Entanglement in the Kondo Spin Chain

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References:

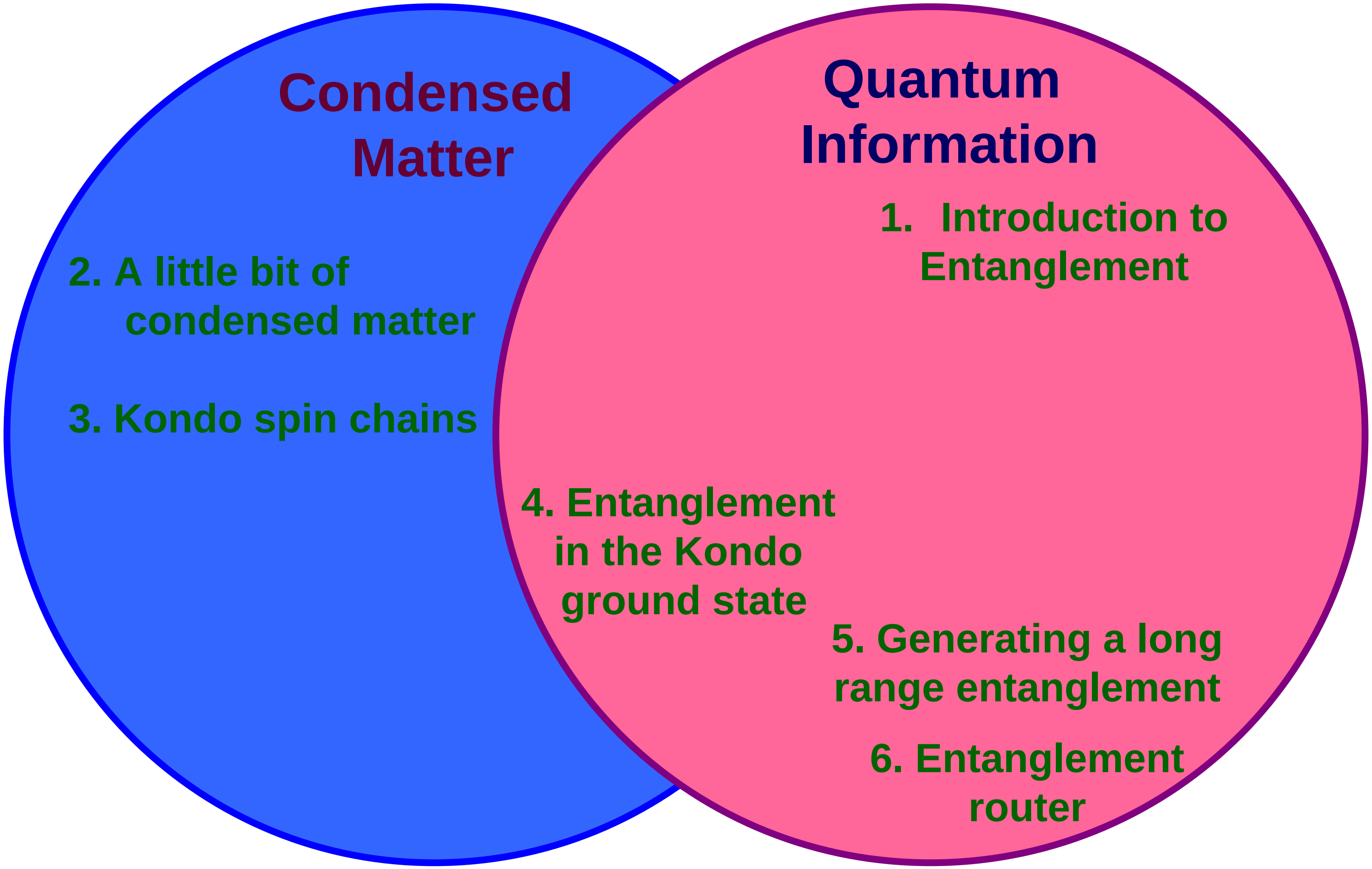
A. Bayat, S. Bose, P. Sodano, PRL 105, 187204 (2010)
A. Bayat, P. Sodano, S. Bose, PRB 81, 100412 (R) (2010)
P. Sodano, A. Bayat, S. Bose, PRB 81, 064429 (2010)

"I feel really fortunate to have met Pasquale in 2008 for the start of what has been an amazing learning, and fruitful research experience for me! I have to say that within the mere span of two years, it already seems that I have known and worked with Pasquale for ages. Pasquale, I have always been highly impressed by your strong physical intuition, during which your take on a problem has repeatedly turned out to be true in the end. I always long for those discussions over the cigarettes, which are very pleasant indeed . I know that I am missing a great meeting but the circumstances were a bit unavoidable. I wish you a very happy 60th birthday and an enjoyable meeting and looking forward to writing several more papers together.

best wishes,

Sougato"

Contents of the Talk



Pure Entangled States

Separable states:

$$|\psi\rangle_{AB} = |\alpha\rangle_A \otimes |\beta\rangle_B$$

$$|\psi\rangle_{AB} = \frac{|00\rangle + |01\rangle}{\sqrt{2}} = |0\rangle_A \otimes \left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right)_B$$

Entangled states:

$$|\psi\rangle_{AB} \neq |\alpha\rangle_A \otimes |\beta\rangle_B$$

$$|\psi\rangle_{AB} = \frac{|00\rangle_{AB} + |11\rangle_{AB}}{\sqrt{2}}$$

$$\rho_B = tr_A |\psi_{AB}\rangle\langle\psi_{AB}| = \frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|1\rangle\langle 1| = \frac{I}{2}$$

In maximally entangled states, state of the subsystem is identity

Non Maximal Entangled States

$$|\psi\rangle_{AB} = \sqrt{\frac{1}{3}}|00\rangle_{AB} + \sqrt{\frac{2}{3}}|11\rangle_{AB}$$

$$\rho_B = tr_A|\psi_{AB}\rangle\langle\psi_{AB}| = \frac{1}{3}|0\rangle\langle 0| + \frac{2}{3}|1\rangle\langle 1|$$

For entangled pure states:

- **State of the subsystem is mixed.**
- **More mixedness in the subsystem more entanglement in the system**
- **Entropy of the subsystem is a unique measure of entanglement**

Mixed Entangled States

UCL

Separable states:

$$\rho_{AB} = \sum_i p_i \rho_i^A \otimes \rho_i^B$$

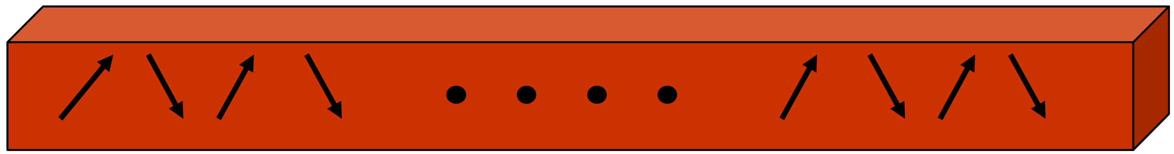
There is no unique measure of entanglement

ρ_{AB} → **Entanglement measure** → $E(\rho_{AB})$

E (Separable states) = 0

E (Maximally Entangled) = 1

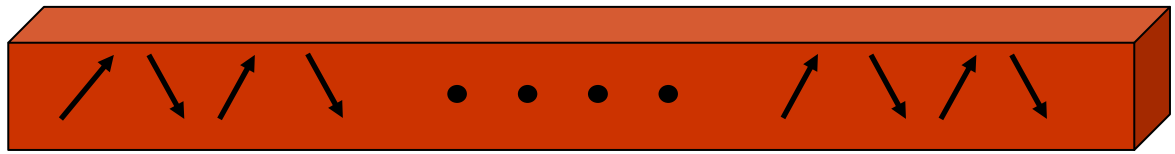
Gapped Systems



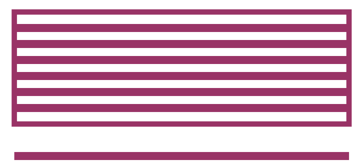
$$\Delta \Leftrightarrow \xi : \qquad \left\langle S_x^i S_x^j \right\rangle \approx e^{-\frac{|i-j|}{\xi}}$$

The intrinsic length scale of the system impose an exponential decay

Gapless Systems



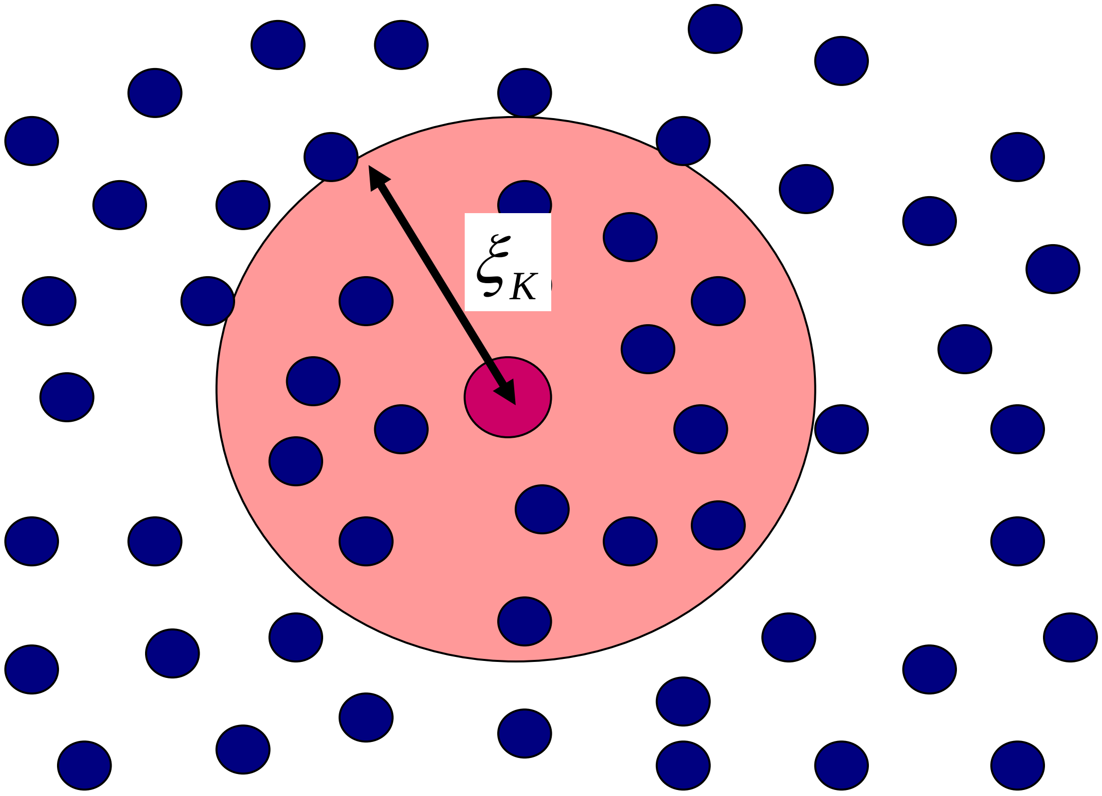
$$N \to \infty : \quad \Delta \to 0$$



$$\langle S_x^i S_x^j \rangle \approx | i - j |^{-\alpha}$$

There is no length scale in the system so correlations decay algebraically

Kondo Physics



$$\xi_K \Leftrightarrow T_K$$

Despite the gapless nature of the Kondo system, we have a length scale in the model

Interesting Issues of the Kondo Physic

1- Size of the cloud

2- Scaling properties in terms of the Kondo length

3- Detecting the Kondo cloud

4- Physical properties (resistively, susceptibility) in the Kondo regime

Realization of the Kondo Effect

Semiconductor quantum dots

D. G. Gordon *et al*. Nature 391, 156 (1998).
S.M. Cronenwett, Science 281, 540 (1998).

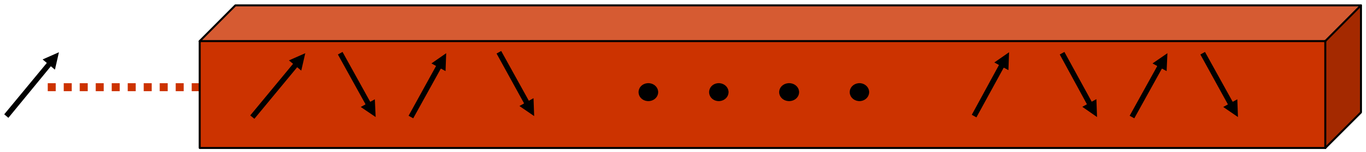
Carbon nanotubes

J. Nygard, *et al*. Nature 408, 342 (2000).
M. Buitelaar, Phys. Rev. Lett. 88, 156801 (2002).

Individual molecules

J. Park, *et al*. Nature 417, 722 (2002).
W. Liang, *et al*, Nature 417, 725–729 (2002).

Kondo Spin Chain



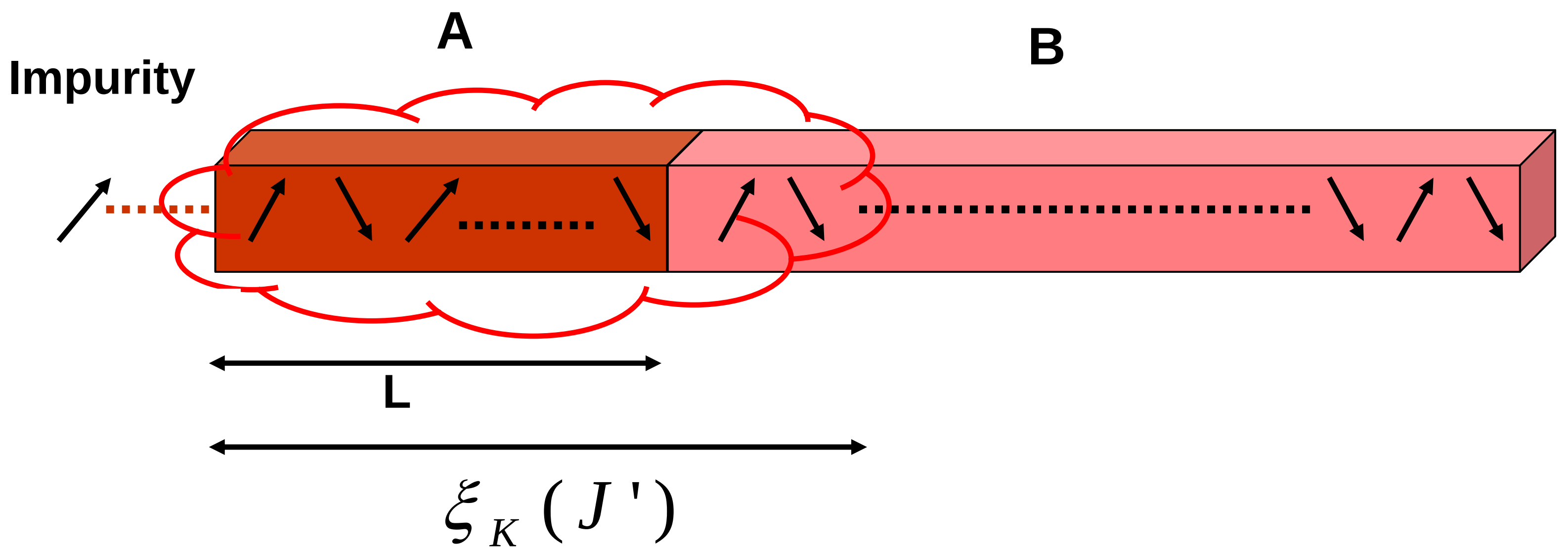
$$H = J'(J_1\sigma_1.\sigma_2 + J_2\sigma_1.\sigma_3) + \sum_{i=2} J_1\sigma_i.\sigma_{i+1} + J_2\sigma_i.\sigma_{i+2}$$

$$\frac{J_2}{J_1} < J_2^c = 0.2412: \quad \text{Kondo (gapless)}$$

$$\frac{J_2}{J_1} > J_2^c: \quad \text{Dimer (gapfull)}$$

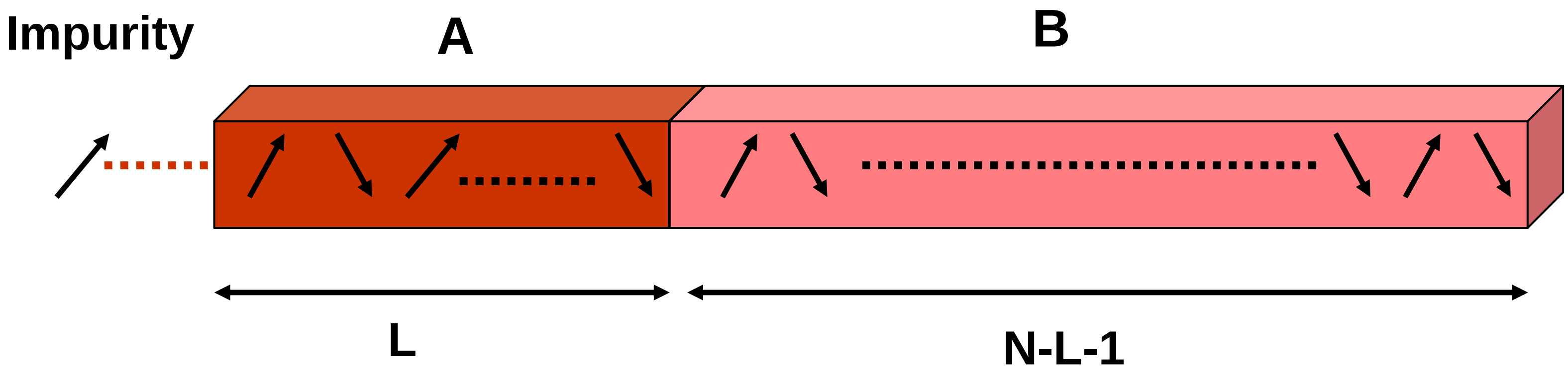
E. S. Sorensen *et al.*, J. Stat. Mech., P08003 (2007)

Entanglement as a Witness of the Cloud



$$
\begin{cases}
L < \xi_K : \quad E_{SA} < 1 \Rightarrow E_{SB} > 0 \\
L = \xi_K : \quad E_{SA} = 1 \Rightarrow E_{SB} = 0 \\
L > \xi_K : \quad E_{SA} = 1 \Rightarrow E_{SB} = 0
\end{cases}
$$

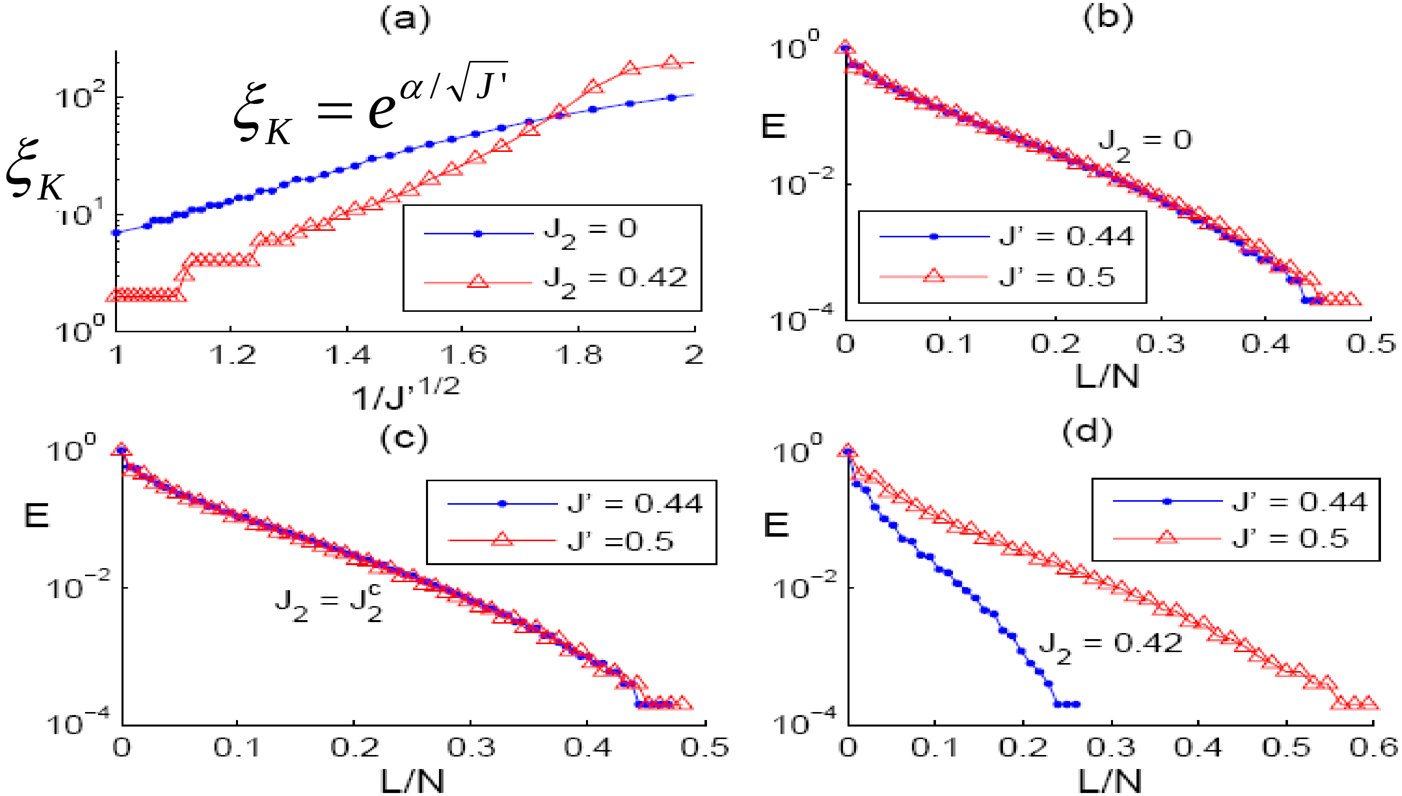
Scaling



Kondo Phase: $$E(L,\xi_K,N) = E\left(\frac{N}{\xi_K},\frac{L}{N}\right)$$

Dimer Phase: $$E(L,\xi,N) \neq E\left(\frac{L}{\xi_K},\frac{N}{L}\right)$$

Scaling Properties of the Kondo regime

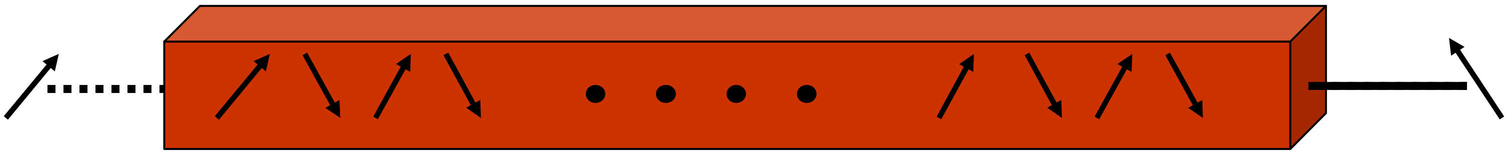


$$\frac{N}{\xi_K} = 4$$

Kondo Phase: $E(L, \xi_K, N) = E\left(\frac{N}{\xi_K}, \frac{L}{N}\right)$

Dimer Phase: $E(L, \xi, N)$

Local Quench



$$H_1 = J'(J_1\sigma_1.\sigma_2 + J_2\sigma_1.\sigma_3) + \sum_{i=2} J_1\sigma_i.\sigma_{i+1} + J_2\sigma_i.\sigma_{i+2}$$

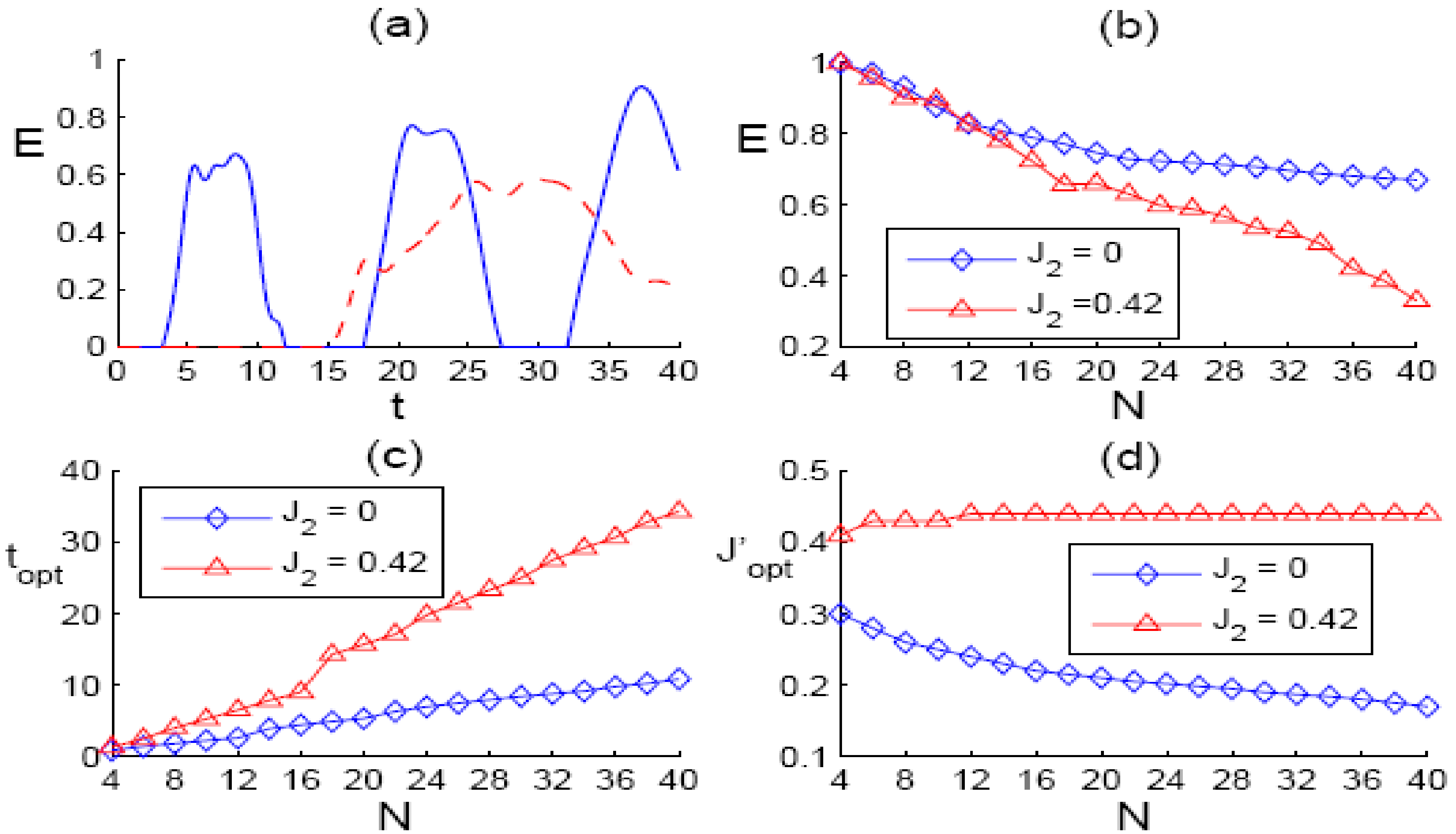
Quench

$$H_2 = J'(J_1\sigma_1.\sigma_2 + J_1\sigma_{N-1}.\sigma_N + J_2\sigma_1.\sigma_3 + J_2\sigma_{N-2}.\sigma_N)$$
$$+ \sum_i J_1\sigma_i.\sigma_{i+1} + J_2\sigma_i.\sigma_{i+2}$$

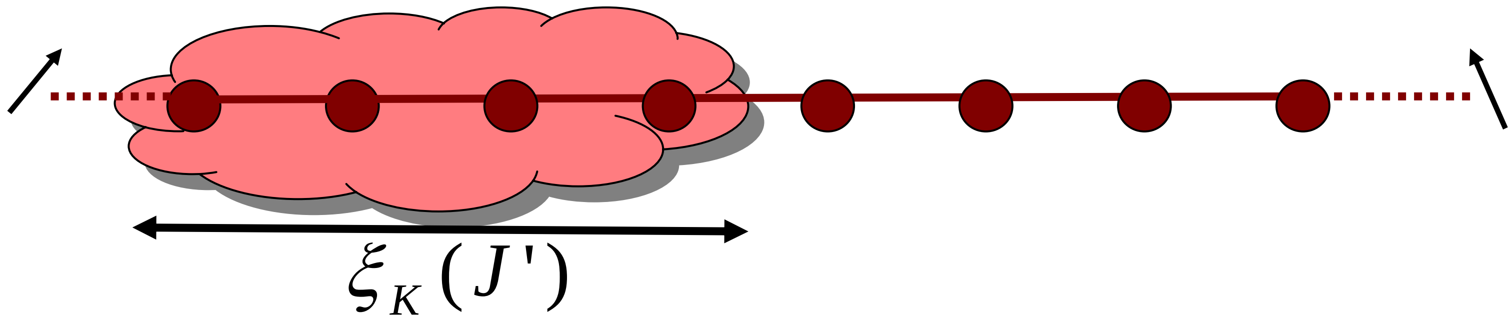
$$|\psi(0)\rangle = |GS_{H1}\rangle$$

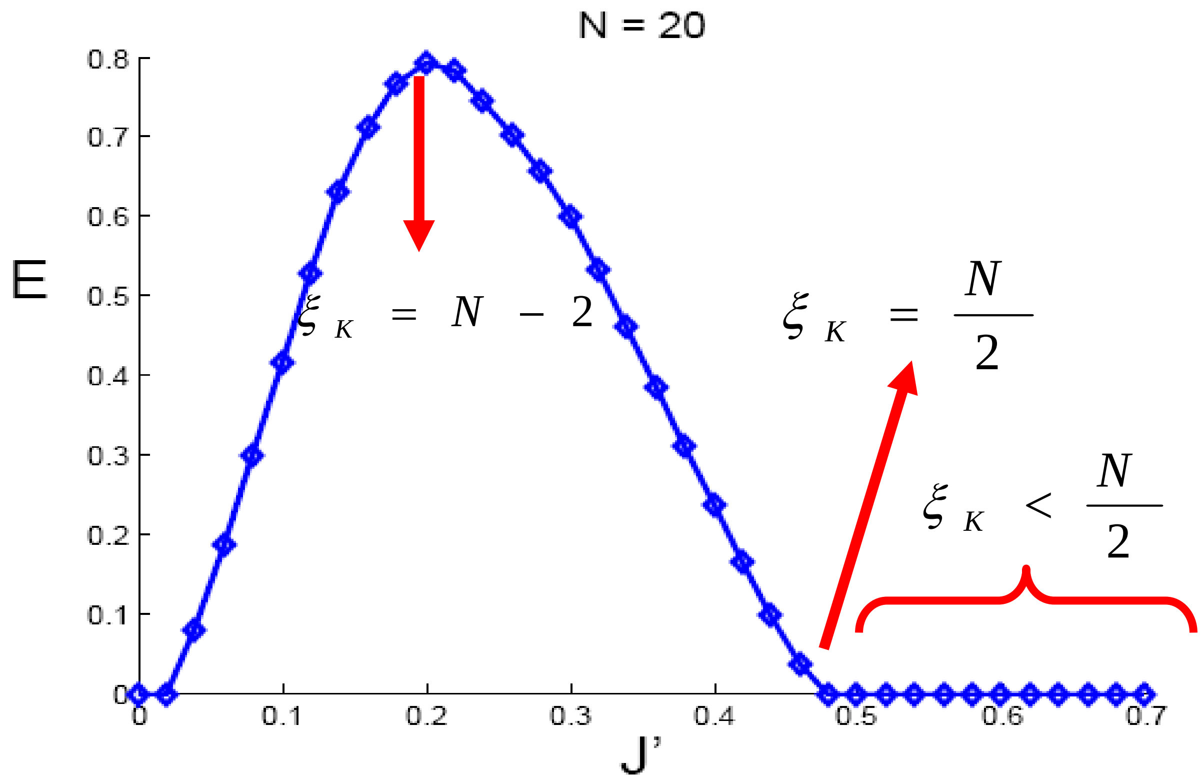
$$|\psi(t)\rangle = e^{-iH_2 t}|GS_{H1}\rangle \quad \rightarrow \quad \rho_{1N}(t) \quad \rightarrow \quad E_{1N}(t)$$

Kondo versus Dimer

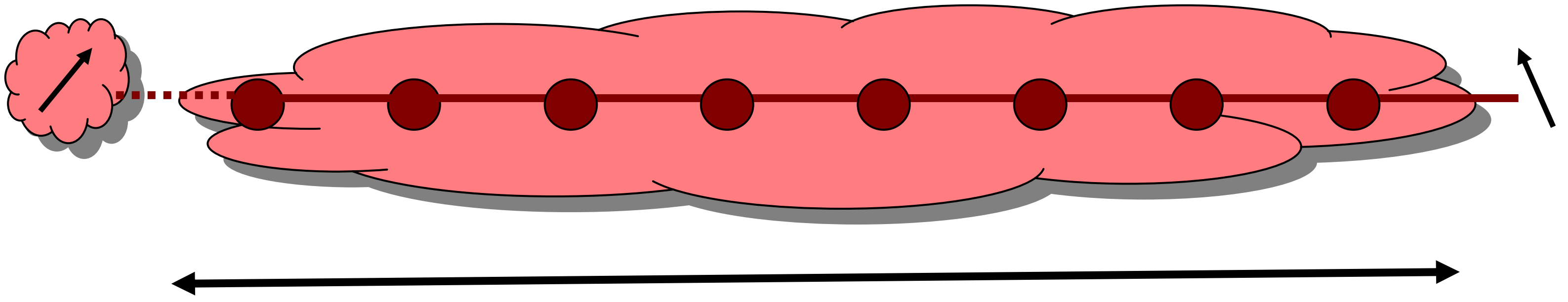


Optimal Parameter





Optimality and Distance independence



$$\xi_K(J'_{opt}) = N - 2$$

Independent of length N, when cloud contains N-2 spins we generate a constant Entanglement

Entanglement in Whole Phase Diagram

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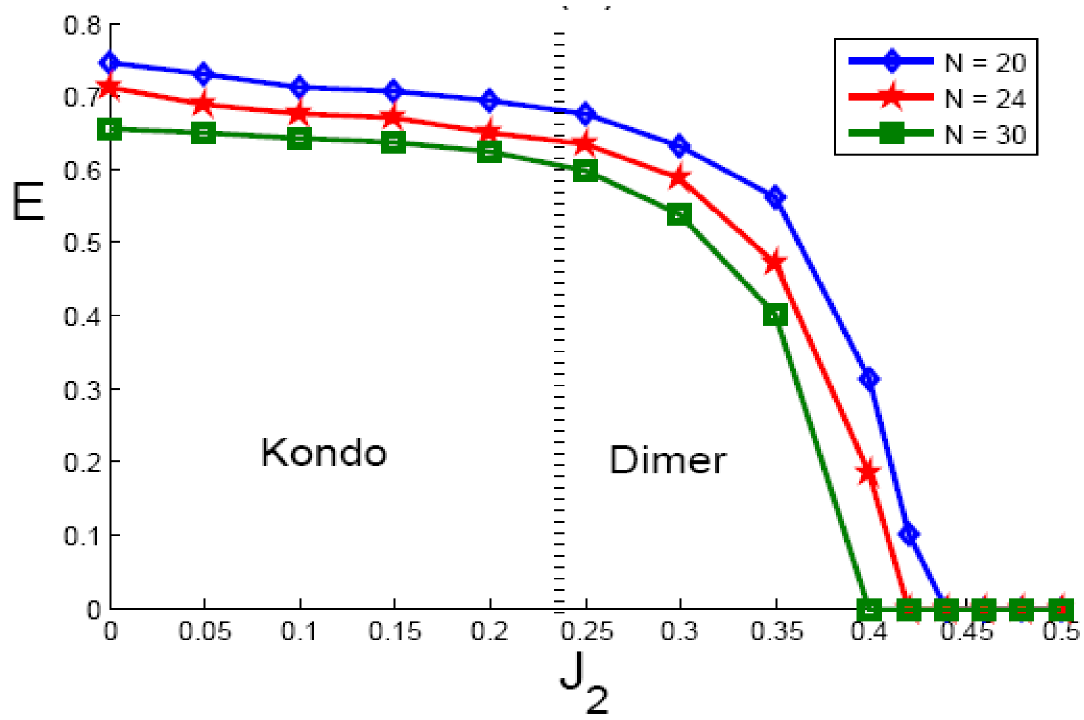
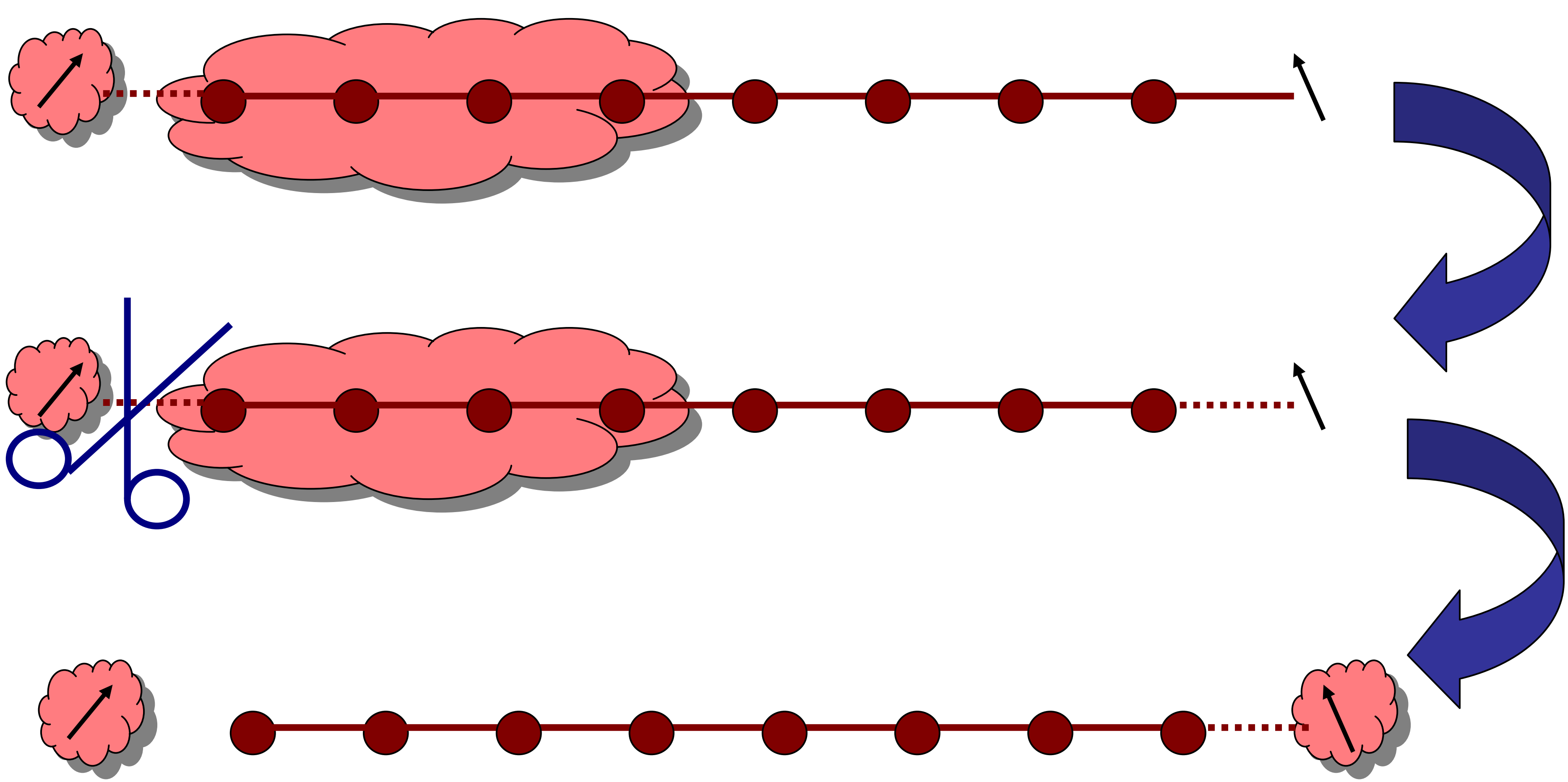
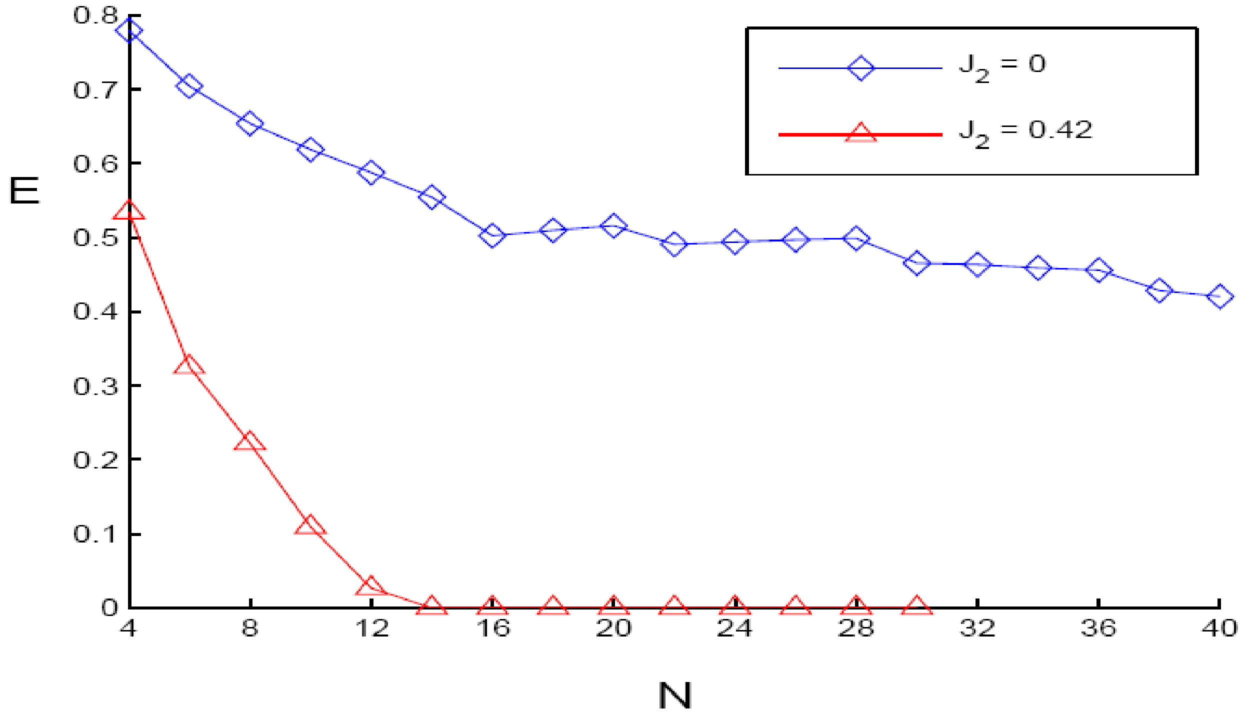


Entanglement drops in the dimer regime

Cloud's Role



Entanglement in the Modified Strategy



Kondo: High value distance independent entanglement mediated by the cloud.

Dimer: Entanglement is just due to an end-end effect which decays exponentially.

More Improvement?

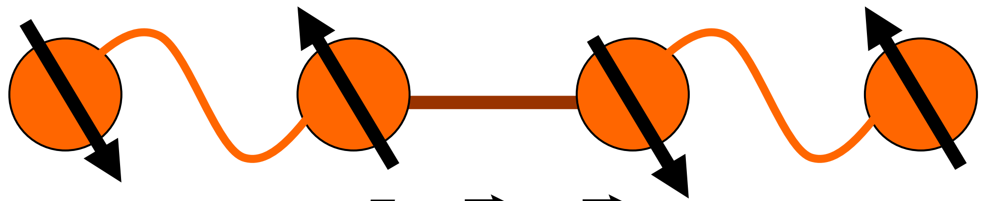
Is there a way to improve the strategy?

1) Higher entanglement

2) A way to route entanglement

Two Spin Singlets

$$J'_L\,\vec{\sigma}_1.\vec{\sigma}_2 \qquad\qquad J'_R\,\vec{\sigma}_3.\vec{\sigma}_4$$



$$J_m\vec{\sigma}_2.\vec{\sigma}_3$$

$$|\psi(0)\rangle = |\psi^-\rangle \otimes |\psi^-\rangle$$

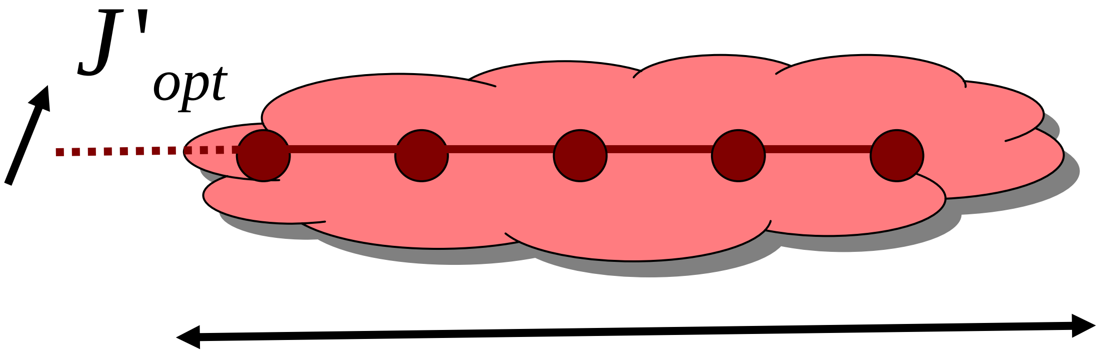
$$|\psi(t)\rangle = e^{-iHt}|\psi(0)\rangle$$

$$J_m = J'_L + J'_R \Longrightarrow E(t) = \max\{0, \frac{1-3\cos(4J_m t)}{4}\}$$

Goal

Can we find some many-body singlets (extended singlets) which play the same role?

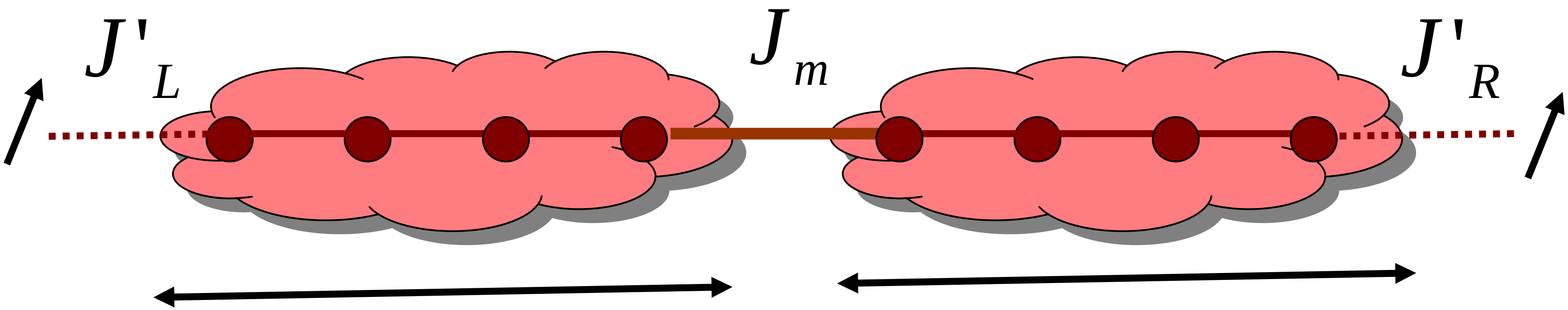
Extended Singlet



$$\xi_K(J'_{opt}) = N - 1$$

With tuning J’ we can generate a proper cloud which extends up to the end of the chain

Quench Dynamics

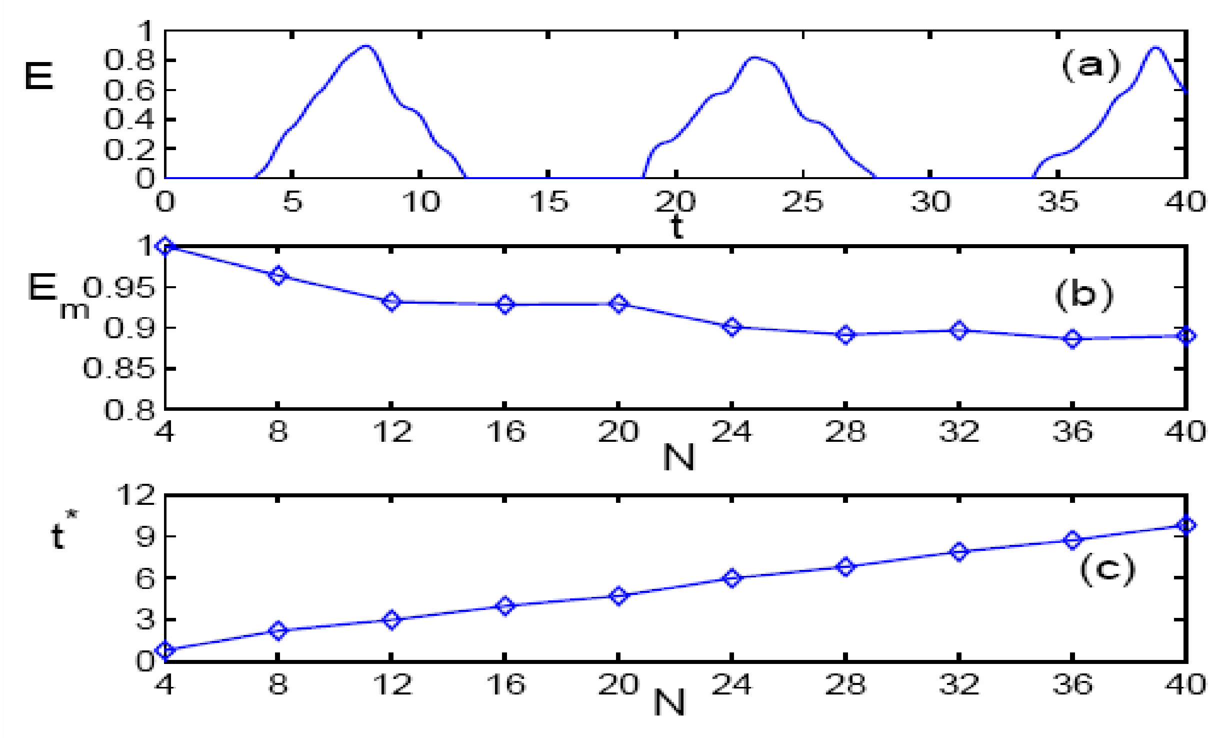


$$\xi_L(J'_L) = N_L - 1 \qquad \xi_R(J'_R) = N_R - 1$$

$$|\psi(0)\rangle = |GS_L\rangle \otimes |GS_R\rangle$$

$$|\psi(t)\rangle = e^{-iH_{LR}t}|\psi(0)\rangle \;\Rightarrow\; \rho_{1N}(t) \;\Rightarrow\; E_{1N}(t)$$

Attainable Entanglement



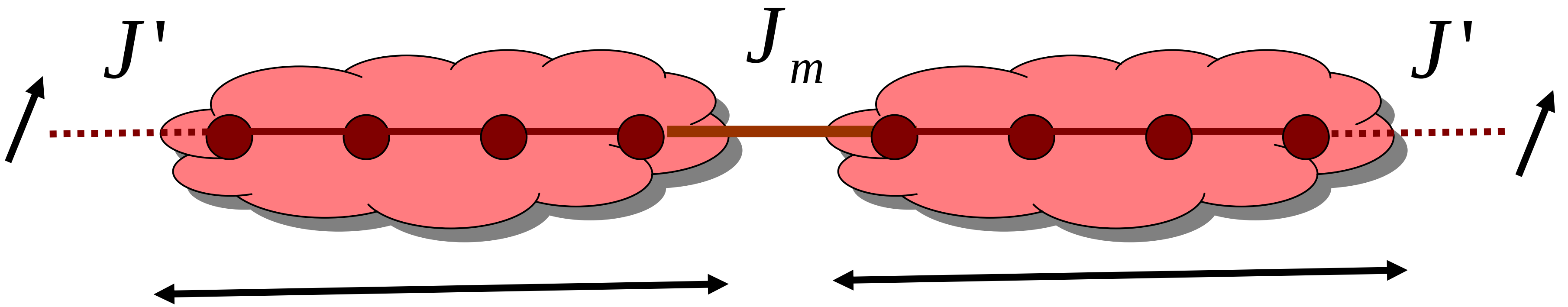
1- Entanglement dynamics is very long lived and oscillatory

2- maximal entanglement attains a constant values for large chains

3- The optimal time which entanglement peaks is linear

Distance Independence

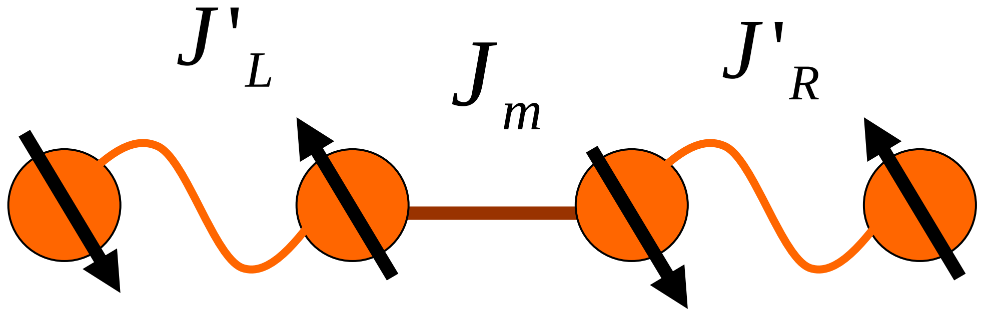
For simplicity take a symmetric composite:



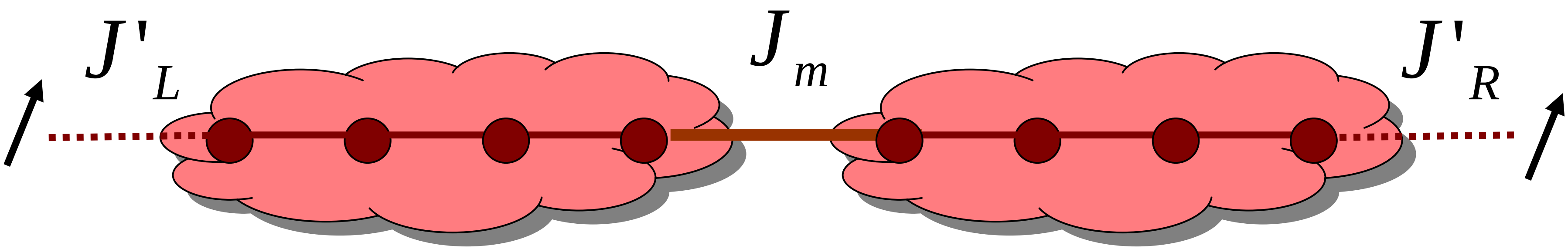
$$\xi_L(J') = (N-2)/2 \qquad \xi_R(J') = (N-2)/2$$

$$E(t,N,J') = E(t,N,\xi) = E(\frac{t}{N},\frac{N}{\xi}) = E(\frac{t}{N},\frac{2N}{N-2})$$

Optimal Quench

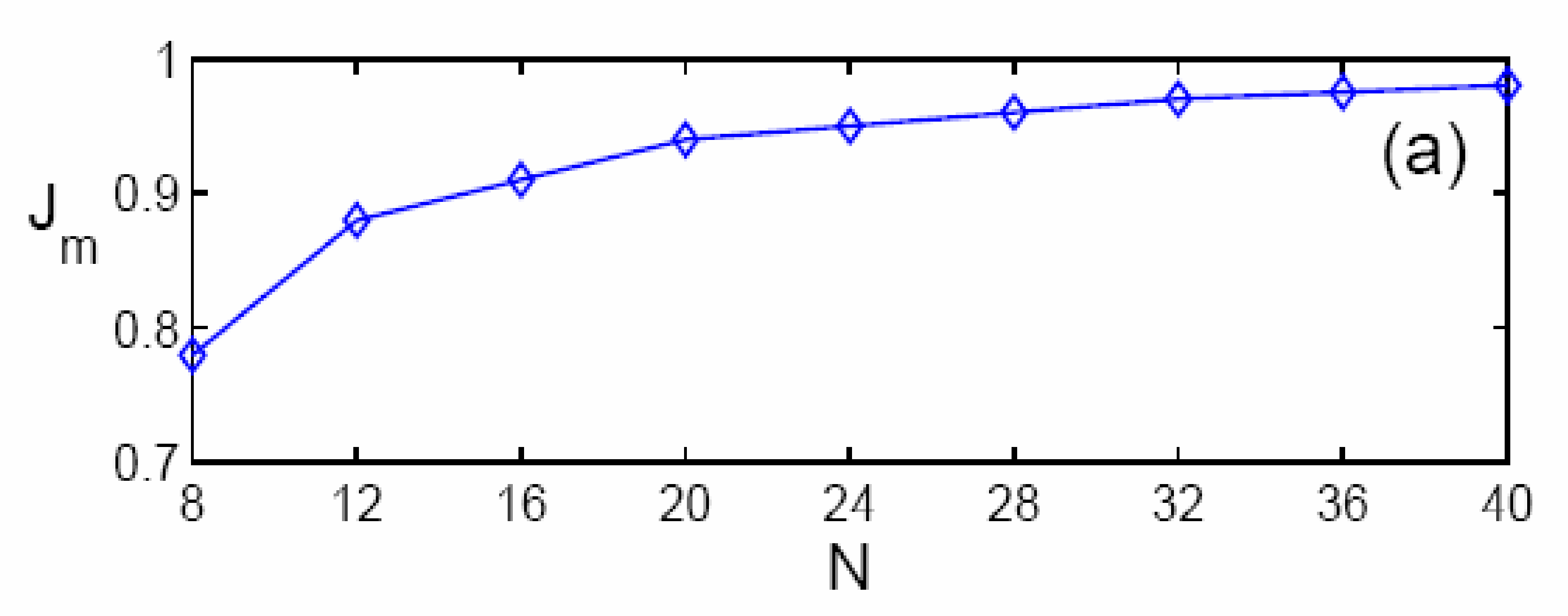


$$J_m = J'_L + J'_R$$



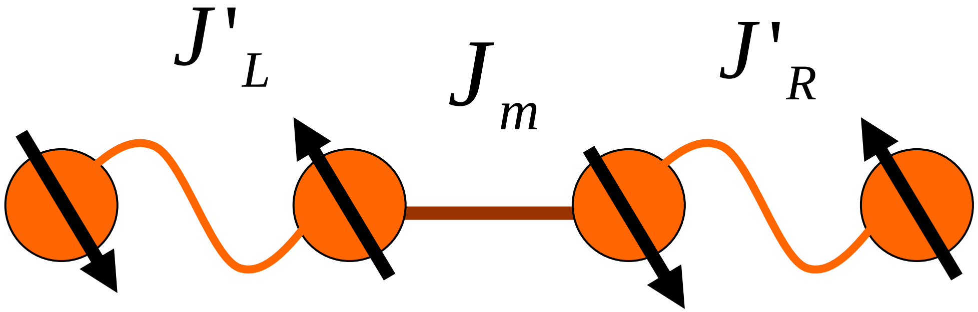
$$J_m = ?$$

Optimal Jm

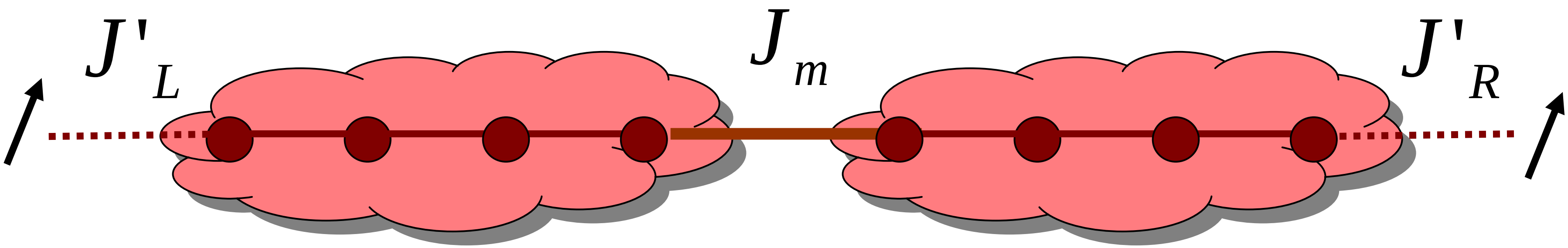


Jm saturates to J1 for large N

Optimal Quench



$$J_m = J'_L + J'_R$$

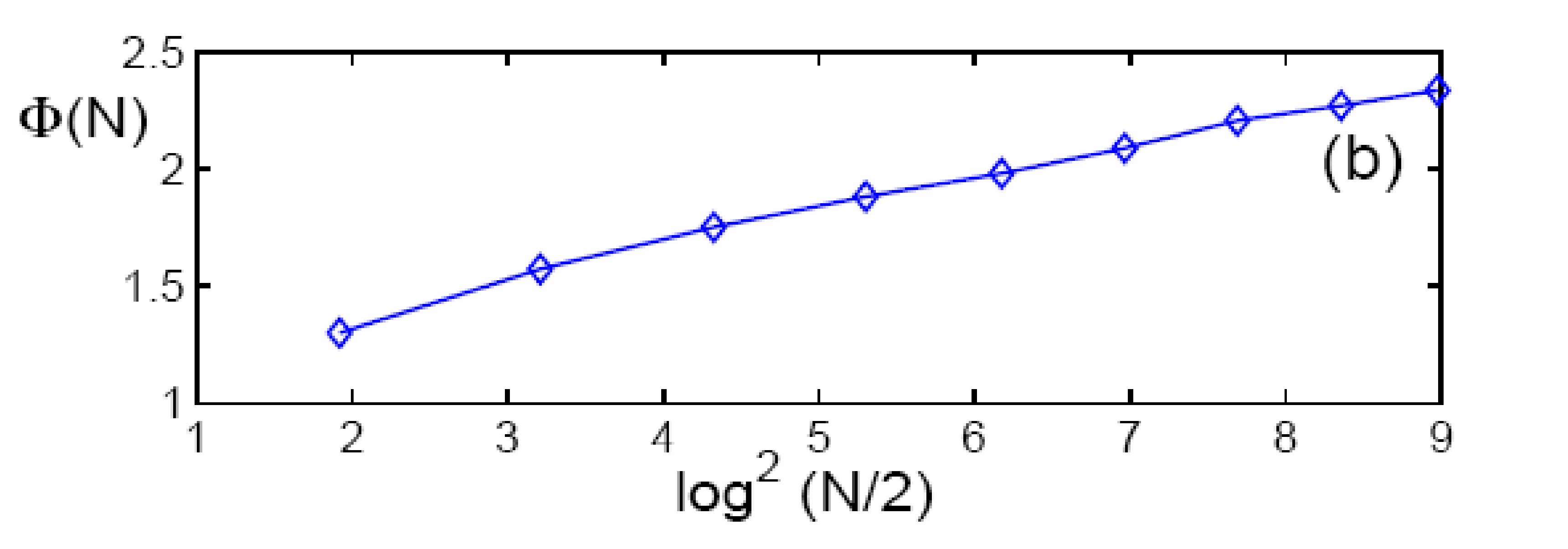


$$J_m = \Phi(N)(J'_L + J'_R)$$

$$J'_{R(L)} = \frac{1}{\log^2(N_{R(L)})}$$

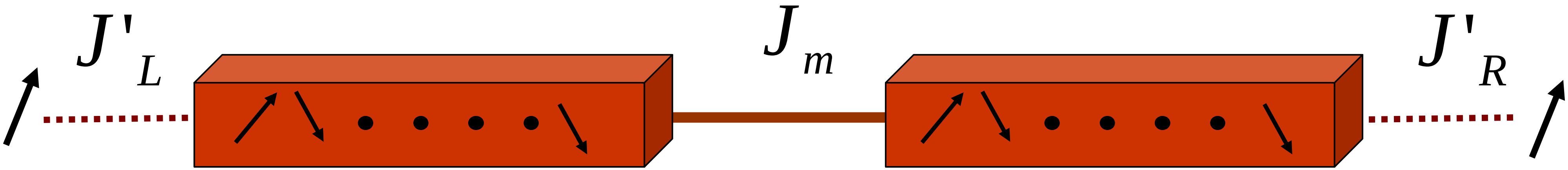
$$\Rightarrow \Phi(N) \approx Log^2(\frac{N}{2})$$

Dependence on N



$$\Phi(N) \approx Log^2(\frac{N}{2})$$

Non-Kondo Singlets (Dimer Regime)

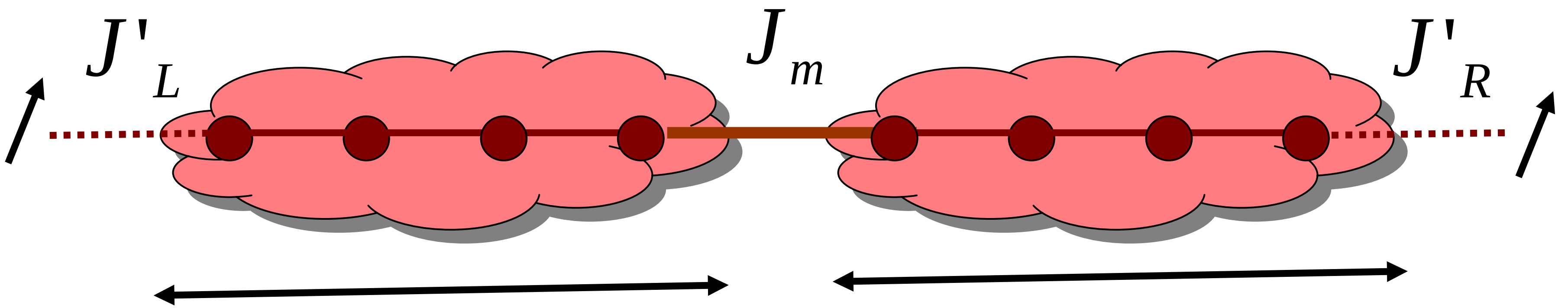


Clouds are absent

N	8	12	16	20	24	28	32	36	40
E_m(K)	0.964	0.932	0.928	0.929	0.901	0.891	0.897	0.886	0.891
E_m(D)	0.957	0.903	0.841	0.783	0.696	0.581	0.468	0.330	0.160
t^*(K)	2.200	2.980	3.980	4.700	5.980	6.800	7.880	8.720	9.800
t^*(D)	3.780	7.290	10.32	13.41	16.89	20.43	24.51	27.12	35.01

K: Kondo (J2=0) **D: Dimer (J2=0.42)**

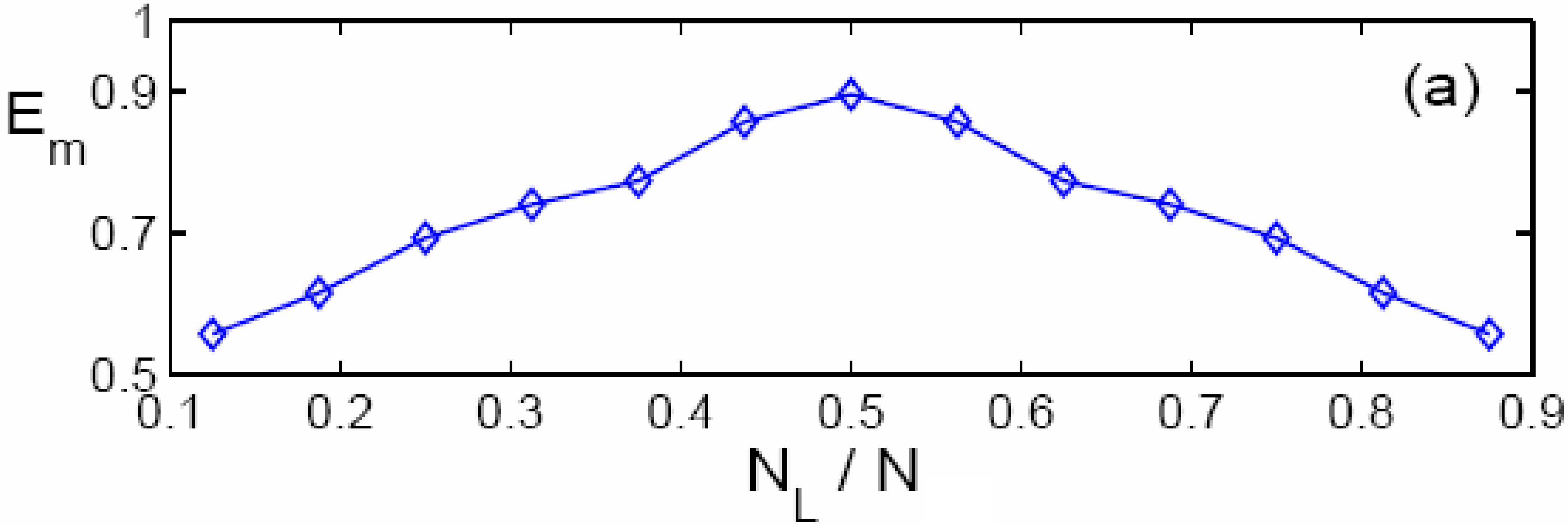
Non-Symmetric Chains



$$\xi_L(J'_L) = N_L - 1$$

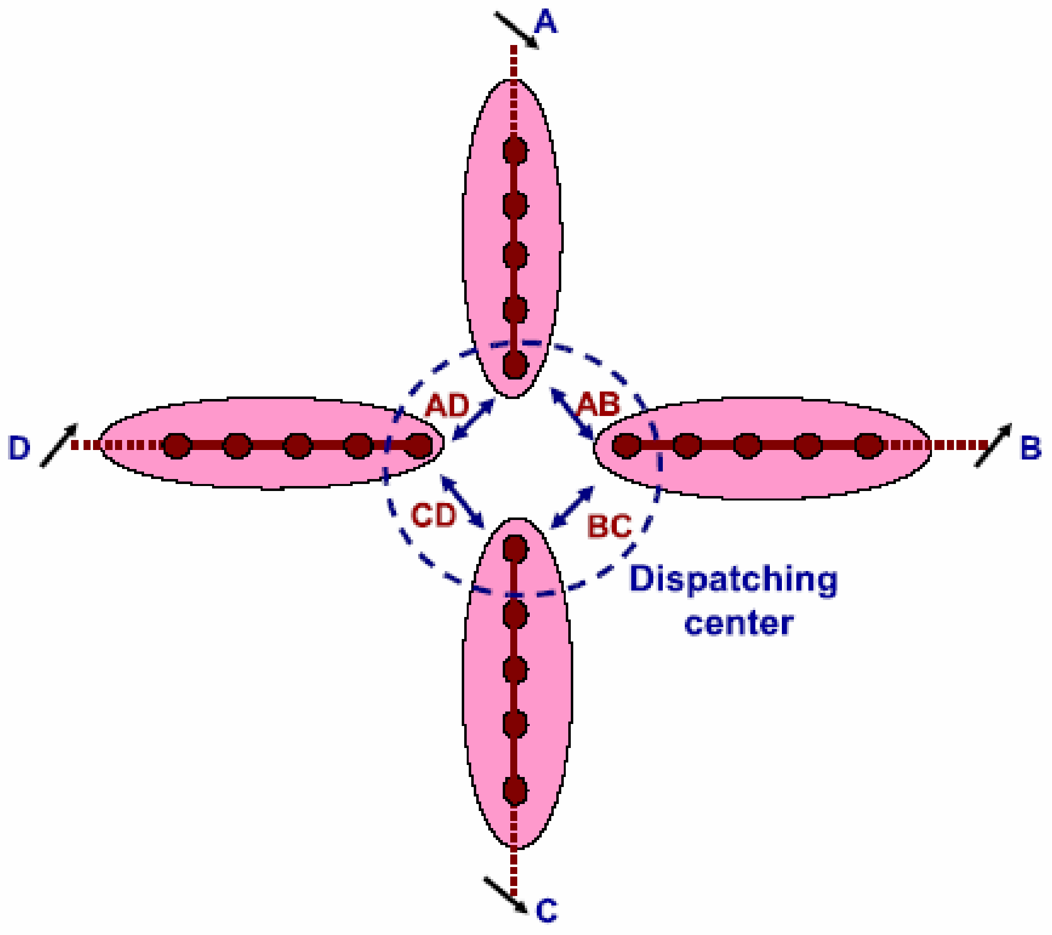
$$\xi_R(J'_R) = N_R - 1$$

Entanglement in Non-Symmetric Chains



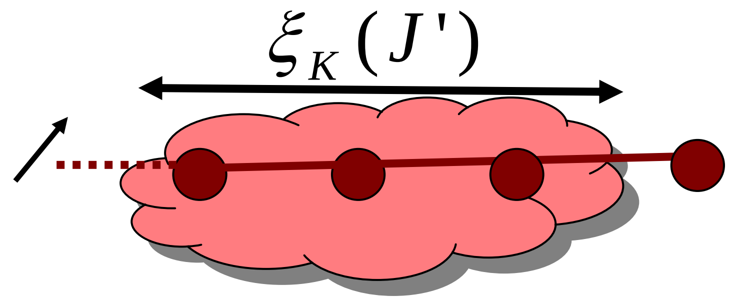
Symmetric geometry gives the best output

Entanglement Router

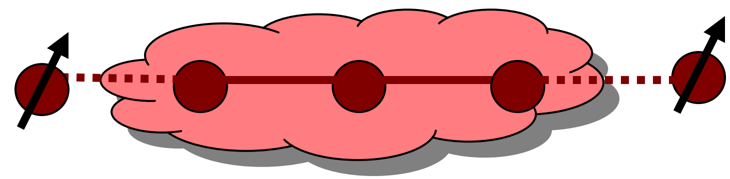


Summary

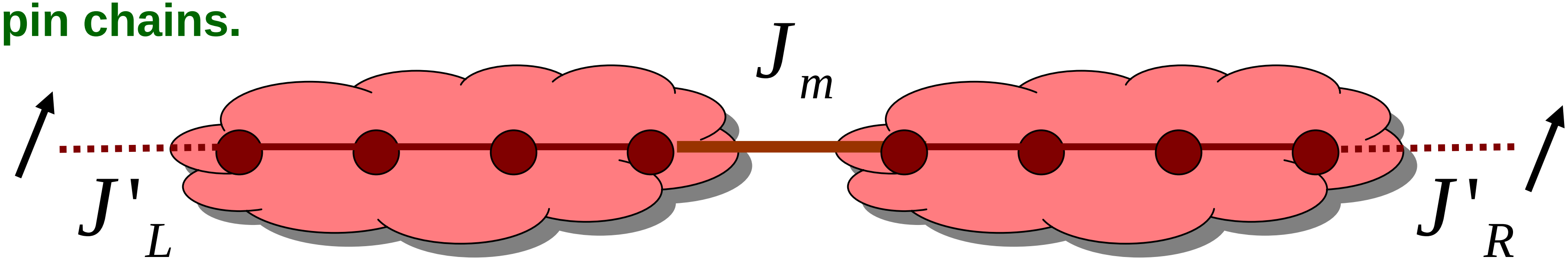
- With using the entanglement measures, one can capture the properties of the Kondo physics.



- In the Kondo regime, Kondo cloud plays the role of the mediator to create a long range "*distance independent*" entanglement between individual ending spins.



- One can make an entanglement router through connecting Kondo spin chains.



Mechanism of Entanglement Generation

$$H_1 : \quad |\psi(0)\rangle = |GS_1\rangle$$

$$H_1 \rightarrow H_2 : \; \{ E_i , |E_i\rangle \}$$

$$|\psi(t)\rangle = \sum_i e^{-iE_i t} \langle E_i | GS_1 \rangle |E_i\rangle$$

Kondo Phase: ***Only two*** **states dominantly involve in the dynamics**

Dimer Phase: ***Many*** **states involve in the dynamics**

Dynamics in the Kondo Regime

$$\left|\psi(t)\right\rangle = \sum_{i=1,2} e^{-iE_i t} \left\langle E_i \middle| GS_I \right\rangle \left| E_i \right\rangle$$

By Quenching a single bond we release some energy into the system:

$$\delta E \propto 1 - J'$$

Energy separation between E1 and E2: $\Delta E(J')$

Optimal quench: $\delta E \propto \Delta E$

Static Entanglement

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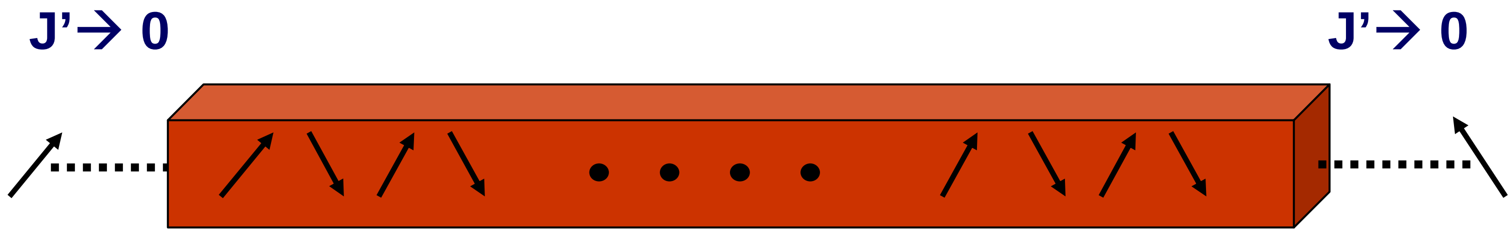


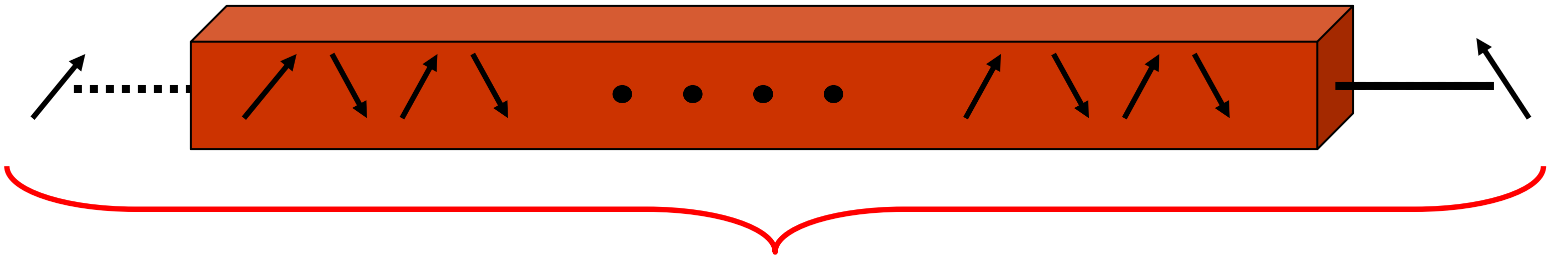
Static strategy creates high amount of entanglement in perturbative regime (J'→0).

$$J' \propto \frac{\varepsilon}{\sqrt{N}} \quad \Rightarrow \quad E \to 1$$

Due to the vanishing gap this entanglement is highly unstable to thermal fluctuations.

$$\Delta = J'^2 = \frac{\varepsilon^2}{N} \quad \Rightarrow \quad KT < \Delta = \frac{\varepsilon^2}{N}$$

Dynamical Entanglement



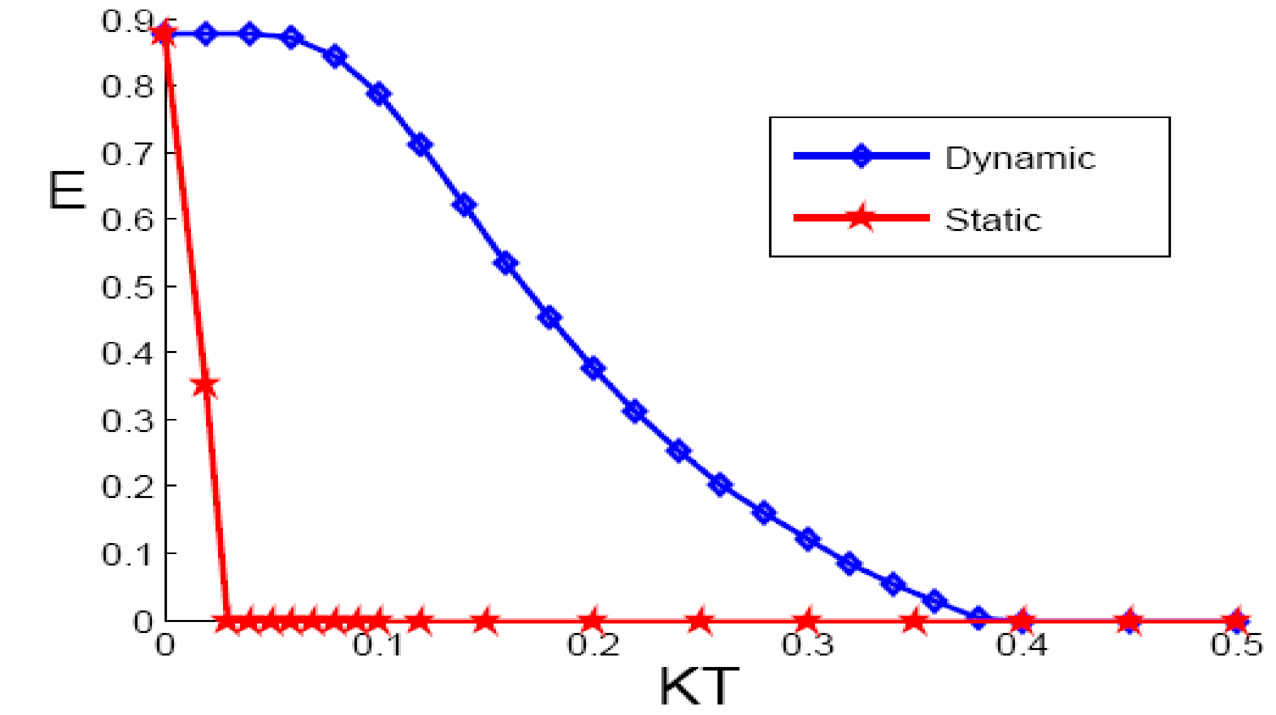
$$\rho_{th} = \frac{e^{-\beta H_1}}{Z}$$

$$H_1 \rightarrow H_2 \Rightarrow \rho(t) = e^{-iH_2 t}\rho_{th} e^{+iH_2 t} \Rightarrow \rho_{1N}(t) \Rightarrow E_{1N}(t)$$

Thermal stability:

$$KT_K = \frac{1}{\xi} = \frac{1}{N-2} \Rightarrow KT < KT_K = \frac{1}{N-2}$$

Thermal Effect on the Entanglement



Dynamical strategy for creating entanglement is more resistive than the static one.