BEAUTY 2005

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B-Meson Mixing and Lifetimes:

- **Mixing Parameters:** $\Delta m_d\,, \Delta m_s$
- **Width Differences:** $\Delta\Gamma_{B_d}\,, \Delta\Gamma_{B_s}$
- **CP-Violation Parameters:** $|\,q\,/\,p\,|_{B_{d,s}}$
- **Lifetime Ratios:** $\tau(B^+)/\tau(B_d), \tau(B_s)/\tau(B_d), \tau(\Lambda_b)/\tau(B_d)$

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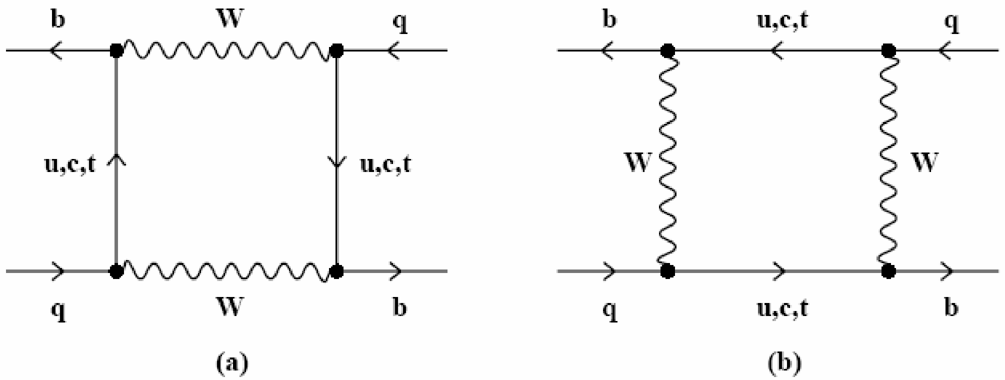
Basic Formalism

Neutral mesons are not eigenstates of the Weak Interactions:



$\Rightarrow$ "particle-antiparticle oscillations":

- Schroedinger equation with an effective 2×2 Hamiltonian

$$i\frac{d}{dt}\begin{pmatrix} B_q \\ \overline{B}_q \end{pmatrix} = \left[\begin{pmatrix} M_{11}^q & M_{21}^{q\,*} \\ M_{21}^q & M_{11}^q \end{pmatrix} - \frac{i}{2}\begin{pmatrix} \Gamma_{11}^q & \Gamma_{21}^{q\,*} \\ \Gamma_{21}^q & \Gamma_{11}^q \end{pmatrix}\right]\begin{pmatrix} B_q \\ \overline{B}_q \end{pmatrix}$$

- Mass eigenstates:

$$|B_q^{L,H}\rangle = (|B_q\rangle \pm (q/p)_q|\overline{B}_q\rangle)/\sqrt{1+|(q/p)_q|^2}.$$

Physical Observables

$$\Delta m_q = 2\,|M_{21}^q|$$

$$\Delta\Gamma_q = -2\,|M_{21}^q|\,\mathrm{Re}\left(\frac{\Gamma_{21}^q}{M_{21}^q}\right)$$

$$\Gamma_q = \tau_q^{-1} = |\Gamma_{11}^q|$$

$$\left|(q/p)_q\right| = 1 + \frac{1}{2}\,\mathrm{Im}\left(\frac{\Gamma_{21}^q}{M_{21}^q}\right)$$

1. Mixing

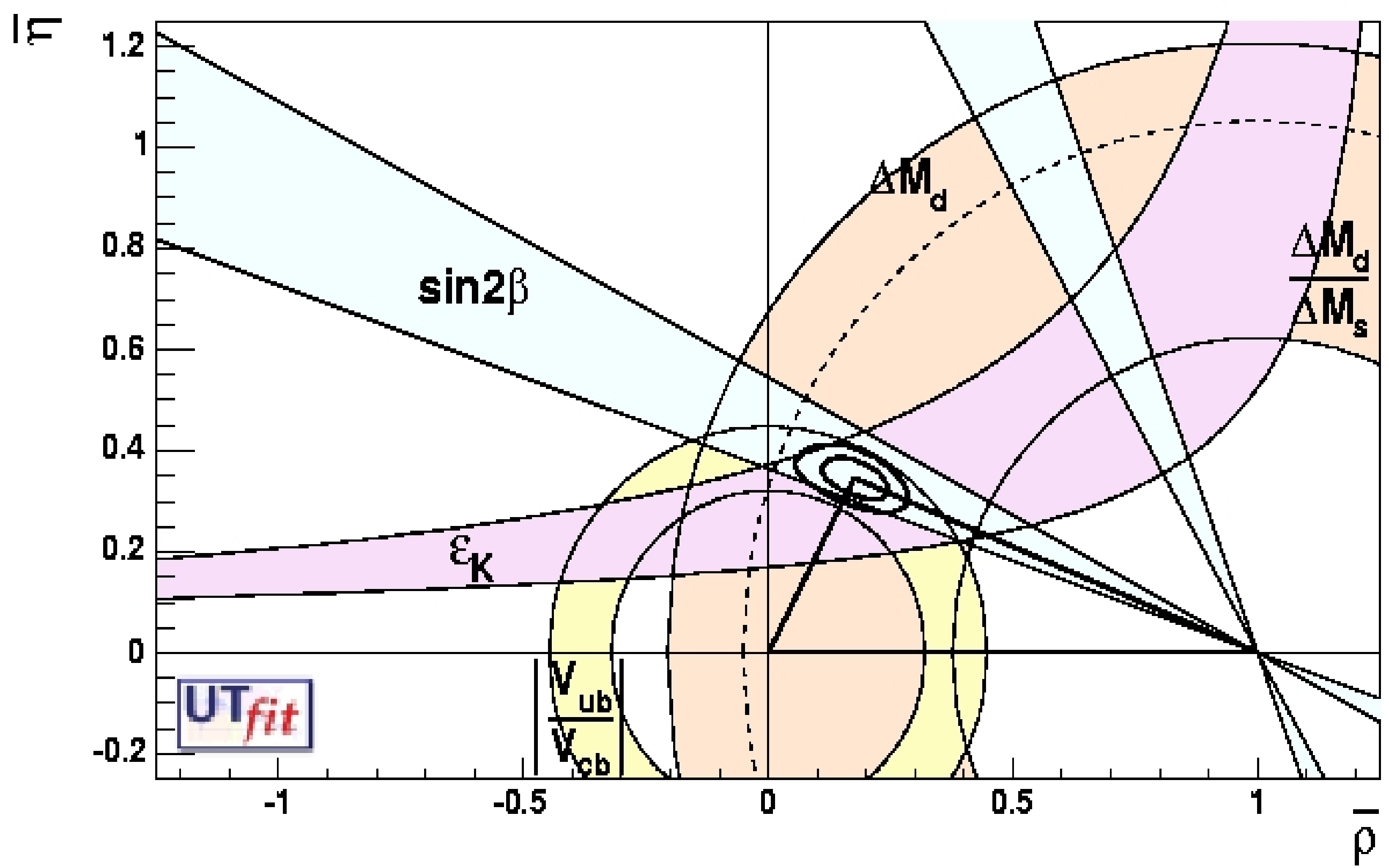
Δm_d, Δm_s

- Phenomenological importance (UTA)
- Theoretical inputs

Unitarity Triangle Analysis

$\Delta m_d = 0.502(7)\, ps^{-1}$ [HFAG 2005]

$\Delta m_s > 14.4\, ps^{-1} (95\% C.L.)$

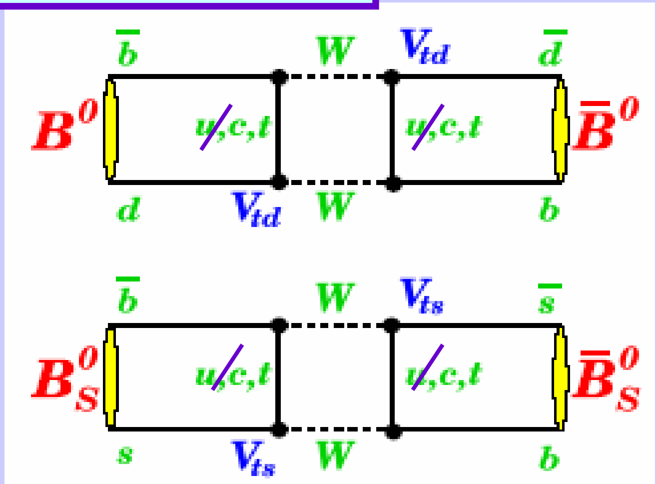


Extracting $|V_{td}|$:

$$\xi^2 = \left(\frac{\hat{B}_{B_s} f_{B_s}^2}{\hat{B}_{B_d} f_{B_d}^2} \right)$$

$$\Delta m_d = \frac{G_F^2 m_W^2}{6\pi^2} m_{B_d} |V_{td}|^2 \eta_B S_0(x_t) \frac{\hat{B}_{B_s} f_{Bs}^2}{\xi^2}$$

$$\Delta m_s = \frac{G_F^2 m_W^2}{6\pi^2} m_{B_s} |V_{ts}|^2 \eta_B S_0(x_t) \hat{B}_{B_s} f_{Bs}^2$$



- **OPE** $(m_t \to \infty, m_W \to \infty)$
- **Dominant top contribution**

- $\eta_B S_0(x_t)$: **short-distance physics, from Perturbation Theory (2%)**
  [at NLO, Buras et al.: Nucl.Phys.B347 (1990)]

- $\hat{B}_{B_s} f_{B_s}^2, \xi^2$: **non-perturbative QCD effects, from Lattice QCD**

- ξ^2 : **lattice uncertainties cancel**

B-Physics on the Lattice

MULTISCALE PROBLEM

$L/a \leq 50$ **(computational cost limit)**

$am_b << 1$ → **HQET, NRQCD, FermiLab approach, extrapolation in the heavy quark mass**

$M_\pi L >> 1$ → Extrapolation in the light quark mass using ChPT

Lattice QCD Actions:

- **Wilson/O(a)-improved Wilson:** problem at light masses (exceptional quenched configurations)
- **Twisted mass:** no exceptional configurations, easier improving, but more expensive
- **Staggered:** light masses, but 4 tastes (fourth root trick)
- **Domain-wall/Overlap:** light masses, but the most expensive

Accurate "quenched" studies + recent Nf=2, Nf=2+1 calculations

Bs-Meson Decay Constant from Lattice QCD

$$\langle 0 | \bar{b} \gamma^\mu \gamma_5 s | B_s \rangle = i p^\mu f_{B_s}$$

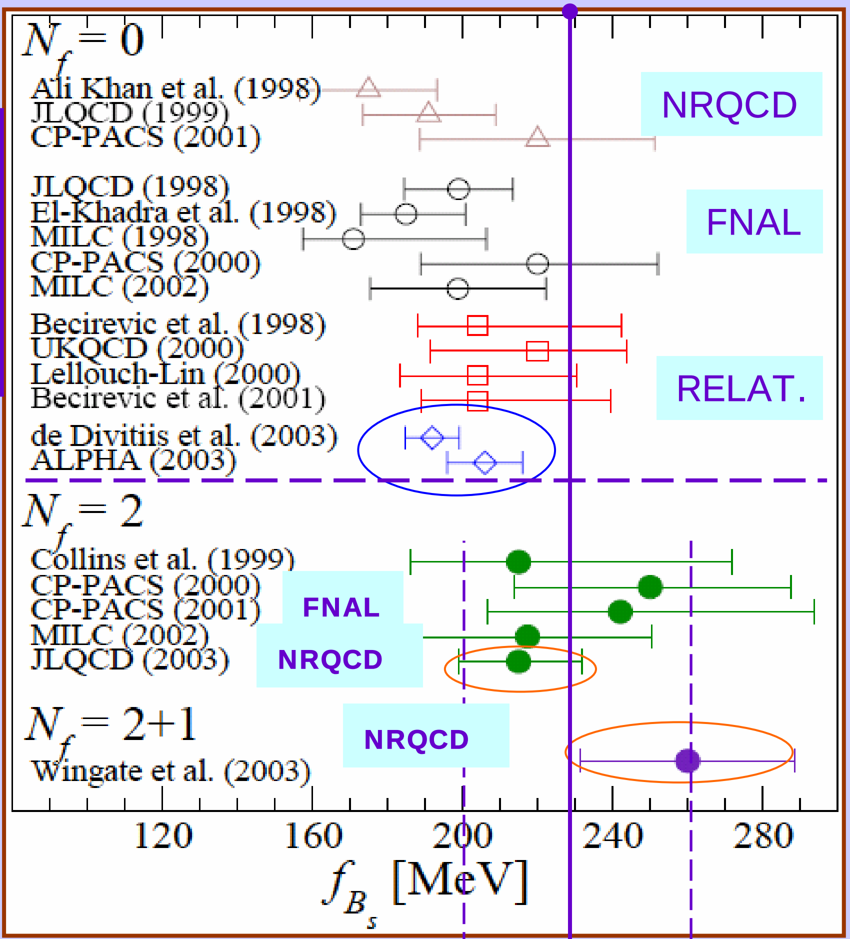
Quenched Simulations

- High level of accuracy;
- Good agreement among different approaches;
- Continnum extrapolation;

[De Divitiis et al.(2003), ALPHA(2003)]

Unquenched Simulations

- Nf=2 studies [JLQCD(2003): high statistics and O(a)-improved action];
- Nf=2+1 preliminary values [Wingate at al. (2003): staggered action];



With the present accuracy:

- **sea quark effects seem at *O*(10%-15%):**
- **But continuum scaling or other systematics (NP-ren., FV effects, diff. actions ...) not completely investigated**

ICHEP 2004:

$$f_{B_s} = 230 \pm 30 \text{ MeV}$$

Bd-Meson Decay Constant from Lattice QCD

$$\langle 0 | \bar{b} \gamma^\mu \gamma_5 d | B_d \rangle = i p^\mu f_{B_d}$$

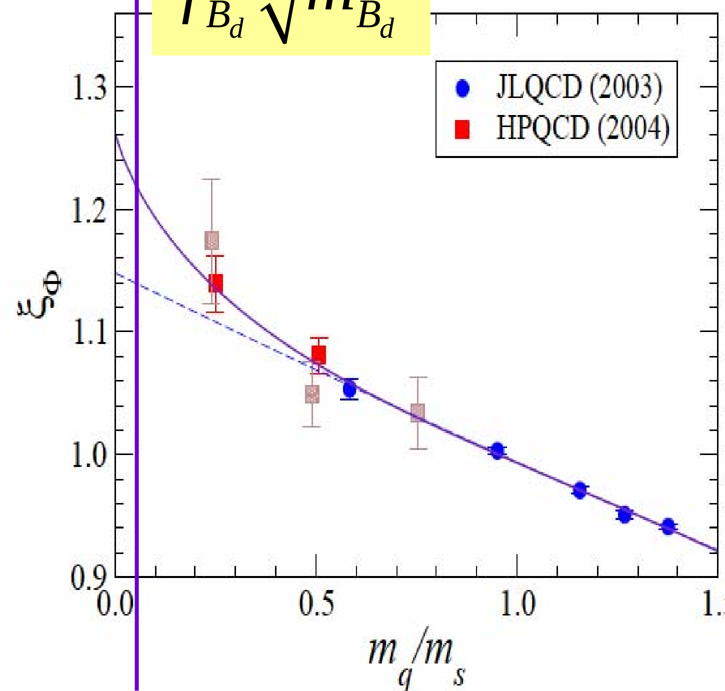
Delicate Issue: Chiral Extrapolation

- Pion Loops could be significant [Kronfeld-Ryan(2002), O(15%)]

$$\frac{f_{B_s}\sqrt{m_{B_s}}}{f_{B_d}\sqrt{m_{B_d}}} \propto 1 + \frac{3(1+3g^2)}{4} \frac{m_\pi^2}{(4\pi f)^2} \log \frac{m_\pi^2}{\mu^2}$$

$$(1+3g^2) \approx 1.8$$

$$\frac{f_{B_s}\sqrt{m_{B_s}}}{f_{B_d}\sqrt{m_{B_d}}}$$



With the present accuracy:

- **Chiral Log Effects roughly estimated**
  [JLQCD (N$_f$=2, m_q/m_s>0.5)
  +HPQCD (N$_f$=2+1, m_q/m_s>0.2)]

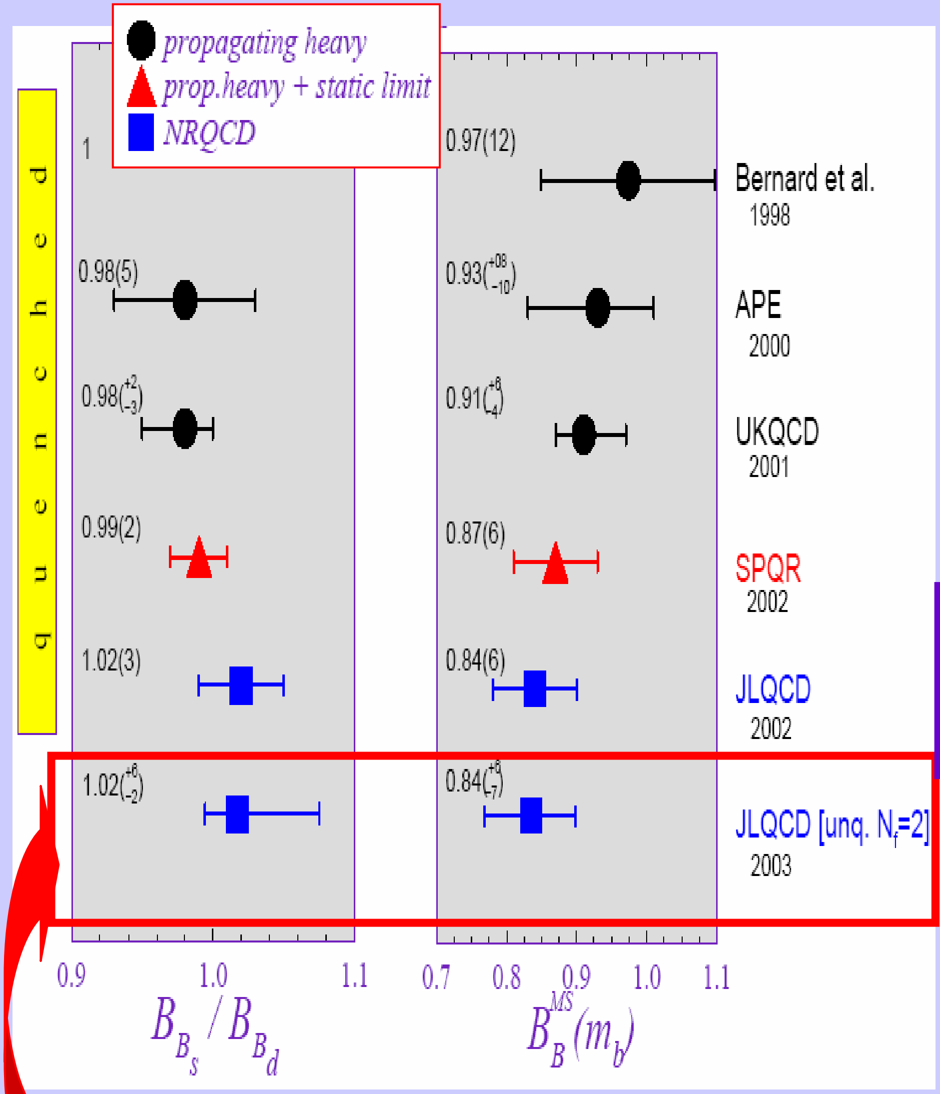
ICHEP 2004:

$$\frac{f_{B_s}}{f_B} = 1.22^{+0.05}_{-0.06}$$

more statistics and lighter quark masses still needed

Final Chiral Loop Effects: *O*(5%)

B-$\overline{\text{B}}$ Mixing on the Lattice



JLQCD(2003): N_f=2 & NRQCD

$$\left\langle \overline{B}_q \left| \left(\overline{b}\,\gamma_L^\mu q\right)\left(\overline{b}\,\gamma_L^\mu q\right) \right| B_q \right\rangle = \frac{8}{3} m_{B_q}^2 f_{B_q}^2 \hat{B}_{B_q}$$

With the present accuracy:

- **Negligible sea quark effects!!**
- **Consistent with diagnosis deduced from QChPT versus ChPT:** **[Booth95,Sharpe,Zhang96]**

$$\hat{B}_{B_d} = 1.287(42)\begin{pmatrix}+86\\-95\end{pmatrix}, \quad \frac{\hat{B}_{B_s}}{\hat{B}_{B_d}} = 1.017(16)\begin{pmatrix}+56\\-17\end{pmatrix}$$

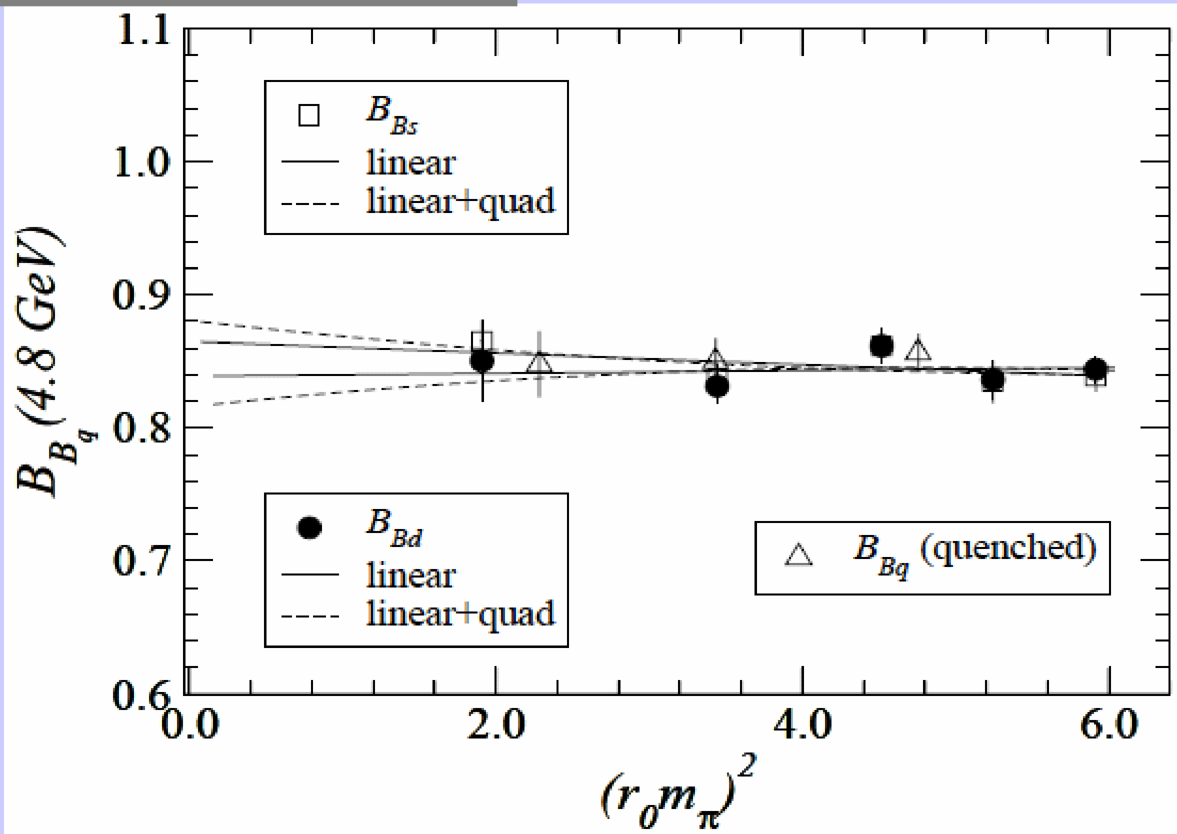
[JLQCD(2003), N_f=2]

B-$\overline{\text{B}}$ Mixing on the Lattice

$$\langle \overline{B}_q | (\overline{b}\gamma_L^\mu q)(\overline{b}\gamma_L^\mu q) | B_q \rangle = \frac{8}{3} m_{B_q}^2 f_{B_q}^2 \hat{B}_{B_q}$$

Chiral Loops not a problem for $\hat{B}_{B_d}$

They are expected small from ChPT



Lattice data are consistent with a constant.

B Mixing from Lattice QCD

	ICHEP 2002 (Lellouch)	BEAUTY 2005 (ICHEP 2004, Hashimoto)
f_{B_d} (MeV)	203(27)(+0-20)	189(27)
f_{B_s} (MeV)	238(31)	230(30)
$f_{B_s}\sqrt{\hat{B}_{B_s}}$ (MeV)	276(38)	262(35)
f_{B_s} / f_{B_d}	1.18(4)(+12-0)	1.22(+5-6)
ξ	1.18(4)(+12-0)	1.23(6)

Now averages include rough ``estimates'' of chiral logs **(m_q/m_s>0.2)** and unquenched effects **(N_f=2+1)**

B Mixing from QCD Sum Rules

1. **Equating phenomenological and theoretical spectral functions;**
2. **Determination of theoretical spectral functions by calculating two or three-point correlators in perturbative QCD, including corrections from the OPE.**

f_B	$O(\alpha_s^2)$ recently calculated
B_B	$O(\alpha_s)$ completed but non-factorisable contributions important

$$f_B = 210(19)\,\mathrm{MeV},$$

$$f_{B_s} = 244(21)\,\mathrm{MeV}$$ (Jamin et al '02)

$$\hat{B}_B = 1.60(3)$$ (Körner et al '03)

Problem: many parameters, loosely constrained!

2. Decay:

$$\Gamma_{21}^{q} \quad (\Delta B = 2)$$

- **Width Differences** $\Delta\Gamma_{B_d}, \Delta\Gamma_{B_s}$
- **CP-Violation Parameters** $|q/p|_{B_{d,s}}$

$$\Gamma_{11}^{q} \quad (\Delta B = 0)$$

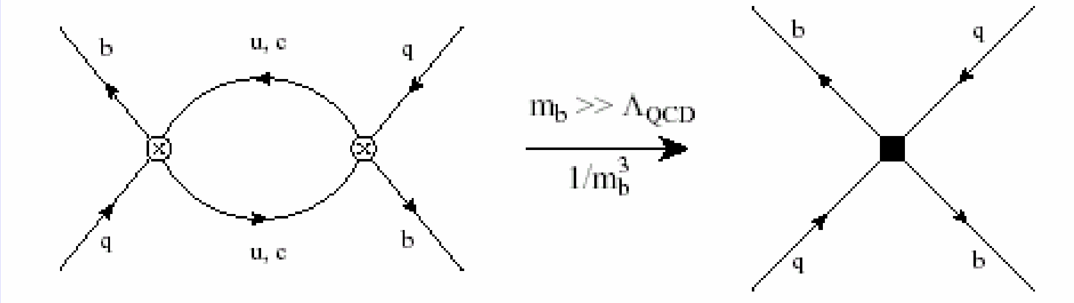
- **Lifetime Ratios** $\tau(B^+)/\tau(B_d), \tau(B_s)/\tau(B_d), \tau(\Lambda_b)/\tau(B_d)$

$$\Gamma^q_{11} \propto Disc\langle B^0_q|\mathcal{T}|B^0_q\rangle \qquad \Gamma^q_{21} \propto Disc\langle \overline{B}^0_q|\mathcal{T}|B^0_q\rangle$$

$$\mathcal{T} = i \int d^4x\, T\left(\mathcal{H}^{\Delta B=1}_{eff}(x)\, \mathcal{H}^{\Delta B=1}_{eff}(0)\right)$$

HQE $(m_b >> \Lambda_{QCD})$

Large energy release ⇔ **contact interaction**



SPECTATOR EFFECTS

$$\Gamma^q_{21} = \sum_k \frac{\vec{c}^{\,q}_k(\mu)}{m_b^k} \langle \overline{B}^0_q|\vec{O}^{q\,\Delta B=2}_k(\mu)|B^0_q\rangle$$

Scale separation

- $\vec{c}_k^{\,q}(\mu)$ **: short-distance (perturbative)**
- $\left\langle \overline{B}_q^{\,0} \middle| \vec{O}_k^{q\,\Delta B=2}(\mu) \middle| B_q^0 \right\rangle$ **: long-distance (non-perturbative)**

Wilson Coefficients from P.T.

- **Lifetime Ratios**

$$\Gamma = \frac{G_F^2 |V_{cb}|^2 m_b^5}{192\pi^3 (2M_B)} \left[c^{(3)} \langle \bar{b} b \rangle + c^{(5)} \frac{g_s}{m_b^2} \langle \bar{b} \sigma_{\mu\nu} G^{\mu\nu} b \rangle + \frac{96\pi^2}{m_b^3} \sum_k \left(c_k^{(6)} \langle O_k^{(6)} \rangle + \frac{c_k^{(7)}}{m_b} \langle O_k^{(7)} \rangle \right) \right]$$

$O(1)$ **(1996)** **[M. Neubert and C.T. Sachrajda]**

$O(\alpha_s)$ **(2002)**

- **[E. Franco, V. Lubicz, F. Mescia and C.T.]**
- **[M. Beneke, G. Buchalla, C. Greub, A. Lenz and U. Nierste]**

$O(1/m_b)$ **(2004)**

- **[F. Gabbiani, A. I. Onishchenko and A. A. Petrov]**

- **Width Differences and ~~CP~~**

$$\Gamma_{21}^s = -\frac{G_F^2 m_b^2}{12\pi(2M_{B_s})} (V_{cb}^* V_{cs})^2 \left[G\langle Q \rangle + G_S \langle Q_S \rangle + \delta_{1/m_b} \right]$$

$O(\alpha_s)$ **(2003)**

- **[M. Ciuchini, E. Franco, V. Lubicz, F. Mescia and C.T.]**
- **[M. Beneke, G. Buchalla, A. Lenz and U. Nierste]**

$O(1/m_b)$ **(1996)**

- **[M. Beneke, G. Buchalla and I. Dunietz]**

Matrix Elements: $\Delta B = 2$ Operators

Leading contribution $O(1/m_b^3)$

$$\mathcal{O}_1^q = (\bar{b}q)_{V-A}\,(\bar{b}q)_{V-A} \leftrightarrow B_1^q, \qquad \mathcal{O}_2^q = (\bar{b}q)_{S-P}\,(\bar{b}q)_{S-P} \leftrightarrow B_2^q.$$

B_q^1, B_q^2 ⇐ **From the *lattice* (with different methods) or QCD -*sum rules* [J.G. Korner et al., 2003]**

Subleading contribution $O(1/m_b^4)$

4 operators $(R_1^q, R_2^q, R_3^q, R_4^q)$ **:**

- R_1^q, R_4^q **: related, through Fierz and eq. of motion, to operators computed on the *lattice***
- R_2^q, R_3^q **: from the *VSA***

B_q^1, B_q^2 on the lattice

- **HQET** $(m_b \to \infty)$

$$B_1^s = 0.83(5)(6),\ \ B_2^s = 0.81(2)(10)$$

[V. Gimenez and J. Reyes, 2000]

- **NRQCD** $\mathcal{O}(1/m_b)$

$$B_1^s = 0.85(3)(11),\ \ B_2^s = 0.82(2)(11)$$

[Hi-KEK (S. Hashimoto et al.), 2000]

- ***unquenched* NRQCD** $n_f = 2$

$$B_1^s = 0.85(2)(6),\ \ B_2^s = 0.84(6)(8)$$

[JLQCD (S. Aoki et al.), 2001-2003]

- **QCD** $(m_c \lesssim m_Q < m_b,\ m_Q \to m_b)$

$$B_1^s = 0.91(3)^{+0}_{-6},\ \ B_2^s = 0.86(2)^{+2}_{-3}$$

[APE (D. Becirevic et al.), 2000]

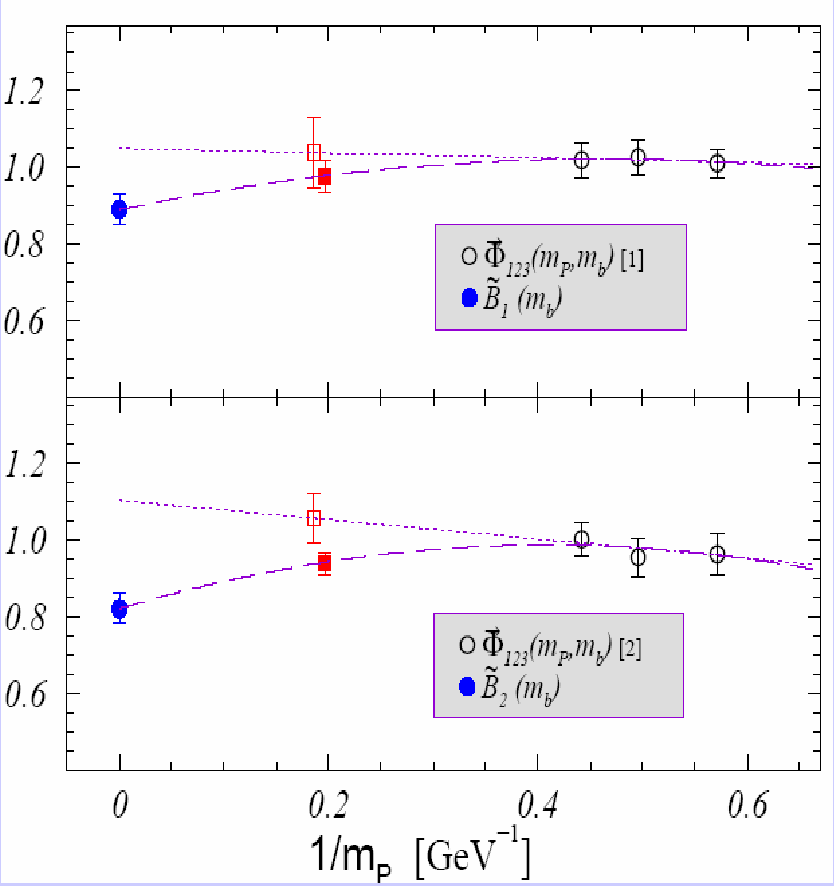
- **QCD +HQET**

$$B_1^s = 0.87(2)(5),\ \ B_2^s = 0.84(2)(4)$$

[APE (D. Becirevic et al.), 2001]

QCD + HQET

SPQcdR, 2001



- Combine the static HQET results for B-parameters with the relativistic lattice QCD ones
  $\Rightarrow$ extrapolation $\rightarrow$ "interpolation"

- Perturbative matching of the anomalous dimensions of 4-f QCD and HQET operators made @ NLO in perturbation theory!

- **So far, the approach has been only applied in the quenched case**

Matrix Elements: $\Delta B = 0$ Operators

Leading spectator effect contribution $O(1/m_b^3)$

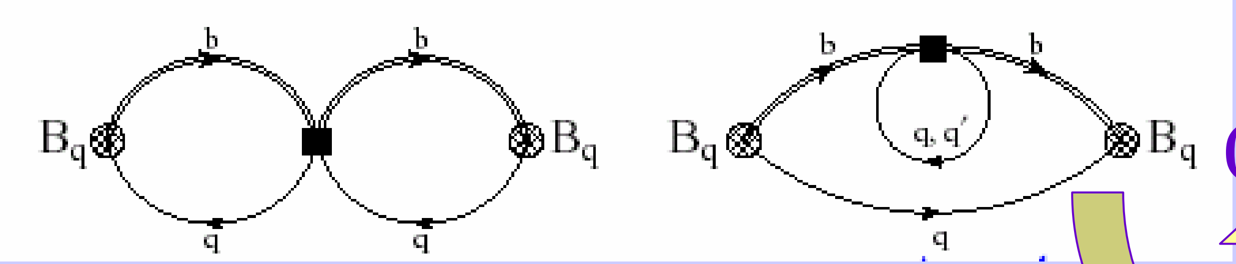
$$\mathcal{O}_1^q = (\bar{b}q)_{V-A}\,(\bar{q}b)_{V-A} \leftrightarrow B_1^q, \qquad \mathcal{O}_2^q = (\bar{b}q)_{S-P}\,(\bar{q}b)_{S+P} \leftrightarrow B_2^q,$$

$$\mathcal{O}_3^q = (\bar{b}T^a q)_{V-A}\,(\bar{q}T^a b)_{V-A} \leftrightarrow \epsilon_1^q, \qquad \mathcal{O}_4^q = (\bar{b}T^a q)_{S-P}\,(\bar{q}T^a b)_{S+P} \leftrightarrow \epsilon_2^q,$$

$$O_P = (\bar{b}T^a b)_V \sum_{q=u,d,s,c} (\bar{q}T^a q)_V$$

(not computed)

$$\left[(\bar{q}q)_{V-A} = \bar{q}\gamma_L^\mu q,\ (\bar{q}q)_{S\pm P} = \bar{q}(1\pm\gamma_5)q,\ (\bar{q}q)_V = \bar{q}\gamma^\mu q\right]$$



(not computed)

B-parameters:

- Λ_b :computed in lattice-HQET, evolved at the LO

[M. Di Pierro,C. T. Sachrajda and C. Michael(UKQCD collaboration), 1999]

$B_d - B_s - B^+$ **B-parameters**

• **Lattice-HQET** $(m_b \to \infty)$

$$B_1^d = 1.06 \pm 0.08, \quad B_2^d = 1.01 \pm 0.07,$$
$$\varepsilon_1^d = -0.01 \pm 0.03, \quad \varepsilon_2^d = -0.03 \pm 0.02.$$

[M. Di Pierro and C.T. Sachrajda, 1998]

• **Lattice-QCD** $(m_c \lesssim m_Q < m_b, m_Q \to m_b)$

$$B_1^d = 1.2 \pm 0.2, \quad B_2^d = 0.9 \pm 0.1,$$
$$\varepsilon_1^d = 0.04 \pm 0.01, \quad \varepsilon_2^d = 0.04 \pm 0.01.$$

[APE (D. Becirevic et al.), 2001]

• **Sum Rules, in HQET**

$$B_1^d = 1.01 \pm 0.01, \quad B_2^d = 0.99 \pm 0.01,$$
$$\varepsilon_1^d = -0.08 \pm 0.02, \quad \varepsilon_2^d = -0.01 \pm 0.03.$$

[M.S. Baek et al., 1998]

Subleading spectator effect contribution $O(1/m_b^4)$

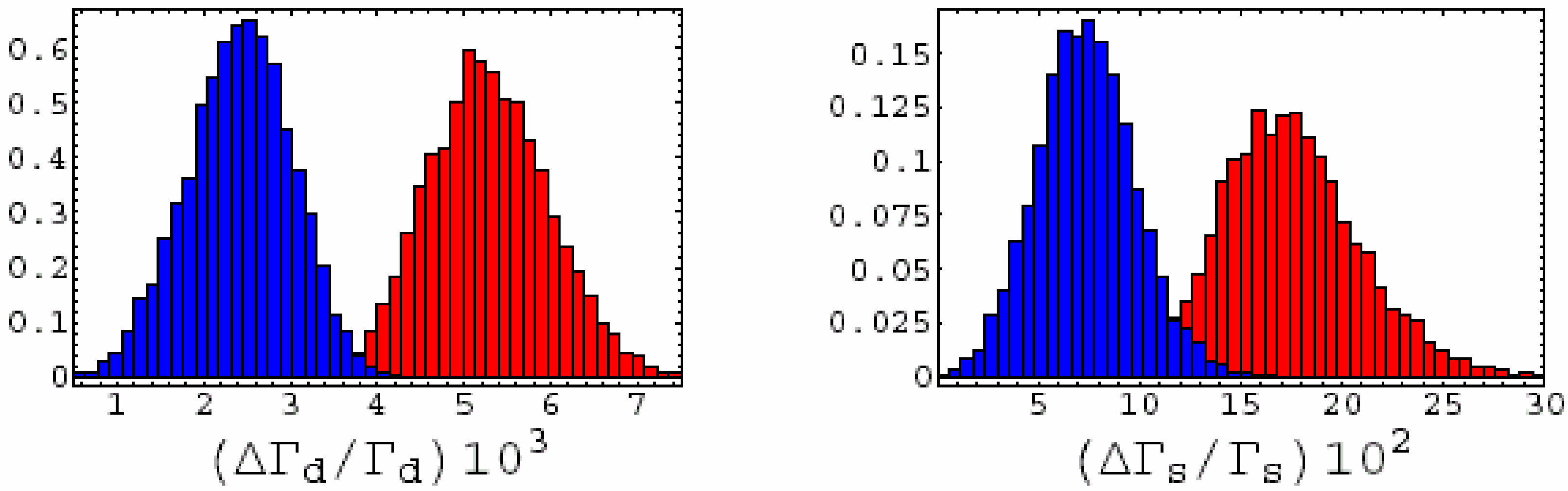
8 operators, from the *VSA* (B-mesons) or the *quark-diquark model* (baryon)

Width Differences

$$\frac{\Delta\Gamma_q}{\Gamma_q} = -\frac{\Delta m_q}{\Gamma_q}\mathcal{R}e\left(\frac{\Gamma_{21}^q}{M_{21}^q}\right)$$

$$(\Gamma_{21}^q/M_{21}^q = \mathcal{O}(m_b^2/m_t^2), \Delta\Gamma_d/\Delta\Gamma_s = \mathcal{O}(\lambda^2))$$

NLO distr. vs LO distr.



Theoretical predictions at the NLO + contribution of $O(1/m_b^4)$:

$$\frac{\Delta\Gamma_d}{\Gamma_d} = (2.42 \pm 0.59)10^{-3} \qquad \frac{\Delta\Gamma_s}{\Gamma_s} = (7.4 \pm 2.4)10^{-2}$$

[M. Ciuchini, E. Franco, V. Lubicz, F. Mescia and C. T., 2003]

Experimental measurements:

$$\left|\frac{\Delta\Gamma_d}{\Gamma_d}\right| = 0.008 \pm 0.037 \pm 0.018 \text{ (BaBar collaboration, 2003)}$$

$$\frac{\Delta\Gamma_s}{\Gamma_s} = 0.07^{+0.09}_{-0.07} \text{ (HFAG, 2004)}$$

[CDF, 2004]

$$\frac{\Delta\Gamma_s}{\Gamma_s} = 0.65^{+0.25}_{-0.33} \pm 0.01 \qquad \approx 2\sigma$$

[D0, CKM2005]

$$\frac{\Delta\Gamma_s}{\Gamma_s} = 0.21^{+0.27}_{-0.40}$$ **(preliminary)**

[THEORY]

$$\frac{\Delta\Gamma_s}{\Gamma_s} = Fact * \left[(c_1^{(0)} + c_1^{(1)}) \left\langle O_1^s \right\rangle + (c_2^{(0)} + c_2^{(1)}) \left\langle O_2^s \right\rangle + \delta_{1/m_b} \right]$$

$$= \left\{ (0.018 - 0.014) + (0.240 - 0.064) - 0.097 \right\}$$

LO	**26%**
LO+ $O(1/m_b^4)$	**16%**
NLO	**18%**
NLO+ $O(1/m_b^4)$	**7%**

$$c_1 << c_2$$

$$\frac{c_2^{(1)}}{c_2^{(0)}} = -27\%$$ **(large cancellations at the NLO)**

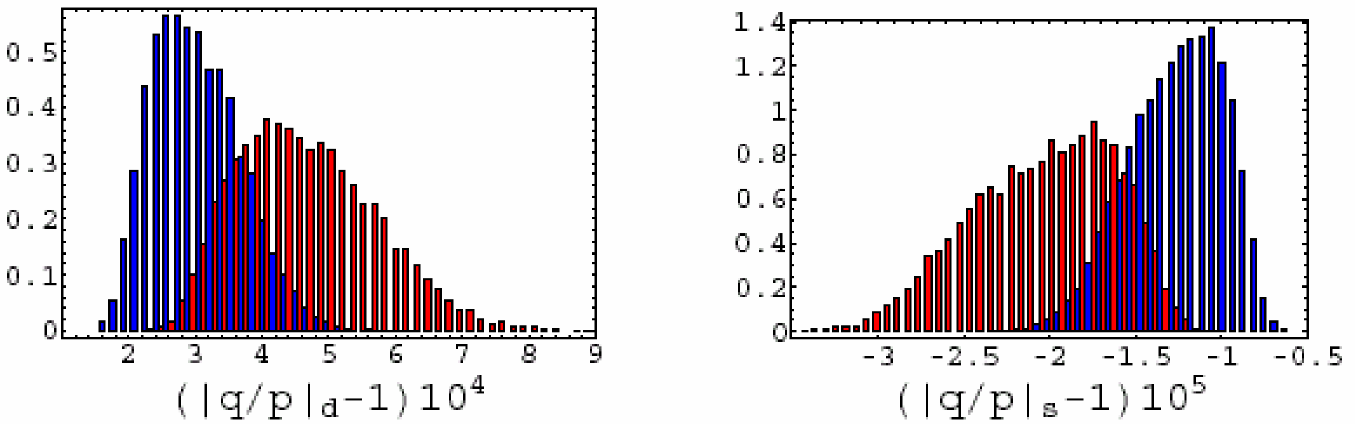
$$\frac{\delta_{1/m_b}}{c_2 \left\langle O_2^s \right\rangle} = -55\%$$ **(large cancellations at** $O(1/m_b^4)$ **)**

CP-Violation Parameters

$$|(\frac{q}{p})_q| - 1 = \frac{1}{2}\mathcal{I}m(\frac{\Gamma_{21}^q}{M_{21}^q})$$

$$(|(q/p)_q| - 1)/\Delta\Gamma_q = \mathcal{O}(m_c^2/m_b^2), (|(q/p)_s| - 1)/(|(q/p)_d| - 1) = \mathcal{O}(\lambda^2)$$

NLO distr. vs LO distr.



Theoretical predictions at the NLO:

$$|(q/p)_d| - 1 = (2.96 \pm 0.67)10^{-4} \quad , \quad |(q/p)_s| - 1 = -(1.28 \pm 0.27)10^{-5}$$

[M. Ciuchini, E. Franco, V. Lubicz, F. Mescia and C. T., 2003]

Experimental measurement:

$$|(q/p)_d| - 1 = 0.0013 \pm 0.0034$$

[HFAG, 2005]

- **Importance of more accurate measurements ⟺ UTA**

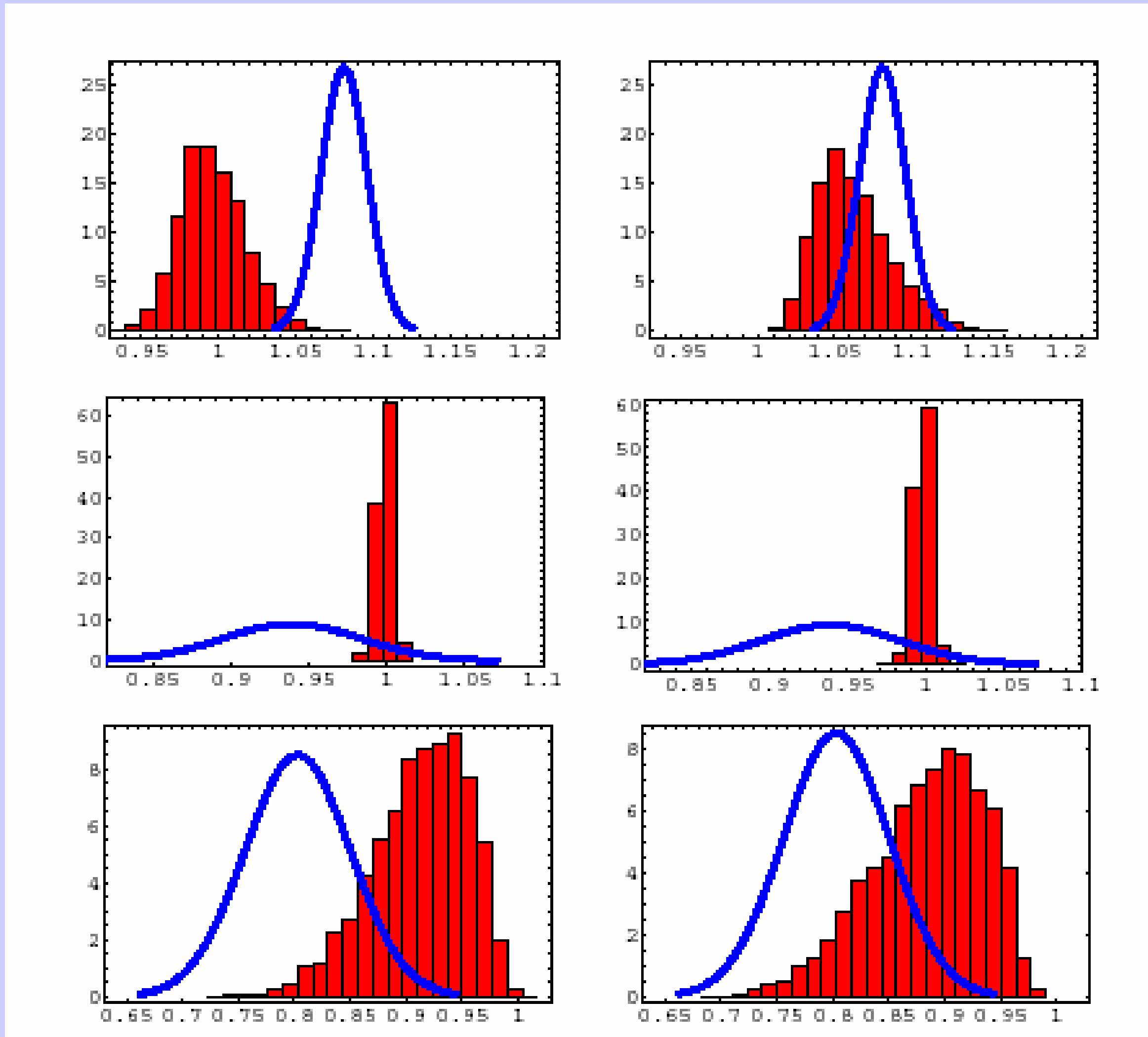
Lifetime Ratios

LO NLO

$$\frac{\tau(B^+)}{\tau(B_d)}$$

$$\frac{\tau(B_s)}{\tau(B_d)}$$

$$\frac{\tau(\Lambda_b)}{\tau(B_d)}$$



	$\frac{\tau(B^+)}{\tau(B_d)}$	$\frac{\tau(B_s)}{\tau(B_d)}$	$\frac{\tau(\Lambda_b)}{\tau(B_d)}$
LO	**1.01(3)**	**1.00(1)**	**0.93(4)**
NLO	**1.06(3)**	**1.00(1)**	**0.90(5)**
NLO+ $O(1/m_b^4)$	**1.06(2)**	**1.00(1)**	**0.88(5)**

Theoretical predictions at the NLO + contribution of $O(1/m_b^4)$:

$$\frac{\tau(B^+)}{\tau(B_d)} = 1.06 \pm 0.02, \quad \frac{\tau(B_s)}{\tau(B_d)} = 1.00 \pm 0.01, \quad \frac{\tau(\Lambda_b)}{\tau(B_d)} = 0.88 \pm 0.05$$

[E. Franco, V. Lubicz, F. Mescia and C. T., 2002-2003]

Experimental measurements:

$$\frac{\tau(B^+)}{\tau(B_d)} = 1.081 \pm 0.015, \quad \frac{\tau(B_s)}{\tau(B_d)} = 0.939 \pm 0.044, \quad \frac{\tau(\Lambda_b)}{\tau(B_d)} = 0.803 \pm 0.047$$

[LEP+CDF+B-factories average, Heavy Flavor Averaging Group (HFAG), 2004]

- **Good agreement at the NLO and $O(1/m_b^4)$** ($\tau(\Lambda_b)/\tau(B_d)$ at 1σ)

Conclusions

MIXING: Phenomenological importance of $\Delta m_d, \Delta m_s$ in the UTA
Theoretical inputs at 5-15% from Lattice QCD

DECAY: Importance of NLO corrections in QCD
and $O(1/m_b^4)$ in the HQE

Width Differences

[CDF, 2004]

$$\frac{\Delta\Gamma_s}{\Gamma_s} = 0.65^{+0.25}_{-0.33} \pm 0.01$$

[D0, Lattice2005]

$$\frac{\Delta\Gamma_s}{\Gamma_s} = 0.21^{+0.27}_{-0.40}$$

(preliminary)

[THEORY]

$$\frac{\Delta\Gamma_s}{\Gamma_s} = 0.07 \pm 0.03$$

CP-Violation Parameters

Importance of more accurate measurements ⟺ UTA

Lifetime Ratios

$\tau(\Lambda_b)/\tau(B_d)$ at 1σ