Electron Fractionalization in 2D Dirac Fermions

Claudio Chamon

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Roman Jackiw - MIT
So-Young Pi - BU
Andreas Schnyder - UCSB

C.Y. Hou, C. Chamon, C. Mudry
Phys. Rev. Lett. 98, 186809 (2007)

C. Chamon, C.Y. Hou, R. Jackiw, C. Mudry, S.-Y. Pi, A. Schnyder
arXiv:0707.0293

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BOSTON UNIVERSITY

Fractionalization in Polyacetelene

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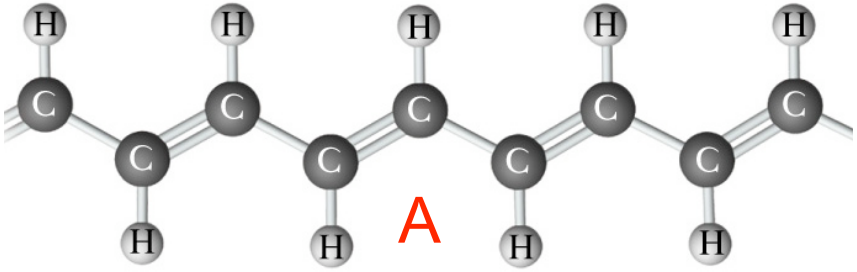

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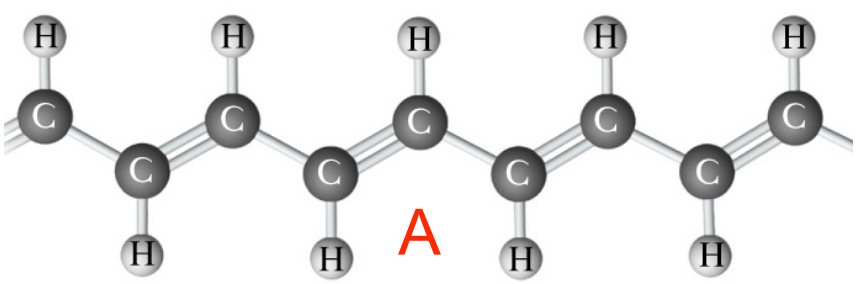

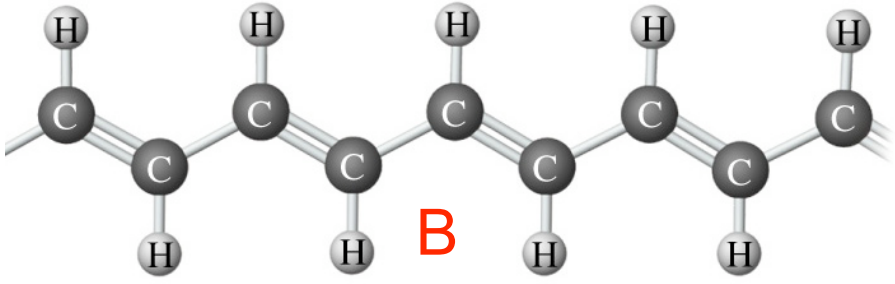

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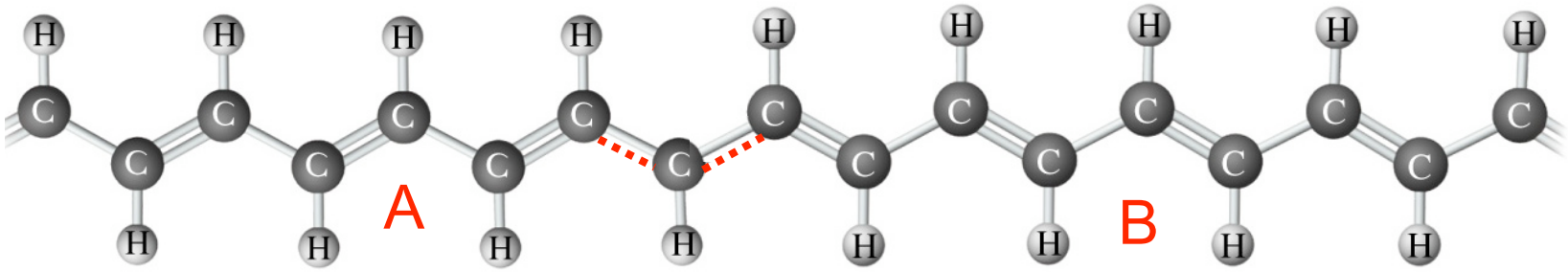

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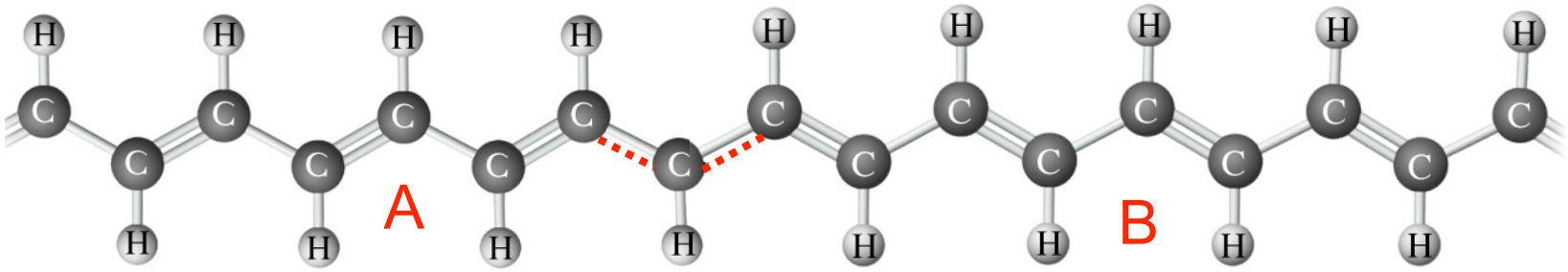

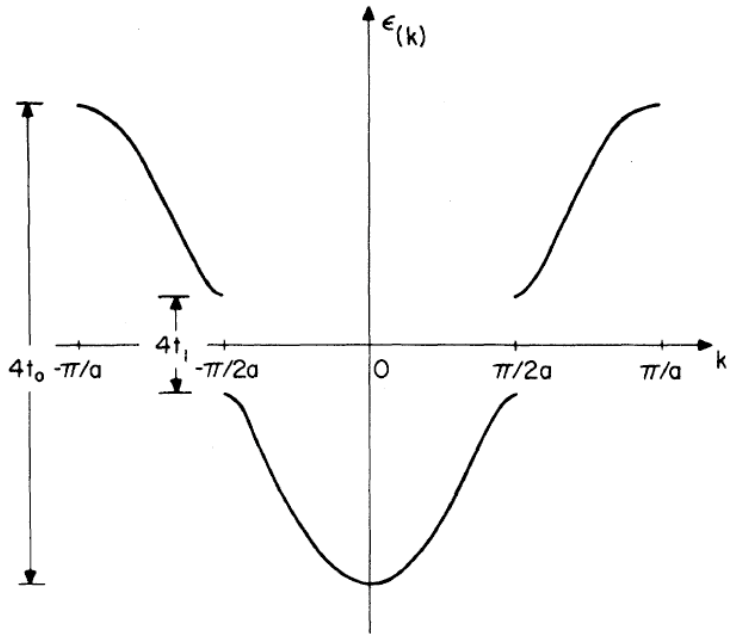

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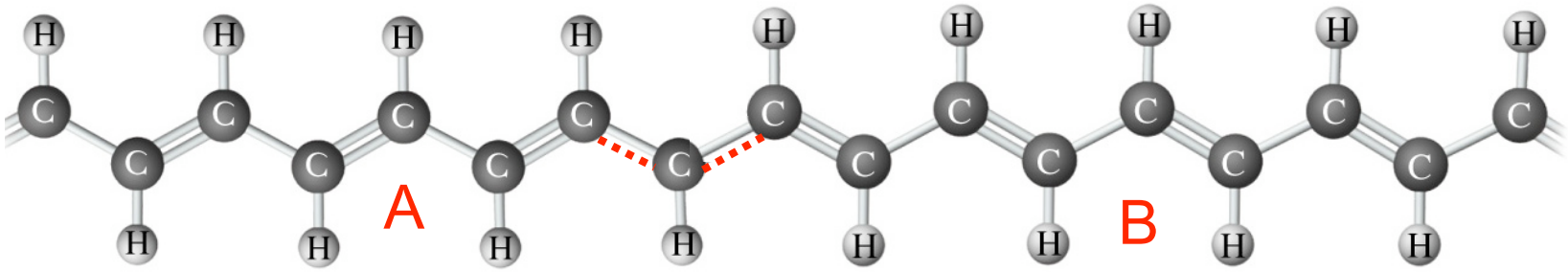

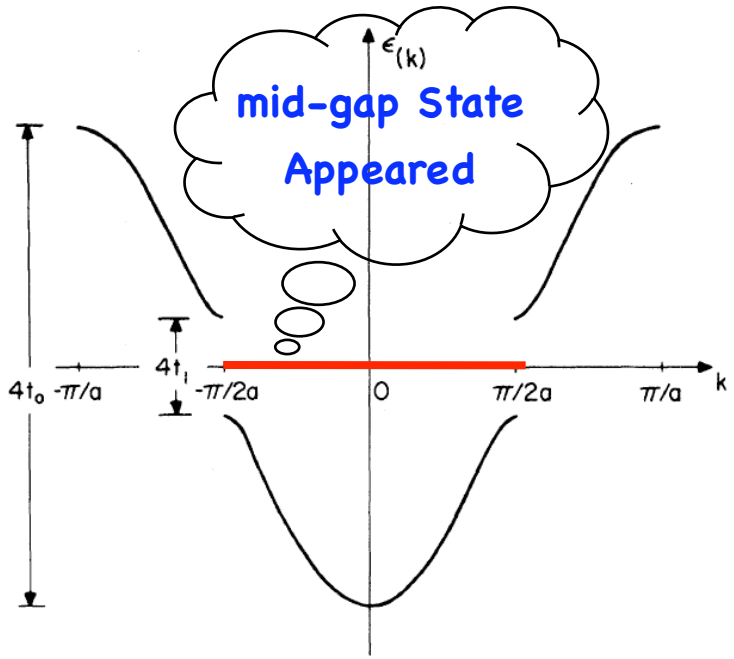

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Ingredients for fractionalization in Polyacetelene

- Polyacetylene has two Fermi points. (The *rule* in 1D but *exceptional* in 2D.)

- A perturbation couples the two Fermi points, opening a single-particle gap at the Fermi energy and stabilizing a bond density wave (BDW) state.

- There are two degenerate BDW states associated to the spontaneous breaking of a $\mathbb{Z}_2$ symmetry.

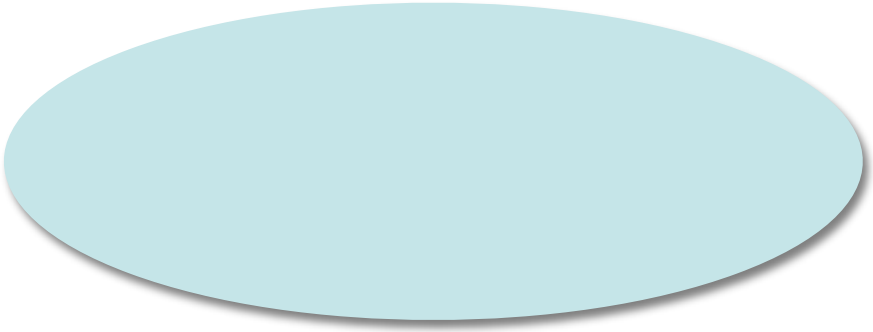
- There are solitons that interpolate between the two degenerate BDW states.

- There are single-particle states at the Fermi energy in the background of the soliton.

- The fractional charge is calculated as the difference between the local single-particle density of states with and without the soliton.

Fractionalization in the Quantum Hall Effect

Laughlin (1983)



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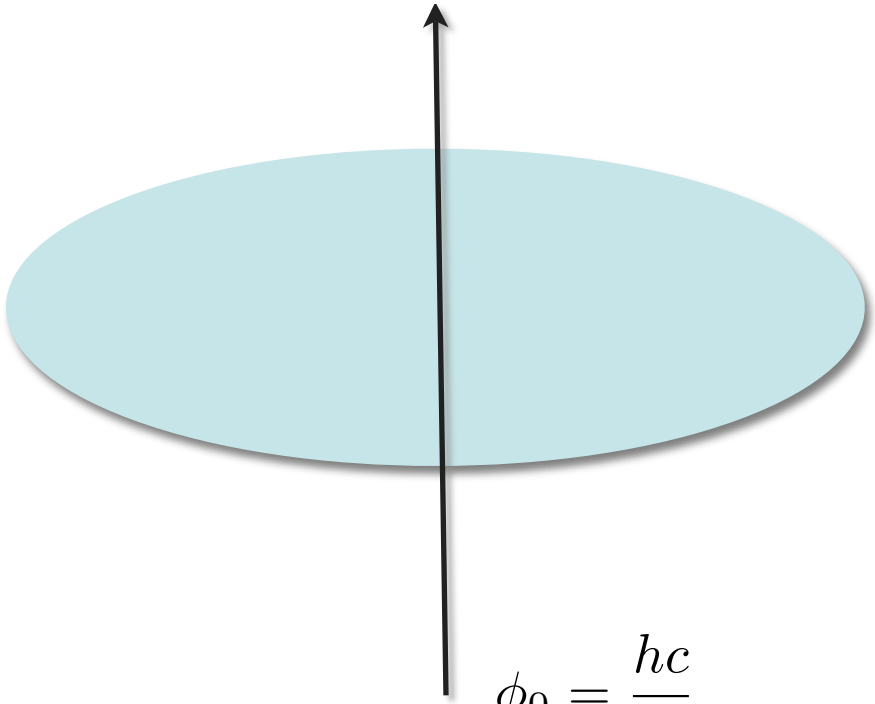

$$\phi_0 = \frac{hc}{e}$$

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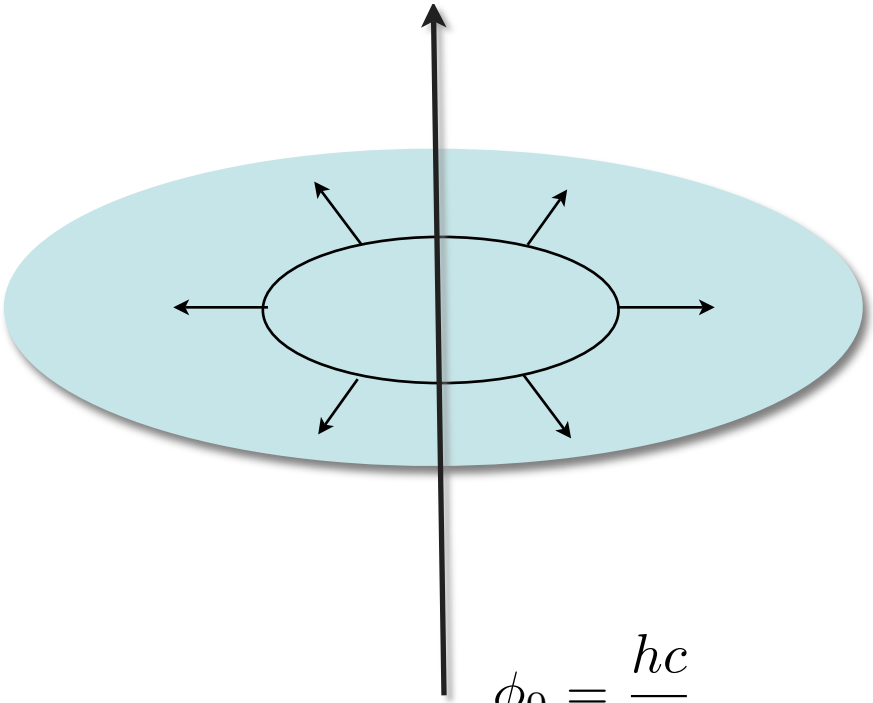

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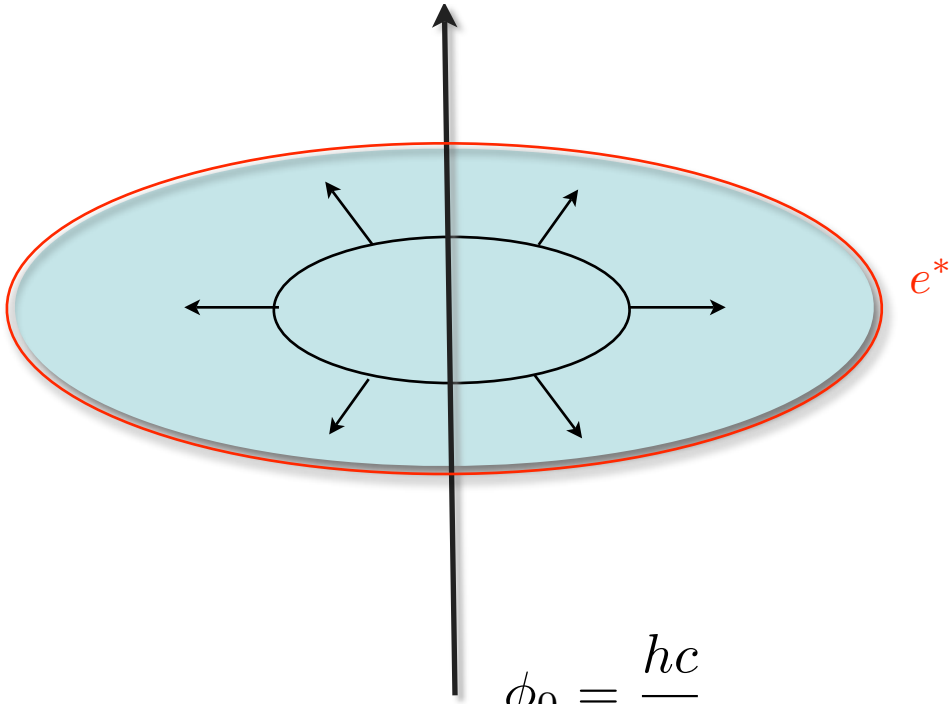

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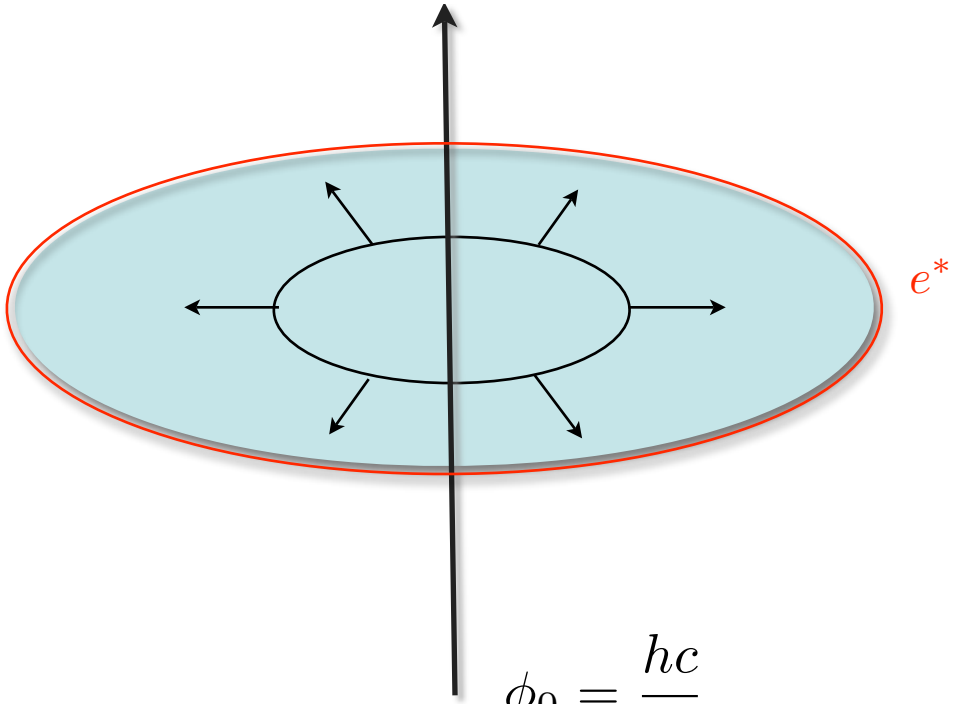

$$\phi_0 = \frac{hc}{e}$$

$$\begin{aligned} Q &= \int dt \int d\theta \; r \; j_r \\ &= \int dt \int d\theta \, r \; \sigma_{xy} \; \frac{1}{r} \frac{1}{c} \frac{d\Phi}{dt} \\ &= \sigma_{xy} \; \phi_0 / c \\ &= \nu \, e = e^* \end{aligned}$$

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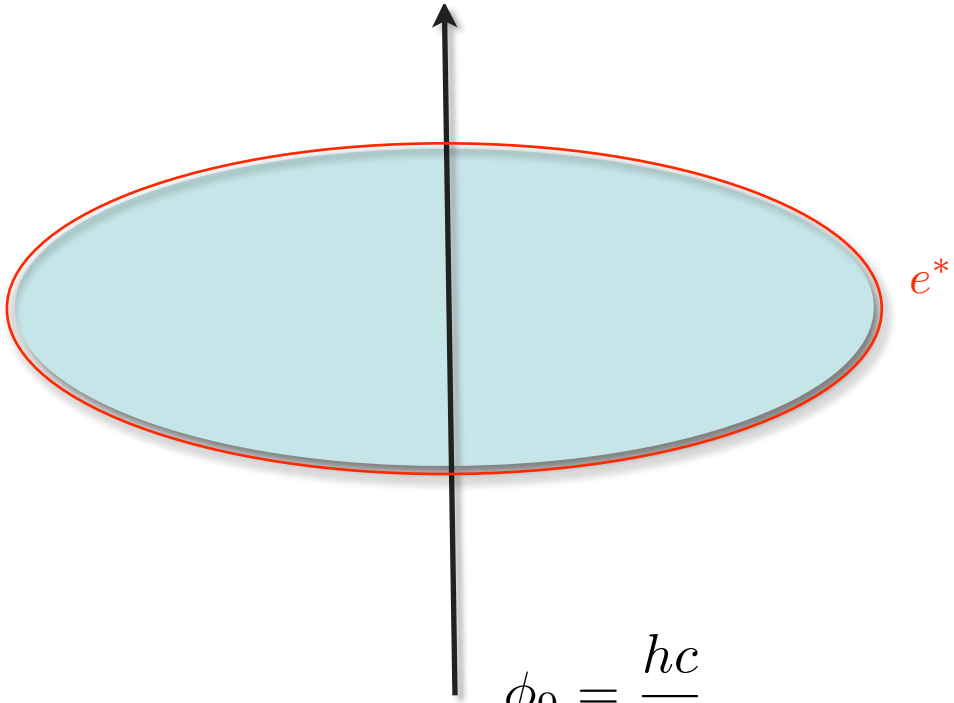
Laughlin (1983)



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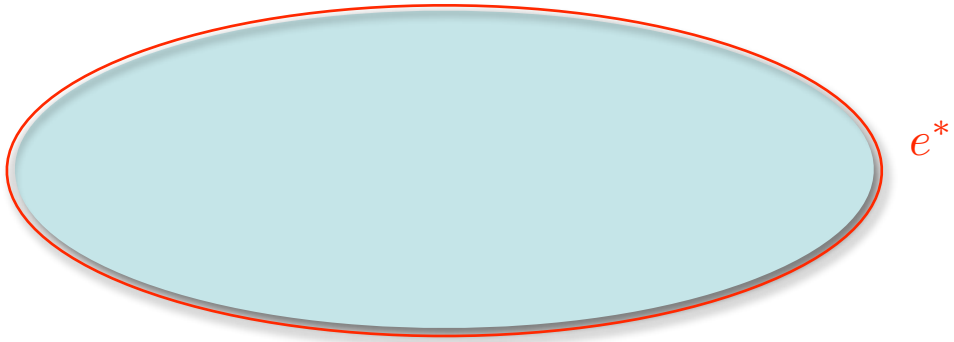

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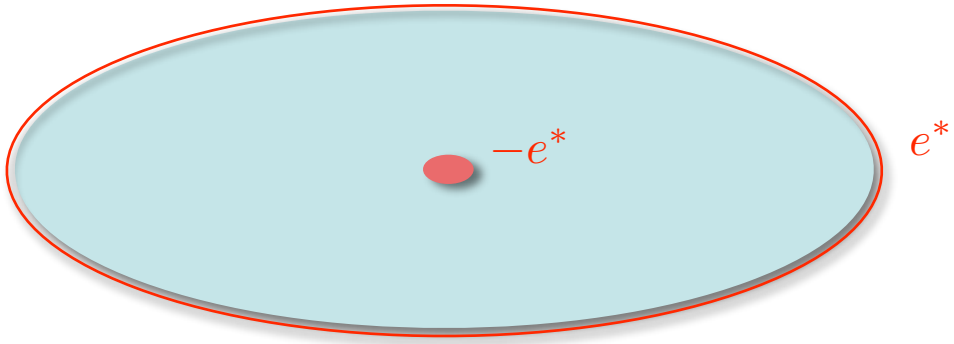

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$$

Ingredients for fractionalization in the QHE

- Time-reversal symmetry is explicitly broken by an external field.

- The Laughlin state is an incompressible quantum liquid that does not spontaneously break any symmetry, and has has a quantized Hall conductance.

The ground state degeneracy depends on the genus of surface, and cannot be lifted by local perturbations: Wen's quantum topological order.

Polyacetelene *vs.* Quantum Hall Fluids

Two different beasts!

Fractionalization in Polyacetylene (Rebbi and Jackiw 1976, Su, Schrieffer, and Heeger 1979)

- holds at the single-particle level in the background of some texture
- that breaks spontaneously a symmetry
- and was believed to be special to 1D.

Fractionalization in the FQHE (Laughlin 1983, Halperin 1984, Wen 1990)

- is a many-body effect
- built on the emergence of quantum topological order.

The notion of quantum topological order has played an essential role in attempts to identify examples of quantum-number fractionalization in space larger than 1D

Taxonomy of fractionalization

1D: Spontaneous symmetry breaking
2D: Topological order

Is this really correct?

Example of fractionalization in 2D via
spontaneous symmetry breaking!

Irrational vs. Rational charge and statistics in 2D

3 steps for 2D fractionalization “à la Polyacetelene”

- We seek a model for non-interacting electrons in 2D with two Fermi points.
- We need to open a gap by spontaneously breaking a symmetry.
- Topological defects on the textured background seen by the electrons.

Step 1: two Fermi points

Systems where the valence and conduction band touch at two points in the Brillouin zone: Ex. square lattice π-flux phase and graphene

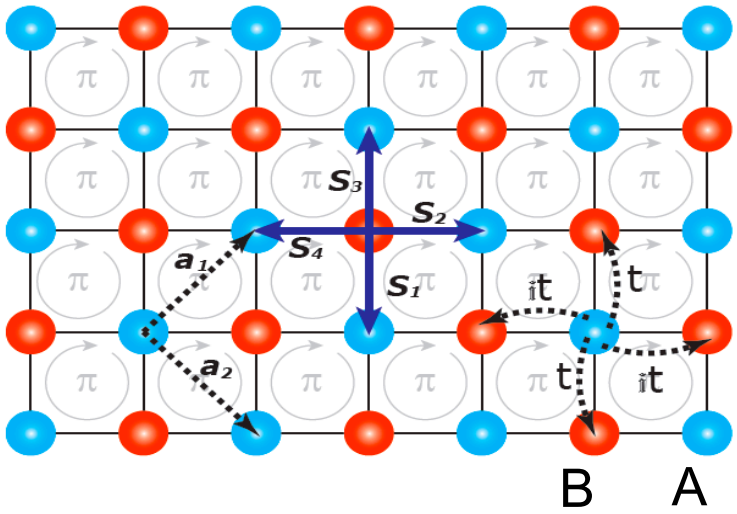
Bipartite lattices A and B - hopping between these

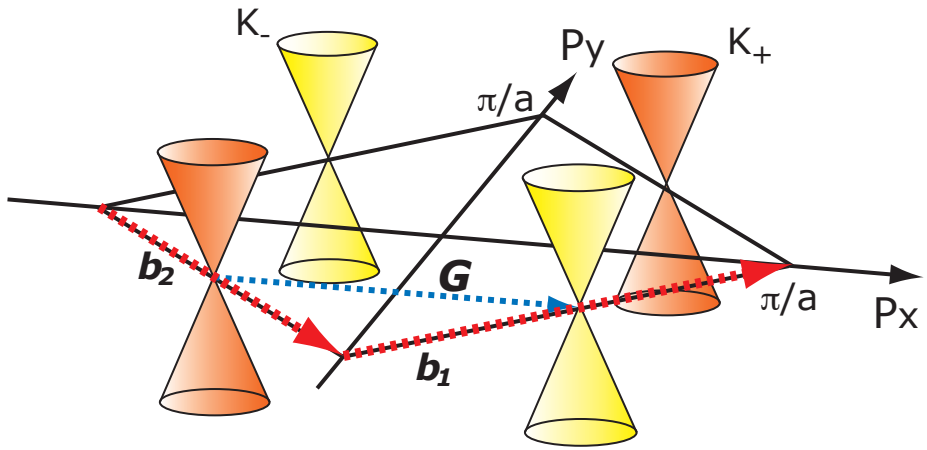
$$H = -\sum_{\boldsymbol{r}\in\Lambda_A}\sum_i t_i\, a^\dagger_{\boldsymbol{r}} b_{\boldsymbol{r}+\boldsymbol{s}_i} + \text{H.c.}$$

Real space

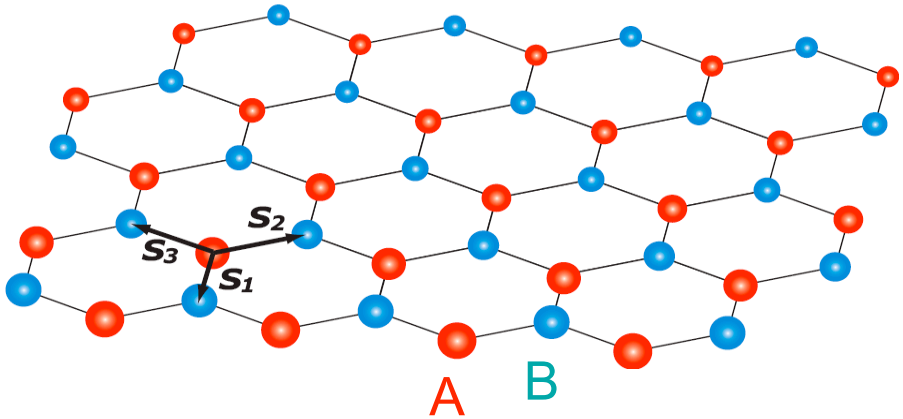
Reciprocal space

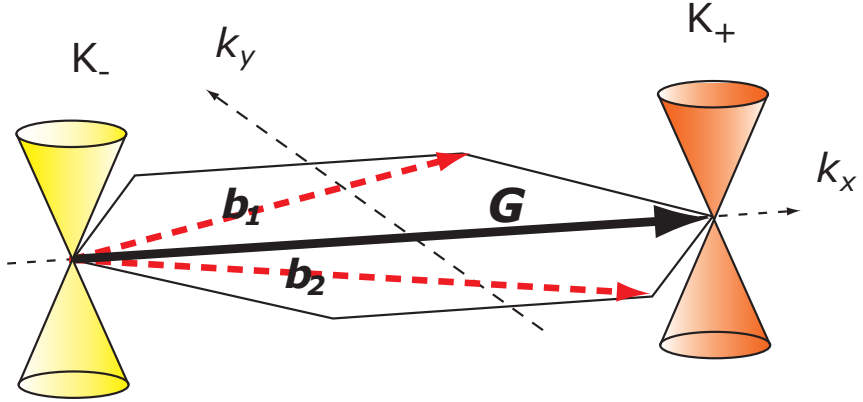
square lattice π-flux



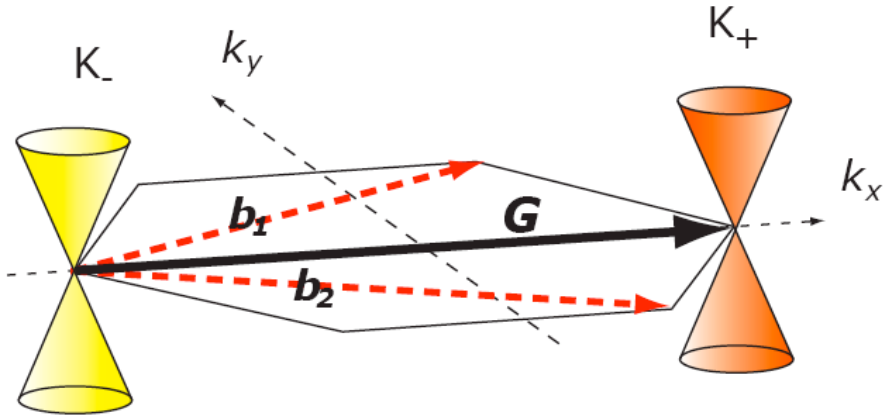


graphene





Low energy effective Hamiltonian



Two independent massless Dirac fermions correspond to two valleys. At half-filling, the Fermi energy is at zero energy -- two Fermi points.

Effective Hamiltonian: $$\mathcal{H} = \int d^2\boldsymbol{r}\, \Psi^\dagger(\boldsymbol{r})\, \mathcal{K}_D(\boldsymbol{r})\, \Psi(\boldsymbol{r})$$

$$\mathcal{K}_D = \begin{pmatrix} 0 & -2i\partial_z & 0 & 0 \\ -2i\partial_{\bar{z}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 2i\partial_z \\ 0 & 0 & 2i\partial_{\bar{z}} & 0 \end{pmatrix} \qquad \Psi(\boldsymbol{r}) = \begin{pmatrix} u_b(\boldsymbol{r}) \\ u_a(\boldsymbol{r}) \\ v_a(\boldsymbol{r}) \\ v_b(\boldsymbol{r}) \end{pmatrix}$$

u_a and v_a correspond to the different valleys, $\boldsymbol{K}_\pm$ respectively. The subscript a and b represent the sublattice A and B, respectively.

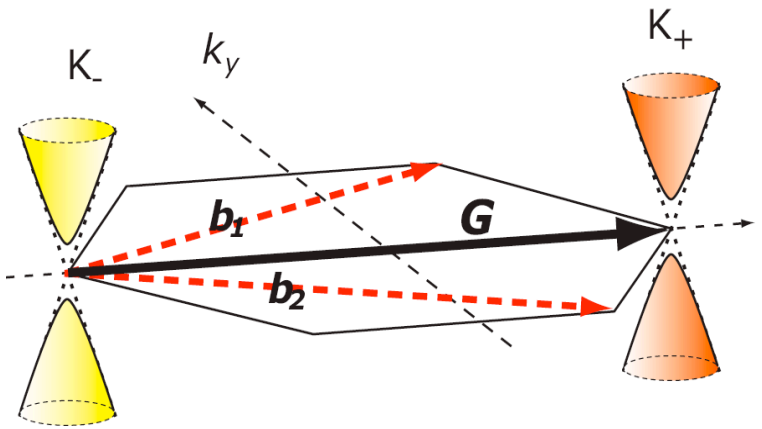
Step II: opening single-particle gap

CDW
(staggered chemical potential):
Does not couple Dirac points

$$\mathcal{K}_D = \begin{pmatrix} \epsilon+\eta & -2i\partial_z & 0 & 0 \\ -2i\partial_{\bar{z}} & -\epsilon-\eta & 0 & 0 \\ 0 & 0 & -\epsilon+\eta & 2i\partial_z \\ 0 & 0 & 2i\partial_{\bar{z}} & \epsilon-\eta \end{pmatrix}$$

G.W Semenoff, PRL 53, 2449 (1984), F.D.M. Haldane, PRL 61, 2015 (1988)

BDW



$$\mathcal{K}_D = \begin{pmatrix} 0 & -2i\partial_z & \Delta(\boldsymbol{r}) & 0 \\ -2i\partial_{\bar{z}} & 0 & 0 & \Delta(\boldsymbol{r}) \\ \bar{\Delta}(\boldsymbol{r}) & 0 & 0 & 2i\partial_z \\ 0 & \bar{\Delta}(\boldsymbol{r}) & 2i\partial_{\bar{z}} & 0 \end{pmatrix}$$

$\Delta = \Delta_0 e^{i\alpha}$: $E_\pm = \pm\sqrt{|\vec{P}|^2 + |\Delta_0|^2}$

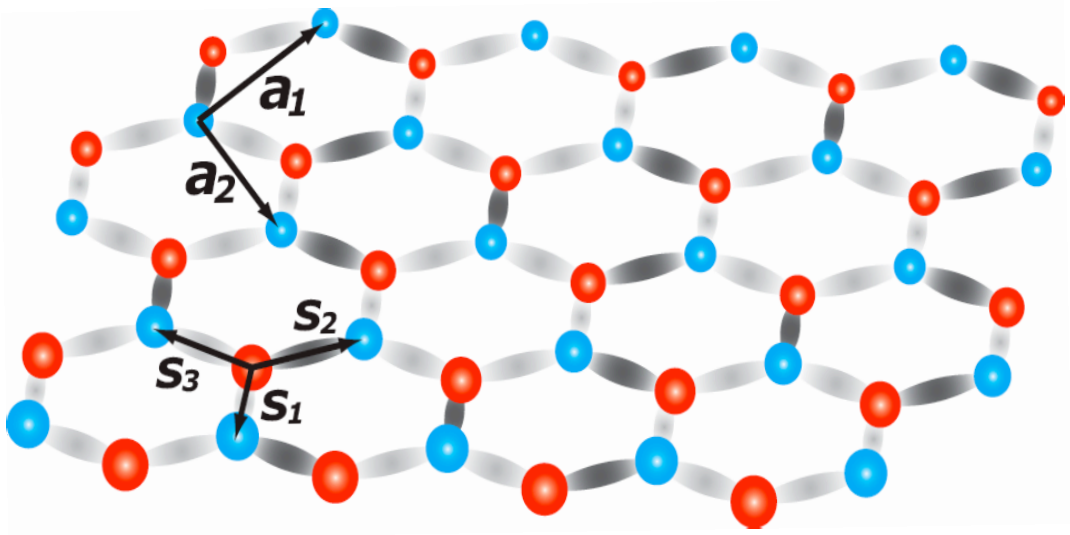
C.-Y. Hou, C. Chamon, M. Mudry, PRL 98, 186809 (2007)

The hopping texture leading to Δ:

Kekule Distortions:

$$H = -\sum_{\boldsymbol{r}\in\Lambda_A}\sum_{i=1}^{3}\left(t+\delta t_{\boldsymbol{r},i}\right)\; a^{\dagger}_{\boldsymbol{r}} b_{\boldsymbol{r}+\boldsymbol{s}_i} + \text{H.c.}$$

$$\delta t_{\boldsymbol{r},i} = \Delta(\boldsymbol{r})\; e^{i\boldsymbol{K}_+\cdot\boldsymbol{s}_i}\; e^{i\boldsymbol{G}\cdot\boldsymbol{r}}/3 + \text{c.c}$$



Step III: topological defects and zero modes

$$\Delta(\boldsymbol{r}) = \Delta_0(r)\, e^{i(\alpha + n\theta)}$$

$$\begin{pmatrix} 0 & -2i\partial_z & \Delta(\boldsymbol{r}) & 0 \\ -2i\partial_{\bar{z}} & 0 & 0 & \Delta(\boldsymbol{r}) \\ \bar{\Delta}(\boldsymbol{r}) & 0 & 0 & 2i\partial_z \\ 0 & \bar{\Delta}(\boldsymbol{r}) & 2i\partial_{\bar{z}} & 0 \end{pmatrix} \begin{pmatrix} u_b(\boldsymbol{r}) \\ u_a(\boldsymbol{r}) \\ v_a(\boldsymbol{r}) \\ v_b(\boldsymbol{r}) \end{pmatrix} = 0$$

Same equations for zero modes were solved in two different contexts:

- When coupling a charge q scalar Higgs field to gauge fields carrying a flux of n/q in 2D (Jackiw and Rossi 1981)

- For mid-gap states in a 2D p-wave superconductor (Read and Green 2000)

Charge is not a conserved quantum number in both cases.

Here it is: quantum numbers must be good (and behave well!) to be fractionalized!

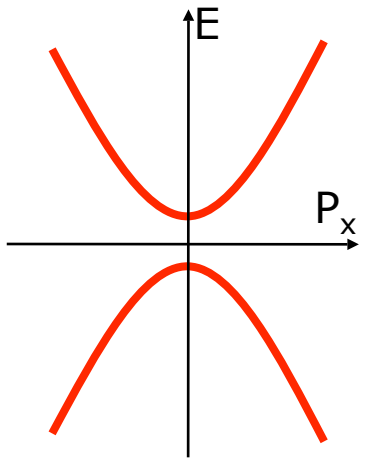
1. n > 0 : n normalizable zero modes at sub-lattice B
2. n < 0 : n normalizable zero modes at sub-lattice A

→ Only one Zero mode for |n|=1 Case

For n=-1 :

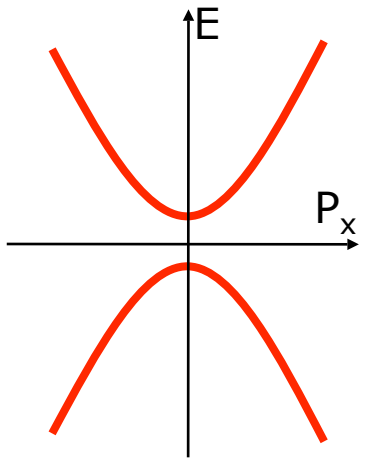
$$u_a \propto Exp[-\int_0^r dr' \Delta_0(r')] \,, \qquad v_a^*(r,\theta) = u_a(r,\theta)$$

Electron fractionalization



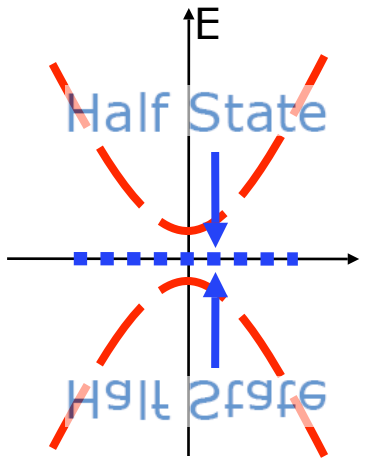
Spectrum Without the Vortex

Electron fractionalization



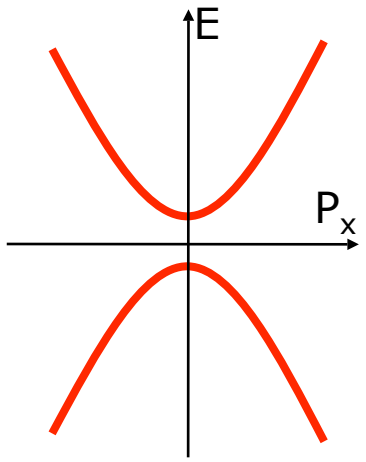
Spectrum Without the Vortex

Mirror Symmetry
respect to E=0
(Particle-Hole Sym)



Spectrum With the n=1 Vortex

Electron fractionalization



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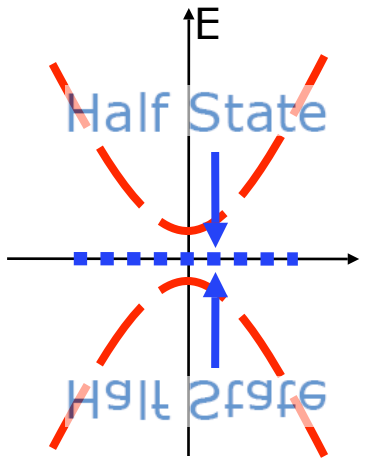
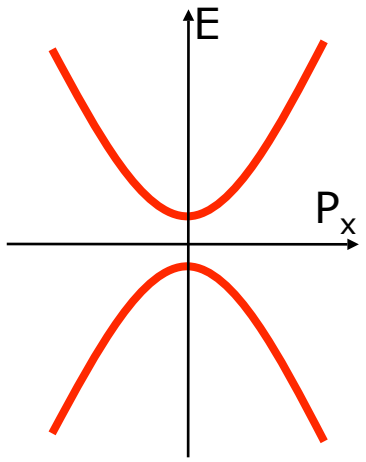


Spectrum With the n=1 Vortex

Half-filling (E_F=0): Filled and unfilled zero mode has the same energy

Electron fractionalization



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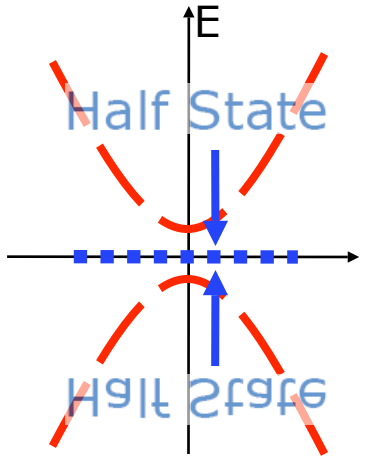

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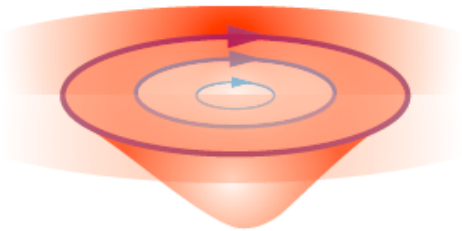
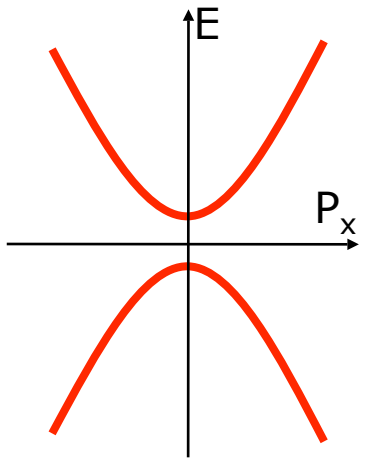


Unfilled State: Charge e/2

Has 1/2 fewer filled state
compared to the no-vortex
background.

Electron fractionalization



Spectrum Without the Vortex

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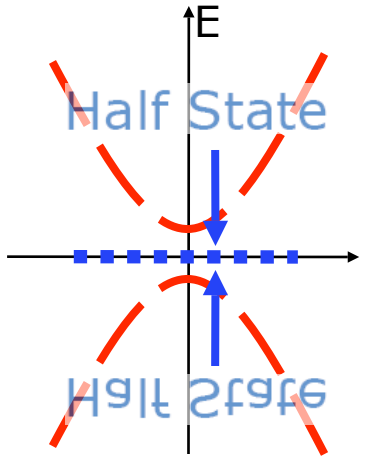

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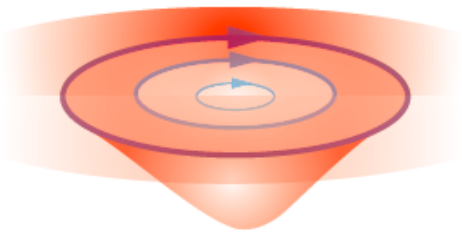
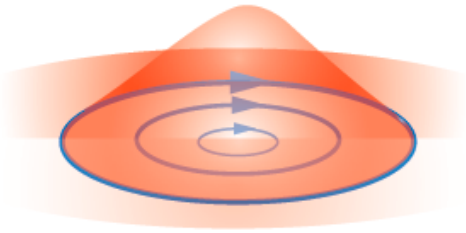


Unfilled State: Charge e/2

Has 1/2 fewer filled state compared to the no-vortex background.



filled State: Charge -e/2

Has 1/2 more filled state compared to the no-vortex background.

Irrational vs. rational charge

C. Chamon, C.Y. Hou, R. Jackiw, C. Mudry, S.-Y. Pi, A. Schnyder
arXiv:0707.0293

Back to massive Dirac equation:

$$\begin{pmatrix} m\,n_0 & +p & m(n_1+\mathrm{i}n_2) & 0 \\ +\bar{p} & -m\,n_0 & 0 & m(n_1+\mathrm{i}n_2) \\ m(n_1-\mathrm{i}n_2) & 0 & -m\,n_0 & -p \\ 0 & m(n_1-\mathrm{i}n_2) & -\bar{p} & m\,n_0 \end{pmatrix} \begin{pmatrix} \psi_+^{\mathrm{B}}(p) \\ \psi_+^{\mathrm{A}}(p) \\ \psi_-^{\mathrm{A}}(p) \\ \psi_-^{\mathrm{B}}(p) \end{pmatrix} = \pm\sqrt{|\boldsymbol{p}|^2+m^2} \begin{pmatrix} \psi_+^{\mathrm{B}}(p) \\ \psi_+^{\mathrm{A}}(p) \\ \psi_-^{\mathrm{A}}(p) \\ \psi_-^{\mathrm{B}}(p) \end{pmatrix}$$

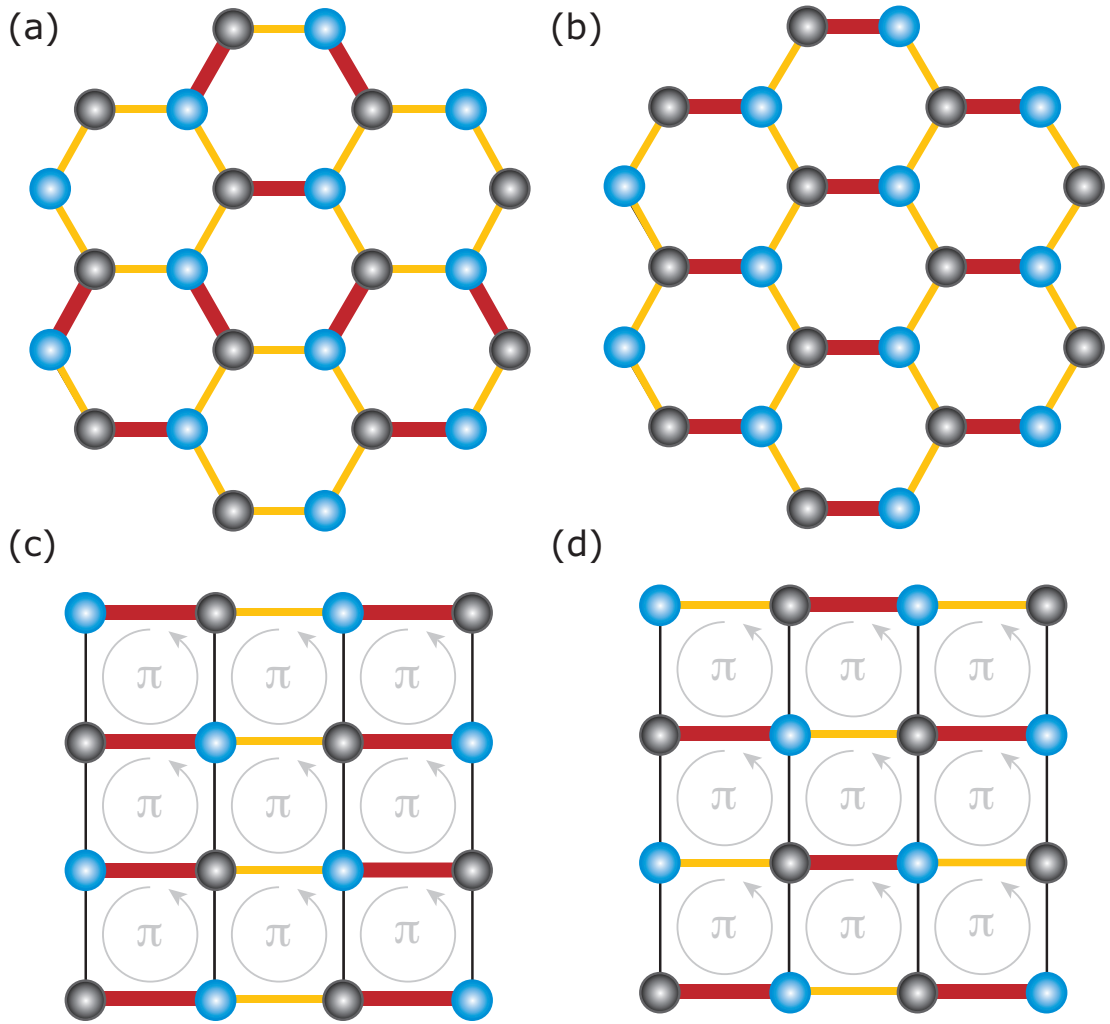
$$\boldsymbol{n}^2 = 1, \qquad \boldsymbol{n} := (n_0, n_1, n_2)$$

$$\mathcal{L} := \bar{\Psi}\left[\left(\mathrm{i}\gamma_\mu\partial^\mu - \gamma_\mu A^\mu - \gamma_\mu\gamma_5 A^{5\mu}\right) + mM_a n_a\right]\Psi$$

Jackiw & Pi
axial gauge field

Masses

On the lattice:



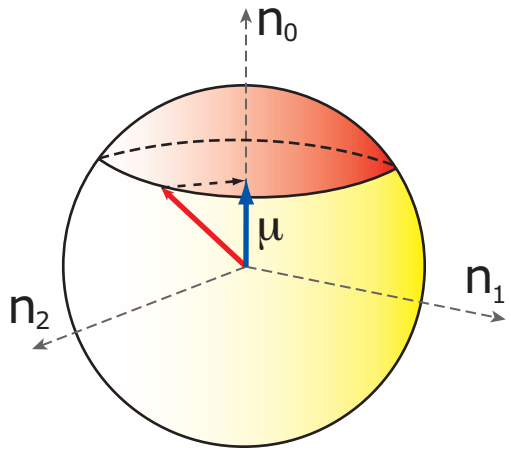
Masses

$\Delta_{1,2}(\text{or } m\, n_{1,2})$

Axial vector potential

$A^5_{1,2}$

What is the induced charge for a mass twist?



$$j^\mu = \frac{1}{8\pi}\epsilon^{\mu\nu\rho}\,\boldsymbol{n}\cdot\left(\partial_\nu \boldsymbol{n}\wedge\partial_\rho \boldsymbol{n}\right)$$

$$Q = \int d^2\boldsymbol{r}\, j_0(t,\boldsymbol{r}) = \int_{\mathcal{S}} \frac{d\Omega}{4\pi}$$

Continuously varying charge (irrational)

In the presence of the axial flux:

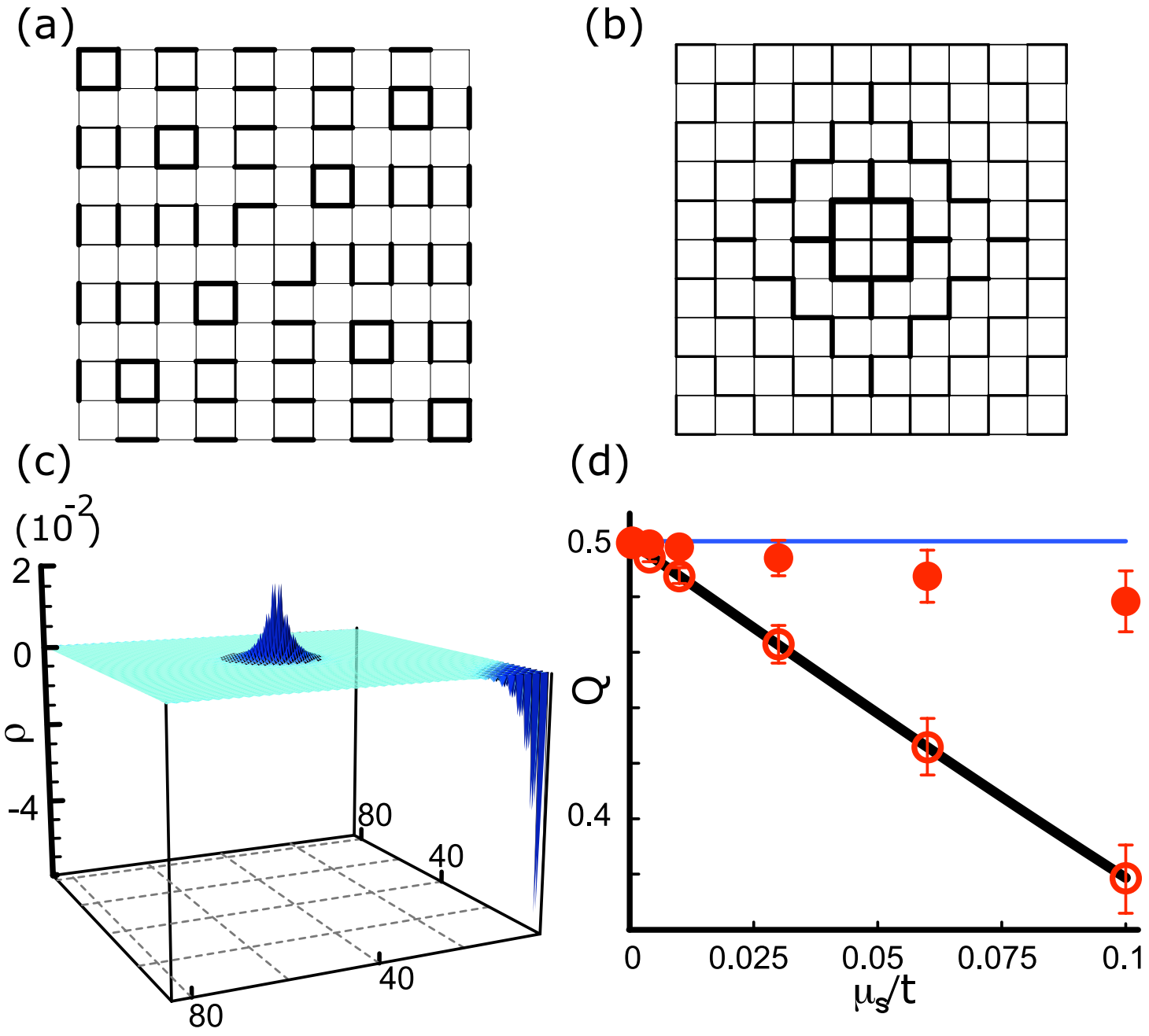
$$J^\mu = j^\mu - \frac{1}{2\pi}\epsilon^{\mu\nu\rho}\partial_\nu\left(n_0 A^5_\rho\right)$$

$$\frac{1}{2\pi}\int_{\mathfrak{D}} d^2\boldsymbol{r}\, n_0 \nabla\times\vec{A}^5 = \frac{1}{2\pi}\oint_{\partial} d\vec{\ell}\cdot\vec{A}^5\, n_0 = 2\Phi^5\left(\int_{\mathcal{S}}\frac{d\Omega}{4\pi} - \frac{1}{2}\right)$$

$$\Phi^5 = 1/2 \quad \Longrightarrow \quad \boxed{Q_{\text{total}} = 1/2}$$

Charge re-rationalizes

Numerical check on the π-flux square lattice:

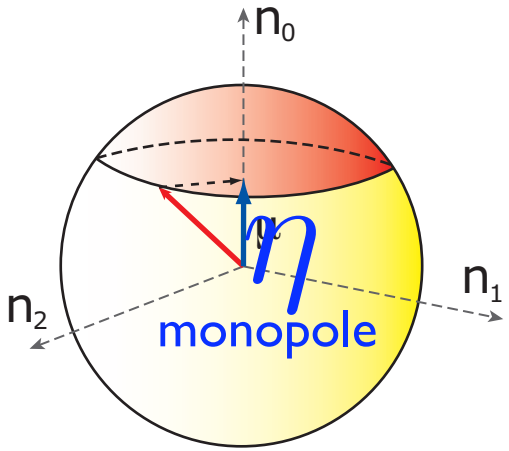


Irrational vs. rational statistics

Square the Dirac Hamiltonian:

$$\mathcal{H}^2 = \left(\boldsymbol{p}^2 + |\Delta|^2 + \epsilon^2 + \eta^2\right)\sigma_0 \otimes \tau_0 + 2\eta\, m \sum_{i=0}^{2} \Sigma_i n_i$$

Haldane mass for graphene breaks TRS



Uniform vector rotates: Berry phase

$$\gamma_{\rm rot}(\boldsymbol{n}) = 2\pi \sin^2 \frac{\phi}{2}$$

Weighted by density:

$$\gamma_{\rm rot} = \int d^2\boldsymbol{r}\, j_0(\boldsymbol{r})\, 2\pi \sin^2 \frac{\phi(\boldsymbol{r})}{2}$$

$$= \int \frac{d\Omega}{4\pi}\, 2\pi \sin^2 \frac{\phi}{2} = \pi\, Q^2(\mu_{\rm s})$$

spin $$S = \frac{1}{2} Q^2(\mu_{\rm s})$$

stat $$\boxed{\frac{\theta}{\pi} = Q^2(\mu_{\rm s})}$$

Summary

- Fractionalization in 2D via spontaneous symmetry breaking
- Irrational charge and exchange statistics
- Deconfinement with an axial half-vortex: re-rationalization

  charge 1/2 and half-semion (θ/π=1/4) statistics

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