

Nucleon elastic and N-to- Δ transition form factors

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6th Intl. Conference on "Perspectives in Hadronic Physics"
Trieste, May 12-16, 2008

Outline

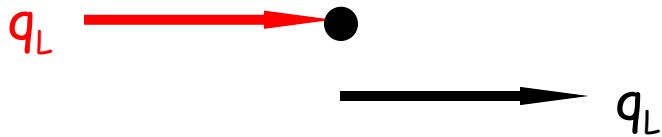
- introduction : nucleon densities and relativity
- transverse densities from nucleon elastic FFs and $N \rightarrow \Delta$ transition FFs
- transverse distance dependence and nucleon GPDs
- large N_c relations
between N elastic and $N \rightarrow \Delta$ FFs

work on densities in coll. with C.E. Carlson : PRL 100, 032004 (2008)

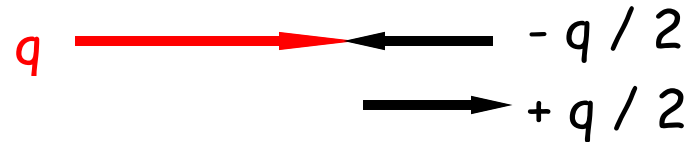
work on large N_c in coll. with V. Pascalutsa : PRD 76, 111501 (R) (2007)

Nucleon densities and relativity

rest frame



Breit frame



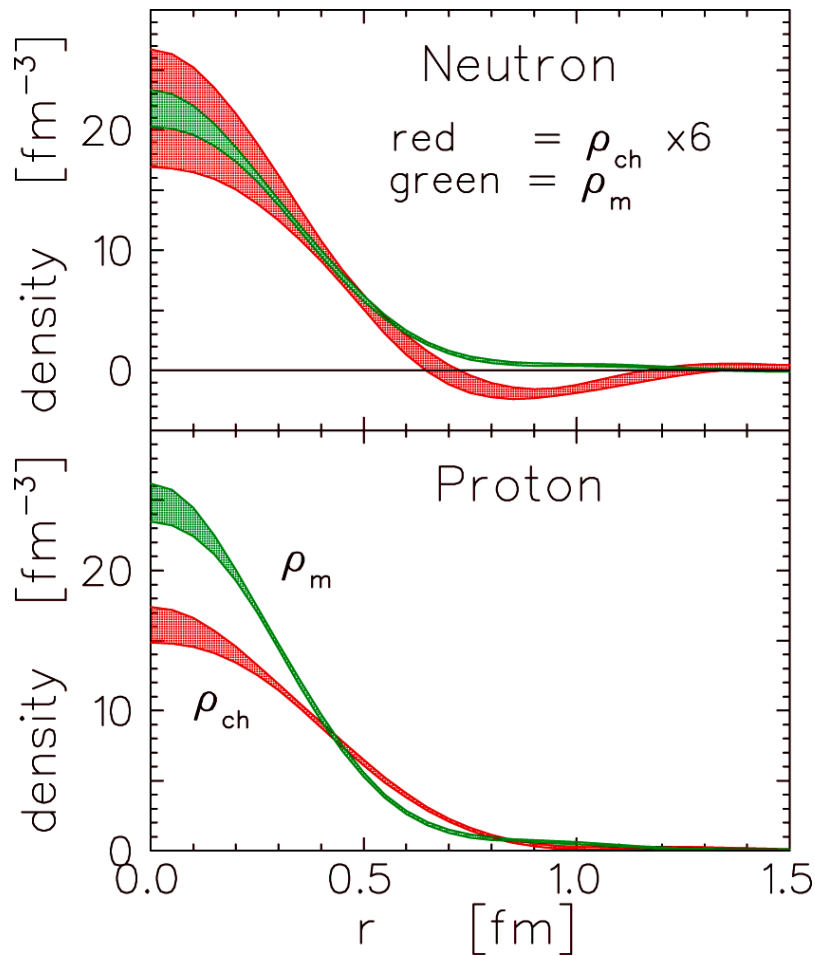
$$\vec{q}^2 = Q^2$$

→ relative velocity between frames :

$$\tau \equiv \frac{Q^2}{4M^2}$$

→ Lorentz contraction factor : $\gamma = \sqrt{1 + \tau}$

Nucleon densities and relativity



Kelly (2000)

$$\rho(r) = \frac{2}{\pi} \int_0^\infty dk k^2 j_0(kr) \tilde{\rho}(k)$$



rest frame density



intrinsic FF

➔ non-rel : $\tilde{\rho}(k) = G(Q^2)$

➔ importance of relativity (with increasing Q^2) : Lorentz contraction of spatial distributions in Breit frame

$$k^2 = Q^2 / (1 + \tau) \quad \tau = Q^2 / (4M^2)$$

$$\tilde{\rho}_{E,M}(k) = G_{E,M}(Q^2) (1 + \tau)^2$$

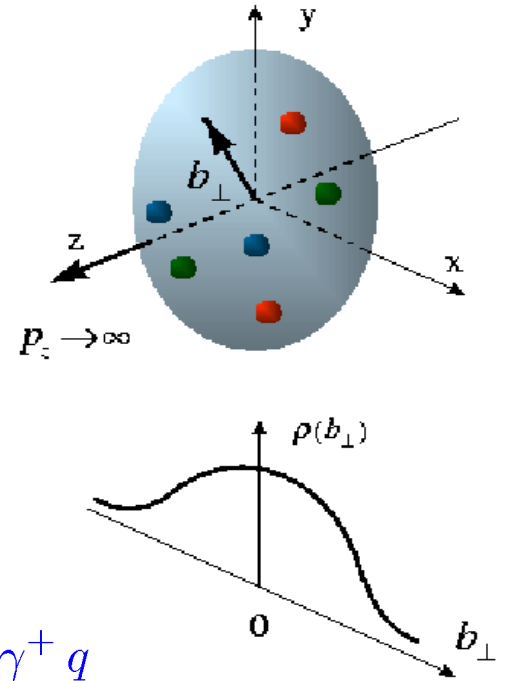
➔ limit : $k = 2M$ (Compton wavelength)

quark transverse charge densities in nucleon (I)

\Rightarrow $q^+ = q^0 + q^3 = 0$ photon only couples to forward moving quarks
 $Q^2 \equiv \vec{q}_\perp^2$

\Rightarrow quark charge density operator

$$J^+ \equiv J^0 + J^3 = \bar{q}\gamma^+q = 2q_+^\dagger q_+, \quad \text{with } q_+ \equiv \frac{1}{4}\gamma^-\gamma^+q$$



★ unpolarized nucleon

$$\begin{aligned}
 \rho_0^N(\vec{b}) &\equiv \int \frac{d^2\vec{q}_\perp}{(2\pi)^2} e^{i\vec{q}_\perp \cdot \vec{b}} \frac{1}{2P^+} \langle P^+, \frac{\vec{q}_\perp}{2}, \lambda | J^+(0) | P^+, -\frac{\vec{q}_\perp}{2}, \lambda \rangle \\
 &= \int_0^\infty \frac{dQ}{2\pi} Q J_0(bQ) F_1(Q^2)
 \end{aligned}$$

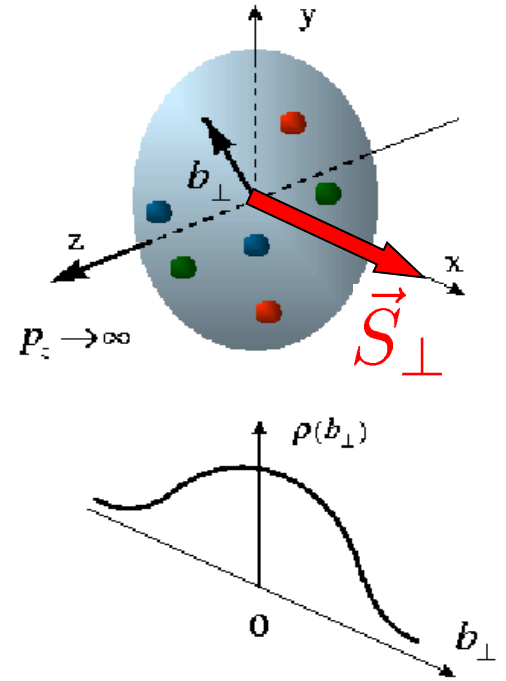
quark transverse charge densities in nucleon (II)

★ transversely polarized nucleon

transverse spin $\vec{S}_\perp = \cos \phi_S \hat{e}_x + \sin \phi_S \hat{e}_y$

e.g. along x-axis : $\phi_S = 0$

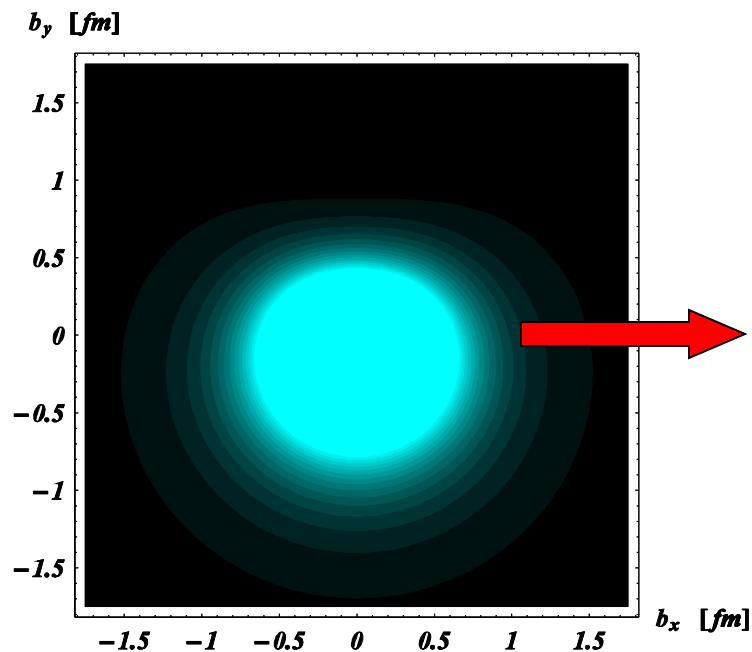
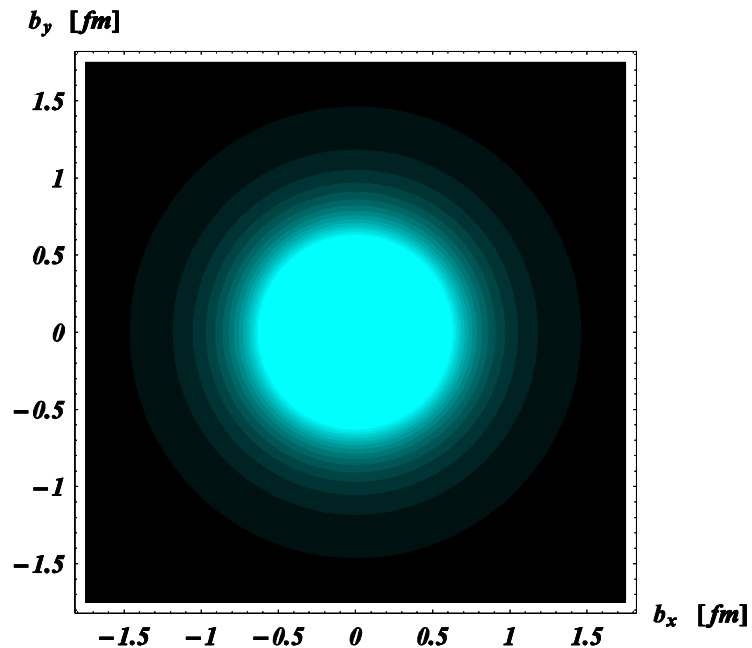
$$\vec{b} = b (\cos \phi_b \hat{e}_x + \sin \phi_b \hat{e}_y)$$



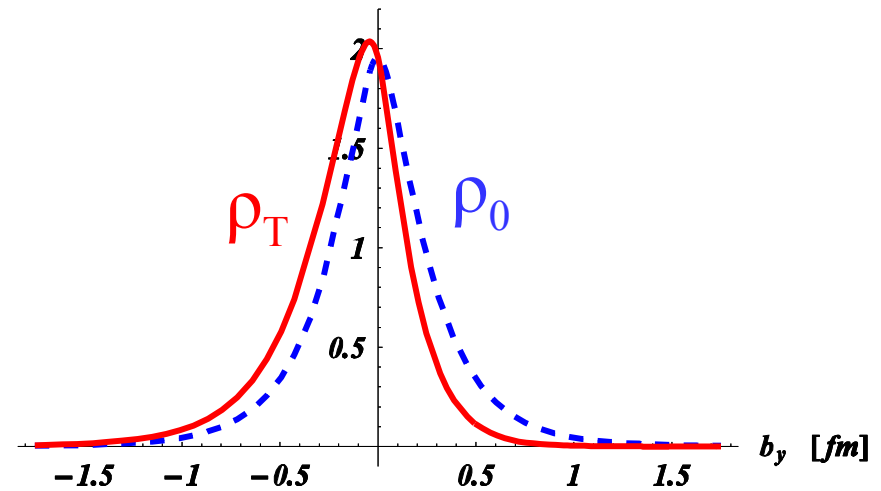
$$\begin{aligned} \rho_T^N(\vec{b}) &\equiv \int \frac{d^2 \vec{q}_\perp}{(2\pi)^2} e^{i \vec{q}_\perp \cdot \vec{b}} \frac{1}{2P^+} \langle P^+, \frac{\vec{q}_\perp}{2}, s_\perp = +\frac{1}{2} | J^+(0) | P^+, -\frac{\vec{q}_\perp}{2}, s_\perp = +\frac{1}{2} \rangle \\ &= \rho_0^N(b) - \sin(\phi_b - \phi_S) \int_0^\infty \frac{dQ}{2\pi} \frac{Q^2}{2M_N} J_1(bQ) F_2(Q^2) \end{aligned}$$

↓
dipole field pattern

empirical quark transverse densities in proton



ρ_0^p, ρ_T^p [$1/\text{fm}^2$]

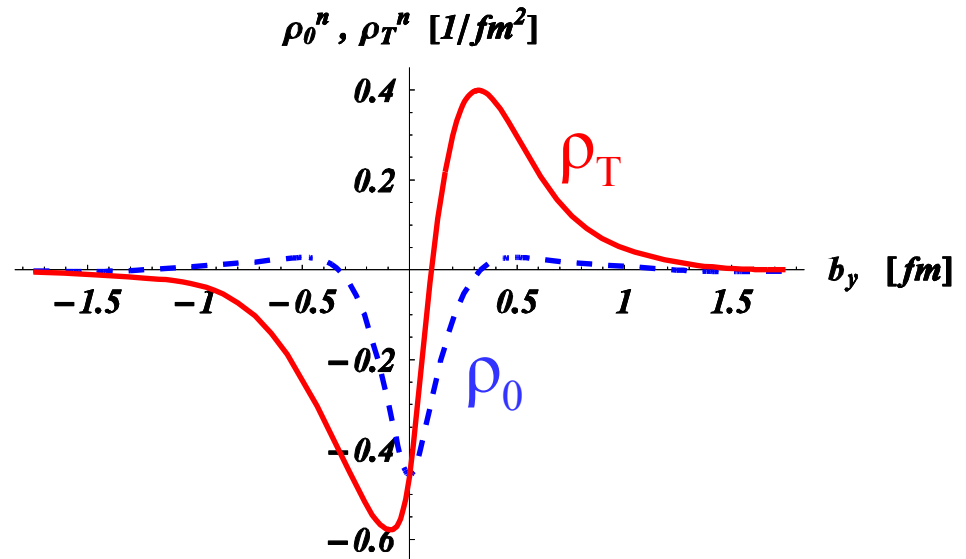
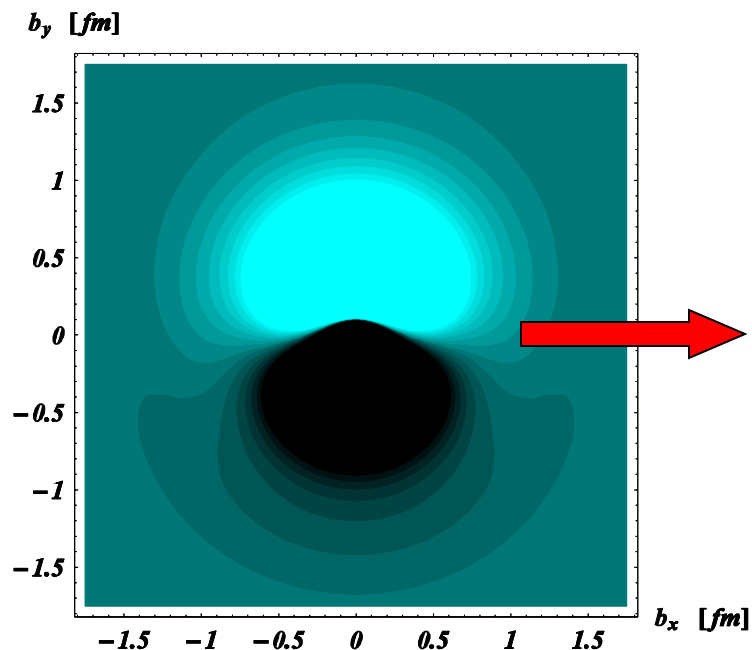
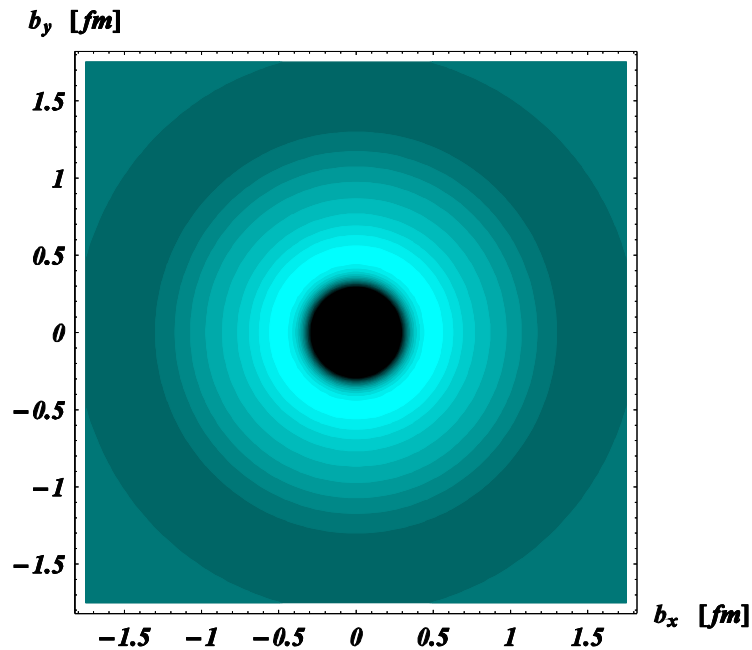


induced EDM : $d_y = - F_{2p}(0) \cdot e / (2 M_N)$

data : Arrington, Melnitchouk, Tjon (2007)

densities : Miller (2007); Carlson, Vdh (2007)

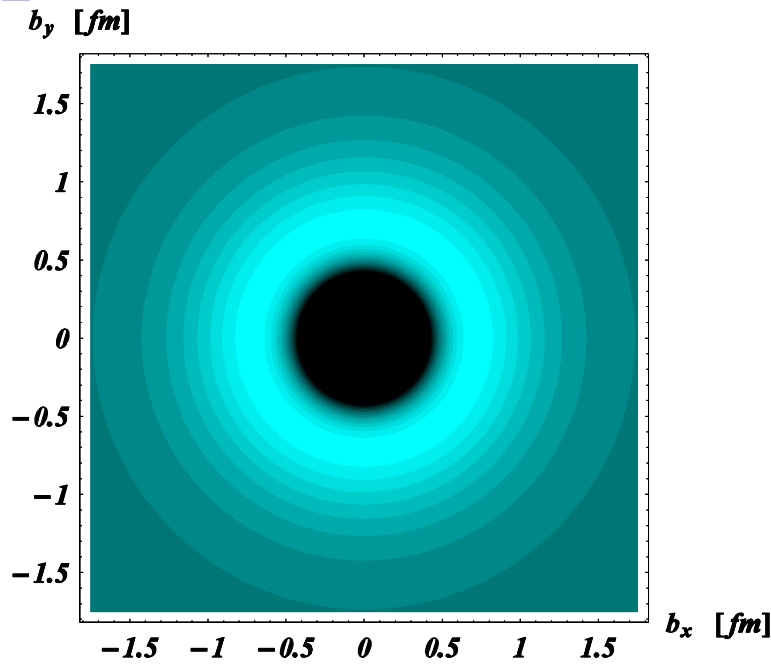
empirical quark transverse densities in neutron



induced EDM : $d_y = - F_{2n}(0) \cdot e / (2 M_N)$

data : Bradford, Bodek, Budd, Arrington (2006)

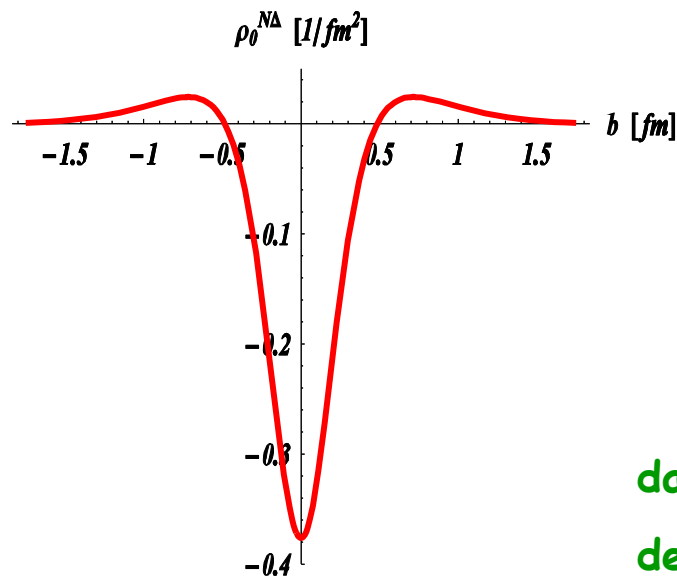
densities : Miller (2007); Carlson, Vdh (2007)



empirical transverse transition densities for $N \rightarrow \Delta$ excitation

$$\langle P^+, \frac{\vec{q}_\perp}{2}, \lambda_\Delta | J^+(0) | P^+, -\frac{\vec{q}_\perp}{2}, \lambda_N \rangle$$

$$= (2P^+) e^{i(\lambda_N - \lambda_\Delta)\phi_q} G_{\lambda_\Delta \lambda_N}^+(Q^2)$$



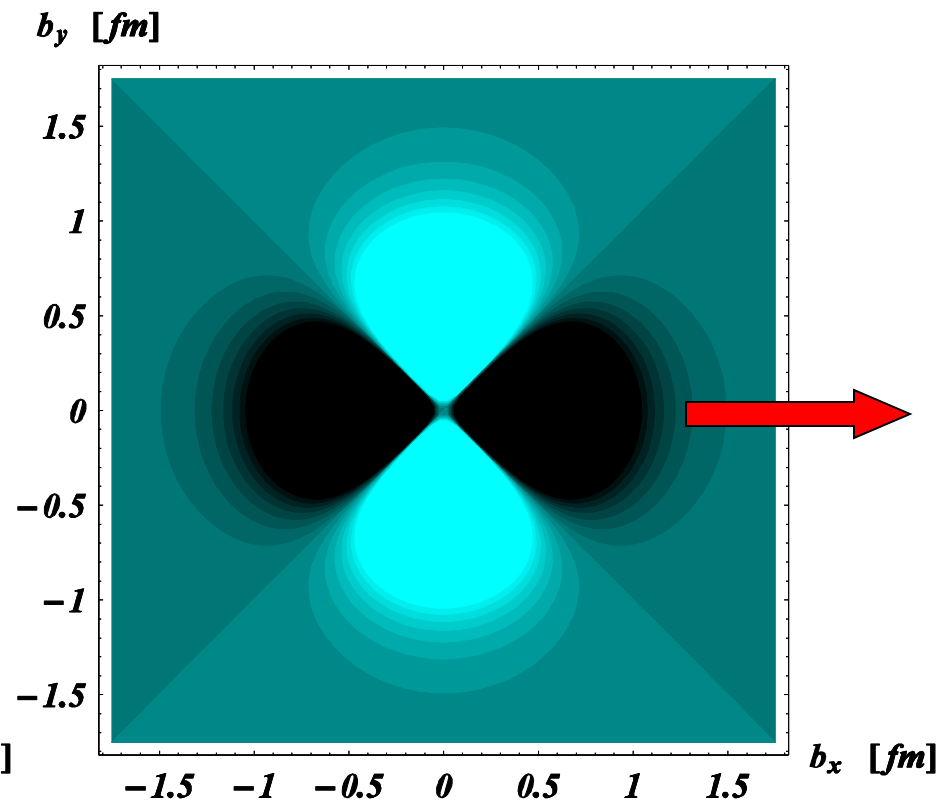
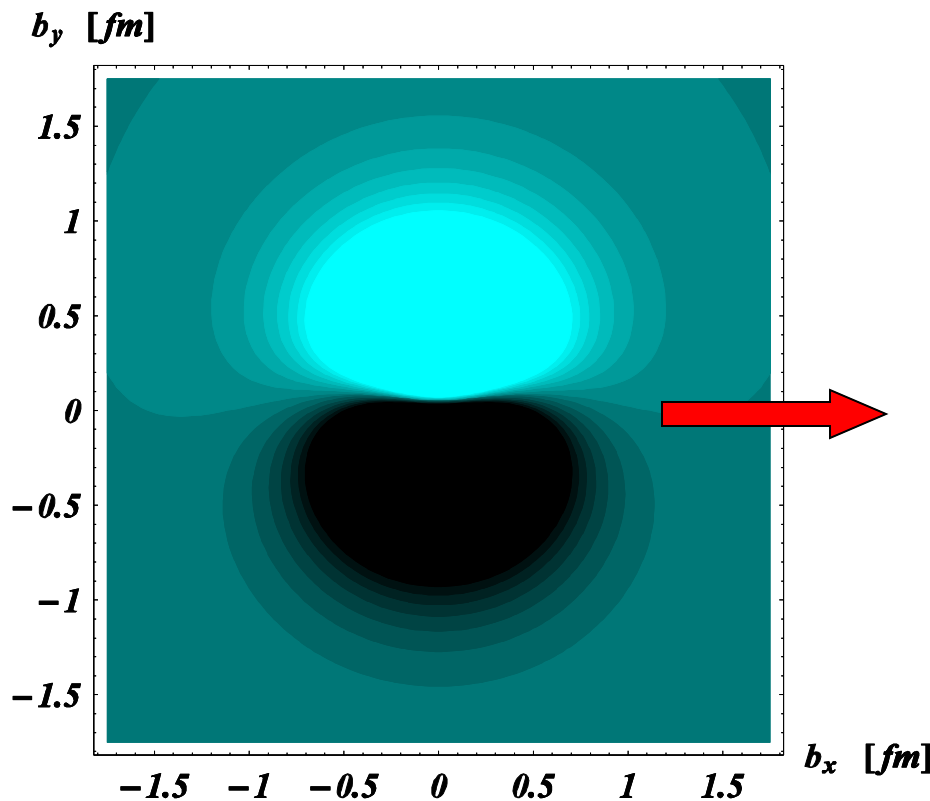
$$\rho_0^{N\Delta}(b) = \int_0^\infty \frac{dQ}{2\pi} Q J_0(bQ) G_{+\frac{1}{2} + \frac{1}{2}}^+(Q^2)$$

combination of **M1**, **E2**, **C2** FFs

data : MAID 2007 , Drechsel, Kamalov, Tiator (2007)

densities : Carlson, Vdh (2007)

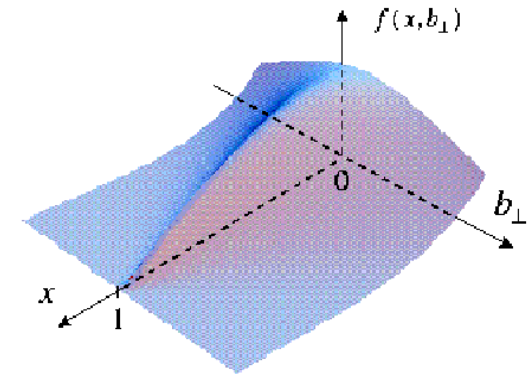
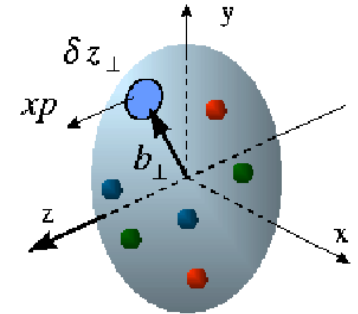
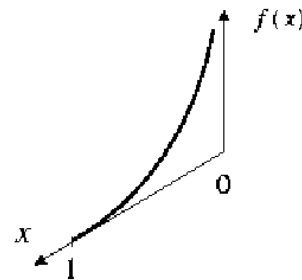
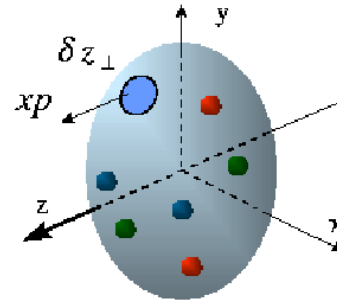
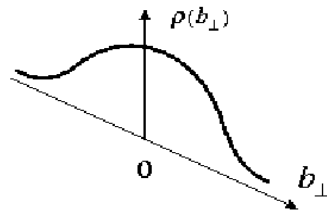
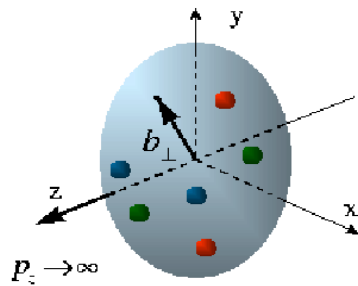
$$\begin{aligned}
\rho_T^{N\Delta}(\vec{b}) &\equiv \int \frac{d^2\vec{q}_\perp}{(2\pi)^2} e^{i\vec{q}_\perp \cdot \vec{b}} \frac{1}{2P^+} \langle P^+, \frac{\vec{q}_\perp}{2}, s_\perp^\Delta = +\frac{1}{2} | J^+(0) | P^+, -\frac{\vec{q}_\perp}{2}, s_\perp^N = +\frac{1}{2} \rangle \\
&= \int_0^\infty \frac{dQ}{2\pi} \frac{Q}{2} \left\{ J_0(bQ) G_{+\frac{1}{2}+\frac{1}{2}}^+ \longrightarrow \text{monopole} \right. \\
&\quad \left. + \sin(\phi_b - \phi_S) J_1(bQ) \left[\sqrt{3} G_{+\frac{3}{2}+\frac{1}{2}}^+ + G_{+\frac{1}{2}-\frac{1}{2}}^+ \right] \longrightarrow \text{dipole} \right. \\
&\quad \left. - \cos 2(\phi_b - \phi_S) J_2(bQ) \sqrt{3} G_{+\frac{3}{2}-\frac{1}{2}}^+ \right\} \longrightarrow \text{quadrupole}
\end{aligned}$$



GPDs yield 3-dim quark structure of nucleon

Burkardt (2000, 2003)

Belitsky, Ji, Yuan (2004)

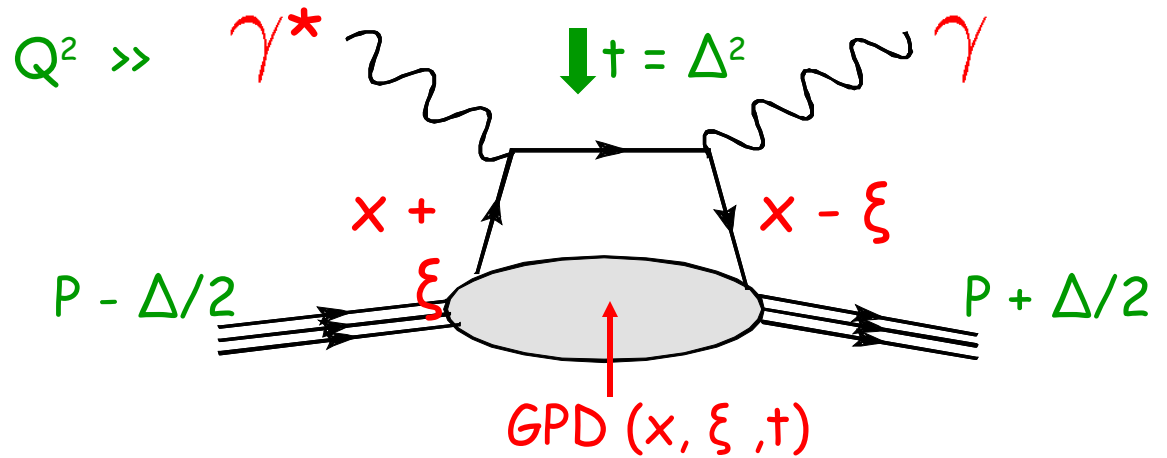


Elastic Scattering
transverse quark
distribution in
coordinate space

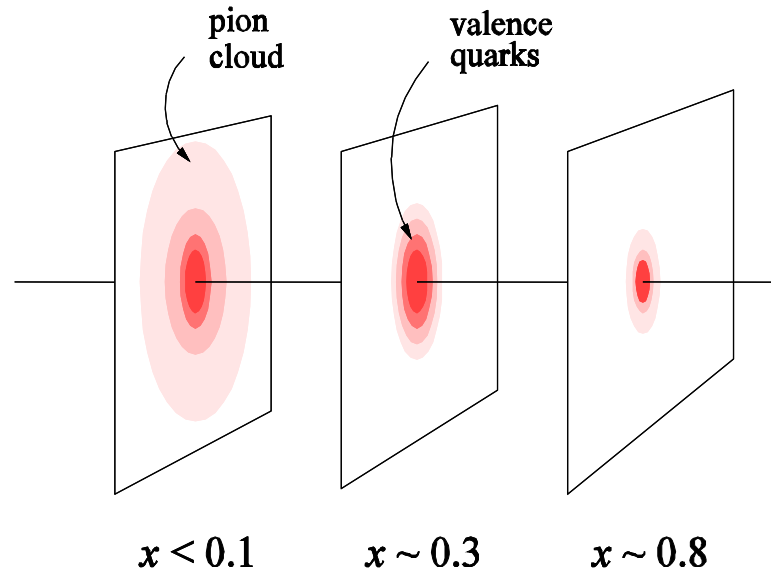
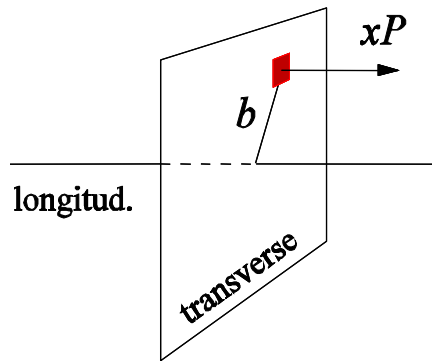
DIS
longitudinal
quark distribution
in momentum space

DES (GPDs)
fully-correlated
quark distribution in
both coordinate and
momentum space

GPDs :



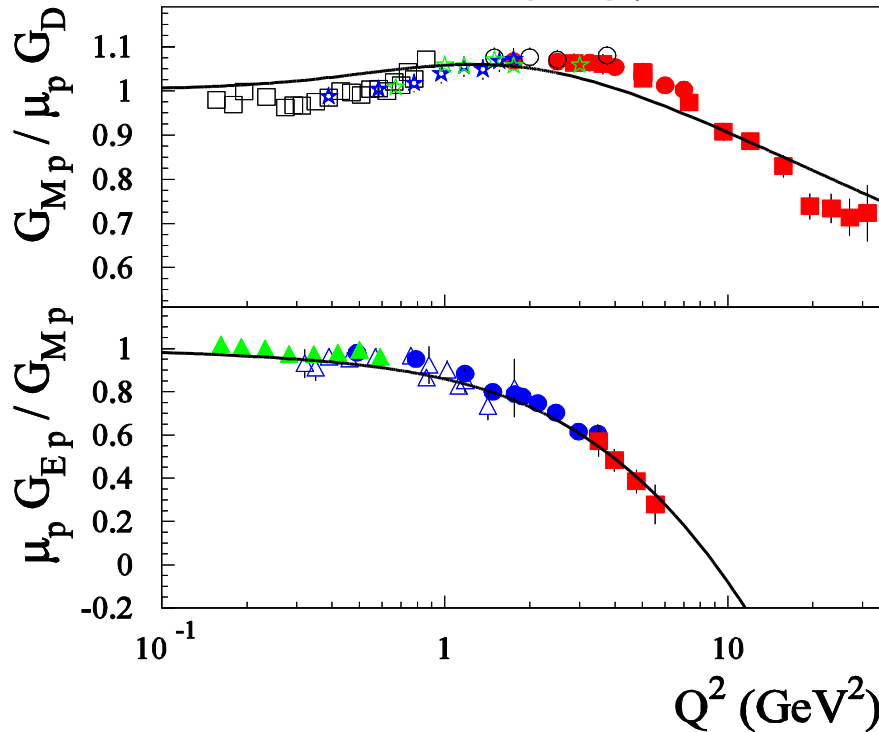
$$\xi = 0$$



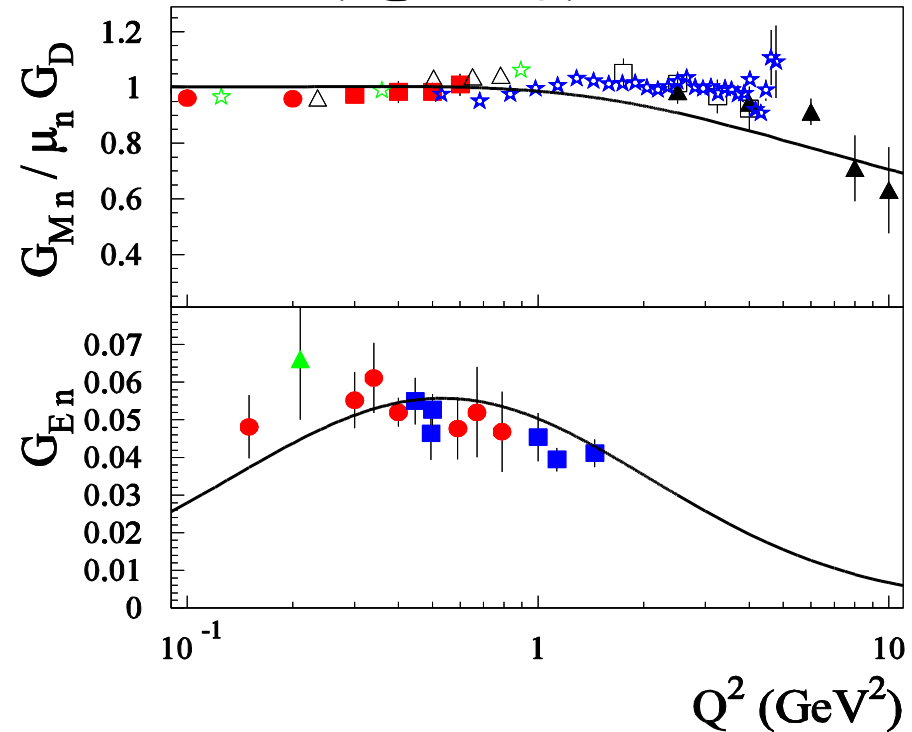
Fourier transform of GPDs : simultaneous distributions of quarks w.r.t. longitudinal momentum xP and transverse position b

electromagnetic form factors

PROTON



NEUTRON



modified Regge GPD parameterization

3-parameter fit $\left\{ \begin{array}{l} 1 : \text{Regge slope} \rightarrow \text{proton Dirac (Pauli) radius} \\ 2, 3 : \text{large } x \text{ behavior of GPD } E^u, E^d \rightarrow \text{large } Q^2 \text{ behavior of } F_{2p}, F_{2n} \end{array} \right.$

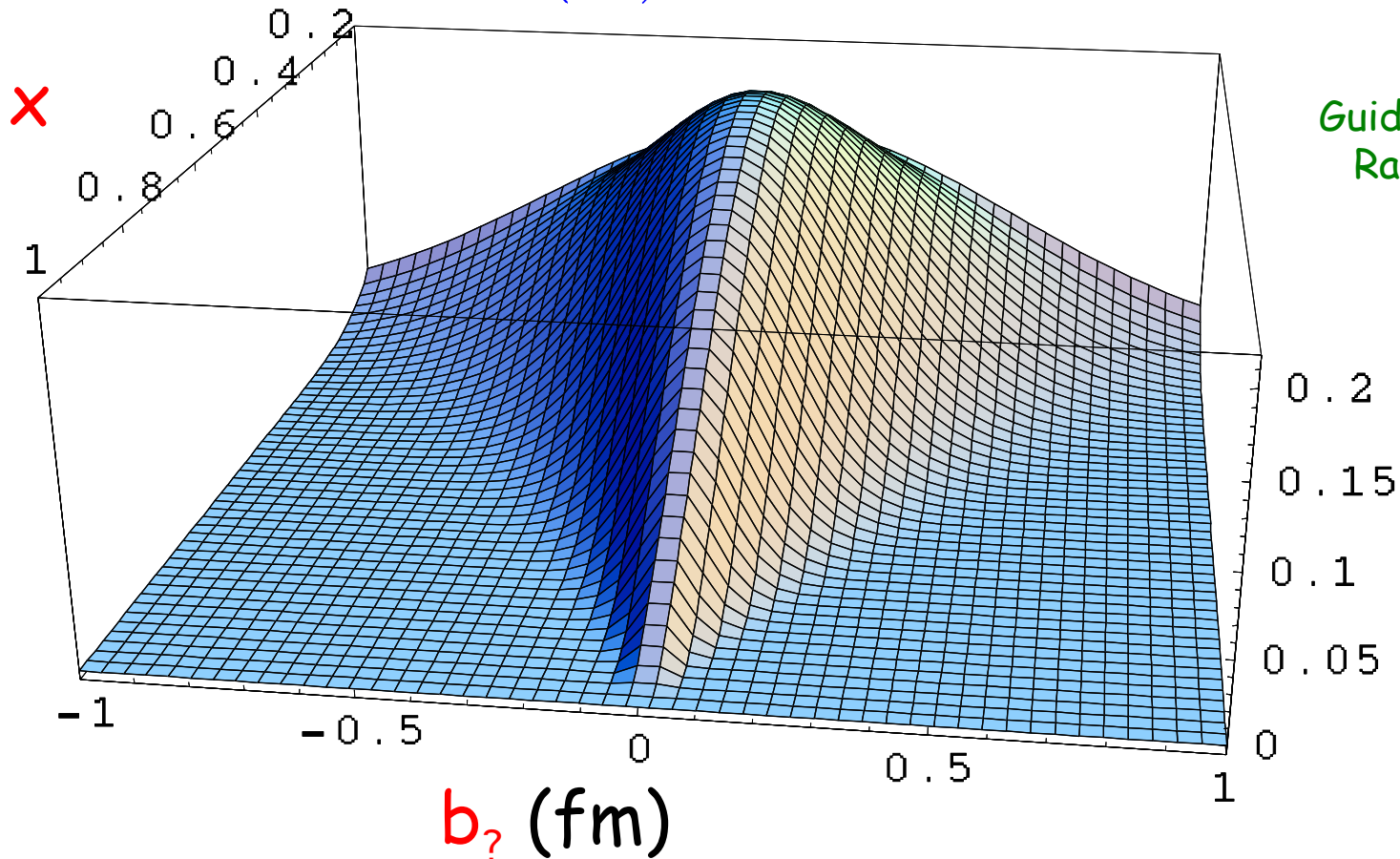
Guidal, Polyakov, Radyushkin, Vdh (2005)

also Diehl, Feldmann, Jakob, Kroll (2005)

world data (2006)

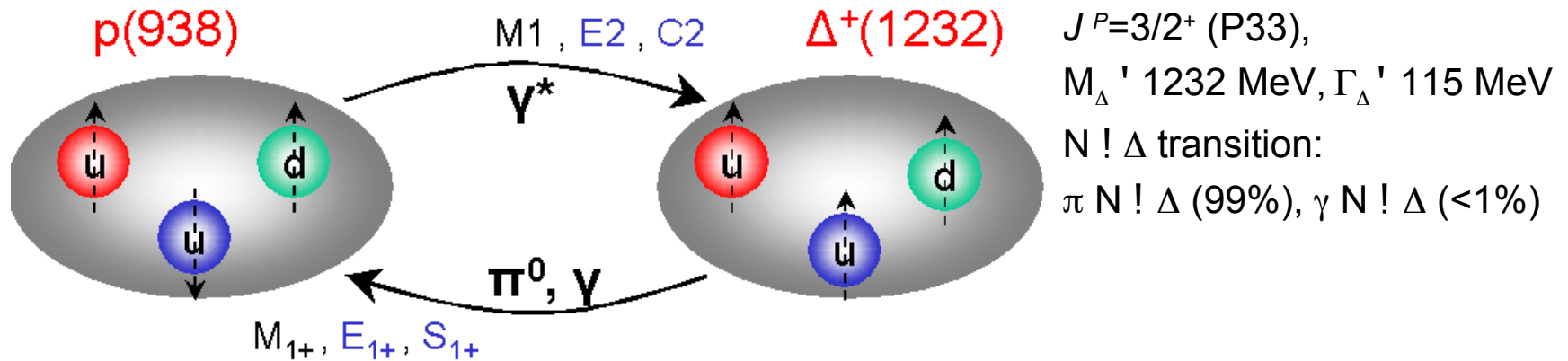
GPDs : transverse image of the nucleon (tomography) $H^u(x, b_\perp)$

$$H^q(x, \mathbf{b}_\perp) = \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{i\mathbf{b}_\perp \cdot \Delta_\perp} H^q(x, \xi = 0, -\Delta_\perp^2)$$

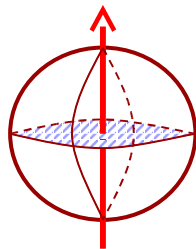


Guidal, Polyakov,
Radyushkin, Vdh
(2005)

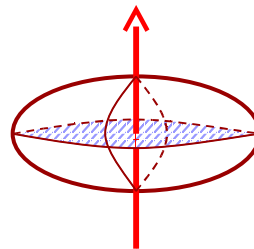
electromagnetic $N \rightarrow \Delta(1232)$ transition



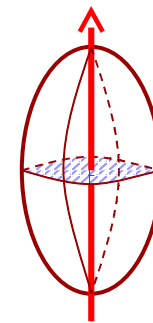
➔ non-zero values for $E2$ and $C2$: measure of non-spherical distribution of charges



Sphere: $Q_{20}=0$



Oblate $Q_{20}/R^2 < 0$



Prolate: $Q_{20}/R^2 > 0$

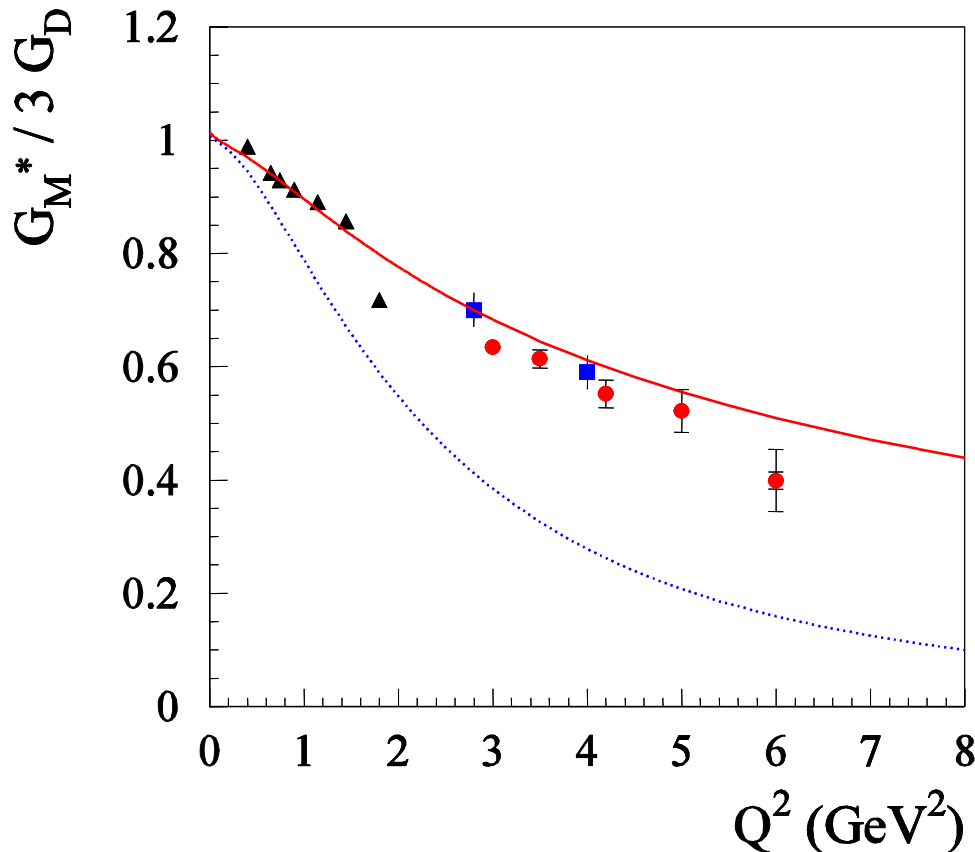
spin 3/2

➔ Role of quark core (quark spin flip) versus pion cloud

N → Δ magnetic dipole form factor

large N_c limit

$$G_M^*(t) = \frac{G_M^*(0)}{\kappa_V} \int_{-1}^{+1} dx \left\{ E^u(x, \xi, t) - E^d(x, \xi, t) \right\} = \frac{G_M^*(0)}{\kappa_V} \left\{ F_2^p(t) - F_2^n(t) \right\}$$



large N_c : $G_M^*(0) = \kappa_V / \sqrt{2} = 2.62$

EXP: $G_M^*(0) = 3.02$

→ modified Regge model

→ Regge model

Guidal, Polyakov, Radyushkin, Vdh (2005)

N → Δ E2 and C2 form factors

→ large N_c limit of QCD :

$$Q_{p \rightarrow \Delta^+} = \frac{1}{\sqrt{2}} r_n^2 \frac{N_c}{N_c + 3} \sqrt{\frac{N_c + 5}{N_c - 1}}$$

Buchmann, Hester, Lebed (2002)

→ $N_c = 3$

EXP : $r_n^2 = -0.113(3) \text{ fm}^2$

$$Q_{p \rightarrow \Delta^+} = \frac{1}{\sqrt{2}} r_n^2$$

large N_c : $Q_{p \rightarrow \Delta^+} = -0.080 \text{ fm}^2$

$$G_E^*(0) = -\frac{1}{6} r_n^2 \frac{1}{\sqrt{2}} \frac{(M_\Delta^2 - M_N^2)}{2}$$

EXP : $Q_{p \rightarrow \Delta^+} = -0.085(3) \text{ fm}^2$

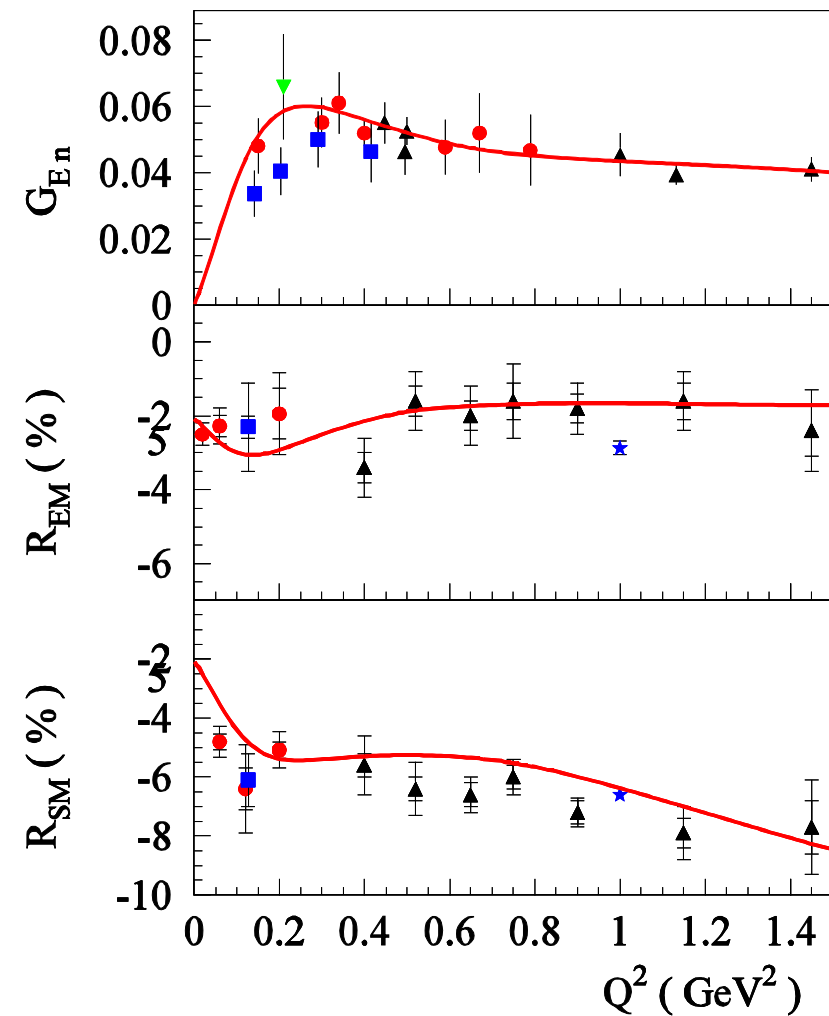
→ finite (low) Q^2 : $G_E^n(Q^2) \approx -r_n^2 Q^2/6$

$$G_E^*(Q^2) \approx \frac{1}{\sqrt{2}} \frac{(M_\Delta^2 - M_N^2)}{2} \frac{G_E^n(Q^2)}{Q^2}$$

$$G_C^*(Q^2) \approx \frac{4M_\Delta^2}{(M_\Delta^2 - M_N^2)} G_E^*(Q^2)$$

Pascalutsa, Vdh (2007)

$N \rightarrow \Delta$ E2 and C2 form factors



$G_{E\Delta}$ fit : Bradford, Bodek, Budd,
Arrington (2006)

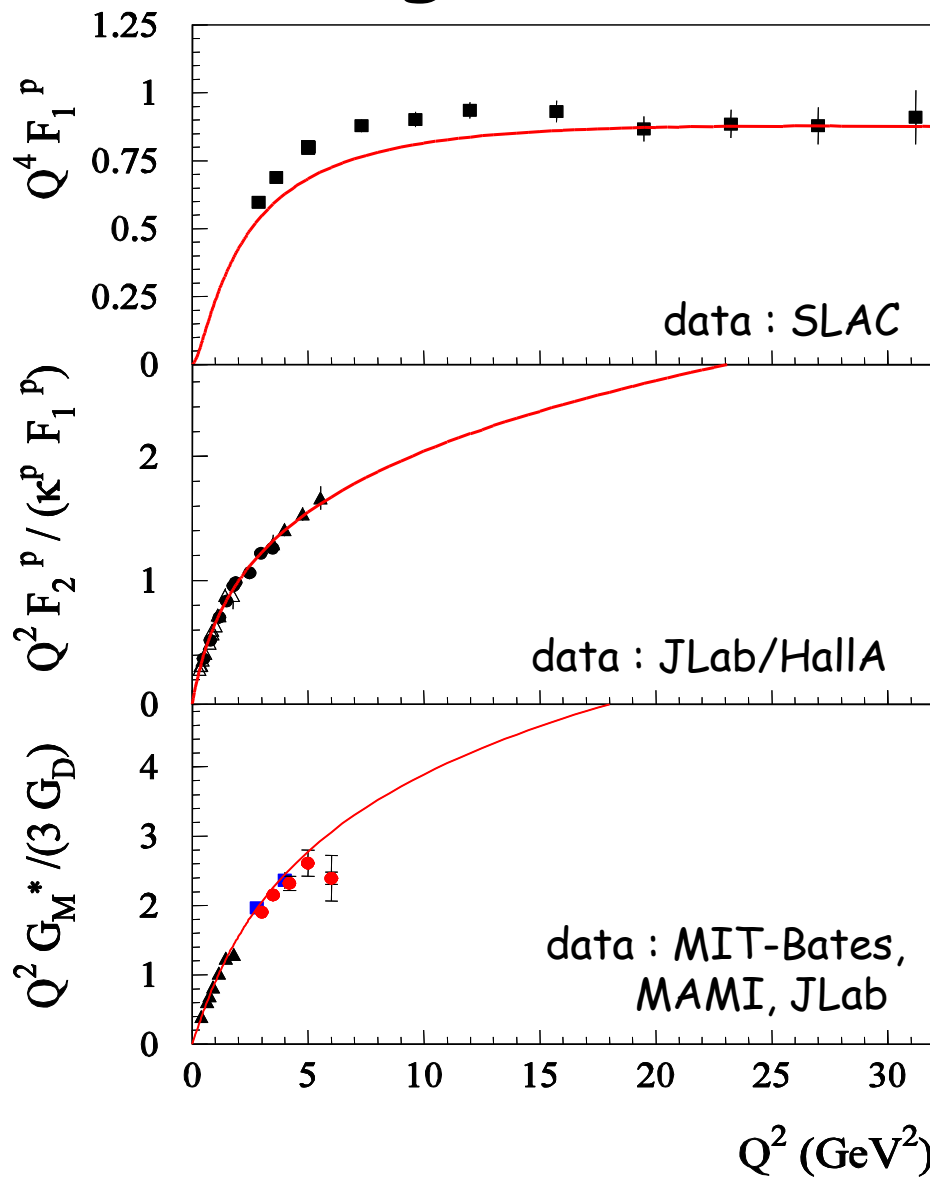
large N_c limit

Pascalutsa, Vdh (2007)

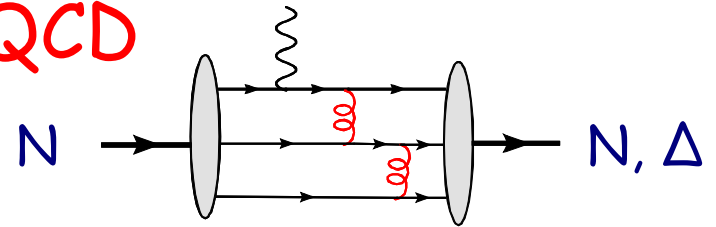
data :

MAMI, BATES, JLab/CLAS, JLab/Hall A

scaling behavior of N and N → Δ F.F.



PQCD



+ collinear quarks

$$F_1^P \sim 1/Q^4$$

$$F_2^P / F_1^P \sim 1/Q^2$$

$$G_M^* \sim 1/Q^4$$

GPD

modified Regge model

Guidal, Polyakov, Radyushkin, Vdh
(2005)

Summary

- for large momentum transfers : relation intrinsic densities and FFs ambiguous
- **transverse densities** (2D) : theoretically well defined relation with **nucleon elastic FFs** :
 F_1 : quark density in nucleon of definite helicity
 F_2 : quark density in transversely polarized nucleon
- transverse densities in the **N \rightarrow Δ** transition :
monopole, dipole, quadrupole field pattern
- **distance** and **GPDs** : correlate information on **transverse longitudinal quark momentum** dependencies
- **Large N_c relations** between **N** elastic and **N \rightarrow Δ** transition FFs