Results on direct CP Violation in $K^{\pm} \rightarrow 3\pi$ decays from the NA48/2 experiment at CERN

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On behalf of the NA48/2 collaboration:
Cambridge, CERN, Chicago, Dubna, Edinburgh, Ferrara, Firenze, Mainz, Northwestern, Perugia, Pisa, Saclay, Siegen, Torino, Vienne

New Trends in High Energy Physics
Yalta, 10-17 September 2005
Outline

- Direct CP violation in $K^\pm \rightarrow 3\pi$ decays
- NA48/2 experimental setup
- Measurement principle
- Systematic effects
- Preliminary result in $K^\pm \rightarrow \pi^\pm \pi^+ \pi^-$ decay
- Outlook for $K^\pm \rightarrow \pi^\pm \pi^0\pi^0$ analysis
- Neutral mode “Cusp effect”
- Conclusions
Brief history of CP violation

1964 - CP violation in $K^0$ (Cronin, Christenson, Fitch, Turlay)
1993-99 - Direct CP violation in $K^0$ (NA31, NA48, KTeV)
2001 - CP violation in $B^0$ mixing (Babar, Belle)
2004 - Direct CP violation in $B^0$ (Belle, Babar)

Direct CP Violation, also known as CPV in decay amplitudes, is the most “straightforward” CP effect:

- Hard to detect experimentally
- Hard to connect to the parameters of the underlying fundamental theory (i.e. SM)

But it is a crucial window to physics beyond SM because possible non-SM enhancements to heavy quark loops are just at the core of DCPV processes.
Direct CPV in $K^0 \to \pi\pi$ decays

Final NA48 result

$\varepsilon' / \varepsilon = (14.7 \pm 2.2) \times 10^{-4}$
CP violation in $K_{3\pi}^{\pm}$ decays

- Potentially large statistics
- Simple selection
- Low background

No absolute K flux measurement

Compare only Dalitz plot shapes

Complementary observables in Kaons: $\varepsilon'/\varepsilon \leftrightarrow A_g \leftrightarrow$ rare decays

Look for direct CP violation in $K^{\pm}$

(only direct CPV in $K^{\pm}$ possible - no mixing)
Direct CP violation observable $A_g$

$|M(u,v)|^2 \sim 1 + gu + hu^2 + kv^2$

$\pi_1^{\text{even}}$  \hspace{1cm} $\pi_2^{\text{even}}$  \hspace{1cm} $\pi_3^{\text{odd}}$

$K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$

$\text{BR} = 5.57\%$; $g = -0.2154 \pm 0.035$

$K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$

$\text{BR} = 1.73\%$; $g = 0.652 \pm 0.031$

$|h|, |k| \ll |g|$

$K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$

$|M(u,v)|^2 \sim 1 + gu + hu^2 + kv^2$

$U = \frac{s_3 - s_0}{m^2_{\pi}}$

$V = \frac{s_2 - s_1}{m^2_{\pi}}$

$s_i = (P_K - p_{\pi_i})^2$

$s_0 = \frac{1}{3} \sum s_i$

$i = 3$ odd pion

$A_g = \frac{g_+ - g_-}{g_+ + g_-} = \frac{\Delta g}{2g} = -\frac{\Delta g}{0.43}$

$A_g < 5 \times 10^{-5}$ compatible with SM

$A_g > 1 \times 10^{-4}$ SUSY/new physics

$A_g \neq 0$  \hspace{1cm} Direct CP violation
Experimental and theoretical status

SM estimates of $A_g$ vary within an order of magnitude (few $10^{-6}$...8 $x10^{-5}$).

Models beyond SM predict substantial enhancements partially within the reach of NA48/2.
( theoretical analyses are by far not exhaustive by now)
Goals and method

- **Primary NA48/2 goals:**
  - Measure slope asymmetries in “charged” and “neutral” modes with precisions $\delta A_g < 2.2 \times 10^{-4}$, and $\delta A_g^0 < 3.5 \times 10^{-4}$, respectively.
  - Statistics required for this measurement: $>2 \times 10^9$ in “charged” mode and $>10^8$ in “neutral” mode.

- **NA48/2 method:**
  - Two simultaneous $K^+$ and $K^-$ beams, superimposed in space, with narrow momentum spectra;
  - Detect asymmetry exclusively considering slopes of ratios of normalized $u$ distributions;
  - Equalise averaged $K^+$ and $K^-$ acceptances by frequently alternating the polarities of the relevant magnets.
**Experimental setup**

- **PK spectra**, $60 \pm 3$ GeV/c

**He tank + spectrometer**

10 cm

- **Front-end achromat**
  - Momentum selection

- **Quadrupole quadruplet**
  - Focusing
  - $\mu$ sweeping

- **Second achromat**
  - Cleaning
  - Beam spectrometer

- **Analysing magnet**

- **Decay volume**

- **K+**
- **K−**

Beams coincide within $\sim 1$ mm all along 114 m decay volume

$\sim 7 \cdot 10^{11}$ ppp

$\cdot$ $K^+$
$\cdot$ $K^−$

- Target
- 0.36 mrad
- 1 cm

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Main detector components:

- **Magnetic spectrometer (4 DCHs):**
  - 4 views: redundancy ⇒ efficiency
  - $\sigma(p)/p = 1.0\% + 0.044\% \ p [GeV/c]$
- **Hodoscope:** fast trigger and precise time measurement (150ps)
- **Liquid Krypton e.m. calorimeter:**
  - High granularity, quasi-homogeneous
  - $\sigma(E)/E = 3.2%/\sqrt{E} + 9%/E + 0.42\% [GeV]$ $e/\pi$ discrimination
- **Hadron calorimeter, photon vetos, muon veto counters**
Data taking: completed

2003 run: ~ 50 days
2004 run: ~ 60 days

Total statistics in 2 years:
• $K^\pm \rightarrow \pi^+ \pi^- \pi^\pm$: $\sim 3.5 \cdot 10^9$
• $K^\pm \rightarrow \pi^0 \pi^0 \pi^\pm$: $\sim 2 \cdot 10^8$

~ 200 TB of data recorded
The result based on 2003 $K^\pm \rightarrow \pi^\pm \pi^+ \pi^-$ sample will be presented here
Data-taking 2003:
$1.61 \times 10^9 \, K^\pm \to \pi^\pm \pi^+ \pi^- \, \text{events}$

- $K^+: \, 1.03 \times 10^9 \, \text{events}$
- $K^-: \, 0.58 \times 10^9 \, \text{events}$

$K^+/K^- \approx 1.8$
Method to extract $A_g$

- Build projections of the Dalitz plot for $K^+$ and $K^-\colon N^+(u),N^-(u)$
- Make the ratio of these two distributions: $R(u)$
- Fit a linear function to this ratio to extract $\Delta g = g^+ - g^-$

$$R(u) = \frac{N^+(u)}{N^-(u)} \propto \frac{1 + g^+ u}{1 + g^- u} \approx 1 + \Delta g u$$

$$A_g = \frac{\Delta g}{2 g}$$

This holds only if the acceptance for $K^+$ and $K^-$ is the same

$$\delta A_g < 2.2 \cdot 10^{-4}$$

$$\delta \Delta g < 0.9 \cdot 10^{-4}$$

The magnetic fields (achromat and spectrometer) are intrinsic sources of charge asymmetry in the detector!!!
Instrumental asymmetries

In real life

- Detector acceptance asymmetries
- Time variation of detector response
- Charge-dependent beam optics
- Time variation of beams’ properties
- Spurious magnetic fields
- Charge-asymmetric interactions
Strategy of data taking

Beam line (achromat) polarity (A) reversed on **weekly** basis
Spectrometer magnet polarity (B) reversed on **daily** basis

**Example**: August 6 to September 7, 2003

<table>
<thead>
<tr>
<th>Week 1</th>
<th>Achromat –</th>
<th>B+ B- B+ B- B+ B-</th>
<th>Supersample 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Week 2</td>
<td>Achromat +</td>
<td>B+ B- B+ B- B+ B-</td>
<td></td>
</tr>
<tr>
<td>Week 3</td>
<td>Achromat –</td>
<td>B+ B- B+ B- B+ B-</td>
<td>Supersample 2</td>
</tr>
<tr>
<td>Week 4</td>
<td>Achromat +</td>
<td>B+ B- B+ B- B+ B-</td>
<td></td>
</tr>
<tr>
<td>Week 5</td>
<td>Achromat –</td>
<td>B+ B-</td>
<td>Supersample 3</td>
</tr>
<tr>
<td></td>
<td>Achromat +</td>
<td>B+ B-</td>
<td></td>
</tr>
</tbody>
</table>

12 subsamples

12 subsamples

4 subsamples
Acceptance cancellation

Detector left-right asymmetry cancels in 4 ratios of $K^+/K^-$ distributions:

(same deviation by spectrometer in numerator and denominator)

$$R_{US} = \frac{N(A+B+K^+)}{N(A+B-K^-)}$$

$$R_{UJ} = \frac{N(A+B-K^+)}{N(A+B+K^-)}$$

$$R_{DS} = \frac{N(A-B+K^+)}{N(A-B-K^-)}$$

$$R_{DJ} = \frac{N(A-B-K^+)}{N(A-B+K^-)}$$

Indexes correspond to:
- beamline polarity ($U / D$)
- direction of kaon deviation in spectrometer ($S / J$)

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Quadruple ratio

\[ R = R_{US} R_{UJ} R_{DS} R_{DJ} \sim 1 + 4 \Delta g \cdot u \]

3-fold cancellation of systematic biases:
1) Global time-variable biases (K\(^+\), K\(^-\) simultaneously recorded)
2) Beam line biases (K\(^+\) beam up / K\(^-\) beam up etc.)
3) Detector asymmetries (K\(^+\) toward Saleve / K\(^-\) toward Saleve etc.)
4) Effects of permanent stray fields (earth, vacuum tank magnetisation) cancel

The result is sensitive only to time variation of asymmetries in experimental conditions with a characteristic time smaller than corresponding field-alternation period (beam-week, detector-day)
Monte Carlo simulation

Due to acceptance cancellations, the analysis does not rely on Monte-Carlo to calculate acceptance.

Still MC is used to study systematics. MC features:

- Based on GEANT
- Full detector geometry and material description
- Local DCH inefficiencies simulated
- Variations of beam geometry and DCH alignment are followed
- Simulated statistics similar to experimental one.

Example of data/MC agreement: mean beam positions @DCH1

<table>
<thead>
<tr>
<th>K+ data</th>
<th>K− data</th>
<th>K+ MC</th>
<th>K− MC</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

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Beam systematics

Time variations of beam geometry

Acceptance largely defined by central beam hole edge.

Acceptance cut defined by (larger) "virtual pipe" centered on averaged beam positions as a function of charge, time and $K$ momentum

Effects due to beam movements and not perfect overlap corrected
**Spectrometer systematics**

**Time variations of spectrometer geometry** - Alignment is fine tuned by forcing mean reconstructed invariant $\pi\pi\pi$ masses to be equal for $K^+$ and $K^-$.

E.g. sensitivity to DCH4 horizontal shift: $\Delta M / \Delta x \approx 1.5$ keV/\(\mu m\)

**Momentum scale**

variation due to limited control of spectrometer magnet current ($10^{-3}$) cancels due to simultaneous beams.

In addition, it is adjusted by forcing mean reconstructed invariant $\pi\pi\pi$ masses to PDG value of $M_{K^+}$.
Trigger systematics

**L1 trigger** (2 hodoscope hits): stable and small inefficiency: $1-e \approx 0.7 \cdot 10^{-3}$, charge-symmetric, flat in $u$ NO CORRECTION NEEDED

**L2 trigger** (online vertex reconstruction on DCH data):

time-varying inefficiency (local DCH inefficiencies) $1-e \approx 0.2\%$ to $1.8\%$, flat in $u$
within measurement precision $u$-dependent CORRECTION APPLIED

<table>
<thead>
<tr>
<th>Subsample</th>
<th>L2 correction $\delta \Delta g \times 10^4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SS0</td>
<td>0.5±1.8</td>
</tr>
<tr>
<td>SS1</td>
<td>1.4±1.0</td>
</tr>
<tr>
<td>SS2</td>
<td>-0.2±1.2</td>
</tr>
<tr>
<td>SS3</td>
<td>-4.5±1.9</td>
</tr>
</tbody>
</table>

**L2 ineff for K+ and K- in subsamples**

**L2 efficiency vs U**

statistical uncertainty from control sample
Other systematics

Residual effects of stray magnetic fields (magnetised vacuum tank, earth field) minimised by explicit field map correction

Further systematic effects studied
- Bias due to resolution in $u$ calculation
- Sensitivity to fitting interval and method
- Effects connected to $\pi \rightarrow \mu \nu$ decay
- Effects due to event pile-up
- $\pi^+ / \pi^-$ interactions in material
- Track charge misidentification

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Systematics summary and result

<table>
<thead>
<tr>
<th>Conservative estimations of systematic errors</th>
<th>Effect on ( \Delta x 10^4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acceptance and beam geometry</td>
<td>0.5</td>
</tr>
<tr>
<td>Spectrometer alignment</td>
<td>0.1</td>
</tr>
<tr>
<td>Analyzing magnet field</td>
<td>0.1</td>
</tr>
<tr>
<td>( \pi^+ \rightarrow \mu \nu ) decay</td>
<td>0.4</td>
</tr>
<tr>
<td>U calculation and fitting</td>
<td>0.5</td>
</tr>
<tr>
<td>Pile-up</td>
<td>0.3</td>
</tr>
<tr>
<td>Syst. errors of statistical nature</td>
<td></td>
</tr>
<tr>
<td>Trigger efficiency: L2</td>
<td>0.8</td>
</tr>
<tr>
<td>Trigger efficiency: L1</td>
<td>0.4</td>
</tr>
<tr>
<td><strong>Total systematic error</strong></td>
<td><strong>1.3</strong></td>
</tr>
</tbody>
</table>

Combined preliminary result:

in \( \Delta g \times 10^4 \) units

(3 independent analyses)

Including L2 trigger correction

<table>
<thead>
<tr>
<th></th>
<th>Raw</th>
<th>Corrected for L2 eff</th>
</tr>
</thead>
<tbody>
<tr>
<td>SS0</td>
<td>0.0±1.5</td>
<td>0.5±2.4</td>
</tr>
<tr>
<td>SS1</td>
<td>0.9±2.0</td>
<td>2.2±2.2</td>
</tr>
<tr>
<td>SS2</td>
<td>-2.8±2.2</td>
<td>-3.0±2.5</td>
</tr>
<tr>
<td>SS3</td>
<td>2.0±3.4</td>
<td>-2.6±3.9</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>-0.2±1.0</strong></td>
<td><strong>-0.2±1.3</strong></td>
</tr>
<tr>
<td>( \chi^2 )</td>
<td>2.2/3</td>
<td>3.2/3</td>
</tr>
</tbody>
</table>
Result stability

\[
\Delta g \times 10^{-4}
\]

\[
\chi^2/\text{ndf} \quad 3.346 \quad / \quad 9
\]

\[
E_K \text{ (GeV)}
\]

\[
\Delta g \times 10^{-4}
\]

\[
\chi^2/\text{ndf} \quad 6.615 \quad / \quad 9
\]

\[
z_{\text{vertex}} \text{ (cm)}
\]
Preliminary result (2003 data)

- **slope difference**
  \[ \Delta g = (-0.2 \pm 1.0)_{\text{stat.}} \pm 0.9_{\text{stat.(trig.)}} \pm 0.9_{\text{syst.}} \times 10^{-4} \]
  \[ \Delta g = (-0.2 \pm 1.7) \times 10^{-4} \]

- **charge asymmetry**
  \[ A_g = (0.5 \pm 2.4)_{\text{stat.}} \pm 2.1_{\text{stat.(trig.)}} \pm 2.1_{\text{syst.}} \times 10^{-4} \]
  \[ A_g = (0.5 \pm 3.8) \times 10^{-4} \]

- This is a preliminary result with conservative estimate of systematic uncertainties.
- Extrapolated statistical uncertainty 2003+2004: \( \delta A_g = 1.6 \times 10^{-4} \)
- Expect smaller systematic effects in 2004 data (due to more frequent polarity alternation, better L2 performance).
Comparison $K^{\pm} \rightarrow \pi^{\pm} \pi^{+} \pi^{-}$

This preliminary result is already an order of magnitude better than previous experiments.
$K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$ analysis

“neutral” mode wrt “charged”

- Same strategy of analysis
- Only the Lkr used to define $u$
- Totally different systematics
- Statistical precision in $A_g^0$ similar
  - Ratio of “neutral” to “charged” statistics: $N^0/N^\pm \sim 1/20(\sqrt{f}=1/4.5)$
  - Ratio of slopes: $|g^0/g^\pm| \sim 3$
  - More favourable Dalitz-plot distribution (gain factor $f\sim 1.5$)
Status of analysis

- Statistics analyzed: 50 × 10^6 events
- Statistical error with analyzed data: \( \delta A_g(\text{stat}) = 1.7 \times 10^{-4} \)
- Extrapolation to 2003+2004 data (115 × 10^6) \( \delta A_g(\text{stat}) = 1.1 \times 10^{-4} \)

Preliminary results will be announced soon
Observation of $\pi\pi$ scattering effect in $K\to 3\pi$ decays

Thanks to the large statistics in the neutral mode we can see (for the first time) the contribution of the charge exchange process $\pi^+\pi^-\to\pi^0\pi^0$ in the $K^\pm\to\pi^0\pi^0\pi^\pm$ decay. This effect stimulated some theoretical work.
(a_0-a_2) determination in K^+\rightarrow\pi^+\pi^0\pi^0

Two processes contribute to K^+ --> \pi^+\pi^0\pi^0
1) Direct emission of \pi^+\pi^0\pi^0
2) \pi^0\pi^0 produced in charged pions rescattering

\[ M_0 = 1 + gu/2 \]
\[ u = 2m_\kappa(m_\kappa/E_{\text{odd}}/m_\pi^2 \]
\[ g = 0.638 \pm 0.020 \]
(present PDG value)

\[ M_1 \propto (a_0 - a_2) \]

\[ \frac{d\Gamma}{dm_{\pi\pi}} \propto |M_0 + M_1|^2 \]

Interference is expected

Fits to the “cusp” effect in $K^\pm \rightarrow \pi^\pm \pi^0\pi^0$

$\Delta = (\text{data-fit})/\text{data}$

$\chi^2 = 133/139$

for $M(\pi^0\pi^0) > 80$ MeV/c$^2$

The best fit obtained with two loops adding a small amount of pionium to improve the $\chi^2$

One-loop exchange: $\chi^2 = 463/149$


One and two loops: $\chi^2 = 159/147$

Cabibbo, Isidori: hep-ph/0502130

Incl. $\pi^+\pi^-$ atoms: $\chi^2 = 144/146$
Preliminary result

the pionium contribution has been fixed to the prediction:
Z.K. Silagadze, hep-ph/9411382

\[
\frac{K^+ \to \pi^+ + \text{pionium}}{K^+ \to \pi^+ \pi^+ \pi^-} \approx 7.4 \times 10^{-6}
\]

\[(a_0 - a_2)m_+ \quad \text{has low sensitivity to pionium}\]

\[(a_0 - a_2)m_+ = 0.281 \pm 0.007 \text{(stat)} \pm 0.014 \text{(syst)} \pm 0.014 \text{(theor)}\]

\[
\chi^2/\text{ndf} = 154/146
\]

No surprise from other parameters

In agreement with theory \[(a_0 - a_2)m_+ = 0.265 \pm 0.004 \text{ (Colangelo 2001)}\]
Conclusions

- Preliminary NA48/2 result (only 2003 data) on direct CP-violating charge asymmetry in $K^\pm \rightarrow \pi^\pm \pi^+ \pi^-$ decays is
  \[ A_g = (0.5 \pm 2.4_{\text{stat.}} \pm 2.1_{\text{stat.(trig.)}} \pm 2.1_{\text{syst.}}) \times 10^{-4} \]
- x 10 times better precision than previous measurements
- Further room to decrease systematic error (trigger efficiency)
- 2004 data contains another $2 \times 10^9 K^\pm \rightarrow \pi^\pm \pi^+ \pi^-$ events, possibly with higher quality → Design goal within reach
- $K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$ neutral asymmetry: complementary, comparable sensitivity
- “Cusp” effect: determination of the $\pi \pi$ scattering lengths
- A lot of other interesting results coming (other CP asymmetries, rare decays)
SPARE SLIDES
$K_{3\pi}^\pm$ decays

\[ \text{BR}(K^\pm \to \pi^\pm \pi^+ \pi^-) = 5.57\% \]

``charged``

Kinematic variables

Lorentz-invariants

\[ u = (s_3-s_0)/m_\pi^2; \]
\[ v = (s_2-s_1)/m_\pi^2; \]
\[ s_i = (P_K-P_{\pi_i})^2, \ i=1,2,3 \ (3=\text{odd } \pi); \]
\[ s_0 = (s_1+s_2+s_3)/3. \]

Centre of mass frame

\[ u = 2m_K \cdot (m_K/3-E_{\text{odd}})/m_\pi^2; \]
\[ v = 2m_K \cdot (E_1-E_2)/m_\pi^2. \]

Matrix element parameterized in terms of slopes

\[ |M(u,v)|^2 \sim 1 + gu + hu^2 + kv^2 \]

Measured quantity sensitive to direct CP violation:

Slope asymmetry:

\[ A_g = (g^+ - g^-) / (g^+ + g^-) \]
## NA48/2 narrow-band beams

**simultaneous, coaxial, focused**

<table>
<thead>
<tr>
<th>Beam</th>
<th>Positive</th>
<th>Negative</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary proton momentum</td>
<td>400 GeV/c</td>
<td></td>
</tr>
<tr>
<td>Duty cycle</td>
<td>5.2 s / 16.8 s</td>
<td></td>
</tr>
<tr>
<td>Protons on target per cycle</td>
<td>$10^{12}$</td>
<td></td>
</tr>
<tr>
<td>Production angle</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Beam acceptance</td>
<td>±0.36 mrad</td>
<td></td>
</tr>
<tr>
<td>Beam momentum</td>
<td>(60 ± 3) GeV/c</td>
<td></td>
</tr>
<tr>
<td>Beamline length</td>
<td>102 m</td>
<td></td>
</tr>
<tr>
<td>$p / \bar{p}$ per cycle ($10^6$)</td>
<td>8.6</td>
<td>0.9</td>
</tr>
<tr>
<td>$\pi^+ / \pi^-$ per cycle ($10^6$)</td>
<td>33.2</td>
<td>24.6</td>
</tr>
<tr>
<td>$K^+ / K^-$ per cycle ($10^6$)</td>
<td>3.1</td>
<td>1.8</td>
</tr>
<tr>
<td>Decay region</td>
<td>115 m</td>
<td></td>
</tr>
</tbody>
</table>

$P_K$ spectra, $60\pm3$ GeV/c

⇒ Pion decay products stay in beam pipe...
### Theoretical predictions of $A_g$

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Standard Model</strong></td>
<td>L. Maiani, N. Paver ’95</td>
<td>$(2.3 \pm 0.6) \times 10^{-6}$</td>
</tr>
<tr>
<td></td>
<td>A. Bel’kov ’95</td>
<td>$&lt; 4 \times 10^{-4}$</td>
</tr>
<tr>
<td></td>
<td>G. D’Ambrosio, G. Isidori ’98</td>
<td>$&lt; 10^{-5}$</td>
</tr>
<tr>
<td></td>
<td>E. Shabalin ’01</td>
<td>$&lt; 3 \times 10^{-5}$</td>
</tr>
<tr>
<td></td>
<td>E. Gamiz, J. Prades, I. Scimemi ’03</td>
<td>$(-2.4 \pm 1.2) \times 10^{-5}$</td>
</tr>
<tr>
<td></td>
<td>E. Shabalin ’05 (La Thuile’05)</td>
<td>$&lt; 8 \times 10^{-5}$</td>
</tr>
<tr>
<td><strong>SUSY</strong></td>
<td>G. D’Ambrosio, G. Isidori, G. Martinelli</td>
<td>$\sim 10^{-4}$</td>
</tr>
<tr>
<td><strong>New physics</strong></td>
<td>E. Shabalin ’98 [Weinberg model of extended Higgs doublet]</td>
<td>$\sim 4 \times 10^{-4}$</td>
</tr>
<tr>
<td></td>
<td>I. Scimemi ’04</td>
<td>$&gt; 3 \times 10^{-5}$</td>
</tr>
</tbody>
</table>
(1) **Double ratio** cancellation of **global time instabilities** (rate effects, *simultaneous beams*):

\[
R_U = R_{US} \times R_{UJ} \quad \Rightarrow \quad R(u) = n \cdot (1 + 2 \Delta g_U u)
\]

\[
R_D = R_{DS} \times R_{DJ} \quad \Rightarrow \quad R(u) = n \cdot (1 + 2 \Delta g_D u)
\]

(2) **Double ratio** cancellation of **beam geometry difference** effects:

\[
R_S = R_{US} \times R_{DS} \quad \Rightarrow \quad R(u) = n \cdot (1 + 2 \Delta g_S u)
\]

\[
R_J = R_{UJ} \times R_{DJ} \quad \Rightarrow \quad R(u) = n \cdot (1 + 2 \Delta g_J u)
\]

(3) **Fit with quadruple ratio**:

\[
R = R_{US} \times R_{UJ} \times R_{DS} \times R_{DJ} \quad \Rightarrow \quad R(u) = n \cdot (1 + 4 \Delta g u)
\]

The fit result is sensitive only to **time variation** of **asymmetries** in experimental conditions on a time-scale of ~ 1 subsample.
### Break down of $K^\pm \rightarrow \pi^\pm \pi^+ \pi^-$ statistics

**Statistics selected for $A_g$ measurement, events $\times 10^6$**

<table>
<thead>
<tr>
<th>Dates</th>
<th>Sub-sample</th>
<th>Achromat A+</th>
<th>Achromat A-</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$K^+$</td>
<td>$K^-$</td>
</tr>
<tr>
<td>0 22.06-25.07</td>
<td>26</td>
<td>229.6</td>
<td>125.9</td>
</tr>
<tr>
<td>1 6.08-20.08</td>
<td>12</td>
<td>122.5</td>
<td>68.1</td>
</tr>
<tr>
<td>2 20.08-3.09</td>
<td>12</td>
<td>147.2</td>
<td>81.8</td>
</tr>
<tr>
<td>3 3.09-7.09</td>
<td>4</td>
<td>40.6</td>
<td>22.6</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>54</strong></td>
<td><strong>Total events selected</strong></td>
<td><strong>1613.2</strong></td>
</tr>
</tbody>
</table>
Invariant $\pi\pi\pi$ mass

$\sigma_M = 1.7$ MeV/c$^2$

$K^+: 1.03 \times 10^9$ events  \hspace{1cm} No significant background  \hspace{1cm} $K^- : 0.58 \times 10^9$ events
Fit linearity - four supersamples

SS0: \( \Delta g = (0.6 \pm 2.4) \times 10^{-4} \) \( \chi^2 = 39.7/38 \)

SS1: \( \Delta g = (2.3 \pm 2.2) \times 10^{-4} \) \( \chi^2 = 38.1/38 \)

SS2: \( \Delta g = (-3.1 \pm 2.5) \times 10^{-4} \) \( \chi^2 = 29.5/38 \)

SS3: \( \Delta g = (-2.9 \pm 3.9) \times 10^{-4} \) \( \chi^2 = 32.9/38 \)
Cancellation of beam spectra

Achromat reversal reverses $K^+$ and $K^-$ beam spectra.

Systematic differences of $K^+$ and $K^-$ acceptance due to beam spectra mostly cancel in $R_U R_D$.

**Systematic check:**
Reweighting $K^+$ events so as to equalise momentum spectra leads to negligible effect $\delta \Delta g = 0.03 \times 10^{-4}$.
Theoretical predictions

Weinberg (1966)
Effective field theory for strong interaction at low E

\[ a_0 m_{\pi^+} = \frac{7 m_{\pi^+}^2}{16\pi f_{\pi}^2} = 0.159 \]
\[ a_2 m_{\pi^+} = \frac{-m_{\pi^+}^2}{8\pi f_{\pi}^2} = -0.045 \]

Most recently
Colangelo (2001)
\( \chi pt \)-theory two loops
Ref: hep-ph/0103088

\[ a_0 m_{\pi^+} = 0.220 \pm 0.005 \]
\[ a_2 m_{\pi^+} = -0.0444 \pm 0.0010 \]
\[ (a_0 - a_2) m_{\pi^+} = 0.265 \pm 0.004 \]

\* 2% level of accuracy: quite unusual for hadronic physics experiments have not yet reached the same level
Experimental status

1977: measurement by Genève/Saclay experiment @ 20% accuracy

2003: BNL E865 extracts $a_0$ at 5% accuracy by measuring the form factors of the decay $K \rightarrow \pi\pi\nu\nu$ with 400,000 events

\[ a_0 m = 0.216 \pm 0.013 \{\text{stat.}\} \pm 0.002 \{\text{syst.}\} \pm 0.002 \{\text{theor.}\} \]

Ref. Pisla et al. (2003) hep-ex/0301040

Present: Cern experiment DIRAC, with a sophisticated technique, aims to measure the pionium lifetime @ 10% accuracy

\[ \tau \sim 40 \cdot (a_0 - a_2)^2 \cdot 10^{-15} \text{sec} \]