The coherent inelastic processes in collisions of particles and nuclei at ultrarelativistic energies

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1. Momentum transfer at ultrarelativistic energies and coherent reactions on nuclei

Processes of inelastic coherent scattering of the type $a \to b$ at collisions of elementary particles (hadrons, γ-quanta) with nuclei (the nucleus remains the same) — in principle, possible at very high energies.

Increase of the role of coherent processes at ultra-relativistic energies → due to the fact that the minimal longitudinal momentum transferred to a nucleus tends to zero.

Impulse approximation ⟹ the following general expression for the effective cross-section of the process $a \to b$:

$$\sigma_{coh}(a \to b) = \int |f_{a+N\to b+N}(\vec{q})|^2 P(\vec{q})\, d\Omega_b \; .$$

Here $f_{a+N\to b+N}(\vec{q}) = \frac{1}{A}\left[Z f_{a+p\to b+p}(\vec{q}) + (A-Z) f_{a+n\to b+n}(\vec{q})\right]$

→ average amplitude of the inelastic process $a+N \to b+N$ on a separate nucleon in the nucleus rest frame;

$\vec{q} = \vec{k}_b - \vec{k}_a$ → momentum transferred to the nucleon;

$\vec{k}_a$, $\vec{k}_b$ → respective momenta of the particles a, b;

$d\Omega_b$ → element of solid angle of flight of the particle b in the laboratory frame;

$P(\vec{q}) = \left| \sum\limits_{i=1}^{A} \langle \exp(-i\vec{q}\vec{r}_i) \rangle \right|^2$ → probability of the event that at the collision with the particle a all the nucleons will remain in the nucleus, i.e. the quantum state of the nucleus will not change

($\langle \ldots \rangle$ → averaging over quantum states of the target nucleus).

Introducing the nucleon density $n(\vec{r})$ $\left(\int\limits_V n(\vec{r}) d^3\vec{r} = A \right)$

⇒ the following representation for $P(\vec{q})$:

$(z \parallel \vec{k}_a)$

$$P(\vec{q}) = \left| \int n(\vec{r}) \exp(-i\vec{q}\vec{r}) d^3\vec{r} \right|^2 =$$

$$= \left| \int\limits_V n(\vec{\rho}, z) \exp(-i\vec{q}_\perp \vec{\rho}) \exp(-iq_\parallel z) d^2\vec{\rho}\, dz \right|^2.$$

$\vec{q}_\perp$, $q_\parallel$ → transverse and longitudinal components of the transferred momentum.

Main contribution to the effective cross-section $\sigma_{coh}(a \to b) \to$ provided by the transferred momenta $\vec{q}$ in the range $|\vec{q}| \lesssim 1/R$ ($R \to$ radius of the nucleus).

At ultrarelativistic energies:

$$E_a \gg m_a, \; E_b \gg m_b, \; E_a \gg 1/R, \; E_b \gg 1/R$$

($m_a, m_b \to$ respective masses of the particles a, b)

$\Rightarrow$ the recoil energy of the nucleon $E_{rec} \approx |\vec{q}|^2 / m_N \lesssim (m_N R^2)^{-1}$ can be neglected (as well as, of course, the much smaller recoil energy of the nucleus).

In doing so, the flight angles θ for the particle b are small:

$$\theta \lesssim 1/kR \ll 1 \quad (k = E_a \approx E_b)$$

$$\Rightarrow |\vec{q}_\perp| = k\theta; \quad q_{\parallel} = q_{min} = \frac{m_a^2 - m_b^2}{2k}$$

($q_{min} \to$ minimal transferred momentum corresponding to the „forward" direction).

Characteristic momentum transferred to the nucleus in the inelastic coherent process $a \to b$ — small as compared with the characteristic momentum transferred to the nucleon in the process $a + N \to b + N$ $\Rightarrow$ the replacement:

$$f_{a+N\to b+N}(\vec{q}) \to f_{a+N\to b+N}(0);$$

$d\Omega_b = \dfrac{d^2\vec{q}_\perp}{k^2}$; integration over transferred momenta $\Rightarrow$

we obtain for the cross-section $\sigma_{coh}(a\to b)$:

$$\sigma_{coh}(a\to b) = \frac{4\pi^2}{k^2}\left|f_{a+N\to b+N}(0)\right|^2 \int \left(\left| \int_{-\infty}^{\infty} n(\vec{\rho}, z)\exp(-iq_{min}z)\,dz \right|^2 \right) d^2\vec{\rho}$$

In particular, for a spherical nucleus with radius R and the constant density of nucleons $n_0 = \dfrac{3A}{4\pi R^3}$:

$$\sigma_{coh}(a\to b) = \frac{32\pi^2}{k^2 q_{min}^2} n_0^2 \left|f_{a+N\to b+N}(0)\right|^2 \int_0^R \sin^2\!\left(q_{min}\sqrt{R^2-\rho^2}\right)\rho\, d\rho$$

Ultrarelativistic limit $\Rightarrow |q_{min}|R \ll 1$

$\Rightarrow$ we obtain: effective solid angle of flight of the final particle b.

$$\sigma_{coh}(a\to b) = \frac{9\pi}{2k^2R^2} A^2 \left|f_{a+N\to b+N}(0)\right|^2.$$

Our consideration relates not only to binary reactions but also to multiparticle coherent processes on nuclei of analogous type:

$$a \to \{b_1 + b_2 + b_3 + \ldots\ldots + b_i\},$$

which become possible as well in the limit of very high energies - essentially higher than for binary processes $a \to b$.

In this general case:

vector $\vec{k}_b$ → has the meaning of the total momentum of the generated multiparticle system $b = \{b_1, b_2, b_3, \ldots b_i\}$ with the effective mass m_b;

magnitude $|f_{a+N \to b+N}(0)|^2$ → determines the cross-section of production of the system b, moving as a whole in the „forward" direction at the collision of particle a with the separate nucleon

2) Effect of the nucleus matter on coherent processes

Previous consideration → neglect of multiple scattering of the initial and final particles on nucleons of the nucleus → justified if the mean lengths of free path of the particles a, b inside the nucleus L_a, L_b strongly exceed the nucleus radius R.

Meantime, for medium and heavy nuclei the role of the nucleus matter may be essential.

Analysis → application of the optical model of the nucleus at high energy, based on the conception of the refraction index

[M. Goldberger, K. Watson. Collision theory, 1964
R. Jastrov, Phys. Rev. 82, 261 (1951)

Renormalized momenta of the ultrarelativistic particles a, b inside the nucleus (the refraction index is close to unity):

$$\vec{\tilde{k}}_a = \vec{k}_a + \frac{\vec{k}_a}{|\vec{k}_a|}\chi_a(\vec{r}); \quad \vec{\tilde{k}}_b = \vec{k}_b + \frac{\vec{k}_b}{|\vec{k}_b|}\chi_b(\vec{r}), \quad \text{where}$$

$$\chi_a(\vec{r}) = \frac{2\pi n(\vec{r})}{k} f_{a+N\to a+N}(0); \quad \chi_b(\vec{r}) = \frac{2\pi n(\vec{r})}{k} f_{b+N\to b+N}(0).$$

Here $f_{a+N\to a+N}(0)$, $f_{b+N\to b+N}(0) \to$ „forward" amplitudes of elastic scattering of the particles a, b on a nucleon in the laboratory frame;

χ_a, $\chi_b \to$ complex magnitudes characterizing phase shifts and absorption of the particles a, b at their passage through the nucleus matter;

$|\chi_a| \ll k$, $|\chi_b| \ll k$ at very high energies.

Using the optical theorem $\to$ the following representation for χ_a, χ_b:

$$\chi_a(\vec{r}) = i\frac{n(\vec{r})}{2}(1-i\alpha_a)\sigma_{aN}; \quad \chi_b(\vec{r}) = i\frac{n(\vec{r})}{2}(1-i\alpha_b)\sigma_{bN};$$

σ_{aN}, $\sigma_{bN} \to$ total cross-sections of interaction of the particles a, b with nucleons, averaged over the protons and neutrons of the nucleus;

$$\alpha_a = \frac{\mathrm{Re}\, f_{a+N\to a+N}(0)}{\mathrm{Im}\, f_{a+N\to a+N}(0)}; \quad \alpha_b = \frac{\mathrm{Re}\, f_{b+N\to b+N}(0)}{\mathrm{Im}\, f_{b+N\to b+N}(0)}.$$

Effect of nucleus matter $\Rightarrow$ introduction of the additional complex phase shift into the integral determining $\sigma_{coh}(a\to b)$:

$$\exp(-iq_{min}z) \to \exp\left(-iq_{min}z + i\delta(\vec{\rho}, z)\right).$$

Here $\delta(\vec{\rho}, z)$ is determined by the quantities χ_a, χ_b;

$$\delta(\vec{\rho}, z) = \int_{-\infty}^{z} [\chi_a(\vec{\rho}, z') - \chi_b(\vec{\rho}, z')] dz' + \int_{-\infty}^{\infty} \chi_b(\vec{\rho}, z') dz'.$$

Spherical nucleus with the uniform distribution of nucleons ($n(\rho, z) = n_0$ for $0 \leq |z| \leq \sqrt{R^2 - \rho^2}$,
$n(\rho, z) = 0$ for other z; $\rho = |\vec{\rho}|$)

$$\Rightarrow \delta(\rho, z) = (\chi_a - \chi_b)(z + \sqrt{R^2 - \rho^2}) + 2\chi_b \sqrt{R^2 - \rho^2}.$$

So, for this case we obtain:

$$\sigma_{coh}(a \to b) = \frac{4\pi}{k^2} |f_{a+N \to b+N}(0)|^2 \int \left(\left| \int_{-\infty}^{\infty} n(\rho, z) \exp\left(-iq_{min} z + i\delta(\rho, z)\right) dz \right|^2 \right) d^2\vec{\rho}$$

$$= \frac{8\pi^3}{k^2} n_0^2 \frac{|f_{a+N \to b+N}(0)|^2}{|q_{min} + \Delta\chi|^2} \int_0^R \Big| \exp[-2i(q_{min} - \chi_a)\sqrt{R^2 - \rho^2}] - \exp[2i\chi_b \sqrt{R^2 - \rho^2}] \Big|^2 \rho \, d\rho;$$

$\Delta\chi = \chi_b - \chi_a$; quantity $\mathrm{Re}\,\Delta\chi \to$ determines the additional longitudinal transferred momentum connected with the presence of matter.

The limit of very high energies $\rightarrow$ the minimal longitudinal transferred momentum $q_{min} = \frac{m_a^2 - m_b^2}{2k}$ tends to zero $\Rightarrow$ the dependence of $\sigma_{coh}(a \to b)$ on the particle masses m_a, m_b vanishes.

③ Dependence of cross-sections of inelastic coherent processes on the nucleus radius.

3 main cases: $\left(L_a = \frac{1}{n_0 \sigma_{aN}}, \; L_b = \frac{1}{n_0 \sigma_{bN}} \right)$

① Both the mean lengths of free path are small as compared with the nucleus radius: $L_a \ll R$, $L_b \ll R$

$\Rightarrow$ neglecting the particle masses, we obtain for this case:

$$\sigma_{coh}(a \to b) = \pi \frac{|f_{a+N\to b+N}(0)|^2}{|f_{b+N\to b+N}(0) - f_{a+N\to a+N}(0)|^2} \times$$

$$\times \left[\frac{L_a^2}{2} + \frac{L_b^2}{2} + 4 L_a^2 L_b^2 \, Re \left(\frac{1}{L_a + L_b + i(L_a \alpha_b - L_b \alpha_a)} \right)^2 \right]$$

if the elastic scattering amplitudes do not coincide

$(f_{b+N\to b+N}(0) \neq f_{a+N\to a+N}(0))$.

2) $L_a \gg R$, but $L_b \lesssim R$ (i.e. $\sigma_{aN} \ll \sigma_{bN}$ → total cross-section of interaction of the initial particle a with nucleons is small

Here we have: $|f_{a+N\to a+N}(0)| \ll |f_{b+N\to b+N}(0)|$

(particular case → coherent production of vector mesons ρ^0, ω, φ at interaction of very high energy photons with nuclei).

Cross-section $\sigma_{coh}(a\to b)$ takes the form (neglecting the particle masses as before):

$$\sigma_{coh}(a\to b) = \pi R^2 \left|\frac{f_{a+N\to b+N}(0)}{f_{b+N\to b+N}(0)}\right|^2 \times$$

$$\times \left\{ 1 + \frac{1}{x^2}\left[\frac{1}{2}(1-e^{-2x}) - 4\frac{1-\alpha^2}{(1+\alpha^2)^2}(1-e^{-x}\cos\alpha x) - \frac{8\alpha}{(1+\alpha^2)^2}e^{-x}\sin\alpha x\right] + \frac{1}{x}\left[\frac{4}{1+\alpha^2}e^{-x}\cos\alpha x - \frac{4\alpha}{1+\alpha^2}e^{-x}\sin\alpha x - e^{-2x}\right]\right\}$$

($\alpha \equiv \alpha_b$; $x = n_0\sigma_{bN}R = R/L_b$).

For $x \gg 1$ (large cross-sections σ_{bN}, heavy nuclei)
→ simple expression:

$$\sigma_{coh}(a \to b) = \pi R^2 \left| \frac{f_{a+N \to b+N}(0)}{f_{b+N \to b+N}(0)} \right|^2$$

⇒ in this limit $\sigma_{coh}(a \to b)$ is proportional to the cross-section πR^2 on the „black" nucleus, despite the smallness of the cross-section of interaction of the initial particle a (e.g. γ-quantum) with a separate nucleon.

For the coherent process $\gamma \to \rho^0$ on the lead nucleus ($R = 1.1 \cdot 10^{-13} \cdot A^{1/3}$ cm ≈ 6.5 Fm, $L_\rho \sim 1.5$ Fm, $\left| \frac{f_{\gamma+N \to \rho+N}(0)}{f_{\rho+N \to \rho+N}(0)} \right|^2 \sim 10^{-3}$) → the above expression is applicable at γ-quantum energies above several tens of GeV in the nucleus rest frame ($k \gg m_\rho^2 L_\rho \sim 4.5$ GeV).

$\sigma_{coh}(\gamma + Pb \to \rho^0 + Pb) \sim 1.3$ mbn.

Inverse situation ($L_b \gg R$, but $L_a < \sim R$) → the same expressions for $\sigma_{coh}(a \to b)$ with the replacement
$f_{b+N \to b+N}(0) \to f_{a+N \to a+N}(0)$.

3. Both the mean lengths of free path L_a, L_b strongly exceed the nucleus radius R:

$L_a \gg R$, $L_b \gg R$, $x = R/L_b \ll 1$

(the nucleus is „transparent" for the initial and final particles).

Expansion of the above formula for $\sigma_{coh}(a\to b)$ into the power series over the parameter x leads, at $x \ll 1$, to the formula obtained above in Section 1, just as one could expect:

$$\sigma_{coh}(a\to b) = \frac{9\pi}{2k^2R^2}A^2 \left| f_{a+N\to b+N}(0) \right|^2$$

$$\left(\sigma_{coh}(a\to b) \sim R^4\right).$$

At $L_a \gg R$, $L_b \ll R$ the coherent process $a \to b$ is conditioned by the interaction of particle a with nucleons located near the surface of the nucleus in the back hemisphere.

The ratio of cross-sections

$$\eta_b = \frac{\sigma_{coh}(a \to b;\ L_a \gg R,\ L_b \ll R)}{\sigma_{coh}(a \to b;\ L_a \gg R,\ L_b \gg R)} = 2\left(\frac{L_b}{R}\right)^2 \frac{1}{1+\alpha_b^2}$$

has the magnitude of the order of the squared ratio of the „transparency" volume for the final particle b in the vicinity of the back hemisphere of the nucleus surface to the total volume of the nucleus.

At $L_a \ll R$, $L_b \gg R$ the coherent process $a \to b$ is conditioned by the interaction of particle a with nucleons located near the surface of the nucleus in the front hemisphere.

The ratio of corresponding cross-sections

$$\eta_a = \frac{\sigma_{coh}(a \to b;\ L_a \ll R,\ L_b \gg R)}{\sigma_{coh}(a \to b;\ L_a \gg R,\ L_b \gg R)} = 2\left(\frac{L_a}{R}\right)^2 \frac{1}{1+\alpha_a^2}$$

has the meaning which is analogous to η_b, but with reference to the initial particle a in the vicinity of the front hemisphere of the nucleus surface.

We have not considered here the reverse transitions at the propagation of the final particle b in the nucleus matter. This is justified under the condition:

$$\left| f_{a+N\to b+N}(0) \right| \ll \left| f_{a+N\to a+N}(0) - f_{b+N\to b+N}(0) \right| .$$

In principle, the contribution of reverse transitions could be studied in the framework of the theory taking into account the distinction of the stationary states in the matter from the stationary states in the vacuum due to the mixing of the vacuum states.

4 Summary

1. Theoretical investigation of the coherent inelastic processes at the interaction of elementary particles with nuclei at ultrarelativistic energies is performed, taking into account that at very high energies the main contribution into the cross-section of coherent processes is provided by very small transferred momenta, being close to the minimal longitudinal momentum transferred to the nucleus.
2. The influence of the nucleus matter on the coherent processes is analyzed, using the conception of the refraction index.
3. Analytical formulas for the effective cross-sections of coherent inelastic processes are obtained, depending on the nucleus radius and the mean lengths of free path of the initial and final particles in the nucleus matter.