Holographic flavors in Chern-Simons-matter theories

A.V. Ramallo
Univ. Santiago

Based on 1105.6045, 1211.0630, 1309.4453, 1311.6265, 1411.3335
with E. Conde, N. Jokela, J. Mas, Y. Bea, D. Zoakos and M. Lippert

Plan of the talk

- Review of the ABJM model
- Addition of flavor
- Backreacted flavored backgrounds
- Flavor effects
- Massive flavors and flows
- Hall states
- Summary&Outlook

ABJM Chern-Simons-matter theories

(Aharony et al. 0806.1218)

Associated to M2-branes in $\mathbb{C}^4/\mathbb{Z}_k$ in M-theory

Field Theory Chern-Simons-matter theories in 2+1 dimensions

gauge group: $U(N)_k \times U(N)_{-k}$

Field content (bosonic)

-Two gauge fields A_μ, $\hat{A}_\mu$

-Four complex scalar fields: C^I $(I = 1, \cdots, 4)$ bifundamentals $(N, \bar{N})$

Action

$$S = k\, CS[A] - k\, CS[\hat{A}] - k\, D_\mu C^{I\,\dagger} D^\mu C^I - V_{\text{pot}}(C)$$

$V_{\text{pot}}(C) \rightarrow$ sextic scalar potential

The ABJM model has $\mathcal{N} = 6$ SUSY in 3d

It has two parameters

$N \longrightarrow$ rank of the gauge groups

$k \longrightarrow$ CS level $(1/k \sim$ gauge coupling$)$

't Hooft coupling $\lambda \sim \frac{N}{k}$

It is a CFT in 3d with very nice properties

-partition function and Wilson loops can be obtained from localization! (Mariño, Putrov, Drukker)

-has many integrability properties (Bethe ansatz, Wilson loop/amplitude relation, ...)

-connection to FQHE?

It is the 3d analogue of N=4 SYM

Sugra description in type IIA

$$AdS_4 \times \mathbb{CP}^3 + \text{ fluxes}$$

$$ds^2 = L^2\, ds^2_{AdS_4} + 4\, L^2\, ds^2_{\mathbb{CP}^3}$$

$$L^4 = 2\pi^2 \frac{N}{k}$$

$$F_2 = 2k\, J \qquad F_4 = \frac{3\pi}{\sqrt{2}} \left(kN \right)^{\frac{1}{2}} \Omega_{AdS_4}$$

$$e^{\phi} = \frac{2L}{k} = 2\sqrt{\pi} \left(\frac{2N}{k^5} \right)^{\frac{1}{4}}$$

Effective description for $N^{\frac{1}{5}} << k << N$

Flavor in Chern-Simons-matter systems in 2+1

Hohenegger&Kirsch 0903.1730

Gaiotto&Jafferis 0903.2175

Flavor branes

D6-branes extended in AdS_4 and wrapping $\mathbb{RP}^3 \subset \mathbb{CP}^3$

Introduce quarks in the $(N, 1)$ and $(1, N)$ representation

$$Q_1 \to (N, 1) \qquad Q_2 \to (1, N) \qquad \tilde{Q}_1 \to (\bar{N}, 1) \qquad \tilde{Q}_2 \to (1, \bar{N})$$

coupling to the vector multiplet

$$Q_1^\dagger \, e^{-V} \, Q_1 \, + \, Q_2^\dagger \, e^{-\hat{V}} \, Q_2 \, + \, \text{antiquarks}$$

$V, \hat{V}$ vector supermultiplets for $A\,, \hat{A}$

coupling to the bifundamentals

$\Longrightarrow$ $C^I \, = (A_1, A_2, B_1^\dagger, B_2^\dagger)$

$$\tilde{Q}_1 \, A_i B_i \, Q_1 \;, \qquad \tilde{Q}_2 \, B_i \, A_i \, Q_2$$

plus quartic terms in $Q, \tilde{Q}$'s

Probe $\rightarrow$ quenched (neglecting quark loops)

Backreaction $\rightarrow$ unquenched (dynamical quarks)

When backreaction is included we have the coupling

$$S_{WZ} = T_{D_6} \sum_{i=1}^{N_f} \int_{\mathcal{M}_7^{(i)}} \hat{C}_7 \rightarrow T_{D_6} \int_{\mathcal{M}_{10}} C_7 \wedge \Omega$$

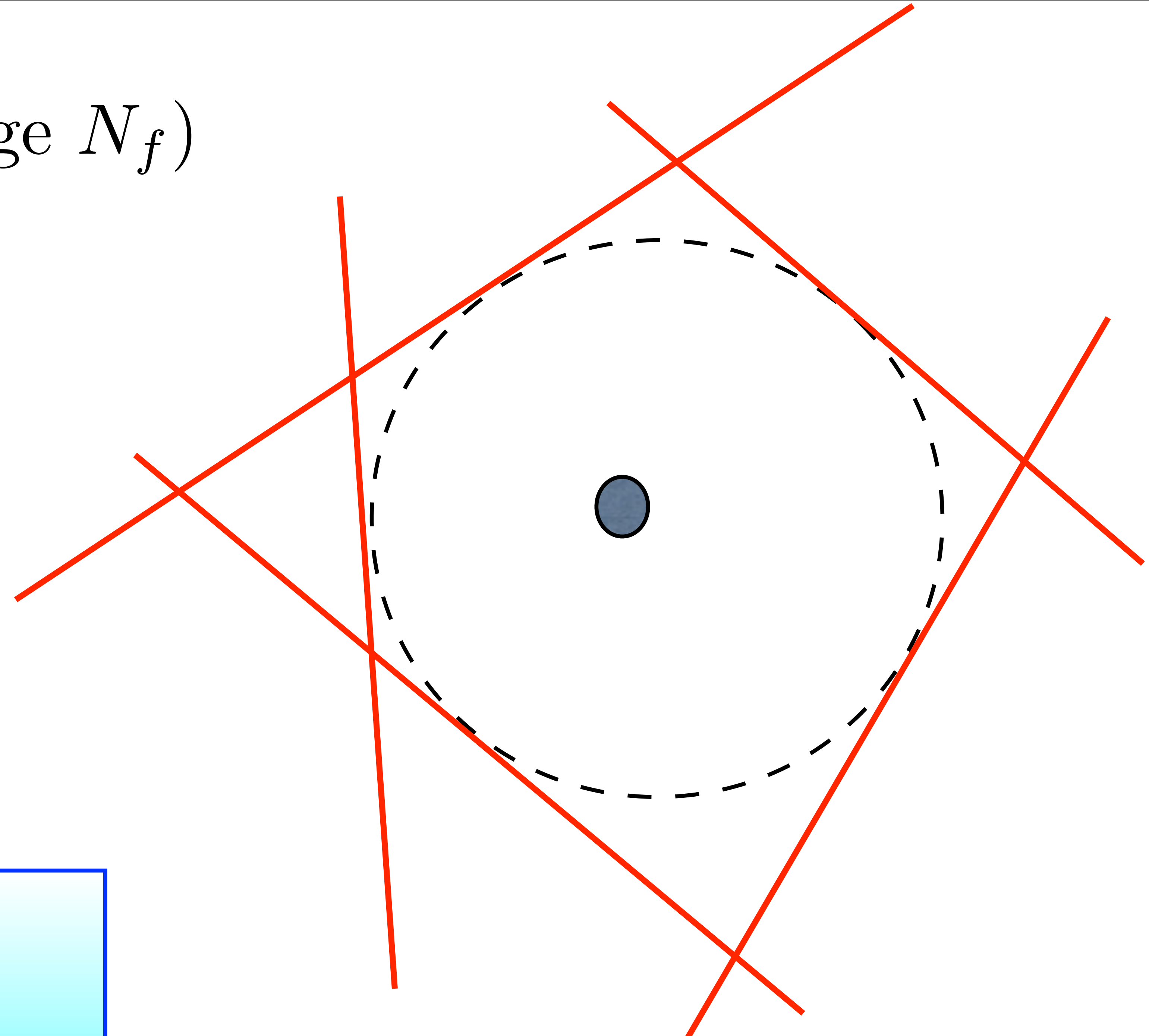
Ω is a charge distribution 3-form

Modified Bianchi identity

$$dF_2 = 2\pi \, \Omega$$

One can keep conformality with massless flavor!

Smeared sources (for large N_f)



-no delta-function sources

-still can preserve (less) SUSY

-much simpler (analytic) solutions

$-\text{flavor symmetry}: U(1)^{N_f}$

how can one find these delocalized solutions?

Backreaction with smearing (massless flavors) (E. Conde and AVR)

Write $\mathbb{CP}^3$ as an $\mathbb{S}^2$-bundle over $\mathbb{S}^4$

$$ds^2_{\mathbb{CP}^3} = \frac{1}{4}\Big[ds^2_{\mathbb{S}^4} + \big(dx^i + \epsilon^{ijk} A^j x^k\big)^2\Big]$$

Fubini-Study metric

$$\sum_i (x^i)^2 = 1$$

$A^i \rightarrow SU(2)$ instanton on $\mathbb{S}^4$

The RR two-form F_2 can be written as:

$$F_2 = \frac{k}{2}\Big(E^1 \wedge E^2 - \big(\mathcal{S}^4 \wedge \mathcal{S}^3 + \mathcal{S}^1 \wedge \mathcal{S}^2\big)\Big)$$

$$\frac{1}{2\pi}\int_{\mathbb{CP}^1} F_2 = k$$

$\mathcal{S}^i \rightarrow$ (rotated) basis of one-forms along $\mathbb{S}^4$

$E^i \rightarrow$ one-forms along the $\mathbb{S}^2$ fiber

Some Killing spinors are constant in this basis ⟹ deform to preserve them

Prescription: squash F_2 and the metric

$$F_2 = \frac{k}{2}\Big[\,E^1\wedge E^2 - \eta\,(\mathcal{S}^4\wedge\mathcal{S}^3 + \mathcal{S}^1\wedge\mathcal{S}^2)\,\Big]$$

Induces violation of Bianchi identity

$$\eta \equiv 1 + \frac{3N_f}{4k}$$

Deformation parameter

$$\epsilon \equiv \frac{N_f}{k} = \frac{N_f}{N}\,\lambda$$

Flavored metric

$$ds^2 = L^2\, ds^2_{AdS_4} + ds^2_6$$

$$ds^2_6 = \frac{L^2}{b^2}\Big[\,q\,ds^2_{\mathbb{S}^4} + \big(dx^i + \epsilon^{ijk}\,A^j\,x^k\,\big)^2\Big]$$

$q \rightarrow \mathbb{C}\mathbb{P}^3$ internal squashing

$b \rightarrow$ relative $AdS_4/\mathbb{C}\mathbb{P}^3$ squashing

$\mathcal{N} = 1$ superconformal SUSY implies

$$q^2 - 3(1+\eta)\,q + 5\eta = 0$$

$$q = 3 + \frac{9}{8}\,\frac{N_f}{k} - 2\sqrt{1 + \frac{3}{4}\,\frac{N_f}{k} + \left(\frac{3}{4}\right)^4 \left(\frac{N_f}{k}\right)^2}$$

Also

$$b = \frac{2q}{q+1}$$

The new AdS_4 radius is:

$$L^4 = 2\pi^2\,\frac{N}{k}\,\frac{(2-q)\,b^4}{q(q+\eta q-\eta)} \equiv 2\pi^2\,\frac{N}{k}\,\sigma$$

$\sigma = \sigma(\epsilon) \rightarrow$ screening function

Dilaton and F_4:

$$e^{-\phi} = \frac{b}{4}\,\frac{\eta+q}{2-q}\,\frac{k}{L}$$

$$F_4 = \frac{3kb}{4}\,\frac{\eta+q}{2-q}\,L^2\,\Omega_{AdS_4}$$

Regime of validity

$$L >> 1\,, \qquad e^{\phi} << 1$$

If $N_f/k \sim 1$ $\Rightarrow$ $N^{\frac{1}{5}} << k << N$ (same as in the unflavored case)

When $N_f >> k$

$$L^4 \sim \frac{N}{N_f} \qquad e^{\phi} \sim \left(\frac{N}{N_f^5}\right)^{\frac{1}{4}} \Rightarrow N^{\frac{1}{5}} << N_f << N$$

Flavor effects

Free energy on the 3-sphere (measures # dof's)

$$F(\mathbb{S}^3) = -\log|Z_{\mathbb{S}^3}| \quad \longrightarrow \quad \boxed{F(\mathbb{S}^3) = \frac{\pi L^2}{2G_N}} \quad \longleftarrow \quad \frac{1}{G_N} = \frac{1}{G_{10}} e^{-2\phi}\, Vol(\mathcal{M}_6)$$

In flavored ABJM

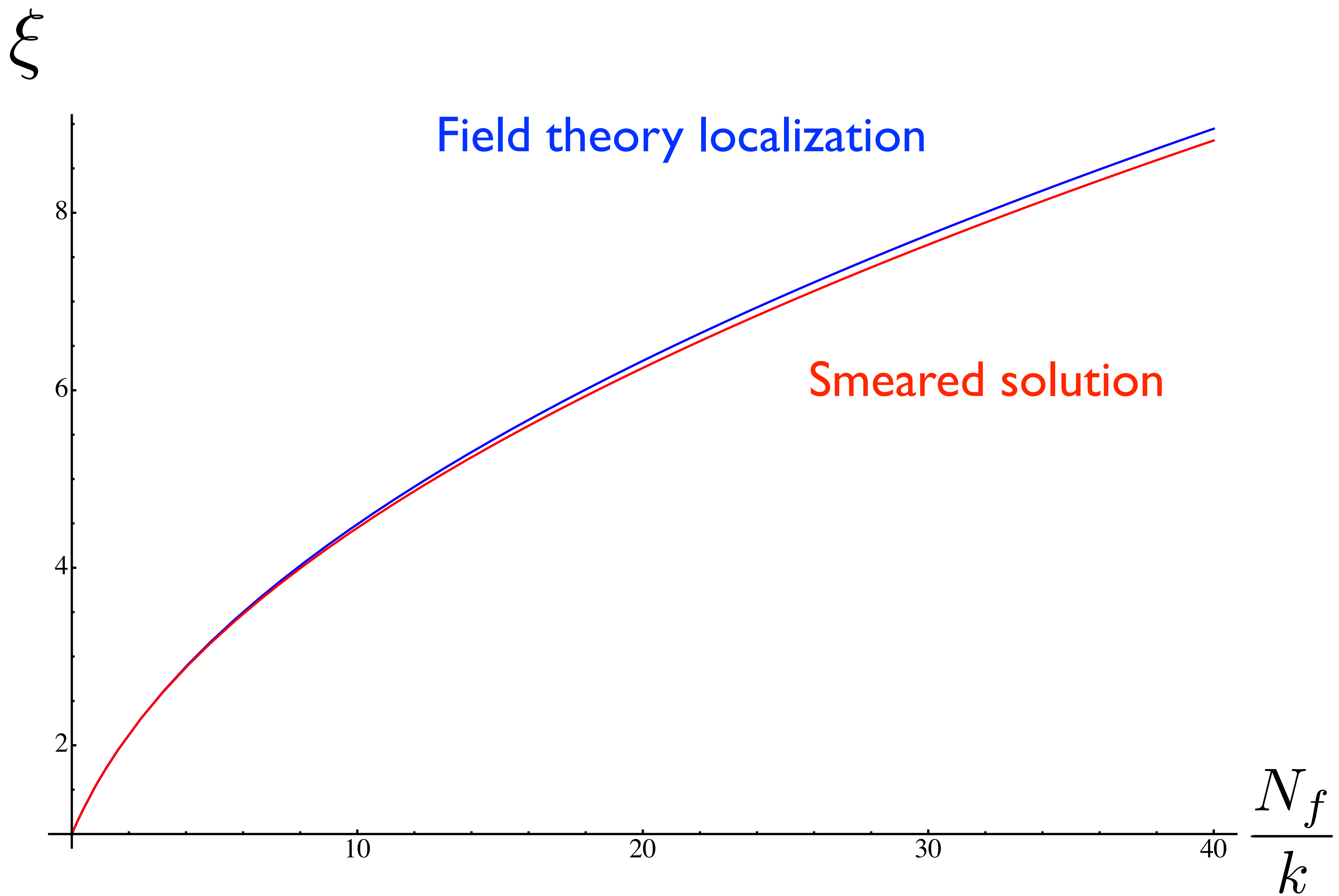
$$\boxed{F(\mathbb{S}^3) = \frac{\pi\sqrt{2}}{3} k^{\frac{1}{2}} N^{\frac{3}{2}} \xi\Big(\frac{N_f}{k}\Big)} \qquad \xi\Big(\frac{N_f}{k}\Big) \equiv \frac{1}{16} \frac{q^{\frac{5}{2}} (\eta+q)^4}{(2-q)^{\frac{1}{2}} (q+\eta q-\eta)^{\frac{7}{2}}}$$

For small N_f/k

$$\xi = 1 + \frac{3}{4}\frac{N_f}{k} - \frac{9}{64}\Big(\frac{N_f}{k}\Big)^2 + \mathcal{O}\Big(\Big(\frac{N_f}{k}\Big)^3\Big)$$

$$\boxed{F(\mathbb{S}^3) = \frac{\pi\sqrt{2}}{3}\frac{N^2}{\sqrt{\lambda}} + \frac{\pi\sqrt{2}}{4} N_f N \sqrt{\lambda} - \frac{3\pi\sqrt{2}}{64} N_f^2 \lambda^{\frac{3}{2}} + \cdots}$$

unflavored term $\sim N^{\frac{3}{2}}$ $\longrightarrow$ amazing field theory match by Drukker et al. (1007.3837) !



Field theory results: Couso-Santamaria et al. 1011.6281 quark-antiquark energy

$$V_{q\bar{q}} = -\frac{Q}{d}$$

$$Q = \frac{4\pi^3\sqrt{2\lambda}}{\left[\Gamma\left(\frac{1}{4}\right)\right]^4}\,\sigma$$

Series expansion

$$\sigma = 1 - \frac{3}{8}\frac{N_f}{k} + \frac{9}{64}\left(\frac{N_k}{k}\right)^2 + \cdots$$

For large deformation parameter

$$\sigma \sim \sqrt{\frac{k}{N_f}} \to 0$$

Dynamical quarks screen the Coulomb interaction

Squashings encode the effect of dynamical flavors

$$\lim_{\epsilon\to\infty} q = \frac{5}{3}$$

$$\lim_{\epsilon\to\infty} b = \frac{5}{4}$$

Flavor brane probes in flavored ABJM

→ D6 extended in x^μ, r, $\mathbb{RP}^3$ → -2 directions inside $\mathbb{S}^4$; -1 direction inside $\mathbb{S}^2$

Embedding → Write the $\mathbb{S}^2$ metric as $ds^2 = d\theta^2 + \sin^2\theta\, d\varphi^2$

D6 $\rightarrow$ extended in φ with a profile $\theta(r)$

Induced metric

$$\frac{ds_7^2}{L^2} = r^2\, dx_{1,2}^2 + \left(\frac{1}{r^2} + \frac{\theta'^2}{b^2}\right) dr^2 + \frac{ds_3^2}{L^2}$$

Line element of the cycle

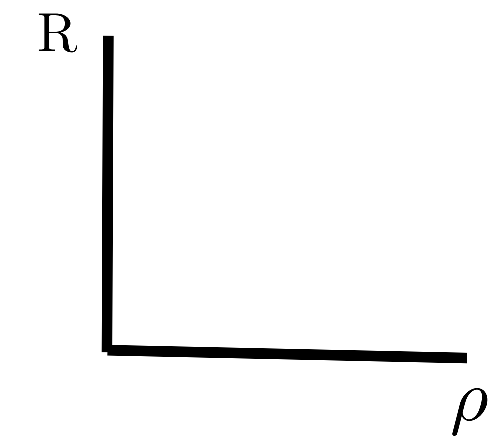
$$\frac{ds_3^2}{L^2} = \frac{1}{b^2}\Big[q^2\, d\alpha^2 + q^2\, \sin^2\alpha\, d\beta^2 + \sin^2\theta\,(d\psi + \cos\alpha\, d\beta)^2\Big]$$

At the tip $\theta = 0 \rightarrow$ non-collapsing $\mathbb{S}^2$

New Cartesian-like coordinates

$$R = r^b \cos\theta \qquad\qquad \rho = r^b \sin\theta$$

$b \rightarrow$ relative $AdS_4/\mathbb{C}\mathbb{P}^3$ squashing



(θ, r) metric $\Longrightarrow$ $\dfrac{L^2}{b^2(\rho^2+R^2)}\left[d\rho^2 + dR^2\right]$

Embeddings $\rightarrow R = R(\rho)$ with the UV behavior

$$R(\rho) \sim m + \frac{c}{\rho^{\frac{3}{b}-2}} \sim m + \frac{c}{r^{3-2b}}$$

$m \rightarrow$ mass

$c \rightarrow$ condensate

Compare with

$$\phi \sim \phi_0\, r^{\Delta-3} + \frac{\langle \mathcal{O} \rangle}{r^{\Delta}}$$

ϕ_0 is the source of $\mathcal{O}$

$\Delta \rightarrow$ dimension of $\mathcal{O}$

In our case $\mathcal{O} \sim \bar{\psi}\psi$

$$\boxed{\Delta = 3 - b}$$

$$\dim(\bar{\psi}\psi) = 3 - b$$

$\longrightarrow$

$$\dim(\bar{\psi}\psi) = 2 - \frac{3}{16}\frac{N_f}{k} + \frac{63}{512}\left(\frac{N_f}{k}\right)^2 + \cdots$$

$$\dim(\bar{\psi}\psi) \to \frac{7}{4} \qquad \left(\frac{N_f}{k} \to \infty\right)$$

Mass anomalous dimension

$$\gamma_m = b - 1$$

γ_m grows with N_f $\longrightarrow$ $\displaystyle\lim_{N_f \to \infty} \gamma_m = \frac{1}{4}$

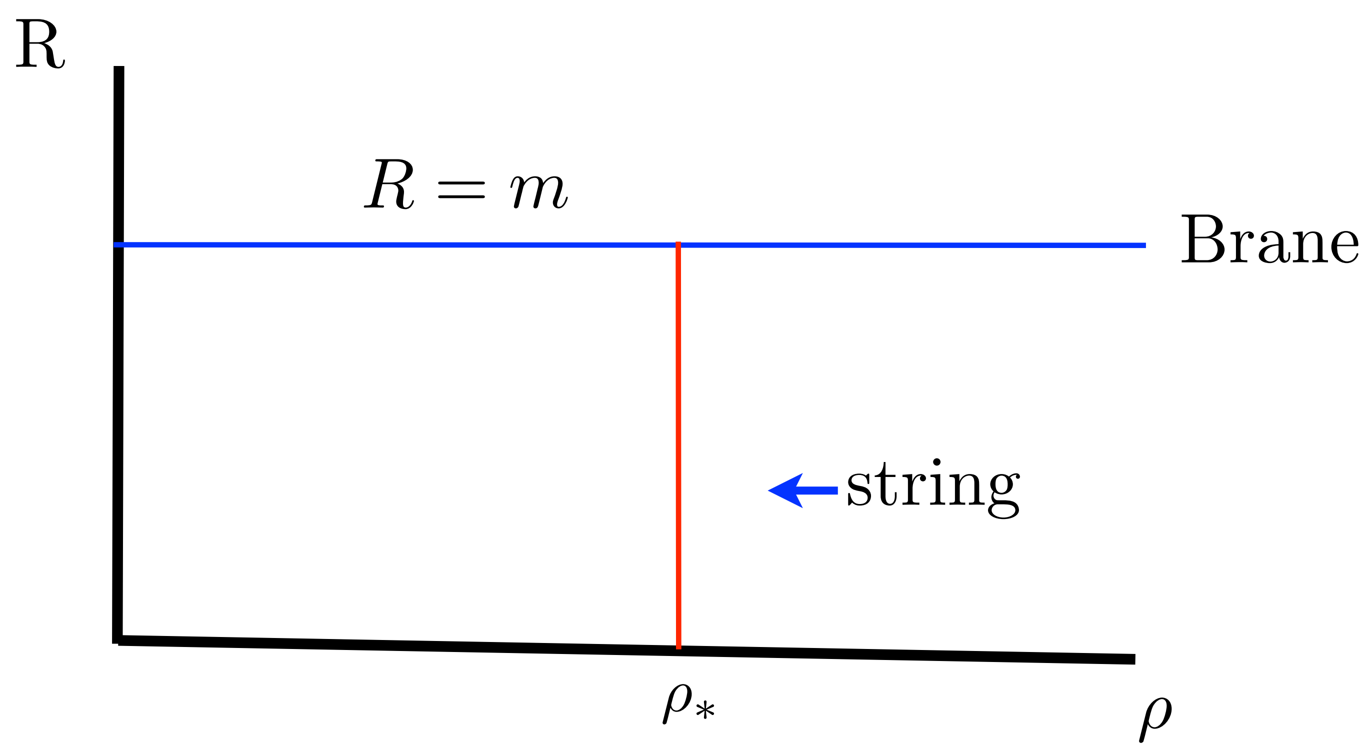
SUSY embeddings

$R = m \qquad c = 0$ $\longrightarrow$

$$\cos\theta(r) = \frac{m}{r^b}$$

Running quark mass

Consider a string extended in the R direction
from $R = 0$ to $R = m$ at the holographic scale $\rho = \rho_*$



Induced metric on the string

$$ds_2^2 = -L^2 \left[R^2 + \rho_*^2\right]^{\frac{1}{b}} dt^2 + \frac{L^2}{b^2} \frac{dR^2}{R^2 + \rho_*^2}$$

Effective quark mass

$$m_q = \frac{1}{2\pi(\alpha')^{\frac{3}{2}}} \int_0^m \sqrt{-\det g_2}\, dR = \sqrt{\frac{\lambda}{2}} \frac{\sigma}{b\sqrt{\alpha'}} \int_0^m \left[R^2 + \rho_*^2\right]^{\frac{1}{2b} - \frac{1}{2}} dR$$

$$m_q = \sqrt{\frac{\lambda}{2}}\,\frac{\sigma}{b\sqrt{\alpha'}}\;m\;\rho_*^{\frac{1}{b}-1}\;{}_2F_1\Big(\frac{1}{2},\frac{\gamma_m}{2b};\frac{3}{2};-\frac{m^2}{\rho_*^2}\Big)$$

$$\text{Energy scale} \rightarrow \Lambda \equiv \rho_*^{\frac{1}{b}} \sim r_*$$

In the UV:

$$\frac{\partial\, m_q}{\partial \log \Lambda} = -\gamma_m\; m_q$$

Callan-Symanzik equation

Massive flavors

In this case $\eta \to \eta(r)$ and the squashings run

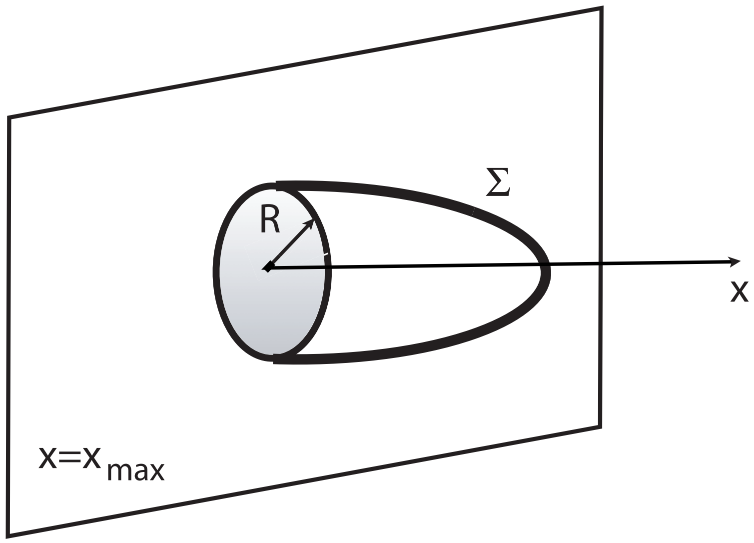
Semi analytic solution interpolating between two AdS

IR $\to$ unflavored ABJM

UV $\to$ massless flavored ABJM

The quark mass is the control parameter of the flow

Entanglement entropy of a disk



$$S_A = \frac{1}{4\,G_{10}} \int_\Sigma d^8\xi\, e^{-2\phi} \sqrt{\det g_8}$$

Ryu-Takayanagi

Finite (topological) part

$$\mathcal{F}(R) \equiv R\,\frac{\partial S}{\partial R} - S$$

Liu-Mezei

$\mathcal{F}(R)$ counts the d.o.f. at the scale R

At a conformal point → $S_{CFT}(R) = \alpha\, R - \beta$

$$\mathcal{F} = \beta = F(\mathbb{S}^3)$$

Casini-Huerta-Myers

UV limit

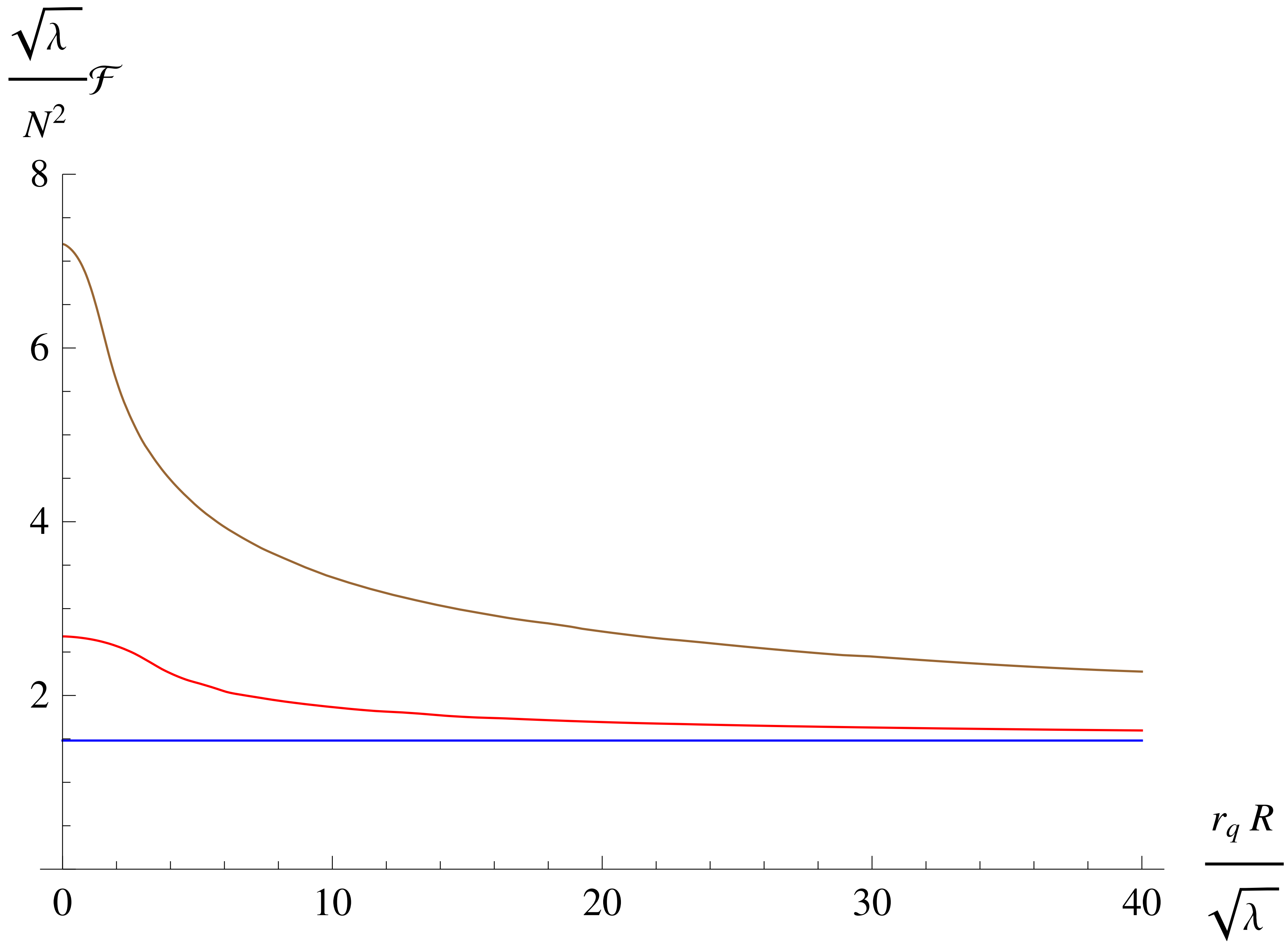
$$\mathcal{F}_{UV} \equiv \lim_{r_q R \to 0} \mathcal{F}(R) = F_{UV}(\mathbb{S}^3) = \frac{2\pi}{3} \frac{N^2}{\sqrt{2\lambda}} \xi\left(\frac{N_f}{k}\right)$$

$$\mathcal{F}(R) = F_{UV}(\mathbb{S}^3) + c_{UV}\,(r_q\,R)^{2(3-\Delta_{UV})} + \cdots$$

$$\Delta_{UV} = 3 - b$$

IR limit

$$\mathcal{F}_{IR} \equiv \lim_{r_q R \to \infty} \mathcal{F}(R) = F_{IR}(\mathbb{S}^3) = \frac{\pi\sqrt{2}}{3} k^{\frac{1}{2}}\, N^{\frac{3}{2}}$$



$\mathcal{F}$ is monotonic $\rightarrow$ F-theorem

Flavor brane with internal flux ➡ $F = F_{int} + \cdots$

WZ action of the D6

$$\int C_1 \wedge F \wedge F \wedge F \sim \int_{AdS_4} F \wedge F$$ ➡ axionic term in AdS_4 CS term on the boundary

($C_1 \wedge F$: internal)

For $N_f = 0$ ➡ Think F as induced by bulk NS-NS flat B_2

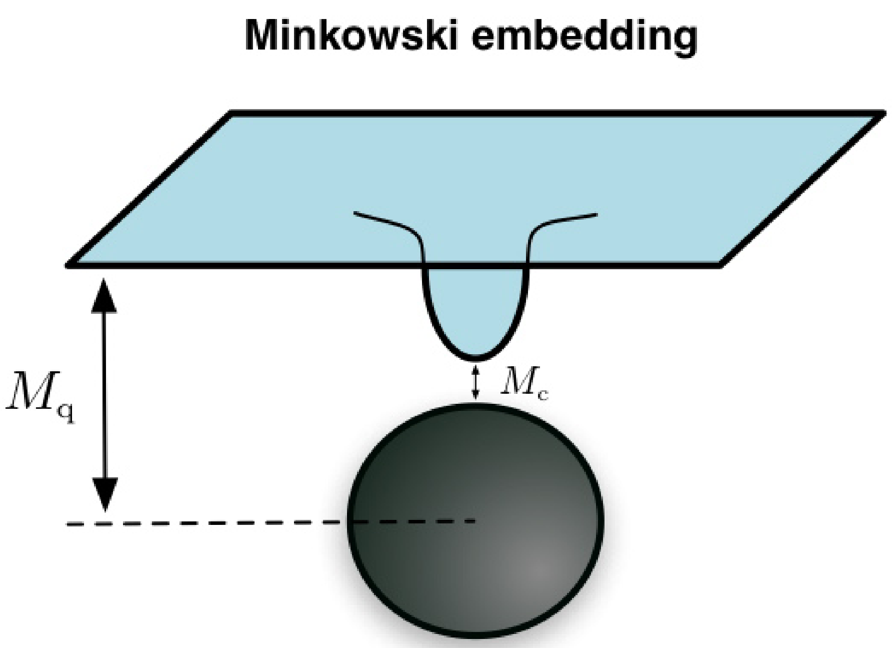
Internal flux of the ABJ model $\rightarrow U(N+M)_k \times U(N)_{-k}$

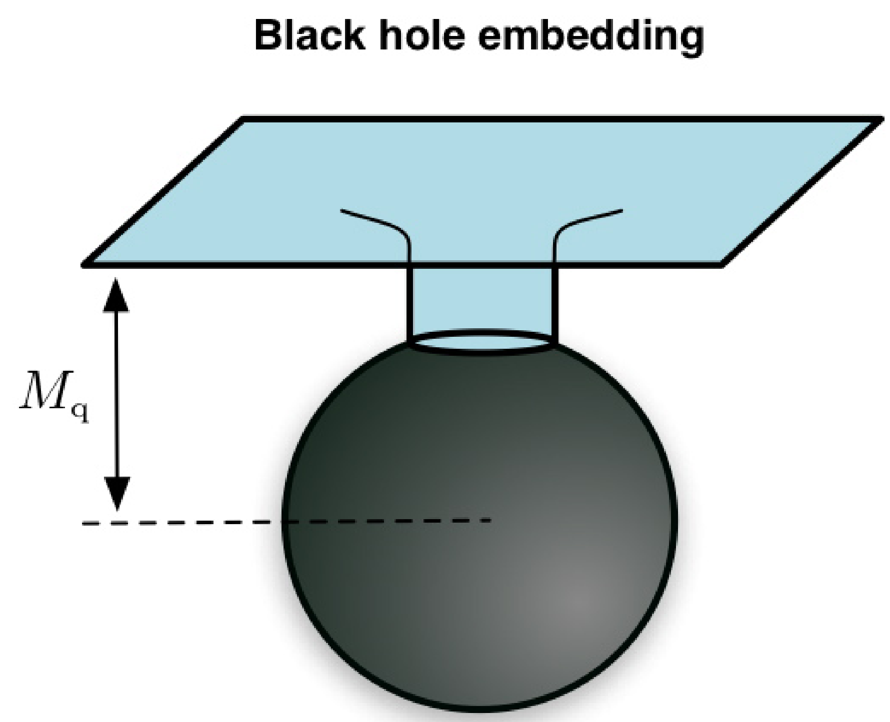
Flat $B_2 \rightarrow$ same SUGRA solution ➡ $B_2 \rightarrow$ couples to the brane

Parity is broken

$$B_2 \sim \frac{M}{k} J$$ ➡ $$\int_{\mathbb{CP}^1} B_2 = (2\pi)^2 \frac{M}{k}$$ quantization condition

Page charge for M fractional D2's





gapped

gappless

Metric at the tip of a Minkowski embedding

$$\frac{ds_7^2}{L^2}\Big|_{r=r_*} = r^2 \left[-h_* dt^2 + dx^2 + dy^2 \right] + \frac{q}{b^2} \underbrace{\left[d\alpha^2 + \sin^2\alpha \, d\beta^2 \right]}_{\text{non-collapsing } \mathbb{S}^2_*} , \qquad h_* = h(r = r_*)$$

Turn on quantized worldvolume flux on $\mathbb{S}^2_*$

$$\frac{1}{2\pi\alpha'} \int_{\mathbb{S}^2_*} F = \frac{2\pi M}{k} , \qquad M \in \mathbb{Z} .$$

Ansatz

$$A_{int} = L^2\, a(r)\, (d\psi + \cos\alpha\, d\beta)$$

$a(r) \rightarrow$ flux function to be determined

$$F\big|_{\mathbb{S}^2_*} = -L^2\, a_*\, \sin\alpha\, d\alpha \wedge d\beta \qquad\qquad a_* \equiv a(r = r_*)$$

$$a_* = -\frac{\pi M}{k L^2} \equiv -Q \quad \Longrightarrow \quad Q = \frac{\sqrt{\lambda}}{\sqrt{2}\,\sigma}\,\frac{M}{N}\,, \qquad\qquad M \in \mathbb{Z}\,.$$

SUSY solution

$$\cos\theta(r) = \left(\frac{r_*}{r}\right)^b \qquad\qquad a(r) = -Q\left(\frac{r_*}{r}\right)^{2-b}$$

General configuration with E, B, d, J_x and J_y

$$A = L^2 \left[a_0(r)\, dt + (Et + a_x(r))\, dx + (B\, x + a_y(r))\, dy + a(r)\, (d\psi + \cos\alpha\, d\beta) \right]$$

Charge density $\rightarrow$ dual to $a_0(r)$

$$\tilde{D} = \frac{2\pi\alpha'}{L^2} \frac{\partial \mathcal{L}_{DBI}}{\partial a_0'} = \frac{N\sigma^2}{4\pi}\, \tilde{d}(r) \rightarrow \text{displacement field}$$

Currents $\rightarrow$ dual to $a_x(r), a_y(r)$

$$J_x = \frac{2\pi\alpha'}{L^2} \frac{\partial \mathcal{L}_{DBI}}{\partial a_x'} = \frac{N\sigma^2}{4\pi}\, j_x \rightarrow \text{constant}$$

$$\tilde{J}_y = \frac{2\pi\alpha'}{L^2} \frac{\partial \mathcal{L}_{DBI}}{\partial a_y'} = \frac{N\sigma^2}{4\pi}\, \tilde{j}_y(r)$$

Charge & currents $\rightarrow$ boundary values

$$d = \lim_{r\to\infty} \tilde{d}(r) \qquad\qquad j_y = \lim_{r\to\infty} \tilde{j}_y(r)$$

Radial evolution

$$\tilde{j}_y(r) = j_y - E\,I(r) \qquad\qquad \tilde{d}(r) = d - B\,I(r)$$

$$I(r) = -\cos\theta(r)\,a(r) + (\eta - 1)\int_r^{\infty} \cos\theta(\bar{r})\,a'(\bar{r})\,d\bar{r}$$

$$\lim_{r\to\infty} I(r) = 0$$

For gapped Minkowski embeddings

Regularity condition at the tip $r = r_*$

$$\tilde{d}(r_*) = j_x = \tilde{j}_y(r_*) = 0 \quad \longrightarrow \quad \frac{j_y}{E} = \frac{d}{B} = I(r_*)$$

Conductivities

$$\sigma_{xx} = \frac{2\pi}{L^2}\frac{J_x}{E}\,, \qquad \sigma_{xy} = \frac{2\pi}{L^2}\frac{J_y}{E}$$

$$\sigma_{xx} = 0$$

$$\sigma_{xy} = \frac{N\sigma}{2\pi\sqrt{2\lambda}}\frac{j_y}{E} = \frac{N\sigma}{2\pi\sqrt{2\lambda}}\frac{d}{B} = \frac{N\sigma}{2\pi\sqrt{2\lambda}}I(r_*)$$

Quantum Hall state

Filling fraction

$$\nu = 2\pi\,\sigma_{xy}$$

$$\nu = \frac{M}{2}\left[1 + \frac{3N_f}{4k}\int_{r_*}^{\infty}\cos\theta(r)\,\frac{a'(r)}{Q}\,dr\right]$$

SUSY Hall state

$$E = B \; , \qquad d = j_y$$

$$\cos\theta(r) = \left(\frac{r_*}{r}\right)^b \qquad a(r) = -Q\left(\frac{r_*}{r}\right)^{2-b}$$

$$a_0' = -a_y' = \frac{(4-3b)(2-b)b\,Q\,B}{r^2}\,\frac{1-\left(\frac{r_*}{r}\right)^2}{1-\left(\frac{r_*}{r}\right)^{2b}}$$

$$d = \frac{q+\eta}{q+1}\,Q\,B$$

Filling fraction

$$\nu = \left[1+\frac{3N_f}{8k}\,(1-\gamma_m)\right]\frac{M}{2} = \frac{q+\eta}{q+1}\,\frac{M}{2}$$

Summary

- We have presented a holographic model of Chern-Simons- matter theory with (unquenched) dynamical flavor
- We studied the effects of quark loops
- We found Hall states in branes with internal flux

Outlook&Generalizations

- Non-zero temperature analysis

MN embeddings
incompressible Hall phase

⟷

BH embeddings
metallic compressible phase

- Hall effect with massive dynamical quarks

Flows with interpolating filling fraction

- Alternative quantization and anyons
- Collective excitations

⋮


Thank You!