

Semileptonic and radiative B decays

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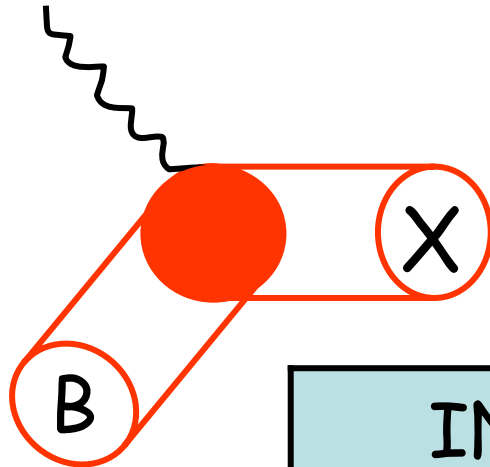
A set of interdependent measurements

$b \rightarrow c \ell \nu$	tree	$BR \sim 10\%$	$ V_{cb} $
$b \rightarrow u \ell \nu$	tree	$\sim 10^{-3}$	$ V_{ub} $
$b \rightarrow s \gamma$	loop	$\sim 3 \cdot 10^{-4}$	new physics, $ V_{ts} $
$b \rightarrow d \gamma$	loop	$\sim 10^{-6}$	new physics, $ V_{td} $

A diagram consisting of two curved arrows on the right side of the table. A red arrow starts from the right edge of the first row ($|V_{cb}|$) and points down to the right edge of the third row ($|V_{ts}|$). A blue arrow starts from the right edge of the second row ($|V_{ub}|$) and points down to the right edge of the third row ($|V_{ts}|$).

There are also $b \rightarrow s, d \ell^+ \ell^-$ to complement the radiative modes
Not only BR are relevant: various asymmetries, spectra etc

What do they have in common?



Simplicity: ew or em currents
probe the B dynamics

INCLUSIVE	EXCLUSIVE
OPE : non-pert physics described by B matrix elements of local operators can be extracted by exp suppressed by $1/m_b^2$	Form factors : in general computed by non pert methods (lattice, sum rules,...) symmetry can provide normalization

Simplicity is almost always destroyed in practical situations...

Determination of A

$$V_{CKM} = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

A can be determined using $|V_{cb}|$ or $|V_{ts}|$

Two roads to $|V_{cb}|$

EXCLUSIVE

INCLUSIVE

$|V_{cb}|$ from $B \rightarrow D^* l \nu$

At zero recoil, where rate vanishes.
Despite extrapolation, exp error $\sim 2\%$

Main problem is form factor $F(1)$

The non-pert quantities relevant for excl
decays cannot be experimentally determined

Must be calculated but HQET helps.

$$F_{B \rightarrow D^*}(1) = \eta_A [1 - O(1/m_b, 1/m_c)^2]$$

Lattice QCD: $F(1) = 0.91^{+0.03}_{-0.04}$

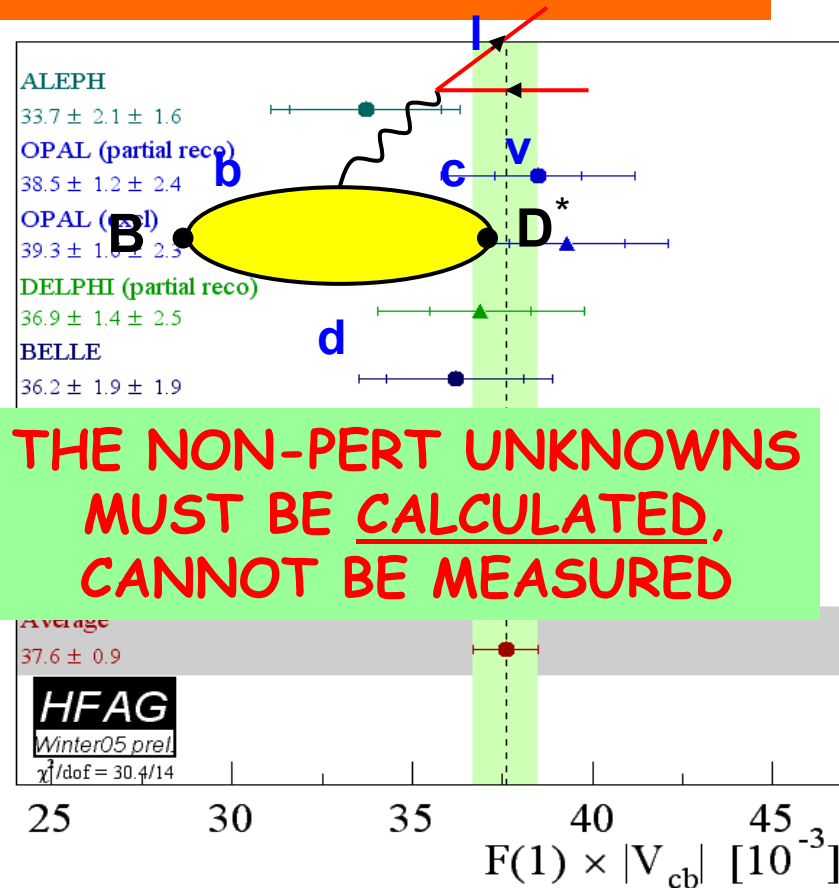
Sum rules give consistent results

Needs unquenching (under way)

Even slope may be calculable...

$\delta V_{cb}/V_{cb} \sim 5\%$ and agrees with inclusive det, despite contradictory exps

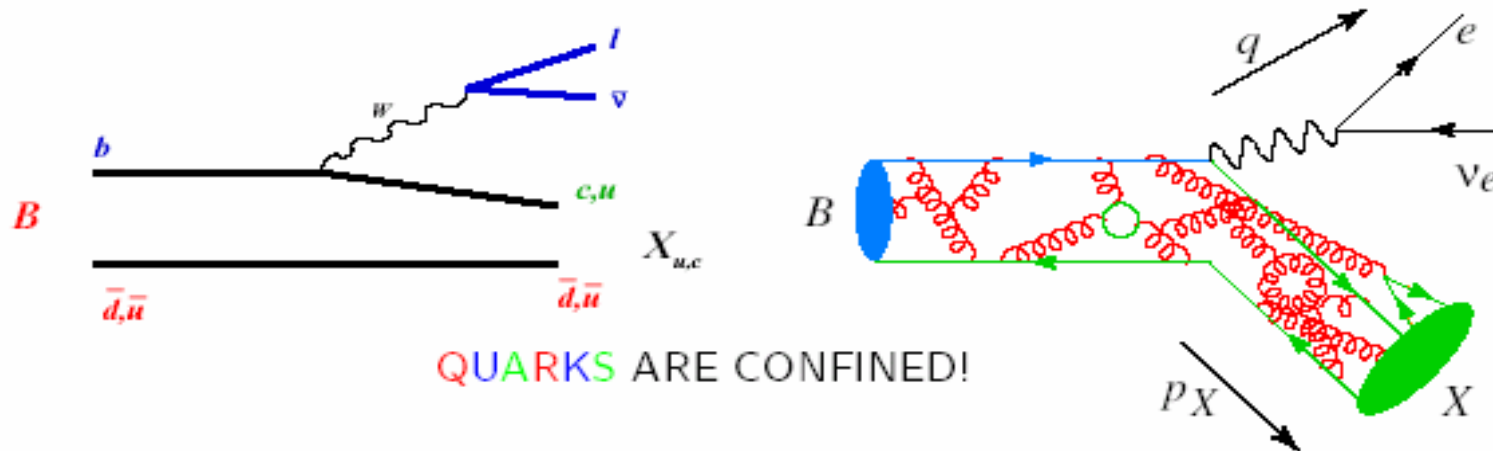
$B \rightarrow D l \nu$ gives consistent but less precise results; lattice control is better



The advantage of being inclusive

$\Lambda_{\text{QCD}} \ll m_b$: inclusive decays admit systematic expansion in Λ_{QCD}/m_b
 Non-pert corrections are generally small and can be controlled

Hadronization probability = 1 because we sum over all states
 Approximately insensitive to details of meson structure as $\Lambda_{\text{QCD}} \ll m_b$
 (as long as one is far from perturbative singularities)



$\frac{d^2 \Gamma}{dE_l dq^2 dq_0}$ can be expressed as double series in α_s and Λ_{QCD}/m_b (OPE)
 with parton model as leading term **No $1/m_b$ correction!**

A double expansion

$$\frac{d^2\Gamma}{dE_l dq^2 dq_0}$$

can be expressed in terms of *structure functions* related to Im of

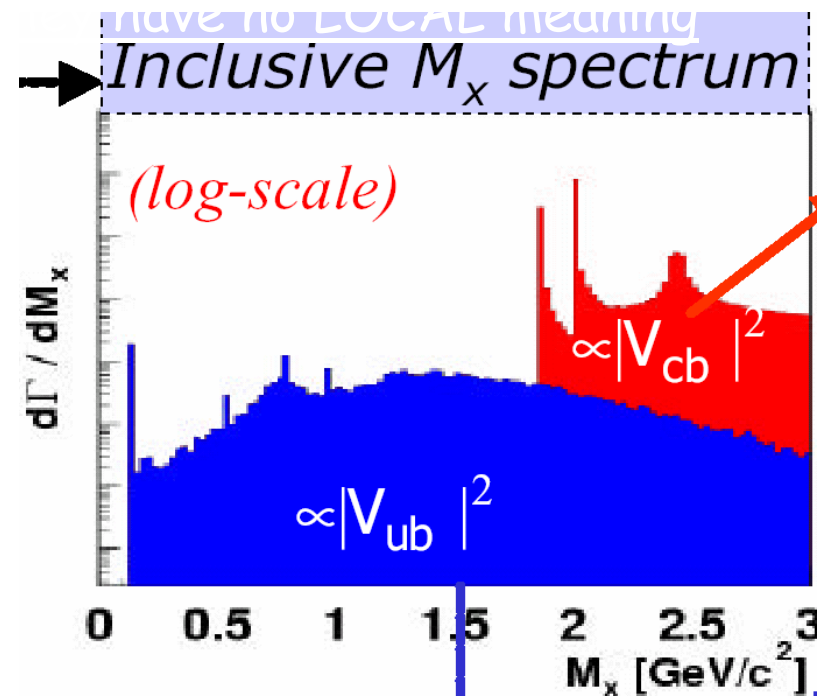
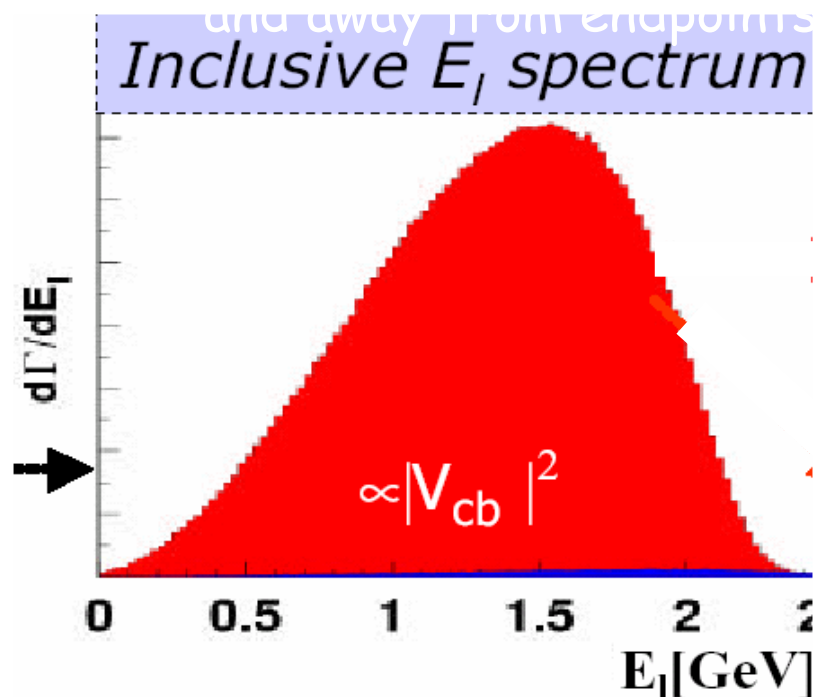
$$h_{\mu\nu}(q^2, q_0) = \frac{1}{2M_B} \langle B | \int d^4x e^{-iqx} iT \{ J_\mu(x), J_\nu^\dagger(0) \} | B \rangle$$

$$\text{OPE (HQE): } T J(x) J(0) \approx c_1 \bar{b}b + c_2 \bar{b} \overrightarrow{D}^2 b + c_3 \bar{b} \sigma \cdot G b + \dots$$

- The leading term is parton model, c_i are series in α_s
- New operators have non-vanishing expectation values in B and are suppressed by powers of the energy released, $\underline{E_r \sim m_b - m_c}$
- **No $1/m_b$ correction!**

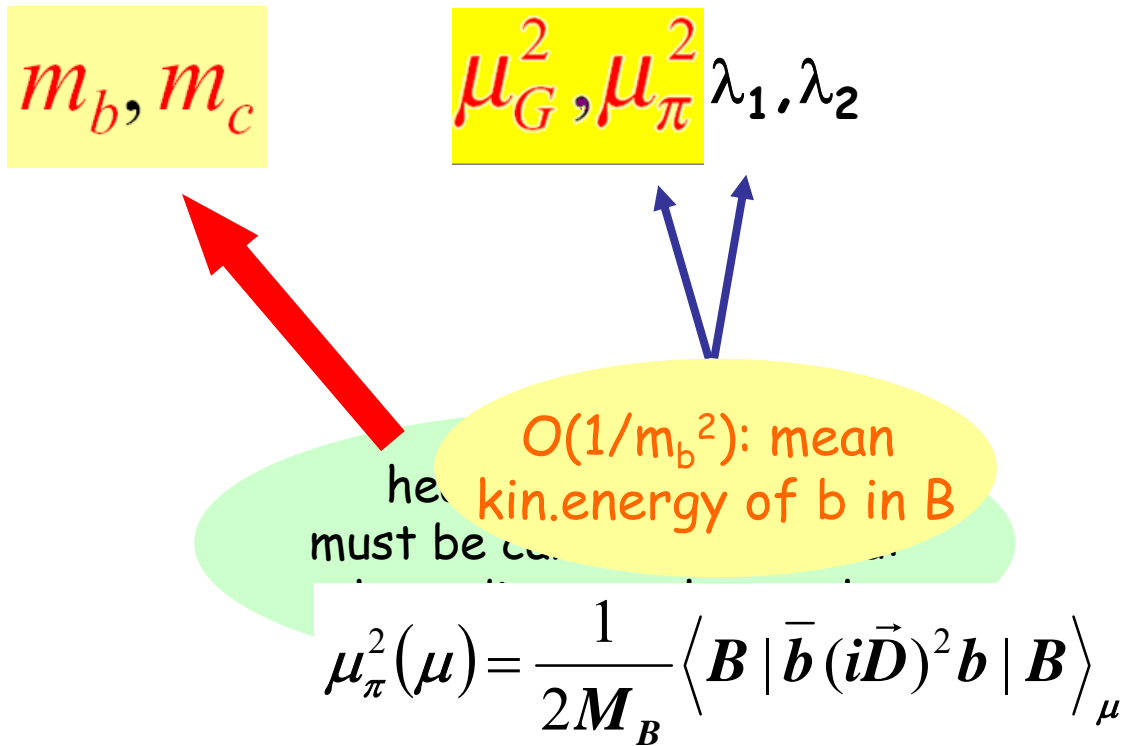
OPE predictions can be compared to exp only after **SMEARING** and away from endpoints: they have no LOCAL meaning

Leptonic and hadronic spectra



Total **rate** gives CKM elmnts; global **shape** parameters
tells us about B structure

State of the art



State of the art

m_b, m_c

μ_G^2, μ_π^2 λ_1, λ_2

ρ_D^3, ρ_{LS}^3 ρ_1, ρ_2
Gremm, Kapustin...

$O(1/m_b^2)$: mean
kin.energy of b in B

$$\mu_\pi^2(\mu) = \frac{1}{2M_B} \langle B | \bar{b} (i\vec{D})^2 b | B \rangle_\mu$$

State of the art

$$m_b, m_c$$

$$\mu_G^2, \mu_\pi^2 \lambda_1, \lambda_2$$

$$\rho_D^3, \rho_{LS}^3 \rho_1, \rho_2$$

Gremm, Kapustin...

$$\Gamma_{cl\nu} = \frac{G_F^2 m_b^5}{192\pi^3} |V_{cb}|^2 A_{ew} z_0(r) \left(1 + a_1(r) \frac{\mu_\pi^2}{m_b^2} + a_2(r) \frac{\mu_G^2}{m_b^2} + a_3(r) \frac{\rho_D^3}{m_b^3} + a_4(r) \frac{\rho_{LS}^3}{m_b^3} \right)$$

Recent implementation for moments of lept and hadronic spectra
including a cut on the lepton energy

Bauer et al., Uraltsev & PG

Perturbative Corrections: full $O(\alpha_s)$ and $O(\beta_0 \alpha_s^2)$ available

For hadronic moments thanks to **NEW** calculations

Trott

Aquila, PG, Ridolfi, Uraltsev

Using moments to extract HQE parameters

We do know something on HQE par.
need to check consistency.

- $M_{B^*} - M_B$ fix $\mu_G^2 = 0.35 \pm 0.03$
- Sum rules: $\mu_G^2 < \mu_\pi^2$, $\rho_D^3 > -\rho_{LS}^3 \dots$

Central moments can be VERY sensitive to HQE parameters

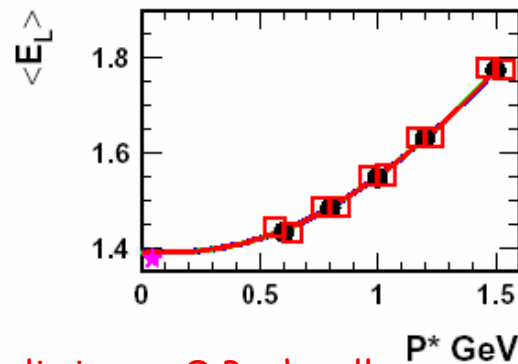
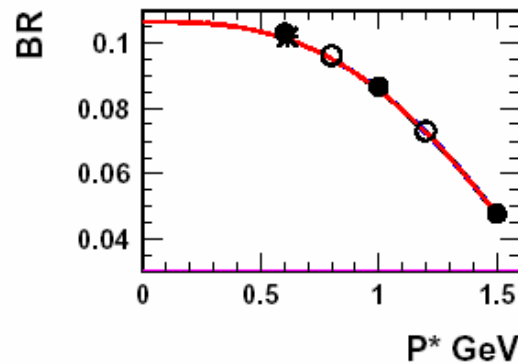
$$\left\langle \left(M_X^2 - \langle M_X^2 \rangle \right)^2 \right\rangle \approx \left[1.3 + 0.4(m_b - 4.6) - (m_c - 1.2) + 5(\mu_\pi^2 - 0.4) - 6(\rho_D^3 - 0.1) + \dots \right] \text{GeV}^4$$

Variance of mass distribution

BUT: OPE accuracy deteriorates for higher moments (getting sensitive to local effects)

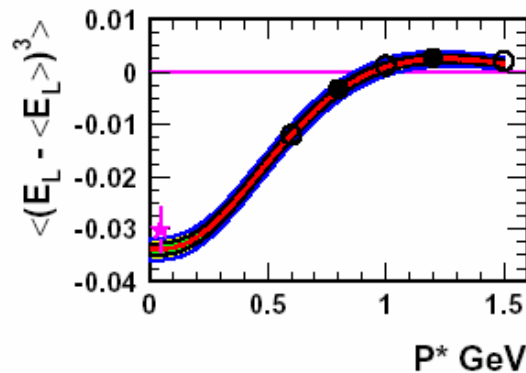
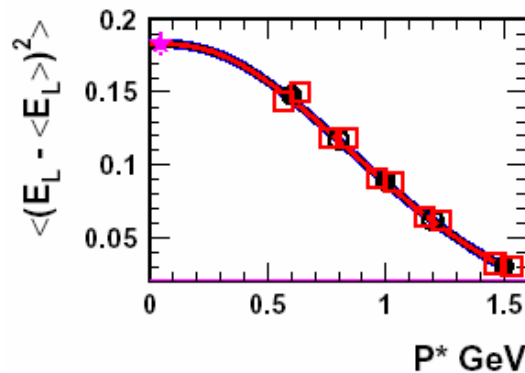
Provided cut is not too severe ($\sim 1.3 \text{ GeV}$)
the cut moments give additional info

Global fit to $|V_{cb}|$, BR_{sl} , HQE parmts



LEPTONIC
MOMENTS

Preliminary, O.Buchmuller

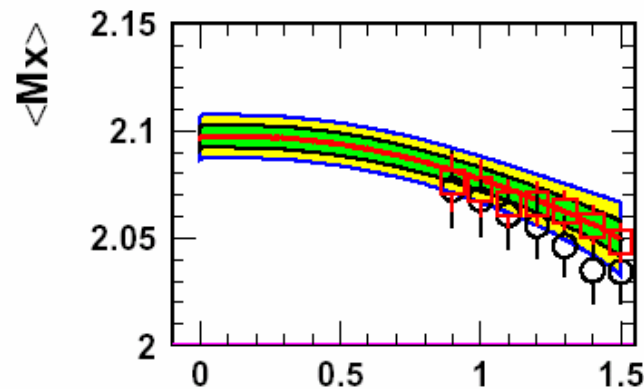


- BABAR ○ (NOT FIT)
- BELLE □ (NOT FIT)
- ▼ CDF
- ▲ CLEO △ (NOT FIT)
- ★ DELPHI ☆ (NOT FIT)
- ✱ HFAG

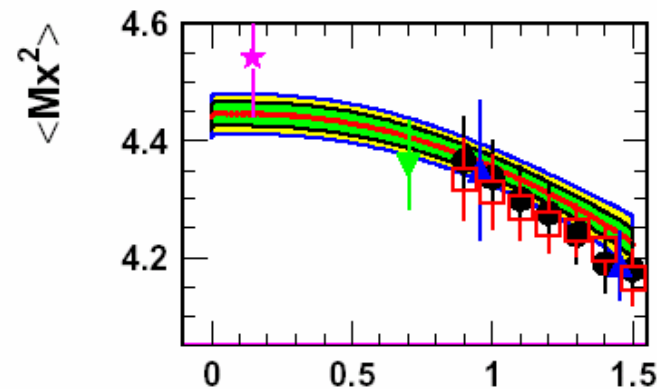
Not all points included
No external constraint

Pioneer work by CLEO & Delphi employed less precise/complete data, some external constraints, and CLEO a different scheme

Global fit to $|V_{cb}|, BR_{sl}, HQE$ parmts

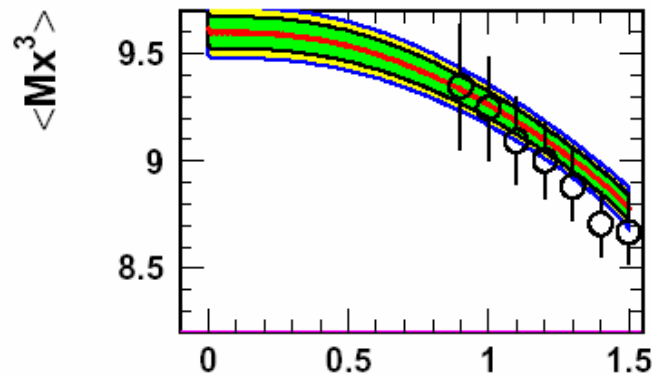


$P^* \text{ GeV}$

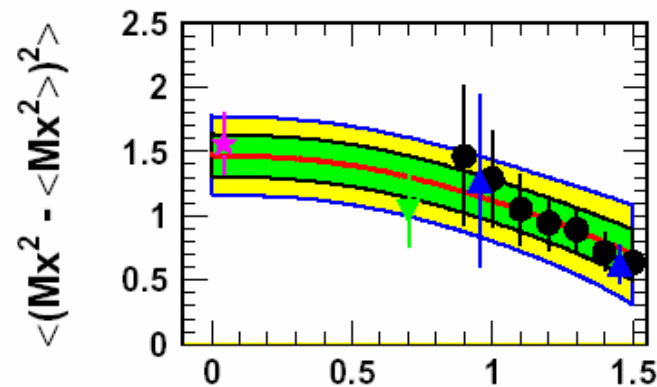


$P^* \text{ GeV}$

**HADRONIC
MOMENTS**



$P^* \text{ GeV}$



$P^* \text{ GeV}$

- BABAR ○ (NOT FIT)
- BELLE □ (NOT FIT)
- ▼ CDF
- ▲ CLEO △ (NOT FIT)
- ★ DELPHI ★ (NOT FIT)

Very similar results in a different approach/scheme, Bauer et al

Excellent agreement within exp and TH errors

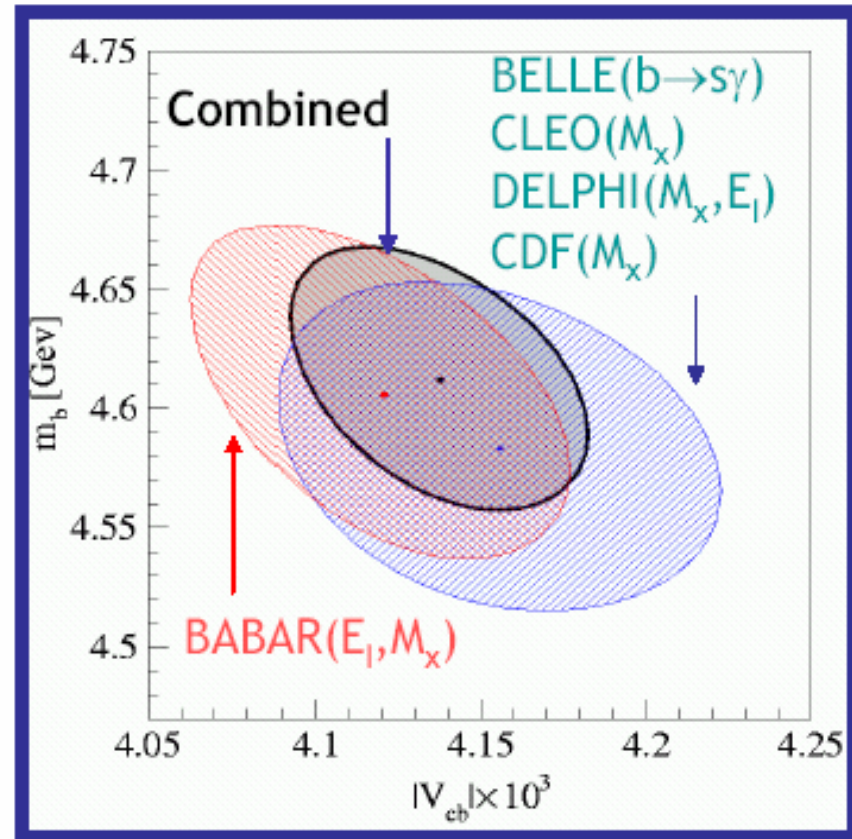
Combined fit in kinetic scheme

Benson, Bigi, Gambino, Mannel, Uraltsev

$$\begin{aligned}
 |V_{cb}| &= 41.38 \pm 0.45 \cdot 10^{-3} \\
 m_b &= 4.61 \pm 0.06 \text{ GeV} \\
 m_c &= 1.17 \pm 0.08 \text{ GeV} \\
 \mu_\pi^2 &= 0.40 \pm 0.04 \text{ GeV}^2 \\
 \mu_G &= 0.29 \pm 0.05 \text{ GeV}^2 \\
 \rho_D &= 0.16 \pm 0.06 \text{ GeV}^3 \\
 \rho_{LS} &= -0.18 \pm 0.09 \text{ GeV}^3 \\
 \text{BR}(B \rightarrow X|v) &= 10.64 \pm 0.14 \%
 \end{aligned}$$

- Stat., syst. and theo. (HQE, α_s) errors included.
- Error from uncertainty in Γ_{SL} (intrinsic charm) not included!
- $|V_{cb}|$ error of $\approx 1\%$

→ Substantial improvement from combination!

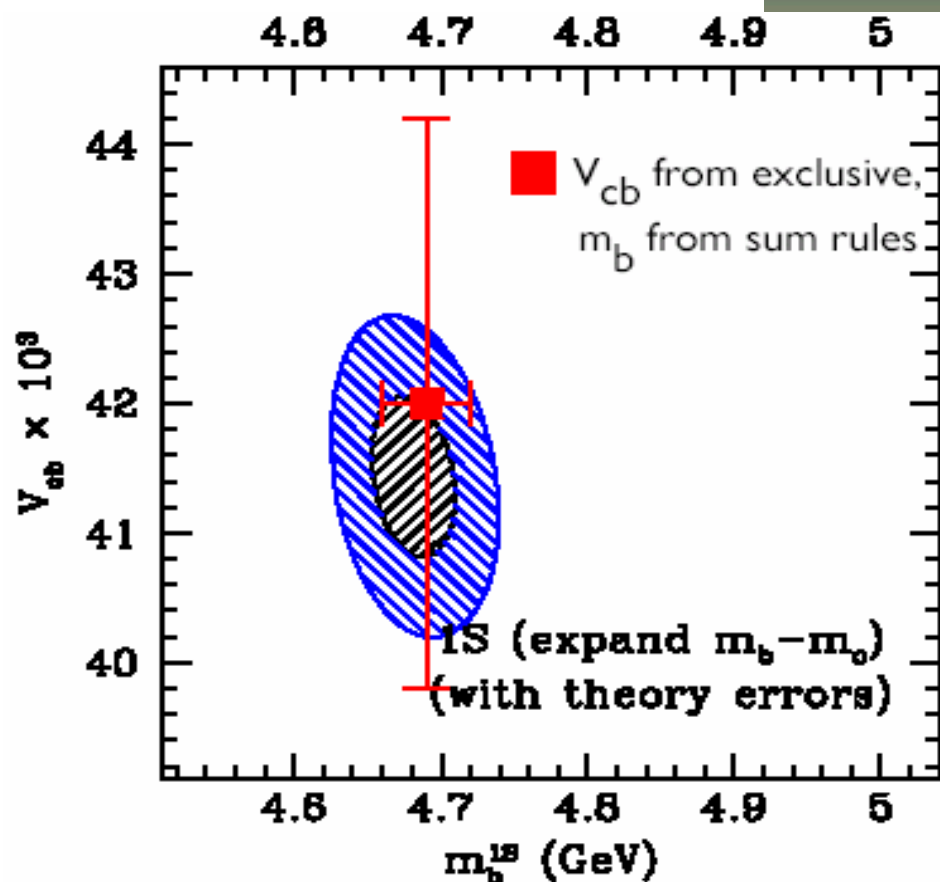


*Consistent description
of all moments by HQE*

Could also be done in
alternative schemes

Results in the 1S scheme

$\chi^2/\nu = 51/86$. (no theory errors $\chi^2 = 158/86$)



$$V_{cb} = (41.4 \pm 0.6 \pm 0.1_{\tau}) \times 10^{-3}$$

$$m_b^{1S} = 4.68 \pm 0.03 \text{ GeV}$$

$$m_b - m_c = 3.41 \pm 0.01 \text{ GeV}$$

$$\lambda_1 = -0.27 \pm 0.04 (\text{GeV})^2$$

$$\bar{m}_c(\bar{m}_c) = 0.90 \pm 0.04 \text{ GeV}$$

$$\bar{m}_c(\bar{m}_c) = 1.07 \pm 0.04 \text{ GeV}$$

There are several differences:

- perturbative quark mass scheme
- expansion in inverse powers of m_c
- handling of higher orders
- estimate of the errors...

Bauer, Manohar, Ligeti, Luke, Trott 2005



Comparison with other Determinations

Measurements and Predictions of the b-Quark Mass (\overline{MS} scheme)

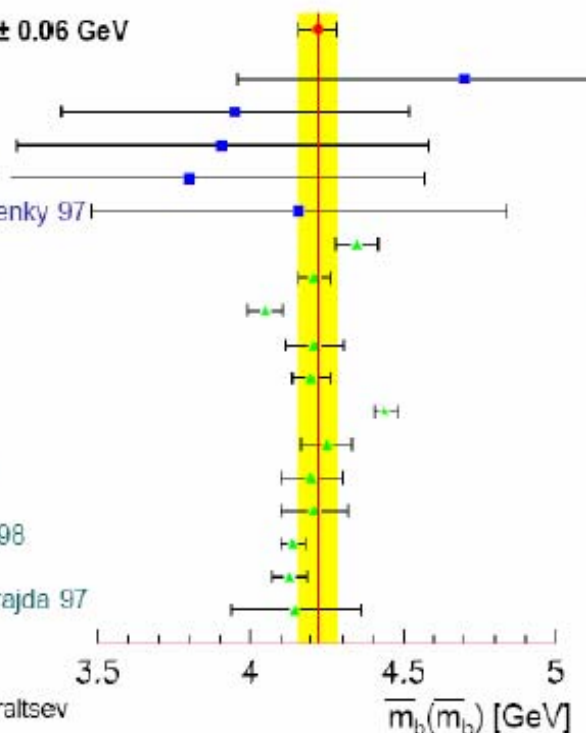
PDG2003

OPE: $B \rightarrow X_c l \nu$

BABAR* $\overline{m}_b(\overline{m}_b) = 4.22 \pm 0.06 \text{ GeV}$

Event Shape
 $Z \rightarrow b\bar{b}$ Decays
 $Z \rightarrow 3\text{-jets}$
 $Z \rightarrow 3\text{-jets}$
 $Z \rightarrow 3\text{-jets}$
 Y spectrum
 $e^+e^- \rightarrow q\bar{q}$
 S.R. for B, D
 $Y(1S)$
 Y : vac. pol. fcn.
 Y spectrum
 $b\bar{b}$ xsec., Y mass
 Sum Rules for Y
 Sum Rules for Y
 Y : vac. pol. fct.
 Y spectrum
 Lattice

*converted to \overline{MS} by N. Uraltsev



$$\overline{m}_b(\overline{m}_b) = 4.22 \pm 0.06 \text{ GeV}$$

Measurements and Predictions of the c-Quark Mass (\overline{MS} scheme)

PDG2003

OPE: $B \rightarrow X_c l \nu$

BABAR* $\overline{m}_c(\overline{m}_c) = 1.33 \pm 0.10 \text{ GeV}$

ν : di-muons
 ν : scattering
 Lattice: D_s mass
 Lattice: D_s mass
 S.R. for Charmonium
 $e^+e^- \rightarrow q\bar{q}$
 $e^+e^- \rightarrow q\bar{q}$
 S.R. for B, D
 $e^+e^- \rightarrow q\bar{q}$
 $Y(1S)$, $m_B - m_D$

*converted to \overline{MS} by N. Uraltsev



$$\overline{m}_c(\overline{m}_c) = 1.33 \pm 0.10 \text{ GeV}$$

Conversion from kinetic mass scheme
 to \overline{MS} scheme with hep-ph/9708372, hep-ph/0302262
 See also report from CKM WS hep-ph/0304132

Theoretical uncertainties are crucial for the fits

- ✓ Missing higher power corrections
- ✓ Intrinsic charm
- ✓ Missing perturbative effects in the Wilson coefficients: $O(\alpha_s^2)$, $O(\alpha_s/m_b^2)$ etc
- ✓ Duality violations

How can we estimate all this?

Different recipes, results for $|V_{cb}|$ unchanged

Testing parton-hadron duality

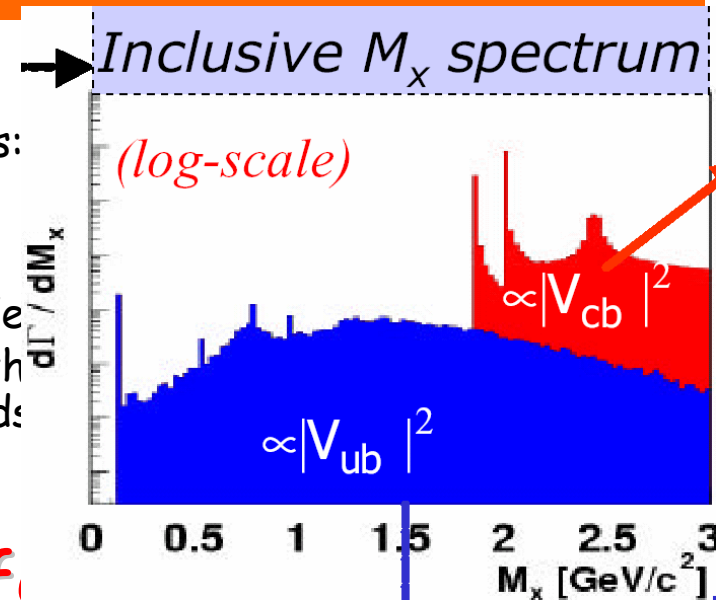
✓ **What is it?** For all practical purposes:
No OPE, no duality

✓ **Do we expect violations?** Yes
because OPE must be continued analytically. the
described by the OPE, like hadronic thresholds:
decays

✓ **Can we constrain them effectively?**

in a self-consistent way: just check the OPE predictions.
E.g. leptonic vs hadronic moments. Models may also give hints of how it works

✓ **Caveats?** HQE depends on many parameters and we know only a few
terms of the double expansion in α_s and Λ/m_b .



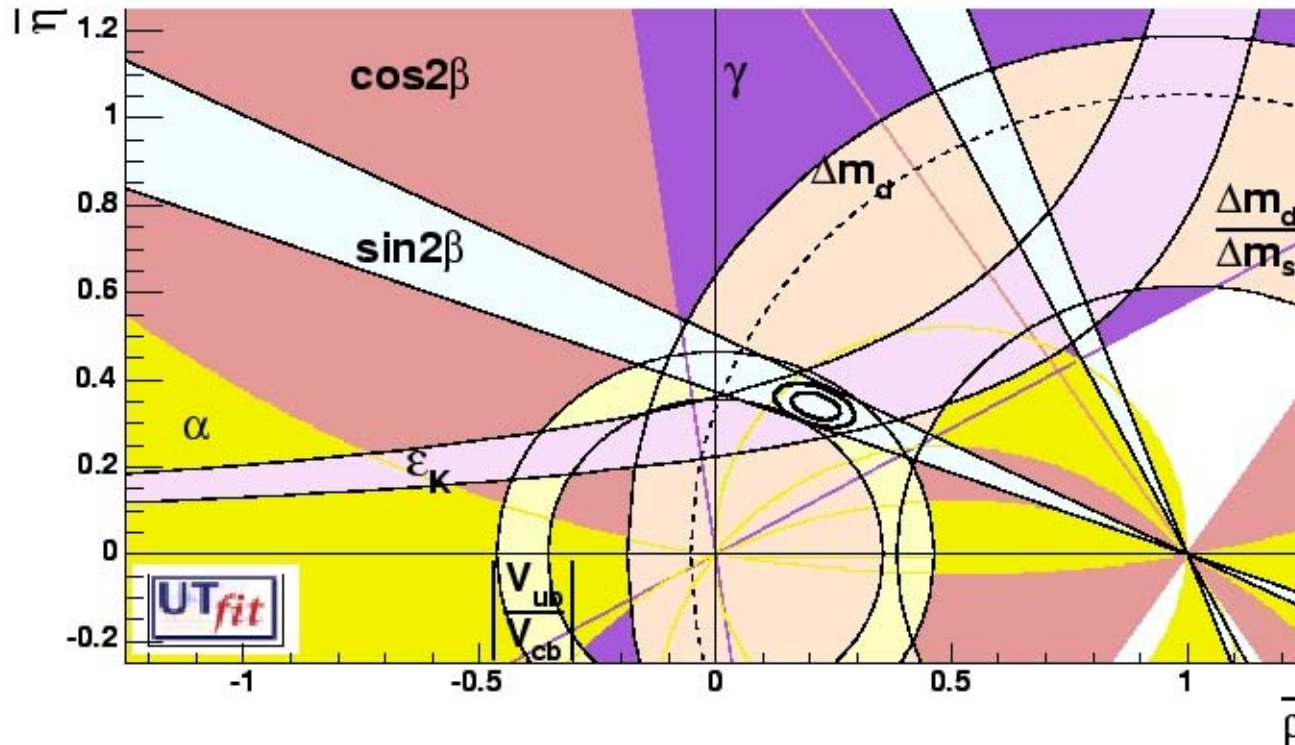
It is not just V_{cb} ...

HQE parameters describe *universal* properties of the B meson and of the quarks

- c and b masses can be determined with competitive accuracy (likely better than 70 and 50 MeV) $m_b - m_c$ is already measured to better than 30 MeV: a benchmark for lattice QCD etc?
- It tests the foundations for inclusive measurements
- most V_{ub} **incl. determinations** are sensitive to a shape function, whose moments are related to μ_π^2 etc,
- Bounds on ρ , the slope of IW function ($B \rightarrow D^*$ form factor)
- ...

Need precision measurements to probe limits of HQE & test our th. framework

$|V_{ub}|$ is the priority now

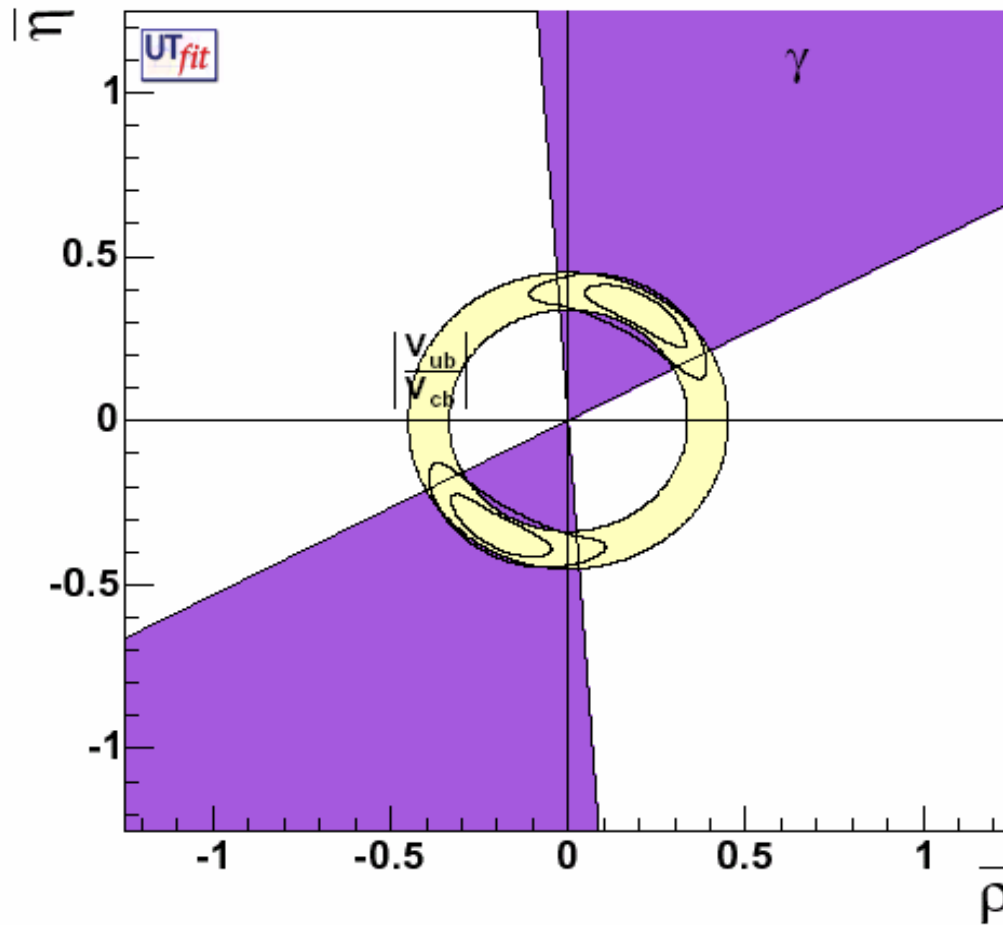


$$\bar{\rho} = 0.210 \pm 0.035$$

$$\bar{\eta} = 0.339 \pm 0.021$$

<http://www.utfit.org>

Strictly tree level



$b \rightarrow ul\nu$ exclusive

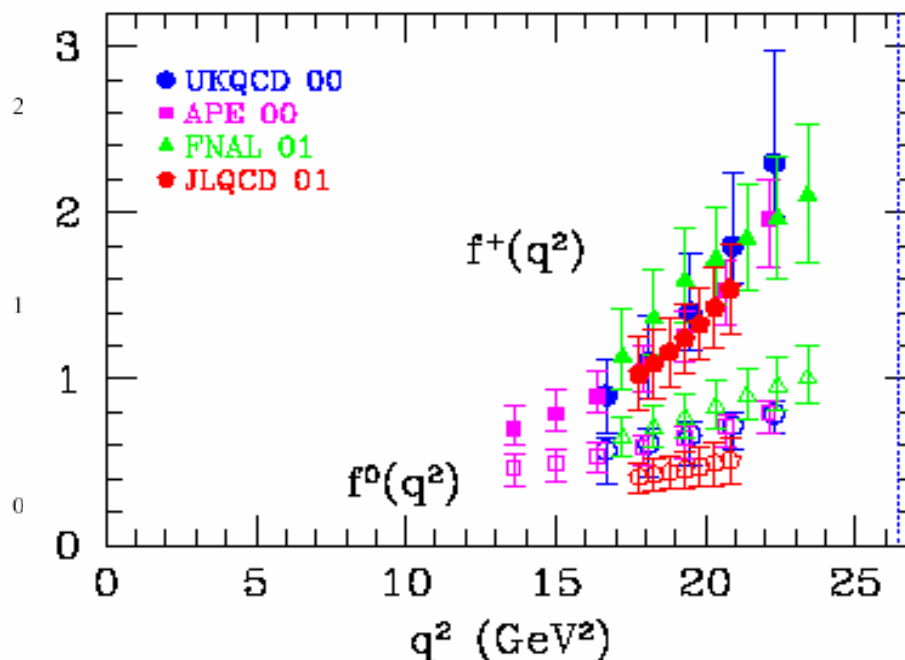
There is NO normalization of form f.s from HQ symmetry

New first unquenched results
lattice errors still ~15%

Sum rules good at low q^2
lattice at high q^2 : complement each other

Lattice (distant) goal is 5-6%

New strategy using combination of rare B,D decays Grinstein& Pirjol



	$ V_{ub} $ Fermilab/MILC	$ V_{ub} $ HPQCD
$16\text{GeV}^2 \leq q^2$	$3.0(4)(6) \times 10^{-3}$	$3.52(44)(73) \times 10^{-3}$
$0 \leq q^2 \leq q_{max}^2$		$3.86(58)(32) \times 10^{-3}$

(CLEO only)

The first error is from the lattice and second from experiment.

$|V_{ub}|$ (not so much) inclusive

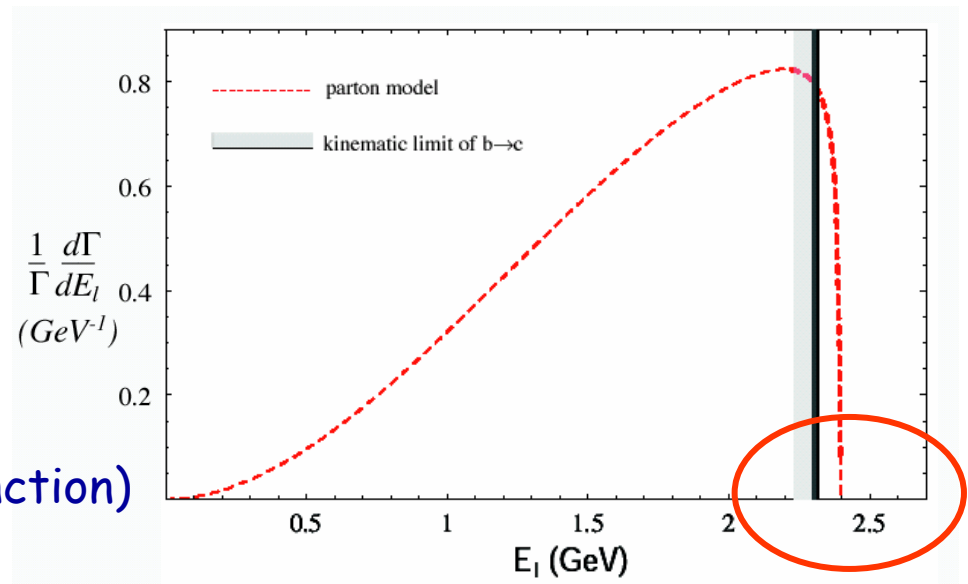
$|V_{ub}|$ from total $\text{BR}(b \rightarrow ul\nu)$ almost exactly like incl $|V_{cb}|$ but we need kinematic cuts to avoid the $\sim 100\times$ larger $b \rightarrow cl\nu$ background:

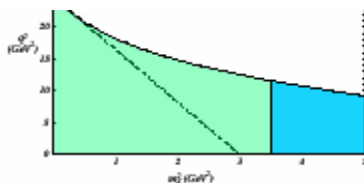
$$m_X < M_D \quad E_l > (M_B^2 - M_D^2)/2M_B \quad q^2 > (M_B - M_D)^2 \dots$$

or combined (m_X, q^2) cuts

The cuts destroy convergence of the OPE, supposed to work only away from pert singularities

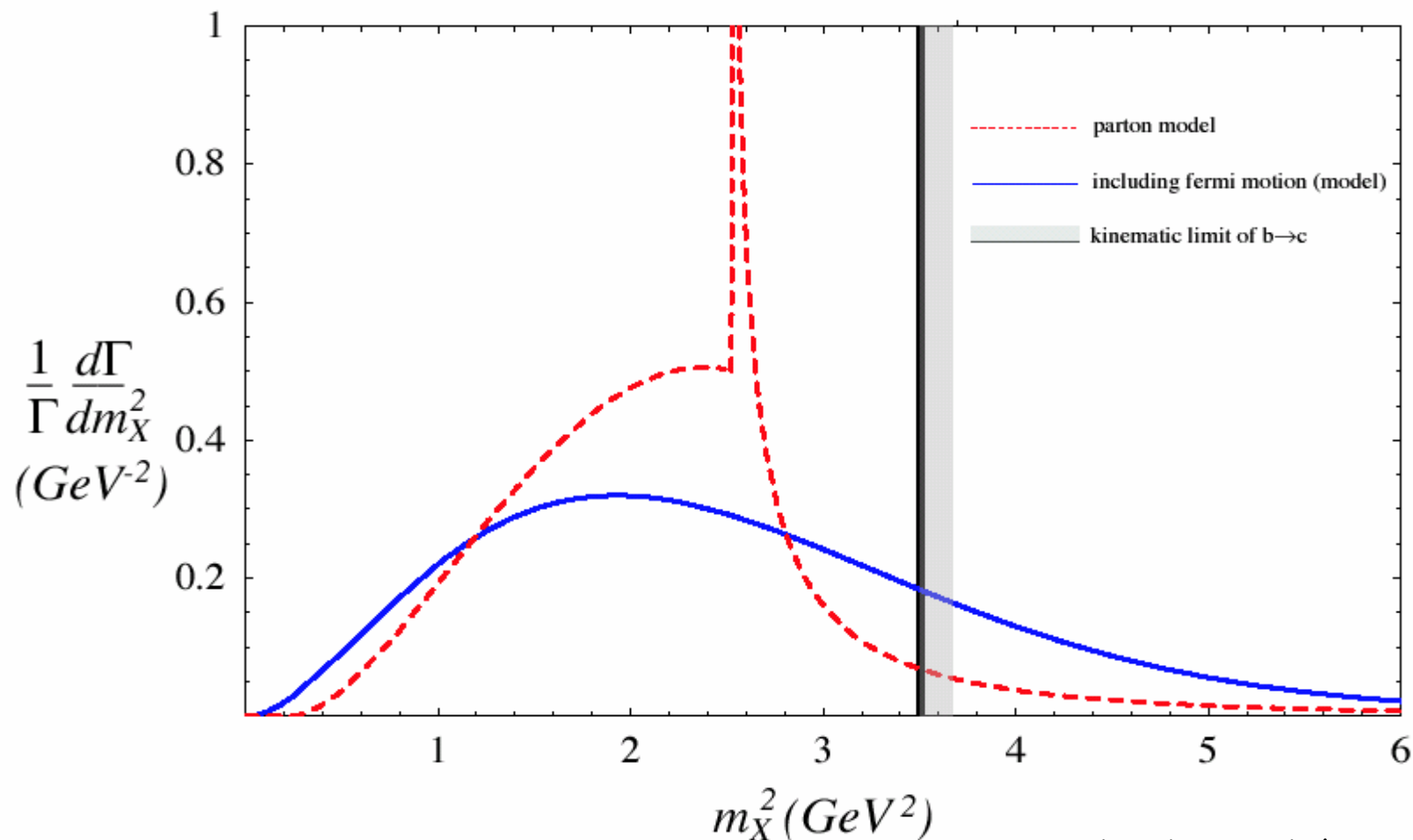
Rate becomes sensitive to "local" b-quark wave function properties (like Fermi motion)
→ at leading in $1/m_b$ SHAPE function)





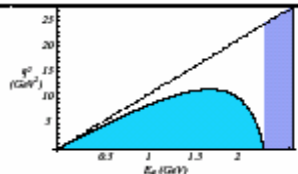
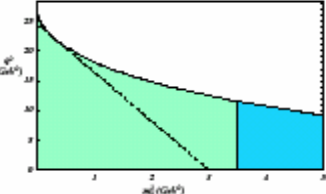
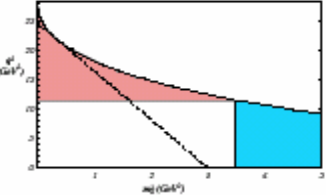
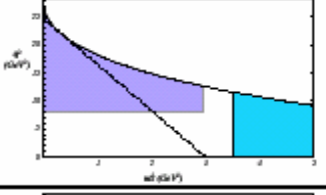
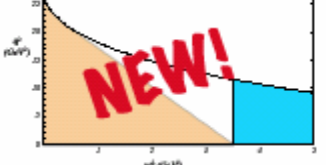
Cutting on the hadronic invariant mass spectrum gives more rate, but has the same problem with fermi motic

(Falk, Ligeti, Wise, Dikeman, Uraltsev)



Luke, CKM workshop 2005

Each strategy has pros and cons

cut	% of rate	good	bad
 $E_l > \frac{m_B^2 - m_D^2}{2m_B}$	~10%	don't need neutrino	<ul style="list-style-type: none"> - depends on $f(k^+)$ (and subleading corrections) - WA effects largest - reduced phase space - duality issues?
 $s_H < m_D^2$	~80%	lots of rate	<ul style="list-style-type: none"> - depends on $f(k^+)$ (and subleading corrections) - need shape function over large region
 $q^2 > (m_B - m_D)^2$	~20%	insensitive to $f(k^+)$	<ul style="list-style-type: none"> - very sensitive to m_b - WA corrections may be substantial - effective expansion parameter is $1/m_c$
 <p>"Optimized cut"</p>	~45%	<ul style="list-style-type: none"> - insensitive to $f(k^+)$ - lots of rate - can move cuts away from kinematic limits and still get small uncertainties 	- sensitive to m_b (need +/- 60 MeV for 5% error in best case)
 $P_+ > m_D^2 / m_B$	~70%	<ul style="list-style-type: none"> - lots of rate - theoretically simplest relation to $b \rightarrow s\gamma$ 	depends on $f(k^+)$ (and subleading corrections)

Luke, CKM workshop 2005

What do we know about $f(k_+)$?

- Its **moments** can be expressed in terms of m.e. of *local* operators, those extracted from the $b \rightarrow c$ moments
- It can be extracted from $b \rightarrow s\gamma$ (see later)
- It can also be studied in $b \rightarrow ul\nu$ spectra (see next)
- It gets **renormalized** and we have learned how (delicate interplay with pert contributions)

V_{ub} incl. and exclusive

Intense theoretical activity:

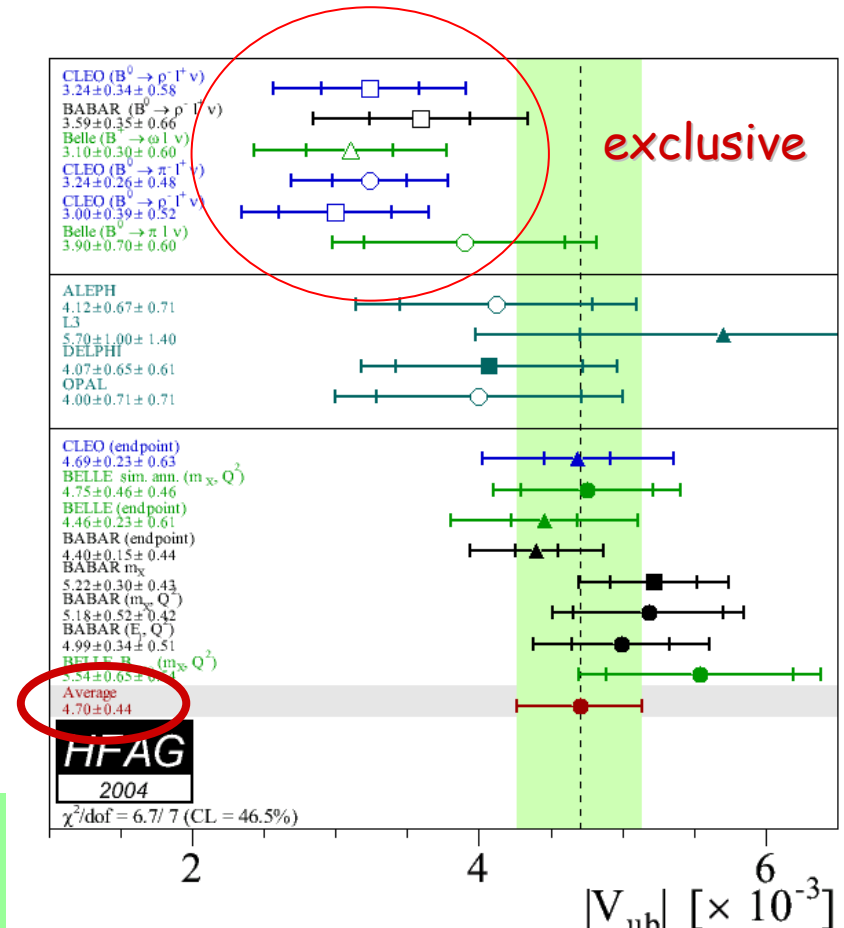
- ✓ subleading shape functions
- ✓ optimization of cuts (P_+ , P_- etc)
- ✓ weak annihilation contriibs.
- ✓ Resum. pert. effects
- ✓ relation to $b \rightarrow s\gamma$ spectrum
- ✓ SCET insight

A lot can be learned from exp

(on shape function from $b \rightarrow s\gamma$, WA, indirect constraints on s.f., subleading effects from cut dependence,...)

REQUIRES MANY COMPLEMENTARY MEASUREMENTS (affected by different uncert.)

There is no Best Method



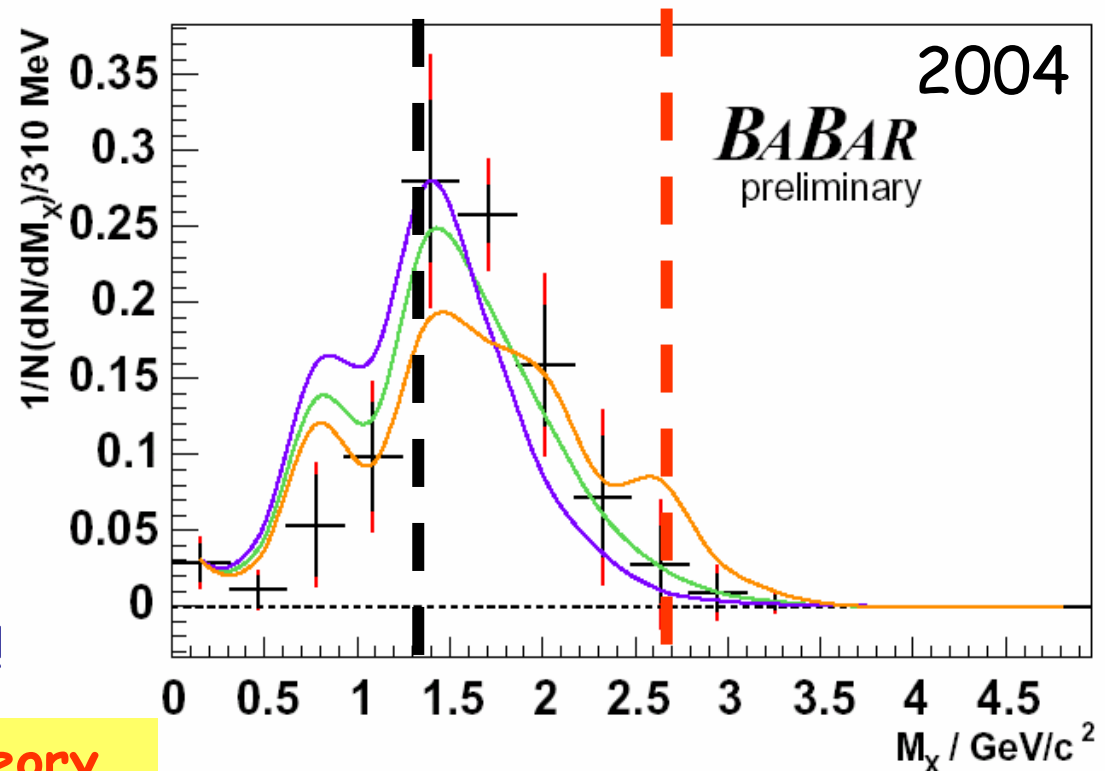
**WE ARE ALREADY AT 10%
New BRECO analyses; new results soon...**

Cutting the cuts...

New exp analyses based on **fully reconstructed** events allow high discrimination of charmed final states

Babar measured M_X moments. Results can be improved by cutting in a milder way than usual

It's time to start using $b \rightarrow u$ data to constrain sf !



Unfolded M_X spectrum

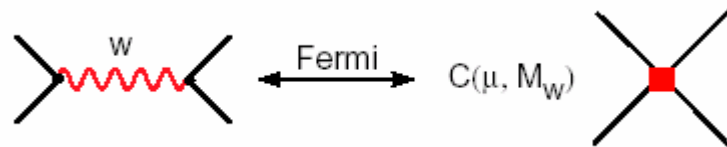
Useful to validate theory
and constrain $f(k_+)$ & WA
PG, Ossola, Uraltsev

$b \rightarrow s$ transitions

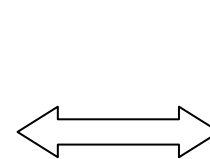
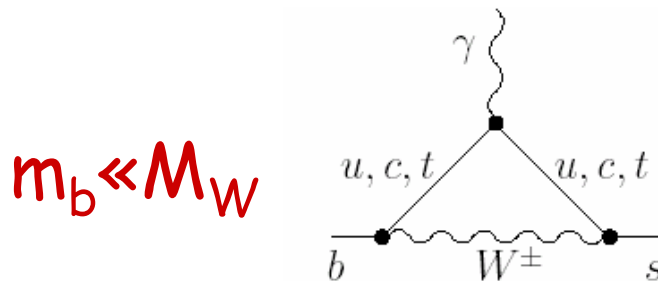
$$\Lambda_{\text{QCD}} \ll m_b \ll M_W$$

Large $L = \log m_b/M_W$ must be resummed.

$$\text{LO: } \alpha_s^n L^n, \quad \text{NLO: } \alpha_s^n L^{n-1}$$



Tower of local ops
OPE



But many more
operators appear
adding gluons

$$O_7 = m_b \bar{b}_R \sigma_{\mu\nu} F^{\mu\nu} S_L$$

The current is not conserved and runs between M_W and m_b
We have AT LEAST 3 scales

Inclusive decays are described by OPE (except charm loop contributions!)

The main ingredients

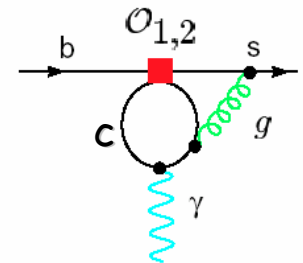
Process independent:

- The Wilson coefficients C_i (encode the short distance information, initial conditions)
- The Anomalous Dimension Matrix (mixing among operators, determines the evolution of the coefficients, allowing to resum large logs)

Process dependent: matrix elements

$B \rightarrow X_s \gamma$: NLO QCD calculation completed, all results checked, EW , power corrections

$B \rightarrow X_s \ell \ell$: NNLO & EW calculation just completed, power corrections



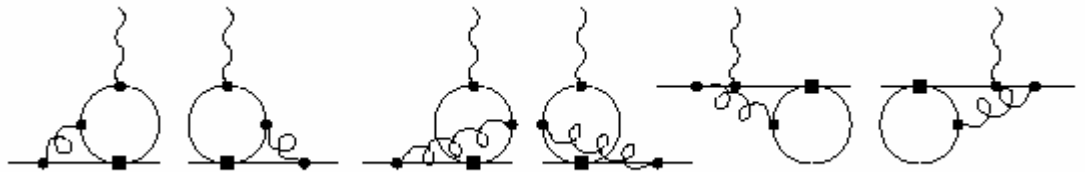
The charm mass problem

m_c enters the phase factor due to normalization

$$C = \left| \frac{V_{ub}}{V_{cb}} \right|^2 \frac{\Gamma[\bar{B} \rightarrow X_c e \bar{\nu}]}{\Gamma[\bar{B} \rightarrow X_u e \bar{\nu}]} = 0.581 \pm 0.017$$

Misiak & PG

and the NLO matrix elements



As the related LO diagrams vanish, the definition of m_c is a NNLO issue. Numerically very important because these are large NLO contributions:

$$m_c(m_c) = 1.25 \pm 0.10 \text{ GeV} \quad m_c(m_b) = 0.85 \pm 0.11 \text{ GeV} \quad m_c(\text{pole}) \sim 1.5 \text{ GeV}$$

But pole mass has nothing to do with these loops

Changing m_c/m_b from 0.29 (pole) to 0.22 (MSbar) increases BR_γ by 11%
 0.22 ± 0.04 gives DOMINANT 6% theory error

Error anatomy of BR_γ

$$\begin{aligned}
 BR \left[\bar{B} \rightarrow X_s \gamma \right]_{E_\gamma > 1.6 \text{ GeV}} &= (3.61 \pm 0.30) \times 10^{-4}, \\
 &= 3.61 \times 10^{-4} \left(1 \pm 0.06_{(m_c/m_b \text{ in } K_c)} \pm 0.04_{(\text{other NNLO})} \right. \\
 &\quad \left. \pm 0.01_{(\text{pert C})} \pm 0.02_{\lambda_1} \pm 0.02_{\Delta} \right. \\
 &\quad \left. \pm 0.02_{\alpha_s(M_Z)} \pm 0.02_{BR(\text{semilept})_{\text{exp}}} \pm 0.01_{m_t} \right)
 \end{aligned}$$

Misiak, PG 2001

Total error 8% dominated by charm mass
 Can be partially resolved by NNLO
 Update under way

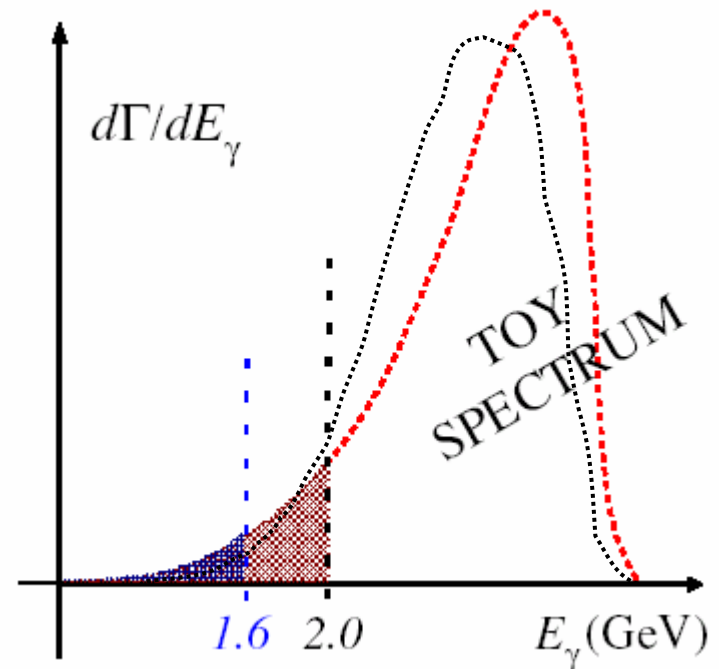
Photon spectrum vs *total*/BR

The OPE does not predict the spectrum, only its global properties: the higher the cut the higher the uncertainty

Conversely, constraining the HQE parameters constrains the possible shape functions

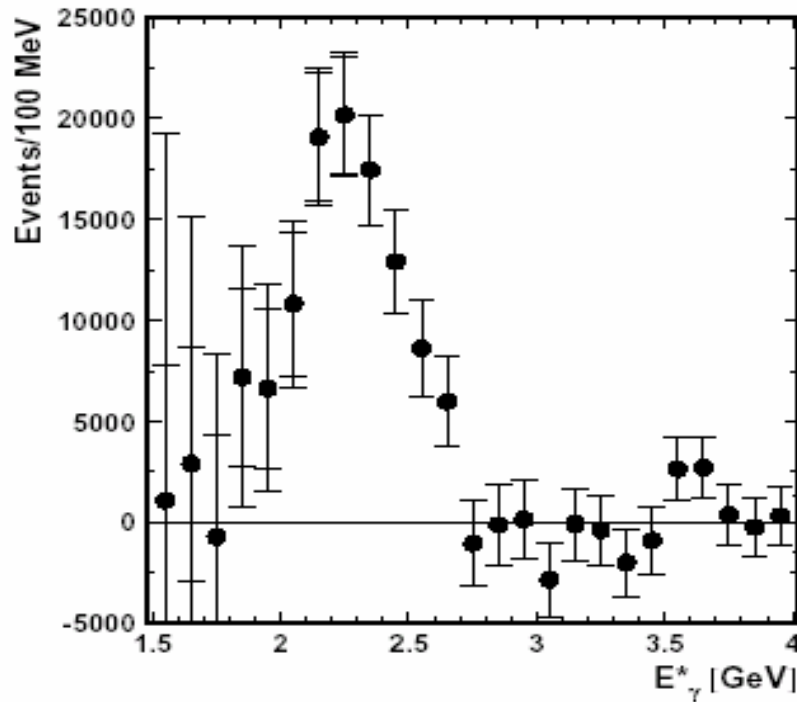
Possible subleading shape functions effects in V_{ub} applications

The shape function gets renormalized by perturbative effects: some complications may be better understood in SCET (Bauer & Manohar, Neubert et al)



Universality: spectrum of $B \rightarrow X_s \gamma$

Motion of b quark inside B and gluon radiation smear the spike at $m_b/2$



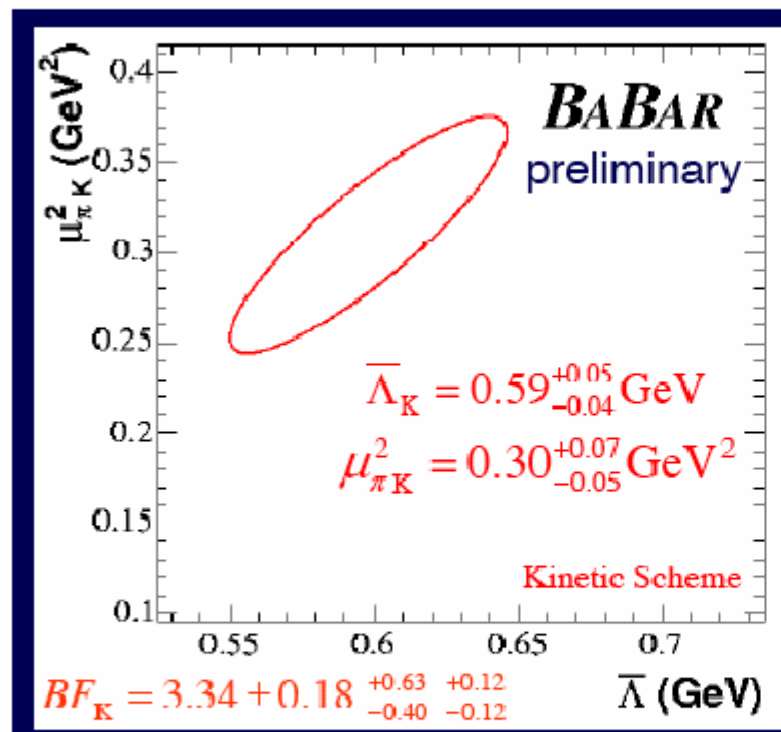
Belle: lower cut at 1.8 GeV

The photon spectrum is very insensitive to new physics, can be used to study the B meson structure

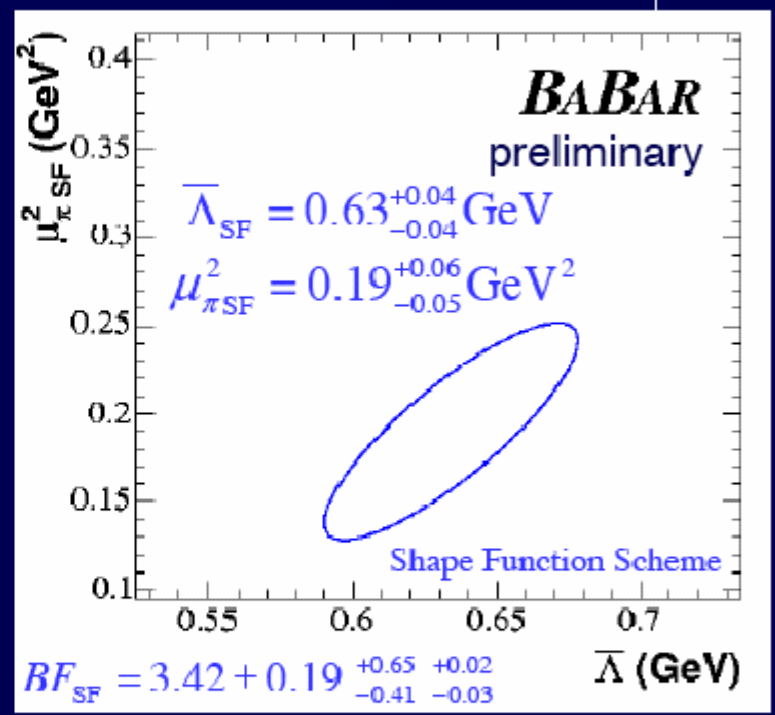
$$\langle E_\gamma \rangle = m_b/2 + \dots \quad \text{var}\langle E_\gamma \rangle = \mu_\Pi^2/12 + \dots$$

Importance of extending to $E_\gamma^{\min} \sim 1.8 \text{ GeV}$ or less for the determination of both the BR AND the HQE parameters
Bigi Uraltsev

Info from radiative spectrum compatible with semileptonic moments $\rightarrow \rightarrow$



Benson-Bigi-Uraltsev



Neubert

CKM 2005, Mar. 15-18, 2005

12

results in two different schemes, agree well with $b \rightarrow cl\nu$

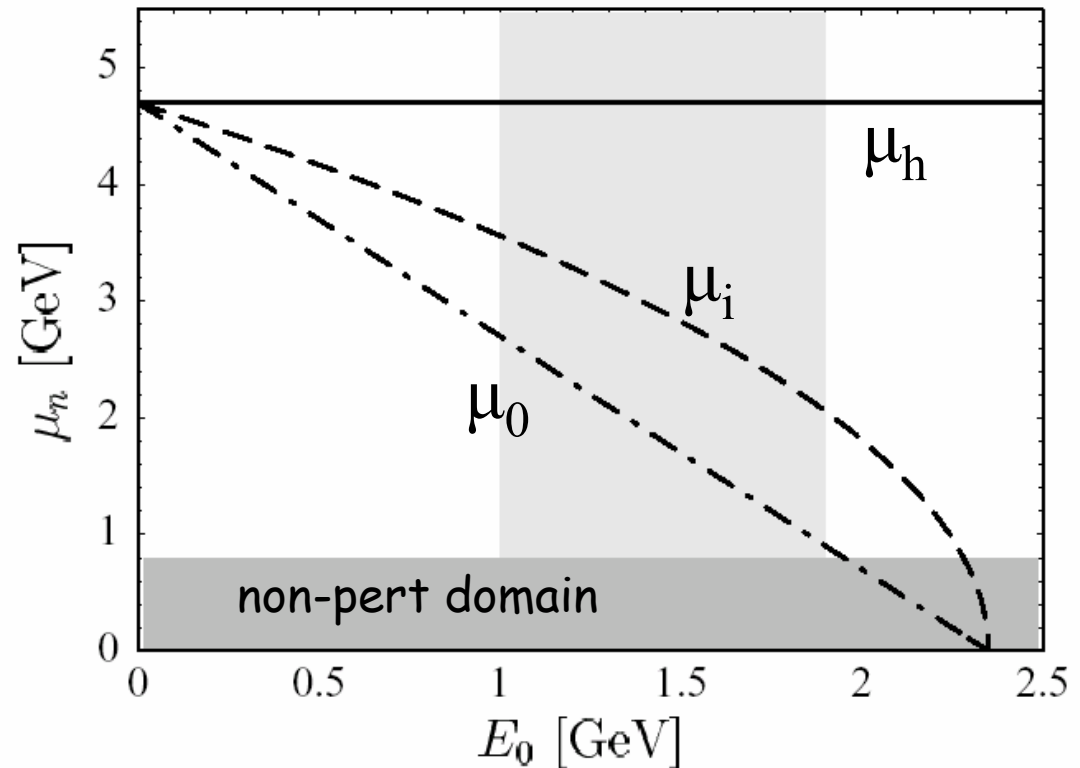
More cuts complications

Neubert 2004

$$\mu_h \sim m_b$$

$$\mu_i \sim \sqrt{\Delta m_b}$$

$$\mu_0 \sim \Delta = m_b - 2E_{\text{cut}}$$



The lower photon energy cut E_{cut} introduces two new scales
EVEN when local OPE works fine \rightarrow terms $\alpha_s(\Delta)$ could be large

Neubert (II)

- Need to disentangle 3 scales \rightarrow MultiScaleOPE

QCD \rightarrow SCET \rightarrow HQET \rightarrow local OPE
 μ_h μ_i μ_0

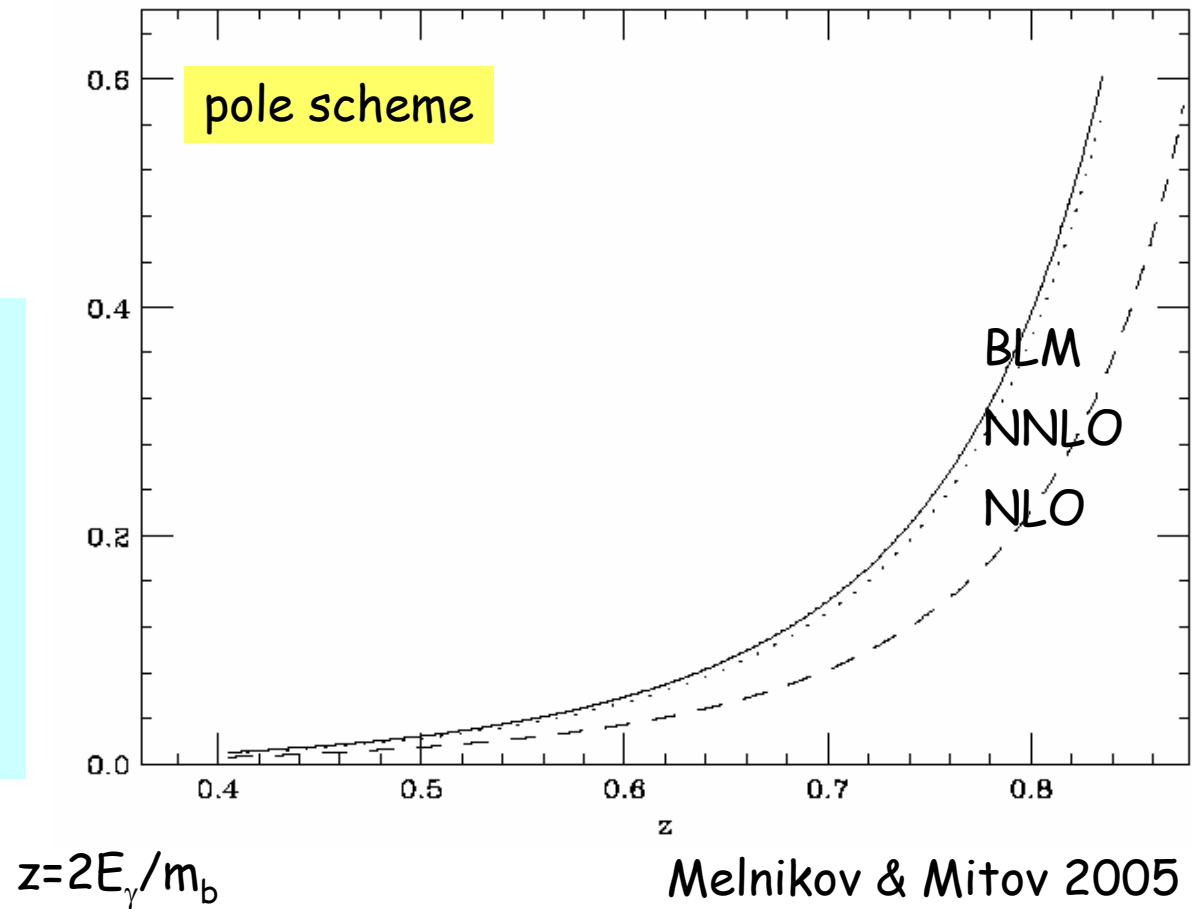
How well can we predict the radiative tail?

- Neubert finds $F(E_\gamma > 1.8 \text{ GeV}) = 0.89 \pm 0.07$, BR 3% lower, and theory error on BR 50% larger

FUNDAMENTAL LIMITATION?

- Main effect due to pert corrections whose scale is determined by higher orders (BLM etc): NNLO is the solution (at least to large extent)
- Sudakov resummation is irrelevant for $E_{\text{cut}} < 1.8 \text{ GeV}$
- New result of dominant 77 photon spectrum at $O(\alpha_s^2)$

The NNLO spectrum (dominant part)

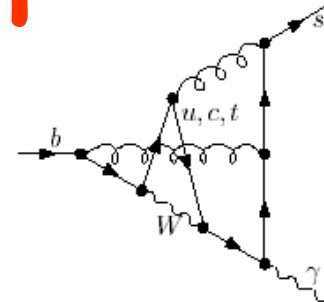


NNLO calculation
very close to BLM

Non-BLM corrections
change BR_γ by 0.5%

Situation seems under
control

NNLO status report

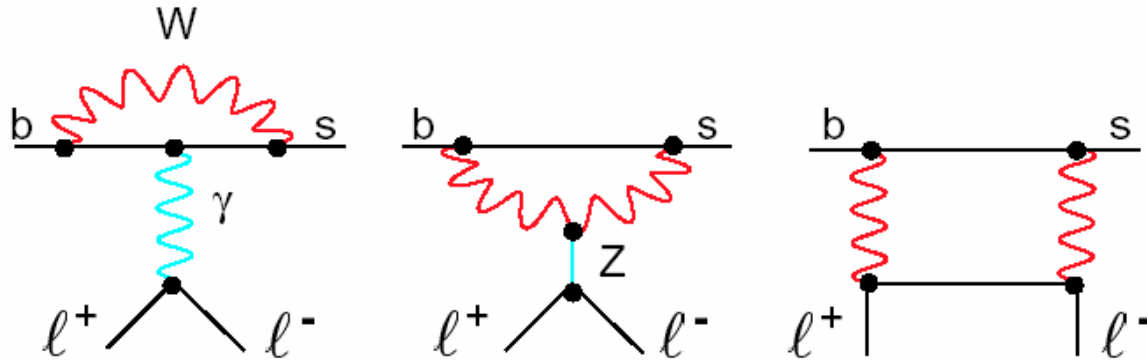


- NNLO $C_{7,8}$ matching completed
Misiak, Steinhauser
- All 3loop NNLO ADM Gorbahn, Haisch, Misiak
- Parts of the 3loop NNLO matrix elements
Bieri et al & Asatryan et al
- 2loop matrix element of Q_7 Czarnecki et al
- Dominant part of NNLO spectrum Melnikov Mitov

Still missing:

- 4loop ADM
- 3loop ME with charm
- subdominant 2loop ME

$b \rightarrow sl^+l^-$: a more complicated case

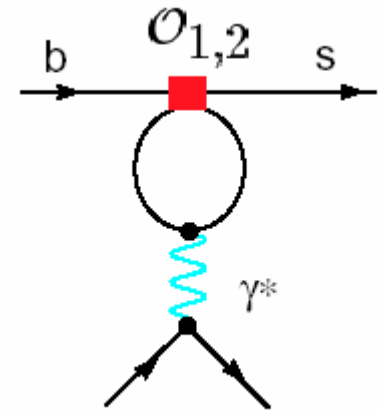


This decay mode is sensitive to different operators, hence to different new physics

Here large logs are generated even without QCD: LO $\alpha_s^n L^{n+1}$, NLO $\alpha_s^n L^n, \dots$

However, numerically the leading log is subdominant, yielding an awkward series:

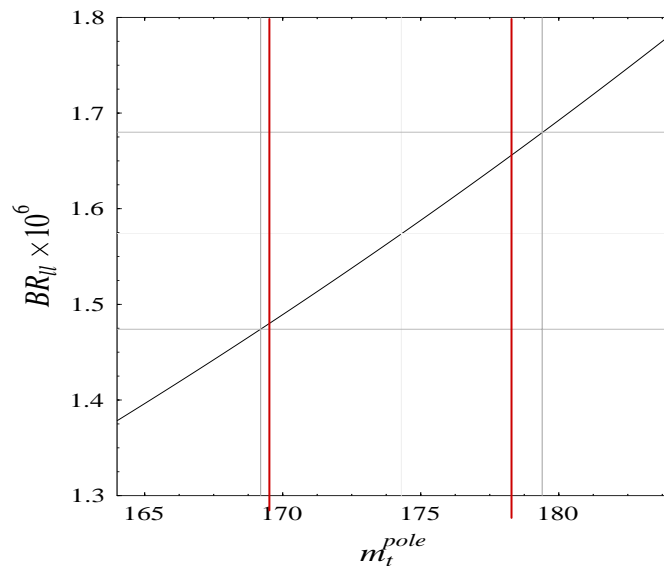
$$\text{in BR} \quad 1 + 0.7 (\alpha_s) + 5.5 (\alpha_s^2) + \dots$$



Error Anatomy for BR_{ll}

$$BR_{\ell\ell} (1 \text{ GeV}^2 \leq q^2 \leq 6 \text{ GeV}^2) = [1.574 \pm_{0.100}^{0.106} | M_t \pm_{0.075}^{0.059} |_{\text{scale}} \pm 0.045_C \pm 0.035_{BR_{st}} \pm_{0.067}^{0.072} | m_b \pm_{0.013}^{0.001} | m_c] \times 10^{-6}$$

Bobeth, PG, Gorbahn, Haisch



- M_{top} dominant error 7%
- scale uncertainty 5%
- $m_b^{\text{pole}} = 4.80 \pm 0.15 \text{ GeV} \rightarrow 5\%$
- phase space factor 3%
- No m_c issue as charm enters at LO

TOTAL ERROR ~10%

BUT: bottom uncertainty is not a fundamental limitation

$\delta m_b^{\text{short distance}} \approx 30\text{-}50 \text{ MeV}$

simply change scheme!

EXP: only inclusive rate,

Belle (140fb^{-1}): $(4.4 \pm 0.8 \pm 0.8) \times 10^{-6}$

Babar (80fb^{-1}): $(5.6 \pm 1.5 \pm 1.3) \times 10^{-6}$

We get $(4.6 \pm 0.8) \times 10^{-6}$ ($m_{ll} > 0.2 \text{ GeV}$)

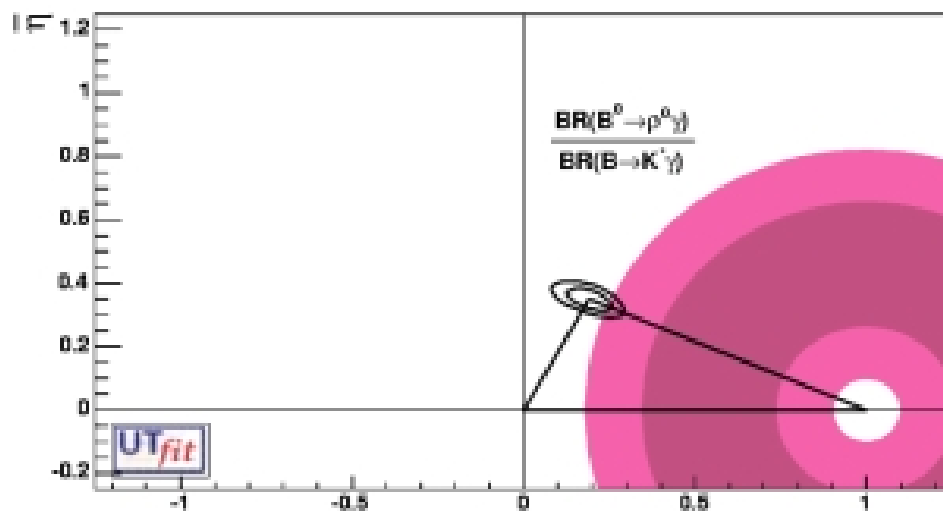
the UT from excl radiative decays

- Inclusive $b \rightarrow d\gamma$ experimentally impossible, but exclusive modes start being accessible
- Ratios of $B \rightarrow \rho\gamma$ / $B \rightarrow K^*\gamma$ allow a determination of $|V_{td}/V_{ts}|$ that is independent of form factors in the limit of SU(3)
- Calculations rely on QCD factorization and on lattice/sum rules for the estimate of SU(3) violation (Beneke et al, Bosch Buchalla)
power corrections apparently suppressed
- Neutral modes don't have WA, $\xi = 1.2 \pm 0.1$ (CKM 2005)
- LC sum rules errors large, Lattice calculations only exploratory...

An interesting deviation?

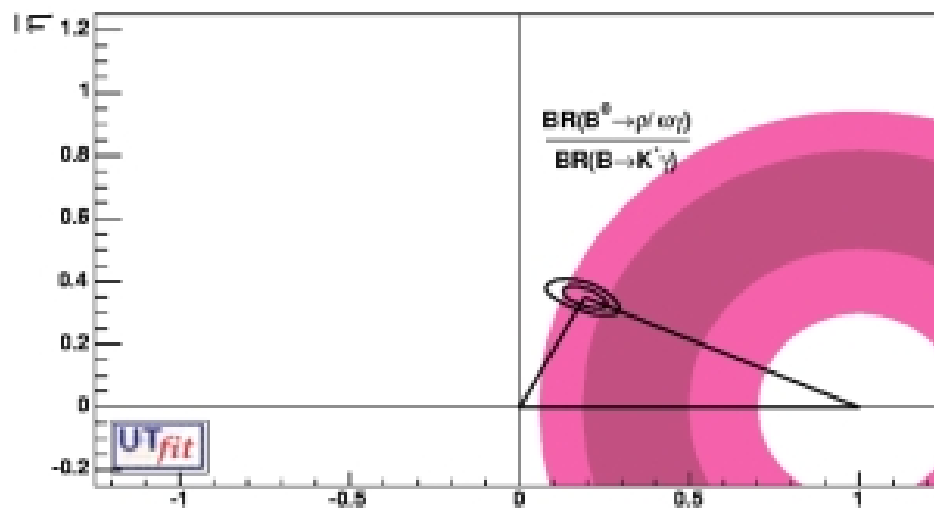
Impact on UT using **only neutral modes**:

$$\text{BR}(B^0 \rightarrow \rho^0 \gamma) = 0.6^{+1.9}_{-1.4} \times 10^{-7}$$



Impact on UT using average of neutral and charged modes:

$$\text{BR}(B \rightarrow \rho/\omega \gamma) = (6.4 \pm 2.7) \times 10^{-7}$$



Summary of main theory limitations

process	quantity	Th error	needs	goal
$B \rightarrow D^* l \nu$	$ V_{cb} $	$\sim 4\%$	New lattice results	1%
$B \rightarrow X l \nu$	$ V_{cb} $	$\sim 1.5\%$	New pert calculations	$< 1\%$
$B \rightarrow \pi l \nu$	$ V_{ub} $	$\sim 15\%$	Lattice developments	6%?
$B \rightarrow X_u l \nu$	$ V_{ub} $	$\sim 10\%$	More data synergy th/exp	5%
$B \rightarrow X_s \gamma$	BR	$\lesssim 10\%$	NNLO, MSOPE?	$< 5\%$
$B \rightarrow \rho^0 \gamma / B \rightarrow K^{*0} \gamma$	$ V_{td} / V_{ts} $	10-20%	Better understanding of th errors, lattice	?



Fit Results

kinetic mass scheme

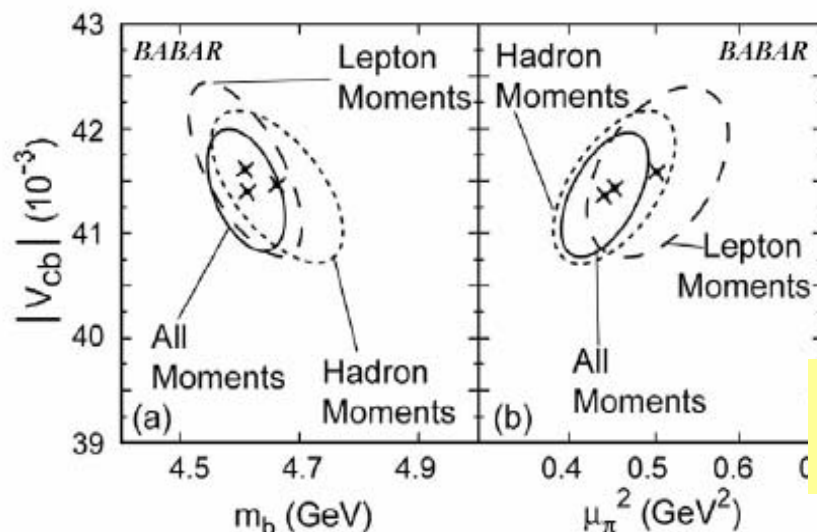
$$\begin{aligned}
 |V_{cb}| &= (41.4 \pm 0.4_{\text{exp}} \pm 0.4_{\text{HQE}} \pm 0.2_{\alpha_s} \pm 0.6_{\Gamma_{\text{SL}}}) \times 10^{-3} \\
 Br(B \rightarrow X_c e \nu) &= (10.61 \pm 0.16_{\text{exp}} \pm 0.06_{\text{HQE}}) \% \\
 m_b(1 \text{ GeV}) &= (4.61 \pm 0.05_{\text{exp}} \pm 0.04_{\text{HQE}} \pm 0.02_{\alpha_s}) \text{ GeV} \\
 m_c(1 \text{ GeV}) &= (1.18 \pm 0.07_{\text{exp}} \pm 0.06_{\text{HQE}} \pm 0.02_{\alpha_s}) \text{ GeV}
 \end{aligned}$$

$$\begin{aligned}
 \mu_\pi^2 &= (0.45 \pm 0.04_{\text{exp}} \pm 0.04_{\text{HQE}} \pm 0.01_{\alpha_s}) \text{ GeV}^2 \\
 \mu_G^2 &= (0.27 \pm 0.06_{\text{exp}} \pm 0.03_{\text{HQE}} \pm 0.02_{\alpha_s}) \text{ GeV}^2 \\
 \rho_D^3 &= (0.20 \pm 0.02_{\text{exp}} \pm 0.02_{\text{HQE}} \pm 0.00_{\alpha_s}) \text{ GeV}^3 \\
 \rho_{LS}^3 &= (-0.09 \pm 0.04_{\text{exp}} \pm 0.07_{\text{HQE}} \pm 0.01_{\alpha_s}) \text{ GeV}^3
 \end{aligned}$$

Kinetic scheme:
Small pert corrections
Minimal set of parmts
No $1/m_c$ expansion
 Uraltsev & PG

Strong correlation between
 m_b **and** m_c :

$$m_b(1 \text{ GeV}) - m_c(1 \text{ GeV}) = (3.44 \pm 0.03_{\text{exp}} \pm 0.02_{\text{HQE}} \pm 0.01_{\alpha_s}) \text{ GeV}$$



2D projections
of the fit result:

$\Delta\chi^2=1$ ellipses

No sign of deterioration
 for higher cuts

