Electromagnetic Productions of $K\Sigma$ on the Nucleons

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Contents

- $\gamma + p \rightarrow K^+\Lambda$ process
- relation with the Gerasimov-Drell-Hearn sum rule
- $\gamma + N \rightarrow K\Sigma$ processes

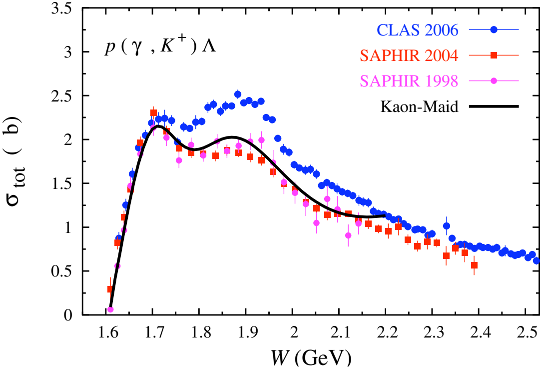
Motivation (traditional)

Elementary Operator is required for:

- On the nucleon:
  - study coupling constants $g_{K\Lambda N}$ and $g_{K\Sigma N}$
  - study N^*, Y^*, and K^* resonances $\rightarrow$ “missing resonances”
  - investigation of hadronic form factors
  - investigation of e.m. form factors
  - Gerasimov-Drell-Hearn sum-rule
- On the deutron:
  - study ΛN and ΣN potentials
- On heavier nuclei:
  - *quasi-free*: study $\Lambda(\Sigma)$-nucleus potentials
  - *bound states*: study hypernuclear production (e.g., hypertriton)

Motivation (new)

- data discrepancy
- missing resonances

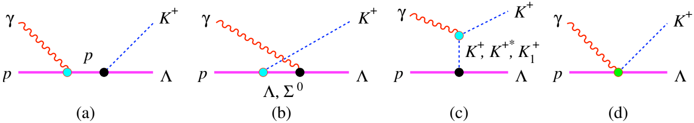


PART 1

The $\gamma + p \rightarrow K^+\Lambda$ process

Formalism

- Background Amplitudes
  - constructed from appropriate Feynman diagrams:



  - gauge method : Haberzettl
  - gauge form factor (Davidson & Workman):

$$\begin{aligned}\widehat{F}(s,t,u) &= F_1(s)+F_1(u)+F_3(t)-F_1(s)F_1(u) \\ &\quad -F_1(s)F_3(t)-F_1(u)F_3(t)+F_1(s)F_1(u)F_3(t)\ ,\end{aligned}$$

with:

$$F_i(x) = \frac{\Lambda^4}{\Lambda^4+(x-m_i^2)^2}\ ,$$

Formalism

- Resonance Amplitudes†

Breit-Wigner form:

$$A^R_{\ell\pm}(W) = \bar{A}^R_{\ell\pm}\, c_{KY}\, \frac{f_{\gamma R}(W)\,\Gamma_{\rm tot}(W) M_R\, f_{KR}(W)}{M_R^2 - W^2 - iM_R\Gamma_{\rm tot}(W)}\; e^{i\phi} \; ,$$

with the electromagnetic vertex:

$$f_{\gamma R} = \left(\frac{k_W}{k_R}\right)^{2\ell'+1} \left(\frac{X^2+k_R^2}{X^2+k_W^2}\right)^{\ell'} \; ,$$

the and hadronic vertex:

$$f_{KR}(W) = \left[\frac{1}{(2j+1)\pi}\,\frac{k_W}{|q|}\,\frac{m_N}{W}\,\frac{\Gamma_{KY}}{\Gamma^2_{\rm tot}}\right]^{1/2} \; , \quad k_W \;=\; \frac{W^2-m_N^2}{2W} \; ,$$

† Drechsel *et al.*, Nucl. Phys. A **645**, 145 (1999)

Formalism

with the partial width:

$$\Gamma_{KY} = \beta_K \Gamma_R \left(\frac{|q|}{q_R}\right)^{2\ell+1} \left(\frac{X^2+q_R^2}{X^2+q^2}\right)^{\ell} \frac{W_R}{W} \; ,$$

and the total width is the sum of Γ_{KY} and the “inelastic” width

$$\Gamma_{\rm tot} = \Gamma_{KY} + \Gamma_{\rm in} \; , \quad \Gamma_{\rm in} = (1-\beta_K)\Gamma_R \left(\frac{q_\pi}{q_0}\right)^{2\ell+4} \left(\frac{X^2+q_0^2}{X^2+q_\pi^2}\right)^{\ell+2} \; .$$

Try to use all nucleon resonances → 15

Resonances up to $\ell = 4$ with the corresponding properties from the Review of Particle Physics.

Resonance	M_R (MeV)	Γ_R (MeV)	β_K	$A_{1/2}(p)$ (10^{-3} GeV$^{-1/2}$)	$A_{3/2}(p)$ (10^{-3} GeV$^{-1/2}$)	Overall status	Status seen in $K\Lambda$
S_{11}	1650	150	0.027 ± 0.004	$+53 \pm 16$	-	****	***
	2090	400	-	-	-	*	-
P_{11}	1710	100	0.050 ± 0.020	$+9 \pm 22$	-	***	**
	2100	200	-	-	-	*	-
P_{13}	1720	150	-	$+18 \pm 30$	-19 ± 20	****	**
	1900	498	0.001 ± 0.001	-	-	**	-
D_{13}	1700	100	-	-18 ± 13	-2 ± 24	***	**
	2080	450	0.002 ± 0.002	-20 ± 8	17 ± 11	**	*
D_{15}	1675	150	-	$+19 \pm 8$	15 ± 9	****	*
	2200	130	-	-	-	**	*
F_{15}	1680	130	-	-15 ± 6	133 ± 12	****	-
	2000	490	-	-	-	**	*
F_{17}	1990	535	-	$+30 \pm 29$	86 ± 60	**	*
G_{17}	2190	450	-	-55	$+81$	****	*
G_{19}	2250	400	-	-	-	****	-

Experimental Data → 3 data sets

EXPERIMENTAL DATA SETS USED IN THE ANALYSIS (INDICATED BY $\surd$).
OTHER DATA ARE ONLY USED FOR COMPARISON.

Name	Observable	Symbol	N	Fit 1	Fit 2	Fit 3
SAPHIR 2004	Differential cross section	$d\sigma/d\Omega$	720	$\surd$	-	$\surd$
	Recoil polarization	P	30	$\surd$	-	$\surd$
	Total cross section	$\sigma_{\rm tot}$	36	-	-	-
CLAS 2006	Differential cross section	$d\sigma/d\Omega$	1377	-	$\surd$	$\surd$
	Recoil polarization	P	233	-	$\surd$	$\surd$
	Total cross section	$\sigma_{\rm tot}$	78	-	-	-
LEPS 2006	Differential cross section	$d\sigma/d\Omega$	54	$\surd$	$\surd$	$\surd$
	Photon asymmetry	Σ	30	$\surd$	$\surd$	$\surd$
OLD	Target asymmetry	T	3	-	-	-
	Total cross section	$\sigma_{\rm tot}$	24	-	-	-
Total data				834	1694	2444

Numerical Results

Contribution to χ^2 shows:

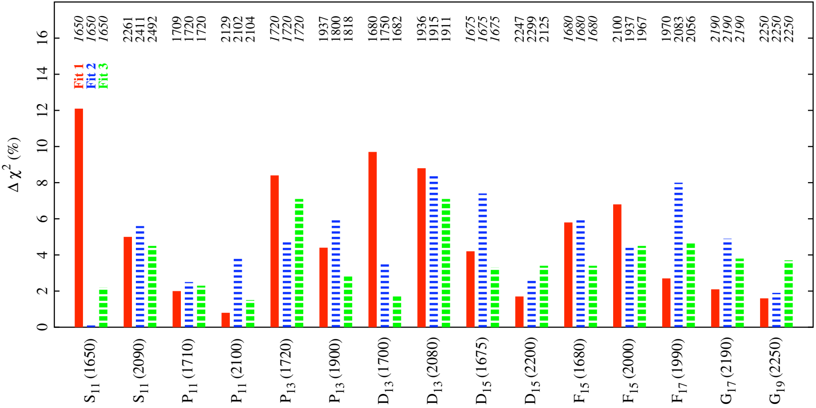
- CLAS data are internally more consistent
- LEPS data is more consistent to the CLAS ones

CONTRIBUTION TO χ^2 (IN %) FROM INDIVIDUAL DATA SETS.

Name	Observable	N	Fit 1	Fit 2	Fit 3
SAPHIR 2004	Differential cross section	720	**84**	-	39
	Recoil polarization	30	3	-	1
CLAS 2006	Differential cross section	1377	-	**74**	45
	Recoil polarization	233	-	17	9
LEPS 2006	Differential cross section	54	10	7	5
	Photon asymmetry	30	3	2	1

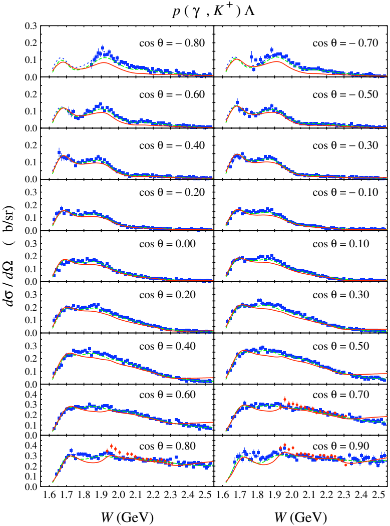
Which resonances are important?

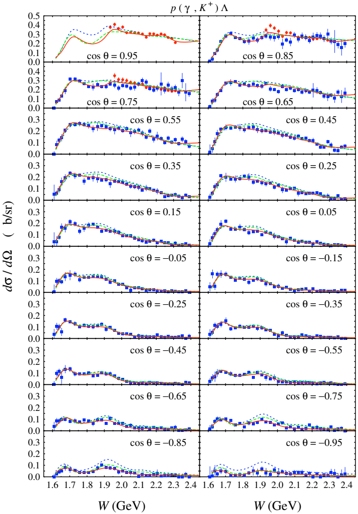
- Define: $\Delta\chi^2 = \frac{|\chi^2_{\rm All} - \chi^2_{{\rm All}-N^*}|}{\chi^2_{\rm All}} \times 100\,\%$ → LARGER IS MORE IMPORTANT
- Different data sets need different resonances
- All data sets require the $D_{13}(2080)$, whereas the $P_{11}(1710)$ is less significant



Differential Cross Sections

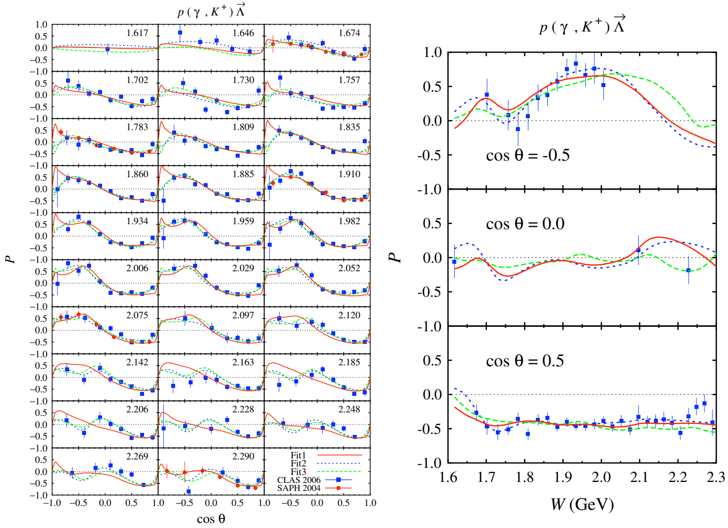
Fit 1, Fit 2, Fit 3





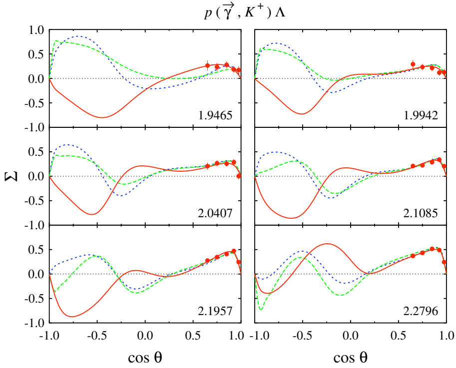
Λ Recoil Polarization

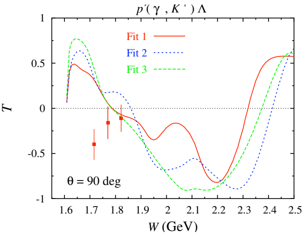
→ not a decisive observable



Beam and Target Polarizations

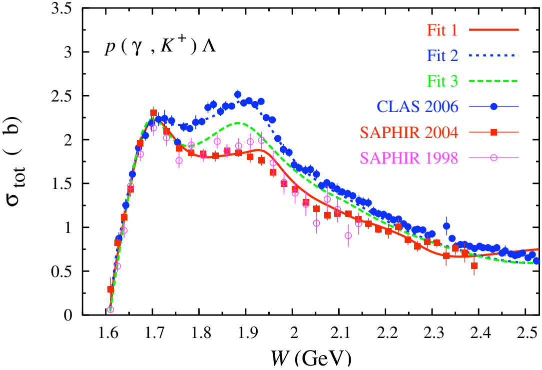
- Beam asymmetry → large variation at backward angles
- Target asymmetry → large variation & models cannot explain the (old) data





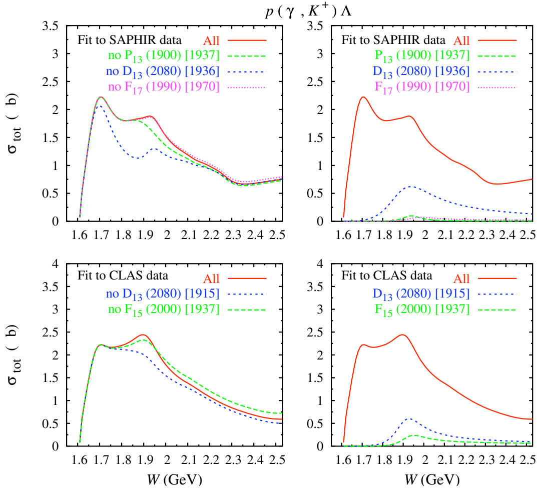
Total Cross Section

- All data shown here were not included in the fits
- Total CS data are consistent with their differential CS
- Including all data sets does not explain all data sets



Where the 2nd peak originates from?

- All data sets agree $\rightarrow$ $D_{13}(2080)$ with a mass of 1915 MeV (CLAS) or 1936 MeV (SAPHIR)



PART 2

Gerasimov-Drell-Hearn Sum Rule

The statistical differences have been also studied [P. Bydzovsky and T.M., *Phys. Rev. C* **76** (2007) 065202]

Question: Is there any observable that can be predicted by the data and can be compared with other measurement/prediction?

Answer: The Gerasimov-Drell-Hearn (GDH) Sum Rule

GDH in one minute

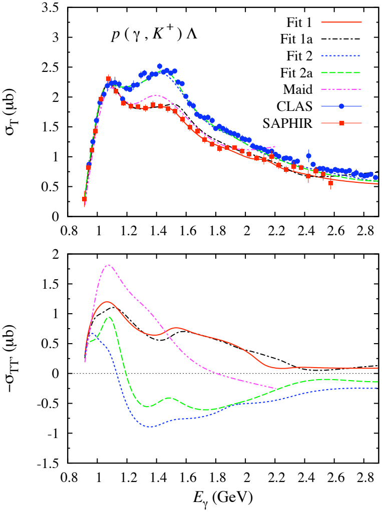
Particle Data Book dictates that $\kappa_p = 1.79284739$
The GDH sum rule says:

$$\kappa_p^2 = \frac{m_p^2}{2\pi^2\alpha}\int_{\nu_0}^{\infty}\frac{d\nu}{\nu}\left[\sigma_{1/2}(\nu) - \sigma_{3/2}(\nu)\right] ,$$

$\sigma_{3/2}$ and $\sigma_{1/2}$ indicate the cross sections of $\gamma + p \rightarrow$ everything for the possible combinations of spins of proton (1/2) and photon (1).
Let's define

$$\begin{aligned} I_{\rm GDH} &\equiv -\frac{\kappa_p^2}{4} = \frac{m_p^2}{8\pi^2\alpha}\int_{\nu_0}^{\infty}\frac{d\nu}{\nu}\left[\sigma_{1/2}(\nu) - \sigma_{3/2}(\nu)\right] \\ &= -\frac{m_p^2}{4\pi^2\alpha}\int_{\nu_0}^{\infty}\frac{d\nu}{\nu}\,\sigma_{\rm TT'}(\nu) . \end{aligned}$$

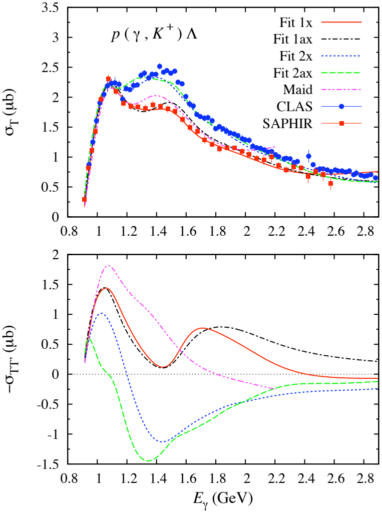
Total Cross Sections and I_{GDH}



Model	I_{GDH} (μb)
MAID	1.247
Fit 1	1.309
Fit 1a	1.274
Fit 2	-0.845
Fit 2a	-0.333

More structures in $\sigma_{TT'}$ indicating that the resonances contribute with the same order of magnitude.

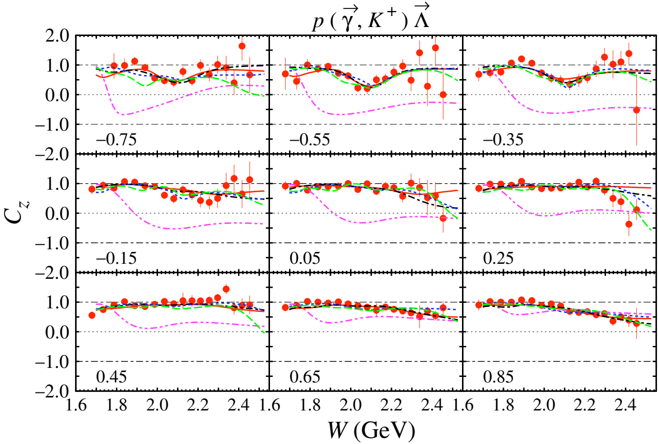
Total Cross Sections after including C_x and C_z



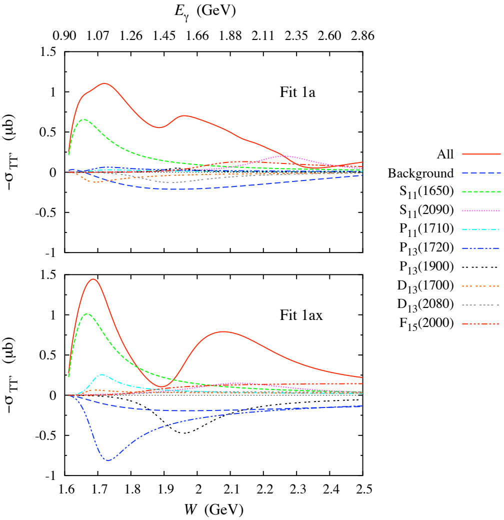
Model	I_{GDH} (μb)
MAID	1.247
Fit 1x	1.140
Fit 1ax	1.380
Fit 2x	-0.642
Fit 2ax	-1.181

Less structures in $\sigma_{TT'}$ indicating that only certain resonances contribute to this process $\rightarrow$ C_x & C_z reveals important resonances.

Example of the C_z data



Resonance Contribution to the Total Cross Sections



T.M., arXiv:0803.0601 [nucl-th]; Few-Body System 2008 (in press)

$S_{11}(1650)$, $P_{11}(1710)$, $P_{13}(1720)$, $P_{13}(1900)$ resonances are important

in agreement with other studies

Our finding shows

- SAPHIR and CLAS data yield very different $I_{\rm GDH}$
- Measurement of $\sigma_{\rm TT'}$ is recommended

PART 3

The $\gamma + N \rightarrow K\Sigma$ processes

Isospin Channels

There are four isospin channels:

- $\gamma + p \rightarrow K^+\Sigma^0$
- $\gamma + n \rightarrow K^0\Sigma^0$
- $\gamma + p \rightarrow K^0\Sigma^+$
- $\gamma + n \rightarrow K^+\Sigma^-$

with the amplitudes

$$\begin{aligned}
A(\gamma p \rightarrow K^+\Sigma^0) &= {}_pA^{(1/2)} + \tfrac{2}{3}\,A^{(3/2)} \\
A(\gamma n \rightarrow K^0\Sigma^0) &= -{}_nA^{(1/2)} + \tfrac{2}{3}\,A^{(3/2)} \\
A(\gamma p \rightarrow K^0\Sigma^+) &= \sqrt{2}\left[{}_pA^{(1/2)} - \tfrac{1}{3}\,A^{(3/2)}\right] \\
A(\gamma n \rightarrow K^+\Sigma^-) &= \sqrt{2}\left[{}_nA^{(1/2)} + \tfrac{1}{3}\,A^{(3/2)}\right]
\end{aligned}$$

where ${}_pA^{(1/2)}$ and ${}_nA^{(1/2)}$ the proton and neutron helicity photon couplings

- Use the same formalism for the multipoles
- LEPS coll. provided for the first time the $K^+\Sigma^-$ channel [H. Kohri, PRL **97**, 082003 (2006)]
- It is now possible to use the above relations (min. 3 channels)

Nucleon Resonances Used = 6

TABLE II: Nucleon resonances up to $l = 4$ considered in this analysis with the corresponding properties from the Review of Particle Physics [9].

Resonance Status	$M_R{}^a$ (MeV)	$\Gamma_R{}^a$ (MeV)	$\beta_i(\%)^b$	$A_{1/2}(N)^c$ (10^{-3} GeV$^{-1/2}$)	$A_{3/2}(N)^c$ (10^{-3} GeV$^{-1/2}$)
$D_{13}(1700)$	1650 - 1750	50 - 150	$5 - 15^{\pi}$	-18 ± 13	-2 ± 24
***	1700	100	$85 - 95^{\pi\pi}$	0 ± 50	-3 ± 44
			$0.0 \pm 1.0^{\eta}$		
			$< 3^{K\Lambda}$		
$P_{11}(1710)$	1680 - 1740	50 - 250	$10 - 20^{\pi}$	$+9 \pm 22$	-
***	1710	100	$40 - 90^{\pi\pi}$	-2 ± 14	-
			$6.2 \pm 1.0^{\eta}$		
			$5 - 25^{K\Lambda}$		
$P_{13}(1720)$	1700 - 1750	150 - 300	$10 - 20^{\pi}$	$+18 \pm 30$	-19 ± 20
****	1720	200	$> 70^{\pi\pi}$	$+1 \pm 15$	-29 ± 61
			$4 \pm 1^{\eta}$		
			$1 \pm 15^{K\Lambda}$		
$G_{17}(2190)$	2100 - 2200	300 - 700	$10 - 20^{\pi}$	-	-
****	2190	500	$0 \pm 1^{\eta}$		
$H_{19}(2220)$	2200 - 2300	350 - 500	$10 - 20^{\pi}$	-	-
****	2220	400			
$G_{19}(2250)$	2200 - 2350	230 - 800	$5 - 15^{\pi}$	-	-
****	2275	500			

Delta Resonances Used = 7

TABLE III: Delta resonances up to $\ell = 4$ considered in this analysis with the corresponding properties from the Review of Particle Physics [9]. Note: ‡Range (first row), average (second row), *decay mode is given.

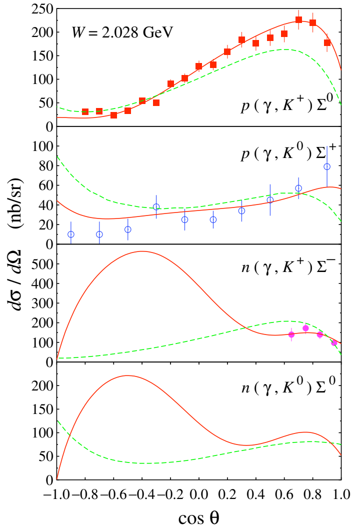
Resonance Status	$M_R{}^a$ (MeV)	$\Gamma_R{}^a$ (MeV)	$\beta_i(\%)^b$	$A_{1/2}(\Delta)^c$ $(10^{-3}\ \mathrm{GeV}^{-1/2})$	$A_{3/2}(\Delta)^c$ $(10^{-3}\ \mathrm{GeV}^{-1/2})$
$D_{33}(1700)$	1670 - 1750	200 - 400	10 - 20^{π}	$+104 \pm 15$	$+85 \pm 22$
****	1700	300	80 - $90^{\pi\pi}$		
$F_{35}(1905)$	1865 - 1915	270 - 400	9 - 15^{π}	$+26 \pm 11$	-45 ± 20
****	1890	330	85 - $95^{\pi\pi}$		
$P_{31}(1910)$	1870 - 1920	190 - 270	15 - 30^{π}	$+3 \pm 14$	-
****	1910	250			
$P_{33}(1920)$	1900 - 1970	150 - 300	5 - 20^{π}	$+40 \pm 14$	$+23 \pm 17$
***	1920	200	$2.1 \pm 0.3^{K\Sigma}$		
$D_{35}(1930)$	1900 - 2020	220 - 500	5 - 15^{π}	-9 ± 28	-18 ± 28
***	1960	360			
$F_{37}(1950)$	1915 - 1950	235 - 335	35 - 45^{π}	-76 ± 12	-97 ± 10
****	1930	285			
$H_{3,11}(2420)$	2300 - 2500	300 - 500	5 - 15^{π}	-	-
****	2420	400			

Experimental data $\rightarrow$ 2816 data points

TABLE IV: Experimental data sets used in the present analysis.

Name	Year	Observable	Channel	Symbol	N	Ref.
SAPHIR	2004	Differential cross section	$\gamma p \to K^+\Sigma^0$	$d\sigma/d\Omega$	660	[15]
		Recoil polarization	$\gamma p \to K^+\Sigma^0$	P	16	[15]
CLAS	2004	Recoil polarization	$\gamma p \to K^+\Sigma^0$	P	168	[16]
SAPHIR	2005	Differential cross section	$\gamma p \to K^0\Sigma^+$	$d\sigma/d\Omega$	120	[17]
		Recoil polarization	$\gamma p \to K^0\Sigma^+$	P	10	[17]
CLAS	2006	Differential cross section	$\gamma p \to K^+\Sigma^0$	$d\sigma/d\Omega$	1280	[18]
LEPS1	2006	Differential cross section	$\gamma p \to K^+\Sigma^0$	$d\sigma/d\Omega$	78	[19]
		Photon asymmetry	$\gamma p \to K^+\Sigma^0$	Σ	30	[19]
LEPS2	2006	Differential cross section	$\gamma p \to K^+\Sigma^0$	$d\sigma/d\Omega$	72	[20]
		Photon asymmetry	$\gamma p \to K^+\Sigma^0$	Σ	36	[20]
		Differential cross section	$\gamma n \to K^+\Sigma^-$	$d\sigma/d\Omega$	72	[20]
		Photon asymmetry	$\gamma n \to K^+\Sigma^-$	Σ	36	[20]
GRAAL	2007	Photon asymmetry	$\gamma p \to K^+\Sigma^0$	Σ	42	[21]
		Recoil polarization	$\gamma p \to K^+\Sigma^0$	P	8	[21]
CLAS	2007	Beam-Recoil Asymmetry	$\gamma p \to K^+\Sigma^0$	C_x	94	[22]
		Beam-Recoil Asymmetry	$\gamma p \to K^+\Sigma^0$	C_z	94	[22]
Total data				2816		

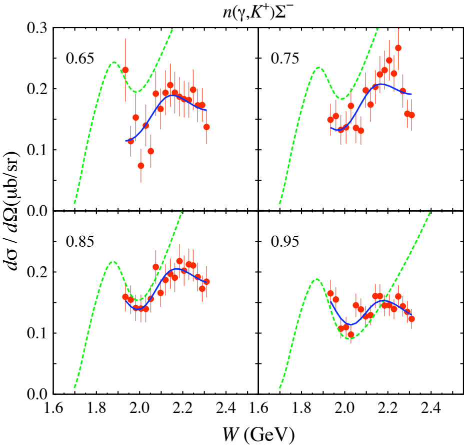
Result



Present work

MAID

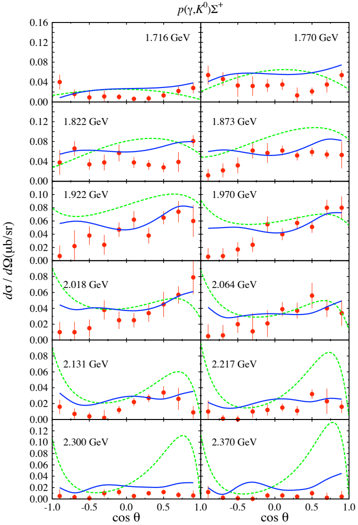
Result



Present work

MAID

Result

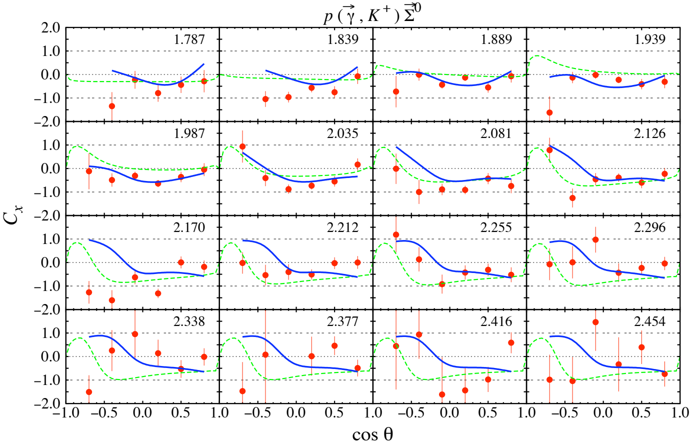


Present work

MAID

indicating that “weihting” is necessary in the fits.

Result


p (γ , K+) Σ0
1.787
1.839
1.889
1.939
1.987
2.035
2.081
2.126
2.170
2.212
2.255
2.296
2.338
2.377
2.416
2.454
2.0
1.0
0.0
−1.0
−2.0
−1.0 −0.5 0.0 0.5 1.0
C_x
cos θ

Summary

- Different data sets require different resonances.
- The *** $P_{11}(1710)$ is found insignificant for the $\gamma p \to K^+\Lambda$.
- All data sets need the $D_{13}(2080)$ with a mass $\sim$ 1915 – 1936 MeV to explain the second peak in cross sections at $W \sim 1900$ MeV.
- SAPHIR and CLAS data yield different $K\Lambda$ contribution to the GDH sum rule.
- Measurement of $\sigma_{\mathrm{TT}'}$ is recommended
- A new model for 4 $K\Sigma$ channels has been created.