin** separation feasible: $\rho^{(2)} = \rho^{pp}_{(2)} + \rho^{pn}_{(2)} + \rho^{nn}_{(2)}$
- We can build **two-body *pp*, *pn* and *nn*** momentum distributions
  (*M. Alvioli, C.Ciofi degli Atti, H.Morita, PRL* ***100*** *(2008)*)

Two-Body Momentum Distributions

$$\boldsymbol{k}_{rel} \equiv \boldsymbol{k} = \frac{1}{2}(\boldsymbol{k}_1 - \boldsymbol{k}_2) \qquad \boldsymbol{r} = \boldsymbol{r}_1 - \boldsymbol{r}_2 \qquad \boldsymbol{r}' = \boldsymbol{r}'_1 - \boldsymbol{r}'_2$$

$$\boldsymbol{K}_{CM} \equiv \boldsymbol{K} = \boldsymbol{k}_1 + \boldsymbol{k}_2 \qquad \boldsymbol{R} = \frac{1}{2}(\boldsymbol{r}_1 + \boldsymbol{r}_2) \qquad \boldsymbol{R}' = \frac{1}{2}(\boldsymbol{r}'_1 + \boldsymbol{r}'_2)$$

we have

$$n(\boldsymbol{k}, \boldsymbol{K}) = \frac{1}{(2\pi)^6} \int d\boldsymbol{r} d\boldsymbol{r}' d\boldsymbol{R} d\boldsymbol{R}' \, e^{-i\,\boldsymbol{K}\cdot(\boldsymbol{R}-\boldsymbol{R}')} \, e^{-i\,\boldsymbol{k}\cdot(\boldsymbol{r}-\boldsymbol{r}')} \rho^{(2)}(\boldsymbol{r}, \boldsymbol{r}'; \boldsymbol{R}, \boldsymbol{R}')$$

and

$$n(\boldsymbol{k}) = \int d\boldsymbol{K} \, n(\boldsymbol{k}, \boldsymbol{K}) = \frac{1}{(2\pi)^3} \int d\boldsymbol{r} d\boldsymbol{r}' d\boldsymbol{R} \, e^{-i\,\boldsymbol{k}\cdot(\boldsymbol{r}-\boldsymbol{r}')} \rho^{(2)}(\boldsymbol{r}, \boldsymbol{r}'; \boldsymbol{R}, \boldsymbol{R})$$

$$n(\boldsymbol{K}) = \int d\boldsymbol{k} \, n(\boldsymbol{k}, \boldsymbol{K}) = \frac{1}{(2\pi)^3} \int d\boldsymbol{r} d\boldsymbol{R} d\boldsymbol{R}' \, e^{-i\,\boldsymbol{K}\cdot(\boldsymbol{R}-\boldsymbol{R}')} \rho^{(2)}(\boldsymbol{r}, \boldsymbol{r}; \boldsymbol{R}, \boldsymbol{R}')$$

$\boldsymbol{K}_{CM} = 0$ corresponds to $\boldsymbol{k}_2 = -\boldsymbol{k}_1$, *i.e. back-to-back* nucleons

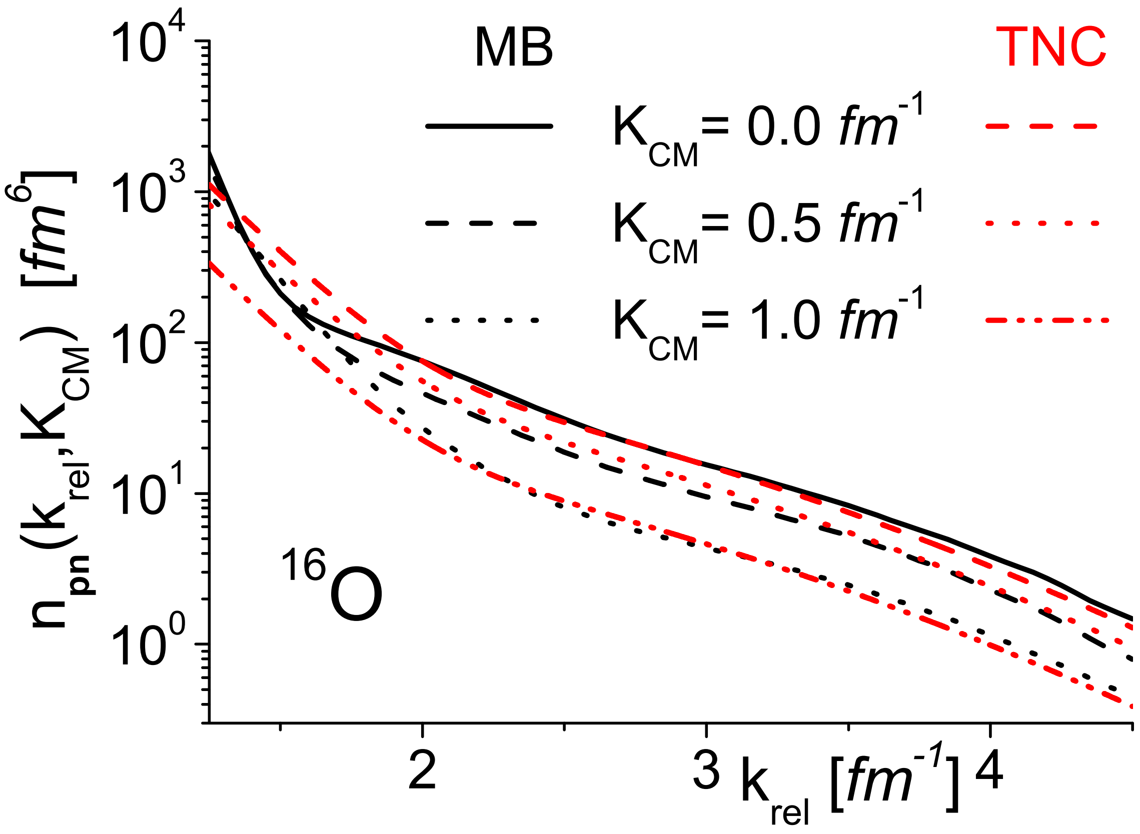
Spectral Function properties at low K_{CM} and high k_{rel}

1-body SF:

$$P_1^A(|\mathbf{k}|,E) = \int d\mathbf{K}_{cm}\; n_{rel}^A(|\mathbf{k}-\mathbf{K}_{cm}/2|)\, n_{cm}^A(|\mathbf{K}_{cm}|)\,\delta\left[E - E_{thr}^{(2)} - \frac{(A-2)}{2M(A-1)}\left(\mathbf{k} - \frac{(A-1)\mathbf{K}_{cm}}{(A-2)}\right)^2\right]$$

2-body SF:

$$P_2^A(\mathbf{k},\mathbf{K}_{cm},E) = n_{rel}^A(|\mathbf{k}-\mathbf{K}_{cm}/2|)\, n_{cm}^A(|\mathbf{K}_{cm}|)\,\delta\left(E - E_{thr}^{(2)}\right)$$

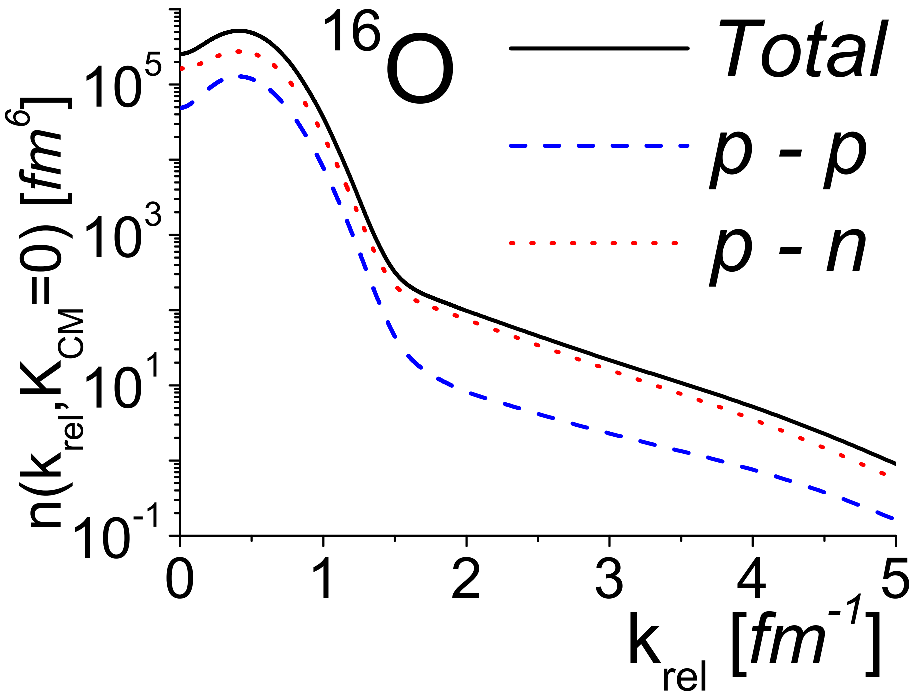


MB: $n_{pn}(\boldsymbol{k}_{rel}, \boldsymbol{K}_{cm})$

TNC: Ciofi, Simula PRC53, (1996)
$C_A\, n_{^2H}(k_{rel})\, n_{cm}(K_{cm})$

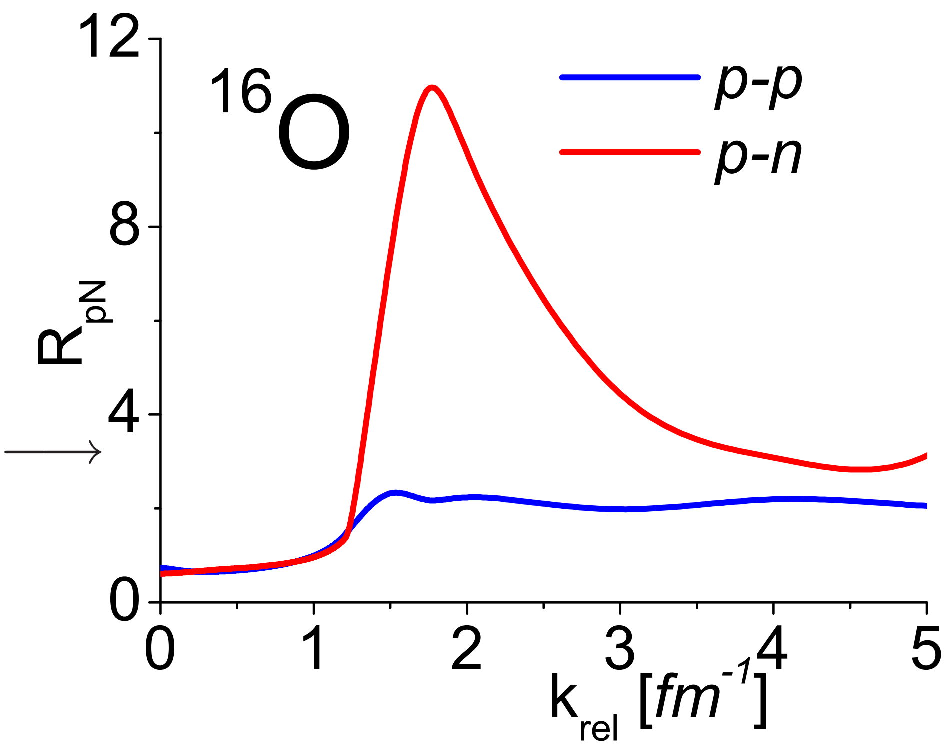
factorization of n_{pn}

Back-to-Back nucleons: *pn* to *pp* ratio



effect of *tensor forces*

$$R_{pN} = n_{pN}(k_{rel},0) \,/\, n_{pN}^{\mathbf{central}}(k_{rel},0) \longrightarrow$$



Back-to-Back nucleons: *pn* and *pp* probabilities

$$P_{pN} = \frac{\int_a^b dk_{rel}\, k_{rel}^2\, n_{pN}(\boldsymbol{k}_{rel},0)}{\int_a^b dk_{rel}\, k_{rel}^2\, \left(n_{pp}(\boldsymbol{k}_{rel},0) + n_{pn}(\boldsymbol{k}_{rel},0)\right)}\,; \qquad 0 < P_{pN} < 1$$

- integration over the whole k_{rel} range: $(a,b) = [0,\infty]$

A	4	12	16	40
P_{pp} (%)	19.7	30.6	29.5	31.0
P_{pn} (%)	81.3	69.4	70.5	69.0

- *correlation region*: $(a,b) = [1.5, 3.0]\ fm^{-1}$

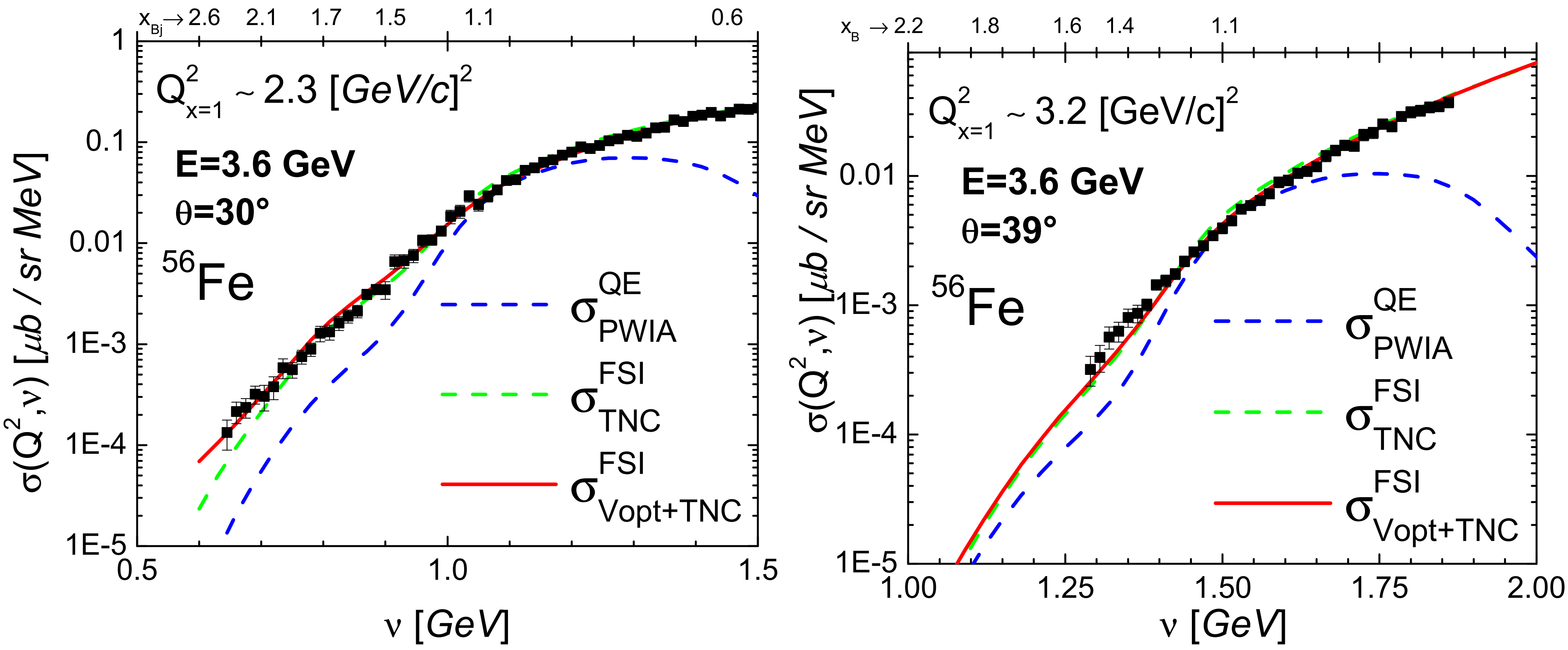
A	4	12	16	40
P_{pp} (%)	2.9	13.3	10.8	24.0
P_{pn} (%)	97.1	86.7	89.2	76.0

(*Alvioli, Ciofi degli Atti, Morita PRL**100** (2008)*)

$P_{pN}^{A=4}$ in agreement with *Schiavilla et al., PRL98 (2007) 132501*
(extracted from published figures, $AV18$: $P_{pp} \simeq 3\%$, $P_{pn} \simeq 97\%$)

$P_{pp} \simeq 10-13\%$ consistent with *Shneor et al., PRL99 (2007) 072501*
(extracted from $^{12}C(e,e'pp)X\ /\ ^{12}C(e,e'p)X$)

3. Effect of NN correlations and FSI in Inclusive Processes

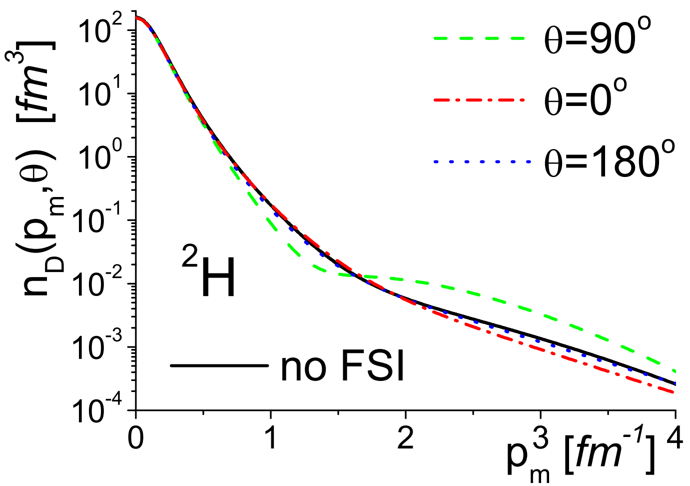


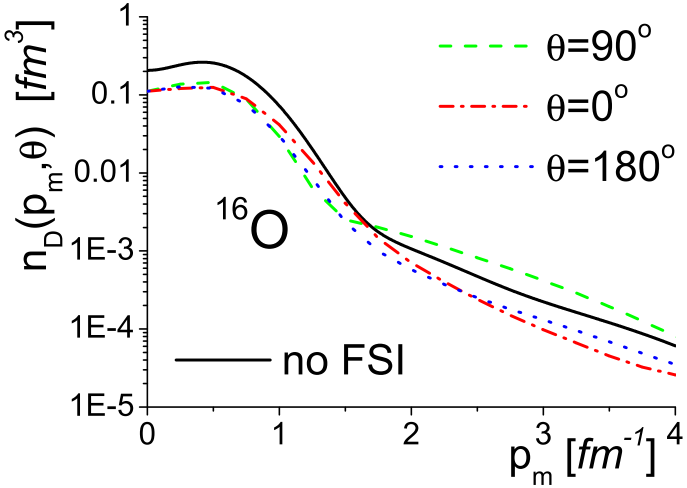
(*Preliminary calculations*)

TNC: OK for $x_{Bj} \leq 2$: three-body correlations are missing.
Calculations with many-body wave functions are in progress

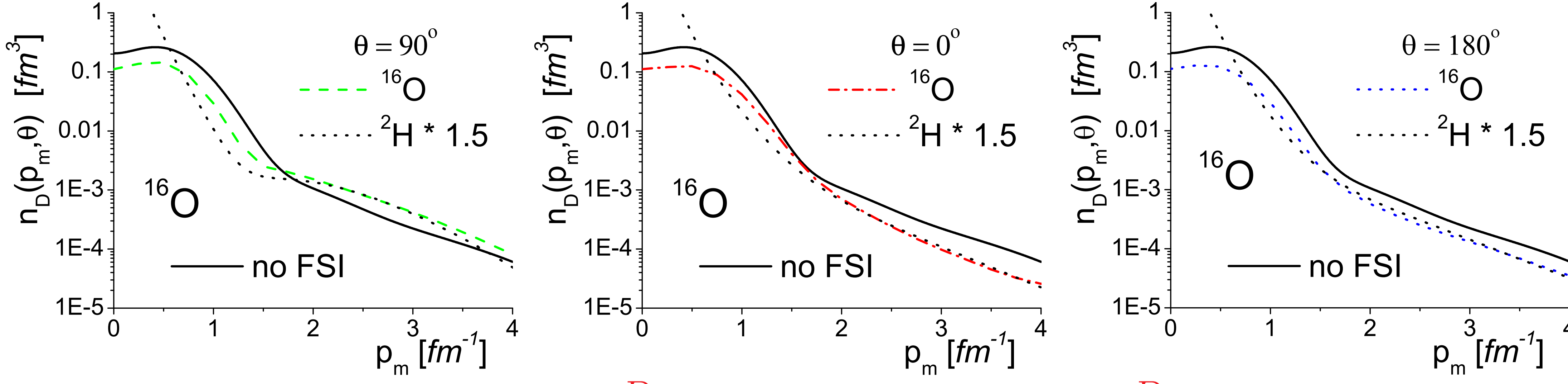
Inclusive *Distorted* Momentum Distribution (Glauber)

well known fact: $n_A(k)$ *proportional to* $n_{^2H}(k)$





$\theta = \boldsymbol{p}_m \cdot \boldsymbol{q} / p_m q$



new result: $n_A^D(k)$ **proportional to** $n_{^2H}^D(k)$ **!**

4. Effects of NN Correlations in High-Energy Processes

- SRC: are they relevant only in dedicated experiments?
- high-energy scattering processes $\longrightarrow$ Glauber multiple scattering
- exact expansion of the many-body WF (*Glauber, Foldy & Walecka*):

$$|\Psi(\boldsymbol{r}_1, ..., \boldsymbol{r}_A)|^2 = \prod_{j=1}^{A} \rho(\boldsymbol{r}_j) + \sum_{i<j=1}^{A} \boldsymbol{\Delta}(\boldsymbol{r}_\mathbf{i}, \boldsymbol{r}_\mathbf{j}) \prod_{k\neq(il)}^{A} \rho_1(\boldsymbol{r}_k) + \sum_{(i<j)\neq(k<l)} \boldsymbol{\Delta}(\boldsymbol{r}_\mathbf{i}, \boldsymbol{r}_\mathbf{j})\, \boldsymbol{\Delta}(\boldsymbol{r}_\mathbf{k}, \boldsymbol{r}_\mathbf{l}) \prod_{m\neq i,j,k,l} \rho_1(\boldsymbol{r}_m) + \dots$$

$$\simeq \prod_{j=1}^{A} \rho(\boldsymbol{r}_j) \longleftarrow \textit{usual approximation}: \textbf{ is it reliable}?$$

our two-body $\boldsymbol{\Delta}(\boldsymbol{r}_\mathbf{i}, \boldsymbol{r}_\mathbf{j}) = \rho_2(\boldsymbol{r}_i, \boldsymbol{r}_j) - \rho_1(\boldsymbol{r}_i)\,\rho_1(\boldsymbol{r}_j)$ provides:

$$\boxed{\int d\boldsymbol{r}_2\, \rho_2(\boldsymbol{r}_1, \boldsymbol{r}_2) = \rho_1(\boldsymbol{r}_1)} \longrightarrow \boxed{\int d\boldsymbol{r}_2 \boldsymbol{\Delta}(\boldsymbol{r}_1, \boldsymbol{r}_2) = 0}$$

Example: total *neutron-Nucleus* cross section at high energies

$$\sigma_{\mathbf{tot}} = \frac{4\pi}{k} Im\left[F_{00}(0)\right] \qquad F_{00}(\boldsymbol{q}) = \frac{ik}{2\pi}\int d^2b_n e^{i\mathbf{q}\cdot\mathbf{b_n}}\left[1 - \mathbf{e}^{\mathbf{i}\,\chi_{\mathbf{opt}}(\mathbf{b_n})}\right]$$

$$\mathbf{e}^{\mathbf{i}\,\chi_{\mathbf{opt}}(\boldsymbol{b}_\mathbf{n})} = \int \prod_{j=1}^{A} d\boldsymbol{r}_j \prod_{j=1}^{A}\left[1 - \Gamma(\boldsymbol{b}_n - \boldsymbol{s}_j)\right] \left|\Psi_0(\boldsymbol{r}_1, ..., \boldsymbol{r}_A)\right|^2 \delta\left(\frac{1}{A}\sum \mathbf{r}_j\right)$$

using the $|\Psi_o|^2$ expansion, with:

$$\boldsymbol{\Delta}(\boldsymbol{r}_\mathbf{i}, \boldsymbol{r}_\mathbf{j}) = \rho_2(\boldsymbol{r}_i, \boldsymbol{r}_j) - \rho_1(\boldsymbol{r}_i)\,\rho_1(\boldsymbol{r}_j)\,;$$

one has:

$$\sigma_{\mathbf{tot}} = \sigma_{\mathbf{G}}^{(\mathbf{1})} + \sigma_{\mathbf{G}}^{(\mathbf{2})} + \boldsymbol{\Delta}\sigma_{\mathbf{in}}$$

$$\sigma_G^{(1)} + \sigma_G^{(2)} =$$

$$= 2\int d\boldsymbol{b}\left\{1 - e^{-A\int d\boldsymbol{r}_1\rho_1(\boldsymbol{r}_1)\Gamma(\boldsymbol{b}-\boldsymbol{b}_1)}\, e^{\frac{A^2}{2}\int d\boldsymbol{r}_1 d\boldsymbol{r}_2 \Delta(\boldsymbol{r}_1,\boldsymbol{r}_2)\Gamma(\boldsymbol{b}-\boldsymbol{b}_1)\Gamma(\boldsymbol{b}-\boldsymbol{b}_2)}\right\}$$

no correlation case $\longrightarrow\ \boldsymbol{\Delta} = 0 \longrightarrow \sigma_G^{(2)} = 0$

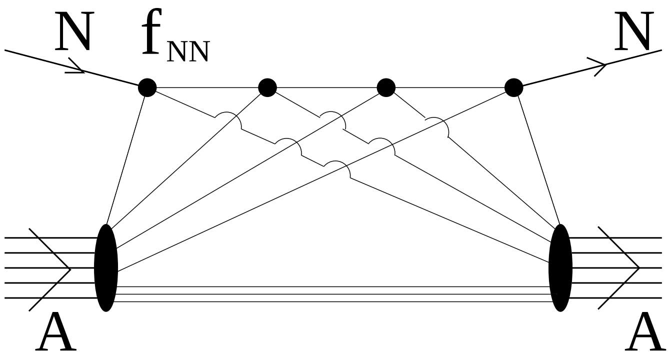
Glauber + Inelastic shadowing

(Diffractive excitation of the projectile)

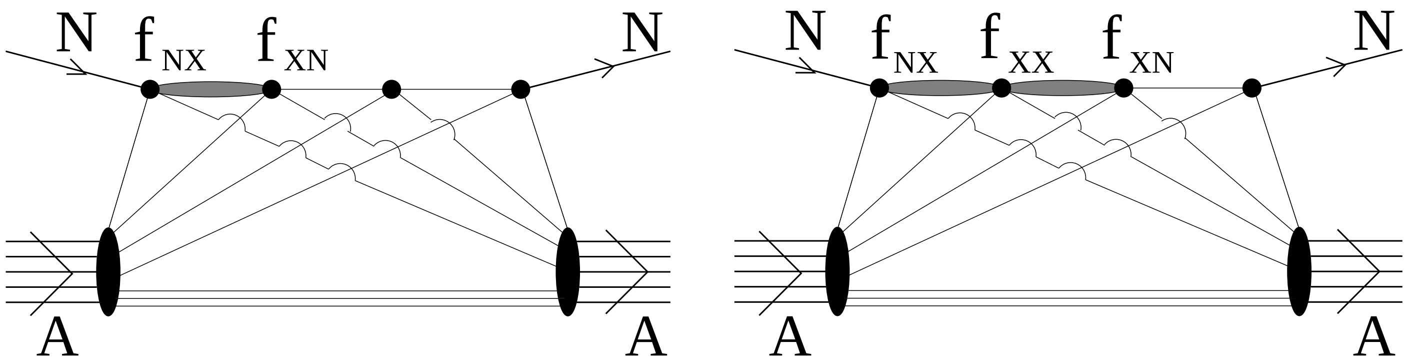
V. N. Gribov, Sov. JETP 29 (1969) 483;

V.A.Karmanov,L.A.Kondratyuk, JETP Lett. 18 (1973) 451;

B. K. Jennings, G. A. Miller, Phys. Rev. C49 (1999) 2637



(Glauber)

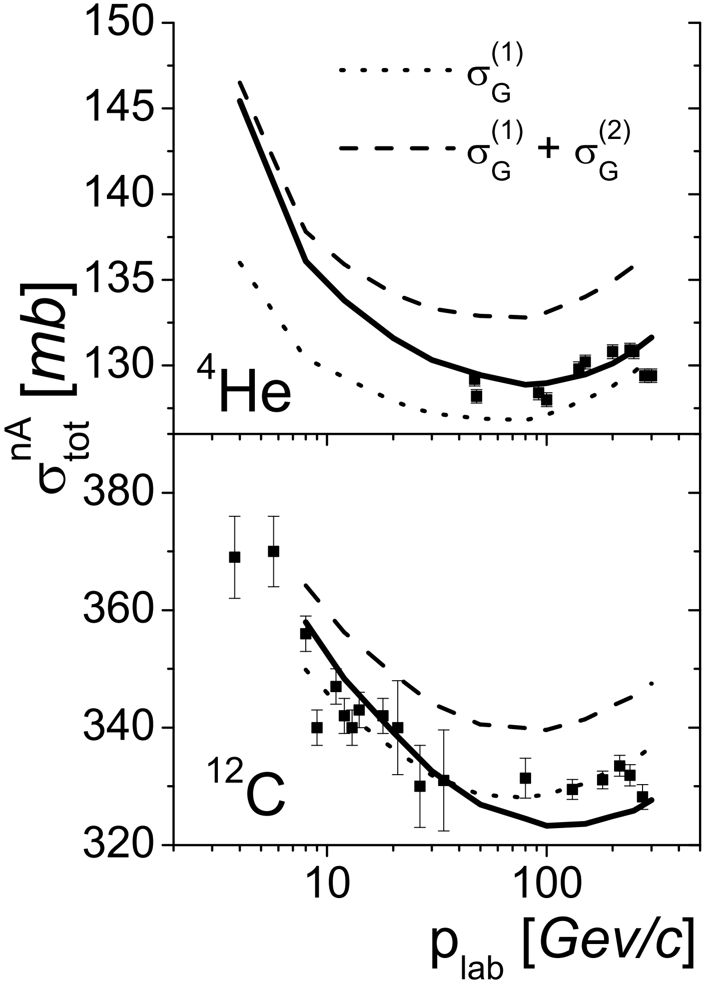


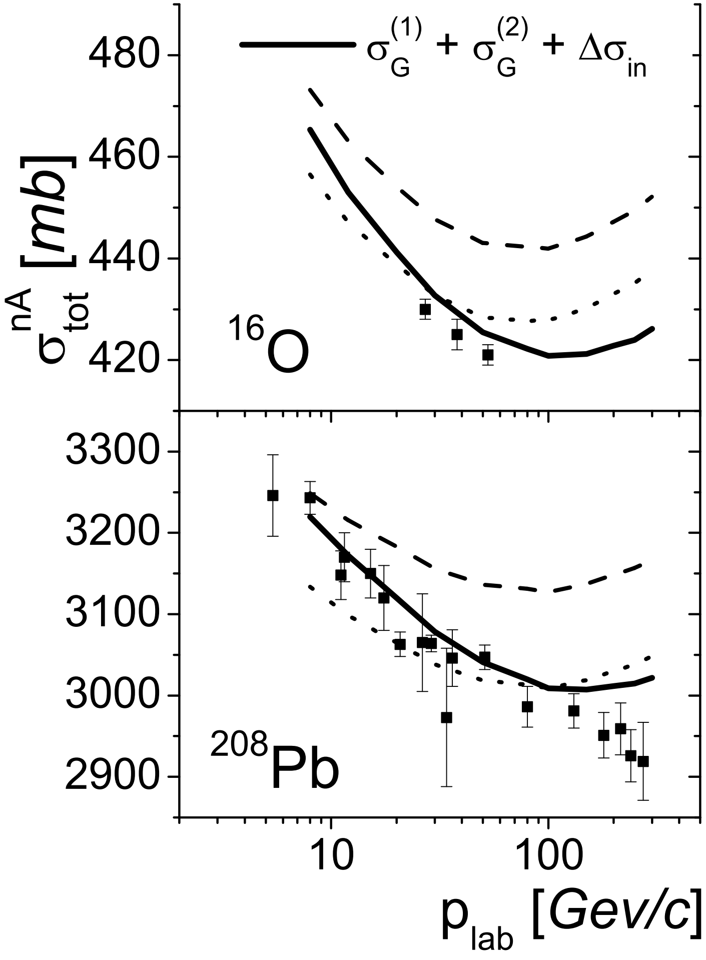
(Inelastic Shadowing)

total neutron-Nucleus cross section:

$$\sigma_{tot} = \frac{4\pi}{k} Im \left[F_{00}^{G}(0) \right] + \Delta\sigma_{in} = \sigma_G^{(1)} + \sigma_G^{(2)} + \Delta\sigma_{in}$$

RESULTS

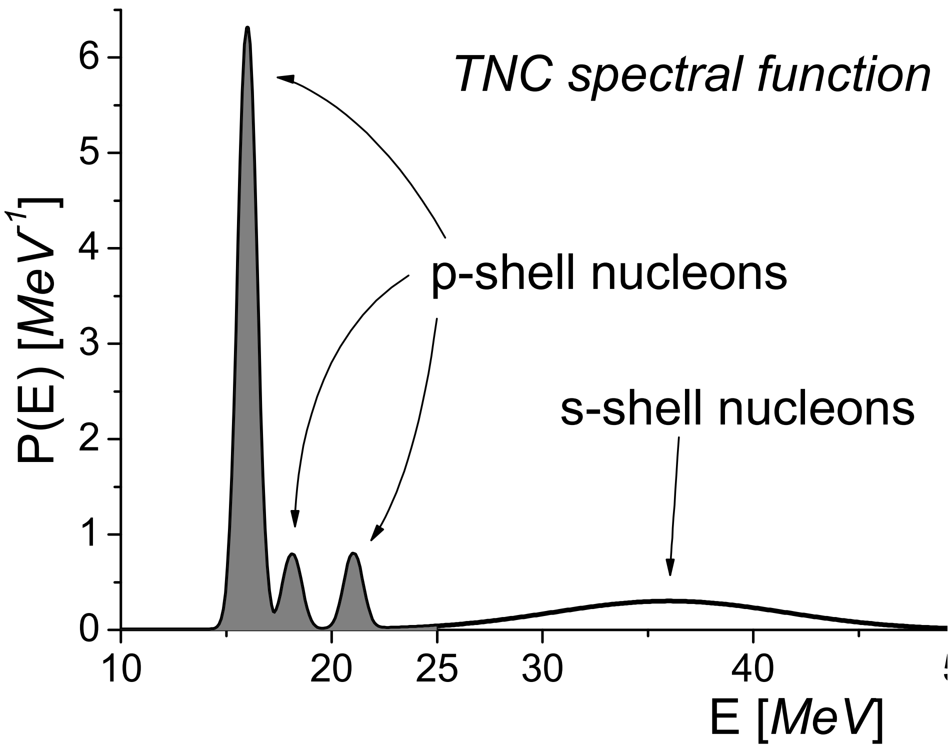


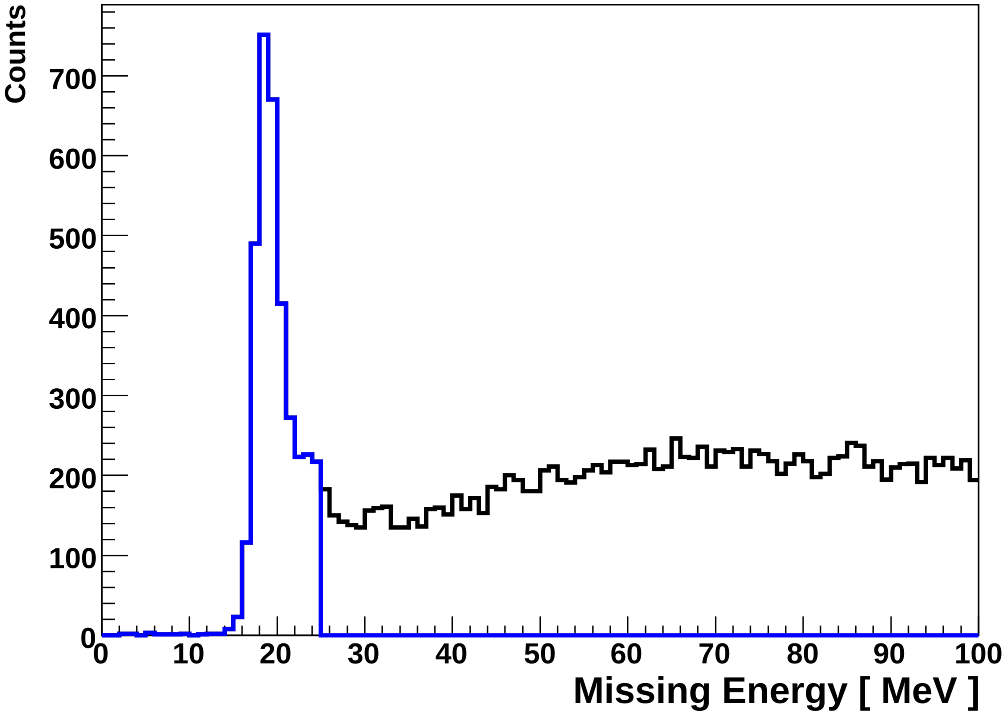


No adjustable parameters

M.Alvioli, Ciofi degli Atti, I.Marchino, H.Morita and V.Palli,
`nucl-th:07053613` - *sumbitted for pubblication*

The process $^{12}C(e,e'p)^{11}B$





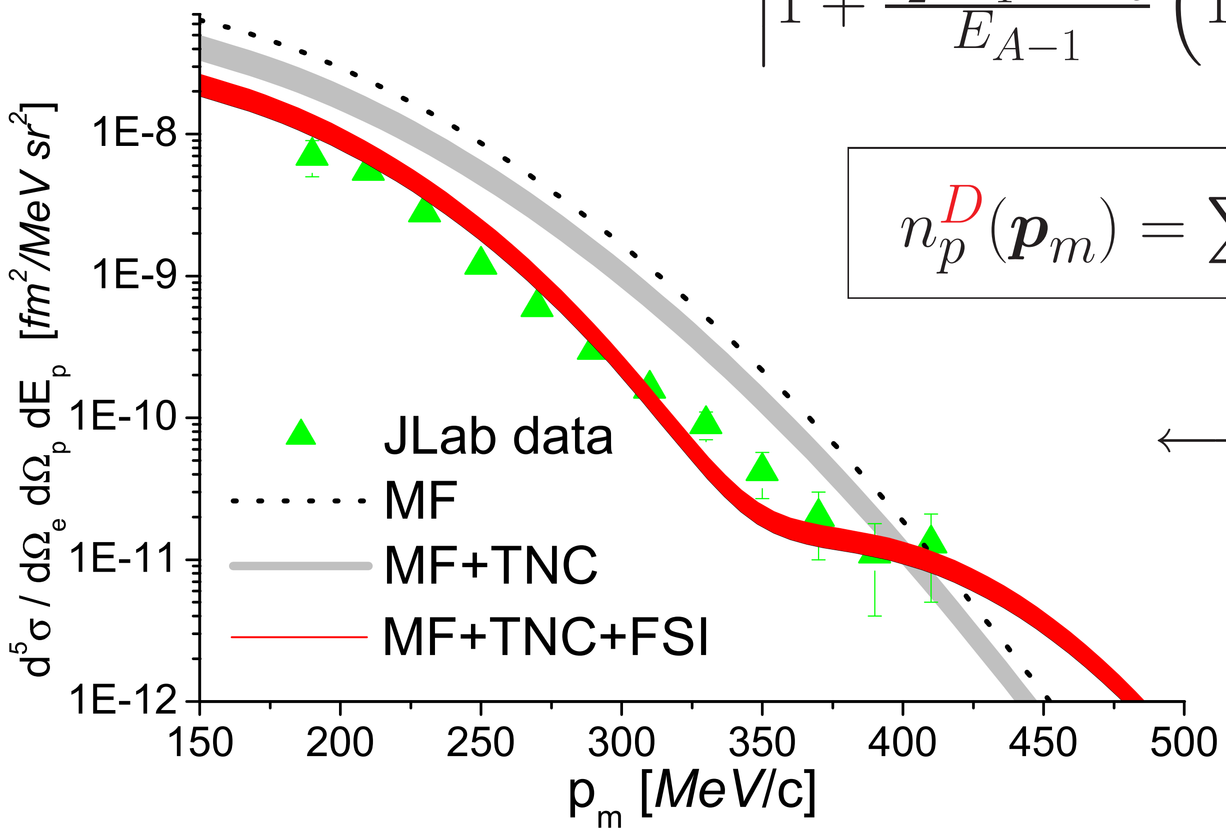
↑ *TNC spectral function* ↑ *JLab experiment*

$$n_p(k) = \int_0^{E_M} dE\, P_0(k,E), \quad \text{with } E_M = 25\ MeV$$

$^{12}C(e,e'p)^{11}B$ factorized cross section

$$\frac{d^5\sigma}{d\Omega_e\, d\Omega_p\, dE_p} = \mathcal{K}\,\sigma_{ep}\, Z_p \int_0^{E_M} dE\, P_0^D(kE) = $$

$$= \frac{E_p\, p\, \sigma_{ep}\, Z_p\, N_p}{\left|1 + \frac{\epsilon_2 - \epsilon_1 \cos\theta_e}{E_{A-1}}\left(1 - \frac{p}{q}\cos\theta_{pq}\right)\right|}\, n_p^D(\boldsymbol{p}_m)$$



$$n_p^D(\boldsymbol{p}_m) = \sum_\alpha \left| \int d\boldsymbol{r}\, e^{i\boldsymbol{p}_m\cdot\boldsymbol{r}} \varphi_\alpha(\boldsymbol{r}) G^{A-1}(\boldsymbol{r}) \right|^2$$

grey and red bands

$\longleftarrow$ due to our present uncertainty

in the calculation of

p-shell spectroscopic factors

$0.5 < N_p < 0.7$

$(Nikhef: \sim 0.5)$

un-factorized *calculation feasible*

Conclusions

- Exclusive processes off few-body nuclei can be reliably described within generalized Glauber FSI and realistic wave functions
- Cross sections and momentum distributions in inclusive reactions suggest that both ground state high momentum components and FSI in complex nuclei can be described with properly *scaled* ^2H corresponding quantities
- Many-body calculations can be reliably performed within a convergent cluster expansion method: any one and two-body quantity can be calculated
- Tensor and Isospin-Tensor correlations appear to be the essential ingredients for a correct description of (one- and two-body) momentum distributions in complex nuclei; at high momentum values TNC model OK
- High-energy processes are also affected by NN correlations and their contribution can be quantitatively be evaluated within the "exact" $|\Psi_o|^2$ expression with two-body correlations

Additional Slides

The Nuclear Many-Body Problem

$$\hat{\mathbf{H}}\,\Psi_n = E_n\,\Psi_n\,, \quad \textit{with}: \quad \hat{\mathbf{H}} = -\frac{\hbar^2}{2\,m}\sum_i \hat{\nabla}_i^2 + \frac{1}{2}\sum_{i<j}\hat{v}_{ij}$$

where

$$\hat{v}_{ij} = \sum_n v^{(n)}(r_{ij})\,\hat{\mathcal{O}}_{ij}^{(n)}$$

$$\hat{\mathcal{O}}_{ij}^{(n)} = \left[1\,,\,\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j\,,\,\hat{S}_{ij}\,,\,(\boldsymbol{L}\cdot\boldsymbol{S})_{ij}\,,\,\ldots\right]\otimes\left[1\,,\,\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j\right].$$

The same operatorial dependence is cast onto Ψ_o:

$$\Psi_o = \hat{\mathbf{F}}\,\phi_o$$

where ϕ_o is the *mean-field* wave function and

$$\hat{\mathbf{F}} = \hat{S}\prod_{i<j}\hat{f}_{ij} = \hat{S}\prod_{i<j}\sum_n f^{(n)}(r_{ij})\,\hat{\mathcal{O}}_{ij}^{(n)}$$

is a *correlation* operator.

• at **first order** of the η−expansion, the **full correlated** `one-body mixed` density matrix expression is as follows:

$$\rho^{(1)}(\boldsymbol{r}_1,\boldsymbol{r}'_1) = \rho_o^{(1)}(\boldsymbol{r}_1,\boldsymbol{r}'_1) + \rho_H^{(1)}(\boldsymbol{r}_1,\boldsymbol{r}'_1) + \rho_S^{(1)}(\boldsymbol{r}_1,\boldsymbol{r}'_1)\,,$$

with

$$\rho_H^{(1)}(\boldsymbol{r}_1,\boldsymbol{r}'_1) = \int d\boldsymbol{r}_2\left[H_D(r_{12},r_{1'2})\,\rho_o^{(1)}(\boldsymbol{r}_1,\boldsymbol{r}'_1)\,\rho_o(\boldsymbol{r}_2) - H_E(r_{12},r_{1'2})\,\rho_o^{(1)}(\boldsymbol{r}_1,\boldsymbol{r}_2)\,\rho_o^{(1)}(\boldsymbol{r}_2,\boldsymbol{r}'_1)\right]$$

$$\rho_S^{(1)}(\boldsymbol{r}_1,\boldsymbol{r}'_1) = -\int d\boldsymbol{r}_2 d\boldsymbol{r}_3\rho_o^{(1)}(\boldsymbol{r}_1,\boldsymbol{r}_2)\left[H_D(r_{23})\rho_o^{(1)}(\boldsymbol{r}_2,\boldsymbol{r}'_1)\rho_o(\boldsymbol{r}_3) - H_E(r_{23})\rho_o^{(1)}(\boldsymbol{r}_2,\boldsymbol{r}_3)\rho_o^{(1)}(\boldsymbol{r}_3,\boldsymbol{r}'_1)\right]$$

and the functions H_D and H_E are defined as:

$$H_{D(E)}(r_{ij},r_{kl}) = \sum_{p,q=1}^{6} f^{(p)}(r_{ij})\,f^{(q)}(r_{kl})\,C_{D(E)}^{(p,q)}(r_{ij},r_{kl}) - C_{D(E)}^{(1,1)}(r_{ij},r_{kl})$$

with $C_{D(E)}^{(p,q)}(r_{ij},r_{kl})$ proper functions arising from spin-isospin traces;

(*Alvioli, Ciofi degli Atti, Morita, PRC72 (2005)*)

• at **first order** of the $\eta-$expansion, the **full correlated** `two-body mixed` density matrix expression is as follows:

$$\rho^{(2)}(\boldsymbol{r}_1,\boldsymbol{r}_2;\boldsymbol{r}'_1,\boldsymbol{r}'_2) = \rho^{(2)}_{\mathbf{SM}}(\boldsymbol{r}_1,\boldsymbol{r}_2;\boldsymbol{r}'_1,\boldsymbol{r}'_2) + \rho^{(2)}_{\mathbf{2b}}(\boldsymbol{r}_1,\boldsymbol{r}_2;\boldsymbol{r}'_1,\boldsymbol{r}'_2) + \rho^{(2)}_{\mathbf{3b}}(\boldsymbol{r}_1,\boldsymbol{r}_2;\boldsymbol{r}'_1,\boldsymbol{r}'_2) + \rho^{(2)}_{\mathbf{4b}}(\boldsymbol{r}_1,\boldsymbol{r}_2;\boldsymbol{r}'_1,\boldsymbol{r}'_2)$$

with:

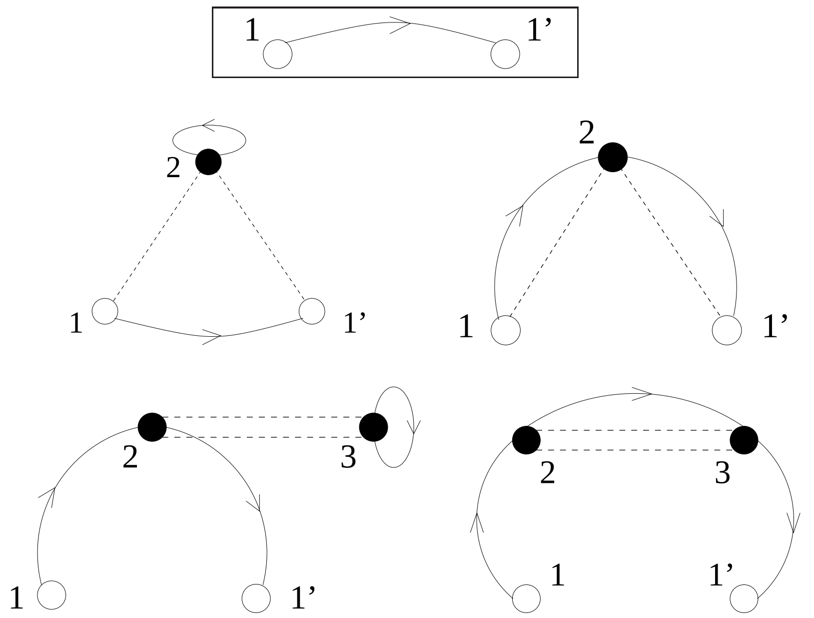
$$\rho^{(2)}_{\mathbf{SM}}(\boldsymbol{r}_1,\boldsymbol{r}_2;\boldsymbol{r}'_1,\boldsymbol{r}'_2) = C_D\,\rho_o(\boldsymbol{r}_1,\boldsymbol{r}'_1)\,\rho_o(\boldsymbol{r}_2,\boldsymbol{r}'_2) - C_E\,\rho_o(\boldsymbol{r}_1,\boldsymbol{r}'_2)\,\rho_o(\boldsymbol{r}_2,\boldsymbol{r}'_1)$$

$$\rho^{(2)}_{\mathbf{2b}}(\boldsymbol{r}_1,\boldsymbol{r}_2;\boldsymbol{r}'_1,\boldsymbol{r}'_2) = \frac{1}{2}\,\hat{\eta}(r_{12},r_{1'2'})\,\rho_o(\boldsymbol{r}_1,\boldsymbol{r}'_1)\,\rho_o(\boldsymbol{r}_2,\boldsymbol{r}'_2) - \frac{1}{2}\,\hat{\eta}(r_{12},r_{1'2'})\,\rho_o(\boldsymbol{r}_1,\boldsymbol{r}'_2)\,\rho_o(\boldsymbol{r}_2,\boldsymbol{r}'_1)$$

$$\begin{aligned}\rho^{(2)}_{\mathbf{3b}}(\boldsymbol{r}_1,\boldsymbol{r}_2;\boldsymbol{r}'_1,\boldsymbol{r}'_2) = \int d\boldsymbol{r}_3\,\hat{\eta}(r_{13},r_{1'3})\,[\,&\rho_o(\boldsymbol{r}_1,\boldsymbol{r}'_1)\,\rho_o(\boldsymbol{r}_2,\boldsymbol{r}'_2)\,\rho_o(\boldsymbol{r}_3,\boldsymbol{r}_3)+\\ &-\rho_o(\boldsymbol{r}_1,\boldsymbol{r}'_1)\,\rho_o(\boldsymbol{r}_2,\boldsymbol{r}_3)\,\rho_o(\boldsymbol{r}_3,\boldsymbol{r}'_2)+\\ &+\rho_o(\boldsymbol{r}_1,\boldsymbol{r}_3)\,\rho_o(\boldsymbol{r}_2,\boldsymbol{r}'_1)\,\rho_o(\boldsymbol{r}_3,\boldsymbol{r}'_2)+\\ &-\rho_o(\boldsymbol{r}_1,\boldsymbol{r}_3)\,\rho_o(\boldsymbol{r}_2,\boldsymbol{r}'_2)\,\rho_o(\boldsymbol{r}_3,\boldsymbol{r}'_1)+\\ &+\rho_o(\boldsymbol{r}_1,\boldsymbol{r}'_2)\,\rho_o(\boldsymbol{r}_2,\boldsymbol{r}_3)\,\rho_o(\boldsymbol{r}_3,\boldsymbol{r}'_1)+\\ &-\rho_o(\boldsymbol{r}_1,\boldsymbol{r}'_2)\,\rho_o(\boldsymbol{r}_2,\boldsymbol{r}'_1)\,\rho_o(\boldsymbol{r}_3,\boldsymbol{r}_3)\,]\end{aligned}$$

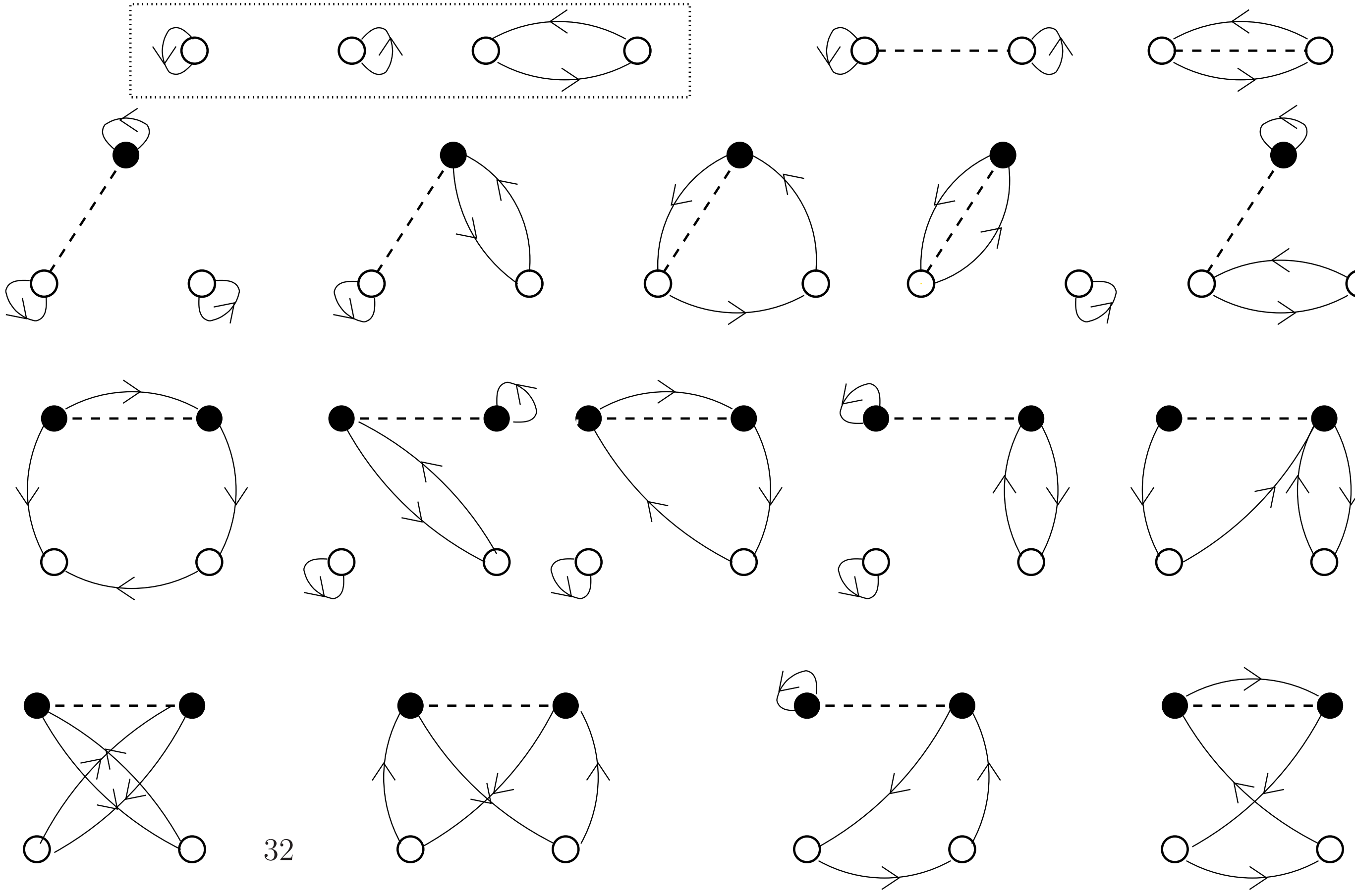
$$\begin{aligned}\rho^{(2)}_{\mathbf{4b}}(\boldsymbol{r}_1,\boldsymbol{r}_2;\boldsymbol{r}'_1,\boldsymbol{r}'_2) = \frac{1}{4}\int d\boldsymbol{r}_3 d\boldsymbol{r}_4\,\hat{\eta}(r_{34})\,\cdot\\ \cdot\sum_{\mathcal{P}\in\mathcal{C}}(-1)^{\mathcal{P}}\,[\,\rho_o(\boldsymbol{r}_1,\boldsymbol{r}_{\mathcal{P}1'})\,\rho_o(\boldsymbol{r}_2,\boldsymbol{r}_{\mathcal{P}2'})\,\rho_o(\boldsymbol{r}_3,\boldsymbol{r}_{\mathcal{P}3})\,\rho_o(\boldsymbol{r}_4,\boldsymbol{r}_{\mathcal{P}4})\,]\end{aligned}$$

(*Alvioli, Ciofi degli Atti, Morita, PRC72 (2005)*)
(*Alvioli, Ciofi degli Atti, Morita,* PRL100 (2008))



one-body, *non-diagonal*
$\longleftarrow \rho(\boldsymbol{r}_1, \boldsymbol{r}'_1)$ *diagrams*

two-body, *diagonal*
$\rho(\boldsymbol{r}_1, \boldsymbol{r}_2)$ *diagrams* $\longrightarrow$

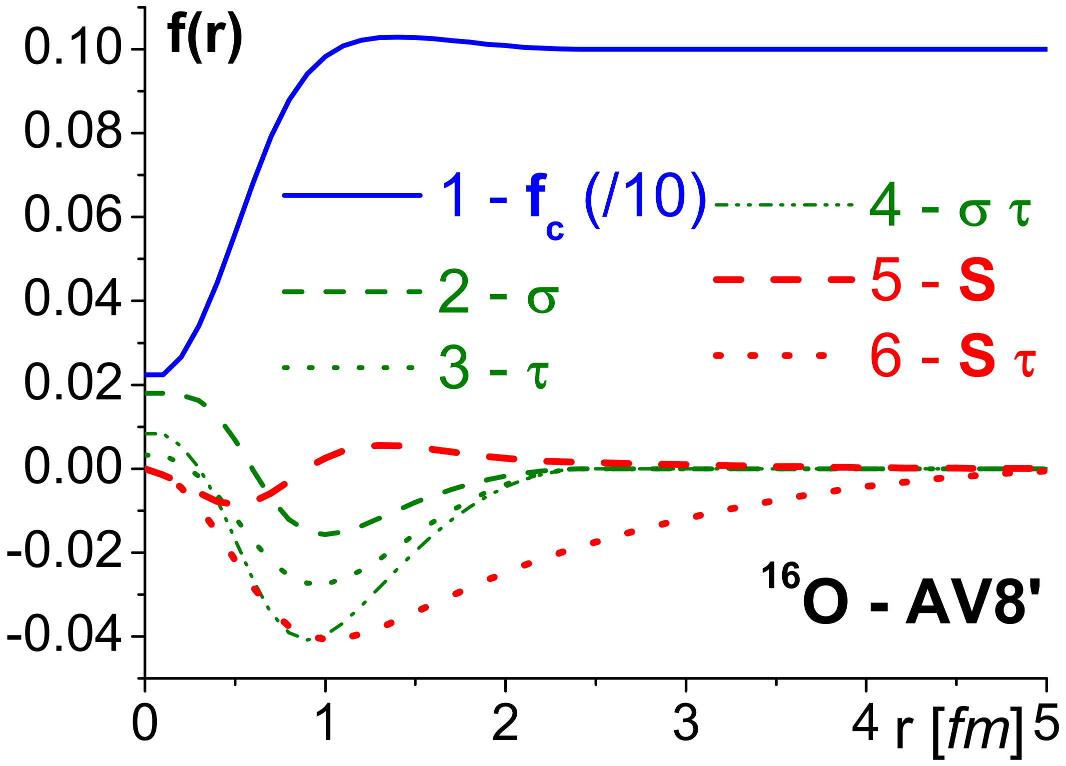


M. Alvioli

Ground state energy: ^{16}O - Argonne $V8'$

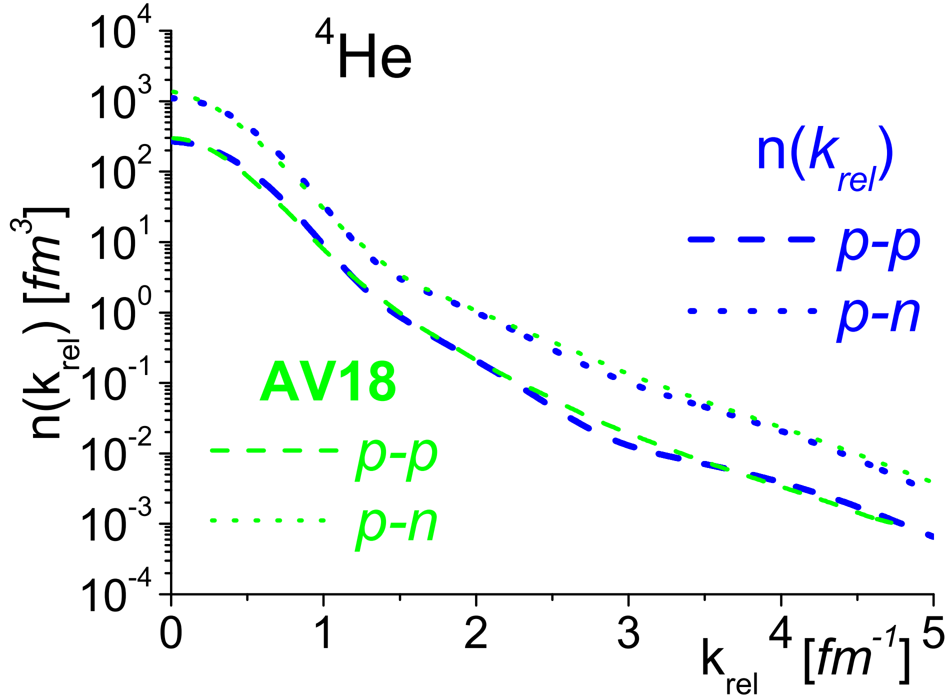
	$< V_c >$	$< V_\sigma >$	$< V_\tau >$	$< V_{\sigma\tau} >$	$< V_S >$	$< V_{S\tau} >$	$< \mathbf{V} >$	$< \mathbf{T} >$	**E**	**E/A** MeV
$\eta-exp$	0.19	-35.88	-9.47	-171.32	-0.003	-172.89	-389.40	323.50	**-65.90**	-4.12
FHNC	0.694	-40.13	-10.61	-180.00	-0.07	-160.32	-390.30	325.18	**-65.12**	-4.07

correlation functions: *Central*, *Spin-Isospin*, *Tensor*

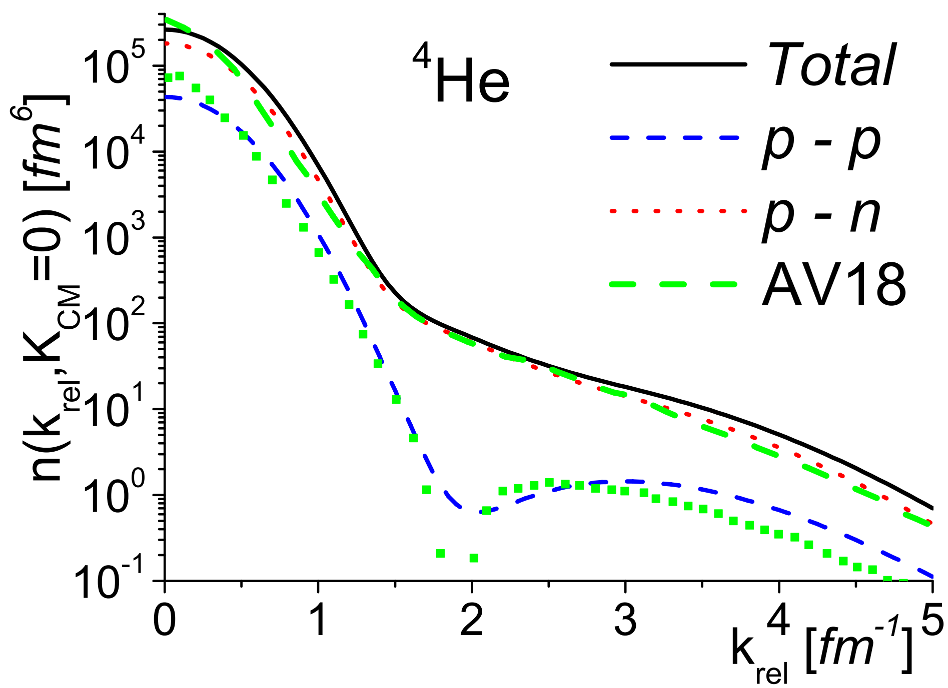


^4He: comparison with VMC

$n_{pN}(k_{rel}) = \int d\boldsymbol{K}_{CM}\, n_{pN}(\boldsymbol{k}_{rel}, \boldsymbol{K}_{CM})$

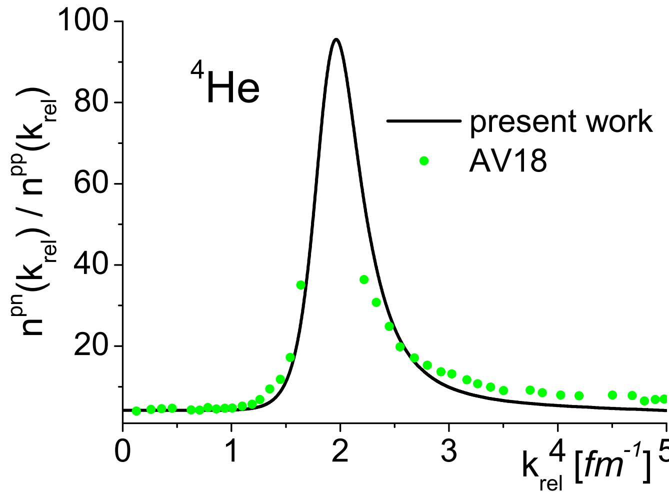


$n_{NN}(\boldsymbol{k}_{rel}, \boldsymbol{K}_{CM} = 0)$



- good agreement with VMC calculations
- $n_{pn}(k_{rel},0)/n_{pp}(k_{rel},0)$ `peak location ok` $\longrightarrow$

(**AV18**: *Schiavilla at al.* **PRL98** (2007))



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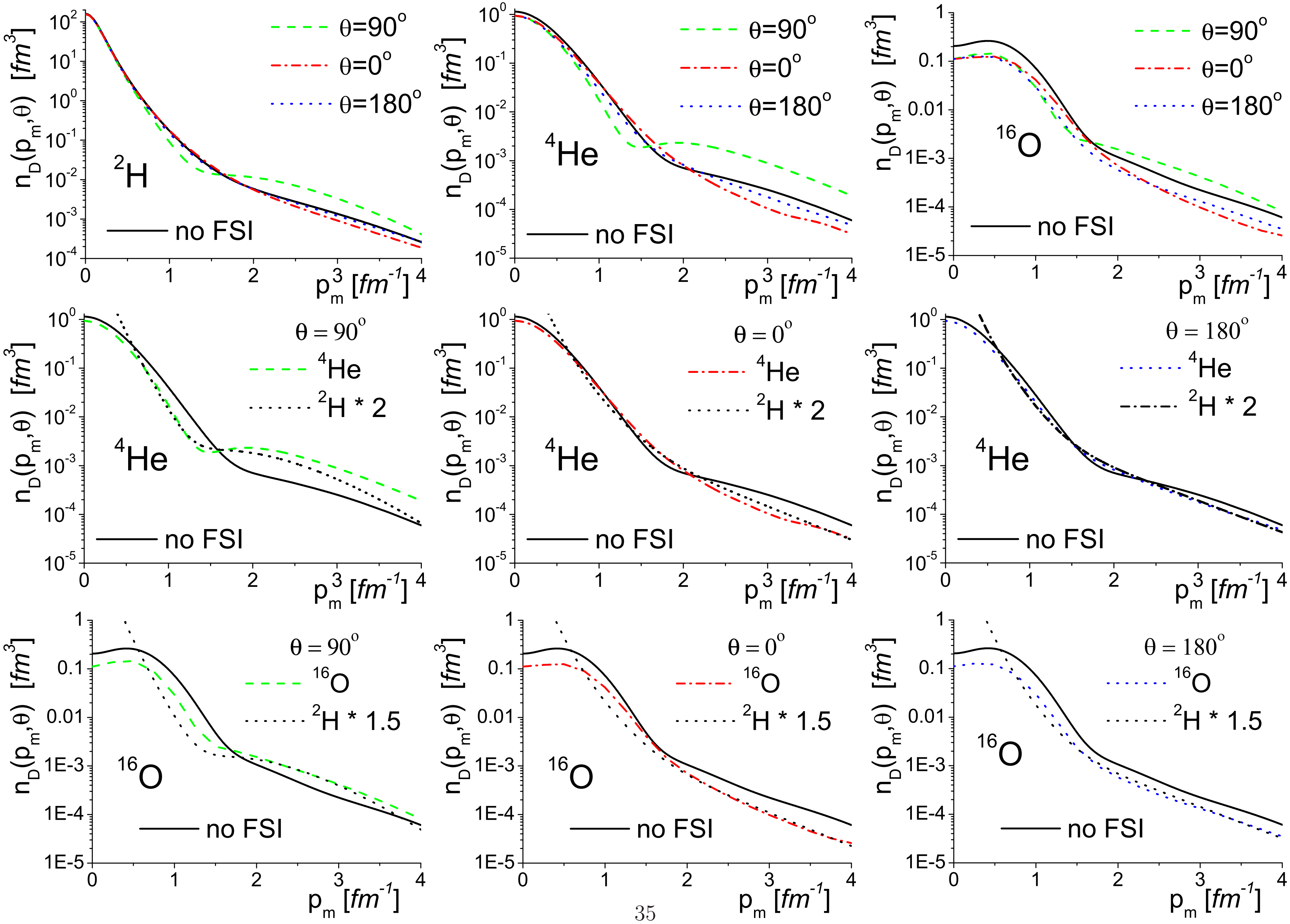
FSI with a correlated pair



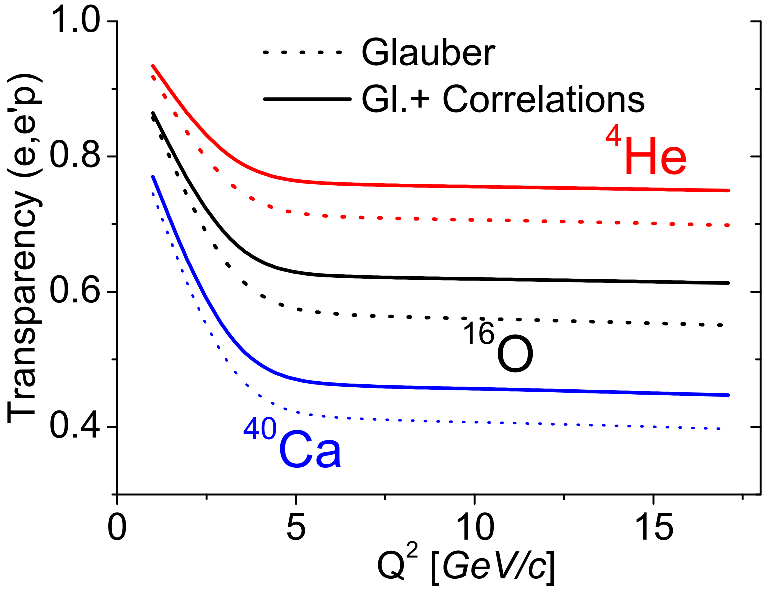
Inclusive Transparency in $A(e,e'p)X$

using again the Foldy-Walecka expansion of $|\Psi_o|^2$ one obtains:

$$T = \int \prod_{j=1}^{A} d\boldsymbol{r}_j\, G(\boldsymbol{b}_1, \boldsymbol{b}_j)\, |\Psi_o(\boldsymbol{r}_1, ..., \boldsymbol{r}_A)|^2 =$$

$$= \int d\boldsymbol{r}_1 \rho(\boldsymbol{r}_1)\, e^{-A\int d\boldsymbol{r}_1 \rho_1(\boldsymbol{r}_1)\Gamma(\boldsymbol{b}-\boldsymbol{b}_1)}\, e^{\frac{A^2}{2}\int d\boldsymbol{r}_1 d\boldsymbol{r}_2 \Delta(\boldsymbol{r}_1,\boldsymbol{r}_2)\Gamma(\boldsymbol{b}-\boldsymbol{b}_1)\Gamma(\boldsymbol{b}-\boldsymbol{b}_2)} +$$

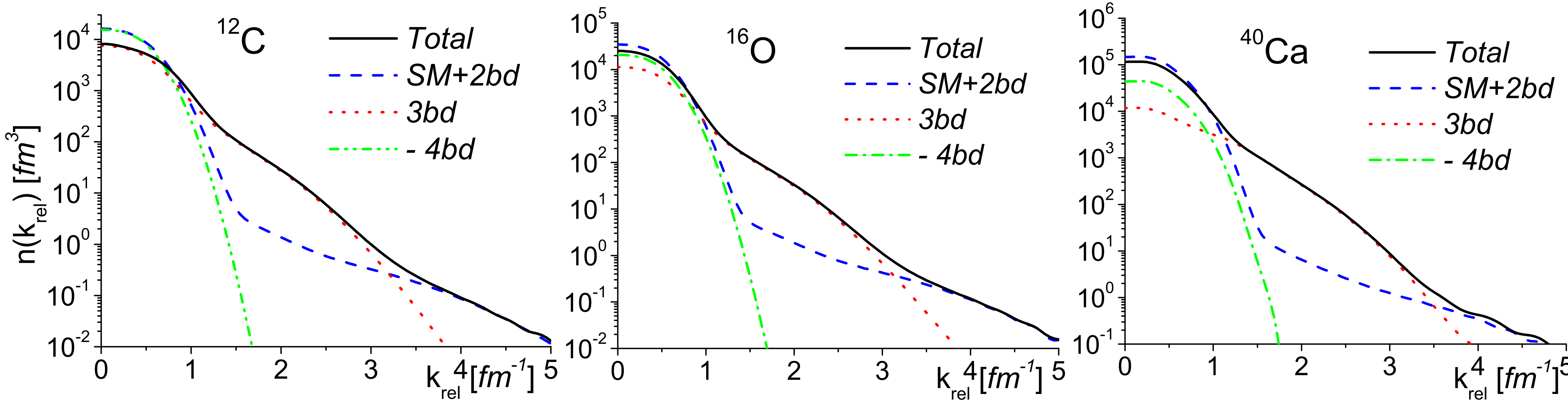
$$-A\int d\boldsymbol{r}_1 \boldsymbol{r}_2 \Delta(\boldsymbol{r}_1, \boldsymbol{r}_2)\, e^{-A\int d\boldsymbol{r}_1 \rho_1(\boldsymbol{r}_1)\Gamma(\boldsymbol{b}-\boldsymbol{b}_1)}\, e^{\frac{A^2}{2}\int d\boldsymbol{r}_1 d\boldsymbol{r}_2 \Delta(\boldsymbol{r}_1,\boldsymbol{r}_2)\Gamma(\boldsymbol{b}-\boldsymbol{b}_1)\Gamma(\boldsymbol{b}-\boldsymbol{b}_2)}$$



Glauber corresponds
to $\Delta(\boldsymbol{r}_1, \boldsymbol{r}_2) = 0$
(*Preliminary calculations*)

$n_{NN}(k_{rel})$ for Complex Nuclei

$$n_{NN}(k_{rel}) = \int d\boldsymbol{K}_{CM}\, n_{NN}(\boldsymbol{k}_{rel}, \boldsymbol{K}_{CM})$$



- normalization (**number of pairs**) conserved by the expansion
- **isospin** separation feasible
- closed **j-shell** nuclei included in the formalism
- *three* and *four-body* diagrams **essential**