Hadronic Physics 08 — Triest, May 2008

Jet correlations from unintegrated parton distributions

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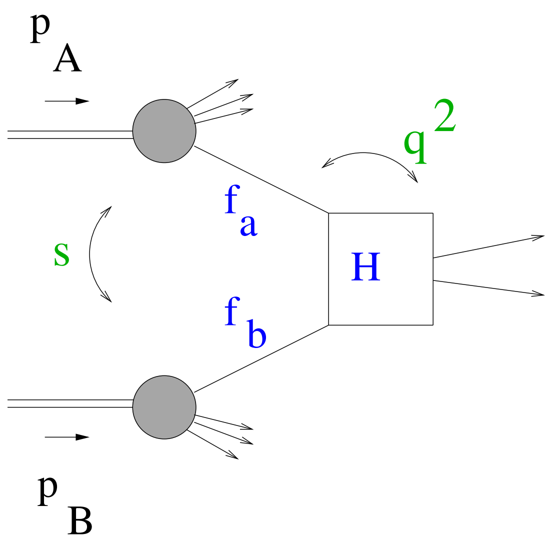
I. Motivation: multiparticle production in high-energy hadron collisions

II. General issues on unintegrated parton distributions

III. Multi-jet correlations in small-x final states

I. INTRODUCTION

Multi-scale hard processes at high-energy hadron colliders



$$\sigma = \int f_{a/A} \otimes f_{b/B} \otimes H_{ab}$$

phase space opening up for large $\sqrt{s}$

$$\Downarrow$$

- large number of events with multiple hard scales: $q_1^2, \cdots, q_n^2$
  - potentially large corrections to all orders in α_s, $\sim \ln^k(q_i^2/q_j^2)$
- parton distributions probed near kinematic boundaries $x \to 0$, $1 - x \to 0$

▷ Part of the effects are universal

$\hookrightarrow$ ex.: high-order corrections in renormalization group evolution

$$\mu \frac{d}{d\mu} f = \gamma \otimes f$$

$$\gamma \simeq \gamma^{(LO)} \left(1 + c_1 \alpha_s + \ldots + c_{n+m} \alpha_s^m (\alpha_s \ L)^n + \ldots\right) \ , \ L = \text{“large log”}$$

▷ Part of them are not universal (final-state correlations, exclusive variables, ...)

$\hookrightarrow$ yet can be summed by techniques that generalize RG factorization

- pdf's unintegrated in both $||$ and $\perp$ components

Examples:

- Sudakov processes
- small-x physics
- reconstruction of fully exclusive final states by Monte-Carlos

Multi-jet final states:

- perturbative calculations at fixed order (NLO, multi-leg)
- parton-shower event simulation (higher orders, hadronization)

Multiple hard scales $\Longrightarrow$ Finite transverse-momentum tail in initial-state radiative processes

important for $x \rightarrow 0$ (high-energy region)

$x \rightarrow 1$ (Sudakov region)

$\downarrow$

- included partially, order-by-order, in perturbative calculations (higher loops)
- amounts to corrections to the angular ordering in standard parton showers (HERWIG, PYTHIA)

$\diamondsuit$ inclusive jet cross sections: NLO (+ standard shower) probably sufficient

$\diamondsuit$ correlations, exclusive final-state structure?

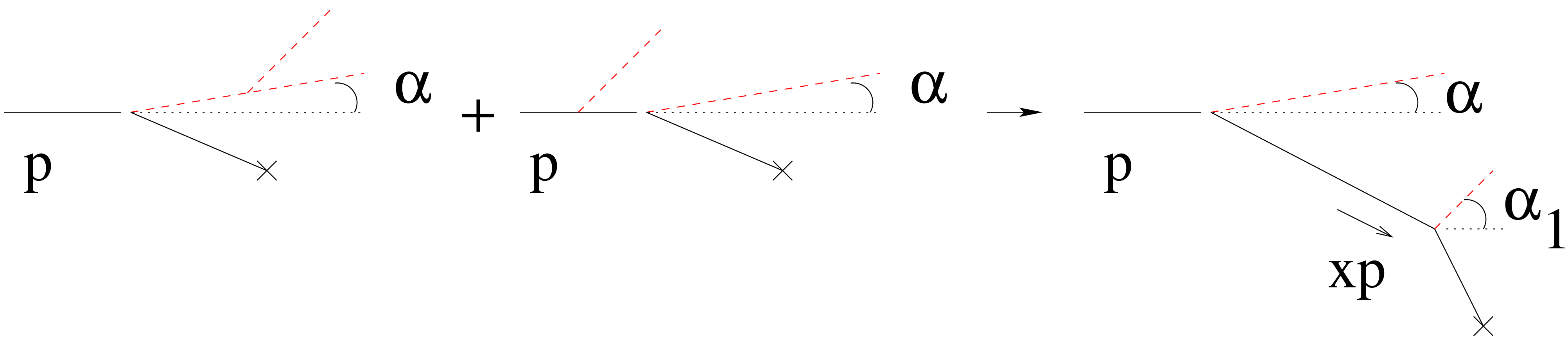
◇ Multiple QCD radiation → parton shower:

▷ HERWIG: ordering in decay angles in space-like shower

($\approx$ $k_\perp$-ordering for $x \ll 1$)

• collinear (incoherent) emission

• soft gluon coherence ($x \sim 1$)



▷ Coherent radiation at $x \ll 1$ → corrections to angular ordering:

MC based on $k_\perp$-dependent unintegrated pdfs and MEs

Outline

i) How to characterize u-pdf's with precision?

$\triangleright$ gauge-invariant operator matrix elements

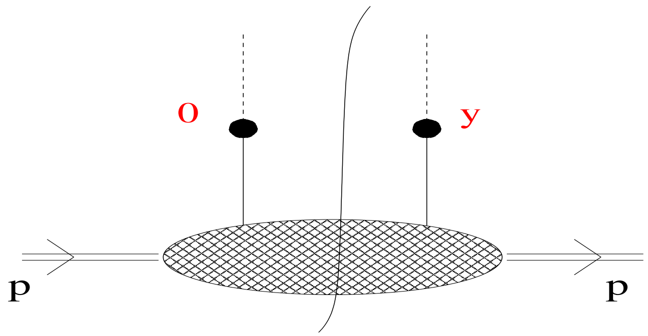
$\triangleright$ lightcone divergences and regularization methods

ii) Applications to x $\ll 1$ parton showers and jets

$\triangleright$ effects of initial-state radiation on multi-jet angular correlations

II. General issues on u-pdf's

Example 1: Ordinary (integrated) pdf



$p = (p^+, m^2/2p^+, 0_\perp)$

$$f_q(x, \mu) = \int \frac{dy^-}{2\pi} \; e^{-ixp^+y^-} \; \widetilde{f}(y)$$

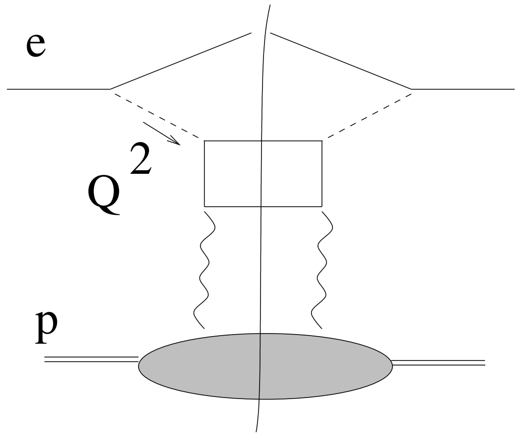
correlation of quark fields at lightlike separation $y = (0, y^-, 0_\perp)$:

$$\widetilde{f}(y) = \langle \; P \mid \overline{\psi}(y) \; V_y^\dagger(n) \; \gamma^+ \; V_0(n) \; \psi(0) \mid P \; \rangle \quad ,$$

$V_y(n) = \mathcal{P} \exp\left(ig_s \int_0^\infty d\tau \; n \cdot A(y + \tau \; n) \right)$ eikonal line in direction $n = (0, 1, 0_\perp)$

- μ-dependence from renormalization of operator product
  - gauge-invariant Wilson line matrix elements

Example 2: Unintegrated pdf from physical cross section (high energy limit). E.g.:



♢ single gluon polarization dominates $s \gg M^2 \gg \Lambda^2_{\rm QCD}$

$\hookrightarrow$ gauge invariance rescued (despite gluon off-shell)

♢ but to define u-pdf gauge-invariantly over the whole phase space

is more difficult

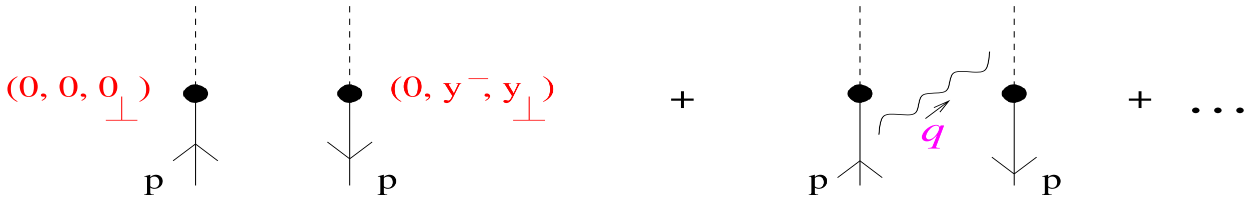
Example 3: Generalize matrix element to non-lightlike distances

$$\widetilde{f}(y) = \langle \, P \mid \overline{\psi}(y) \; V_y^\dagger(n) \; \gamma^+ \; V_0(n) \; \psi(0) \mid P \, \rangle \quad , \qquad\qquad y = (0, y^-, y_\perp)$$

- works at tree level [Mulders, 2002; Belitsky et al., 2003; Collins, 2004]
- subtler at level of radiative corrections $\hookrightarrow$

◇ Suppose a gluon is absorbed or emitted by eikonal line:

$n = (0,\ 1,\ 0_\perp)$



$$f_{(1)} = P_R(x,k_\perp) - \delta(1-x)\,\delta(k_\perp)\int dx'\,dk'_\perp\,P_R(x',k'_\perp)$$

$$\text{where} \qquad P_R = \frac{\alpha_s\,C_F}{\pi^2}\left[\underbrace{\frac{1}{1-x}}_{endpoint\ \ singularity}\ \frac{1}{k_\perp^2+\rho^2} + \{\text{regular at } x \to 1\}\right]$$

ρ=IR regulator

$(q^+ \longrightarrow 0,\ \forall\ k_\perp)$

◇ Physical observables:

$$\mathcal{O} = \int dx\ dk_\perp\ f_{(1)}(x,k_\perp)\ \varphi(x,k_\perp)$$

$$= \int dx\ dk_\perp\ [\varphi(x,k_\perp) - \varphi(1,0_\perp)]\,P_R(x,k_\perp)$$

inclusive case: φ independent of $k_\perp \Rightarrow 1/(1-x)_+$ from real + virtual

general case: endpoint divergences (incomplete KLN cancellation)

• Distributions at fixed $k_\perp$ are no longer protected by KLN mechanism against uncancelled lightcone divergences

• Only after supplying matrix element with a regularization prescription is distribution well defined.

• Note: regularization of endpoint divergences also affects distributions integrated over $k_\perp$ and UV subtractions

[H, hep-ph/0702196]

$$\text{Ex. :} \quad \int dk_\perp \ f(x, k_\perp, \mu) \ \Theta(\mu - k_\perp) \ \stackrel{?}{=} \ f^{\overline{\text{MS}}}(x, \mu)$$

$=$ holds only at tree level: full relation involves coefficient function R

$$\int^\mu dk_\perp \ f(x, k_\perp, \mu) \ = \ R(x) \ \otimes \ f^{\overline{\text{MS}}}(x, \mu)$$

♢ R calculable as a power series in α_s, $R(x) = \delta(1-x) + \sum_k \ r_k \ \alpha_s^k$

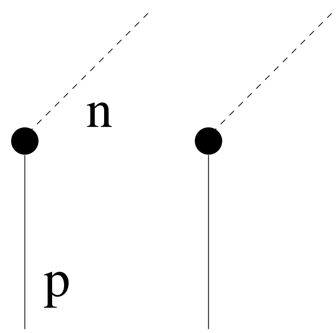
• Applications: <u>Cut-off regularization</u> vs. <u>Subtractive regularization</u>

CUT-OFF REGULARIZATION

▷ cut-off in Monte-Carlo generators using u-pdf's

CASCADE	www.quark.lu.se/~hannes/cascade
SMALLX	Marchesini & Webber, 90's
LDCMC	www.thep.lu.se/~leif/ariadne

▷ cut-off from gauge link in non-lightlike direction n:



$$\eta = (p \cdot n)^2 / n^2$$

Collins, Rogers & Stasto, arXiv:0708.2833

Ji, Ma & Yuan, 2005, 2006

earlier work from 80's and 90's

finite $\eta \Rightarrow$ singularity is cut off at $1 - x \gtrsim \sqrt{k_\perp / 4\eta}$

- Note: lightcone limits $y^2 \rightarrow 0$ and $n^2 \rightarrow 0$ do not commute $\Rightarrow$

$$\Rightarrow \int dk_\perp \; f(x, k_\perp, \mu, \eta) = F(x, \mu, \eta) \neq \text{ ordinary pdf}$$

UPDF's WITH SUBTRACTIVE REGULARIZATION

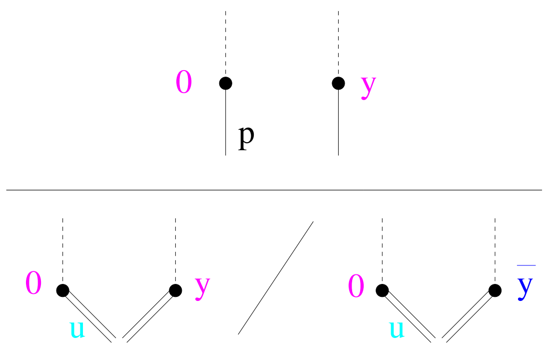
- Endpoint divergences $x \to 1$ from incomplete KLN cancellation

Subtractive method: more systematic than cut-off. Widely used in NLO calculations.

Formulation suitable for eikonal-operator matrix elements: Collins & H, 2001.

- gauge link still evaluated at n lightlike, but multiplied by "subtraction factors"

$$\widetilde{f}^{(\mathrm{subtr})}(y^-, y_\perp) = \frac{\overbrace{\langle P|\overline{\psi}(y)V_y^\dagger(n)\gamma^+ V_0(n)\psi(0)|P\rangle}^{\text{original matrix element}}}{\underbrace{\langle 0|V_y(u)V_y^\dagger(n)V_0(n)V_0^\dagger(u)|0\rangle \;/\; \langle 0|V_{\bar{y}}(u){V_{\bar{y}}}^\dagger(n)V_0(n)V_0^\dagger(u)|0\rangle}_{\text{counterterms}}}$$



$\bar{y} = (0, y^-, 0_\perp)$; $u =$ auxiliary non-lightlike eikonal $(u^+, u^-, 0_\perp)$

H, arXiv:0708.1319

$\diamondsuit$ u serves to regularize the endpoint; drops out of distribution integrated over $k_\perp$

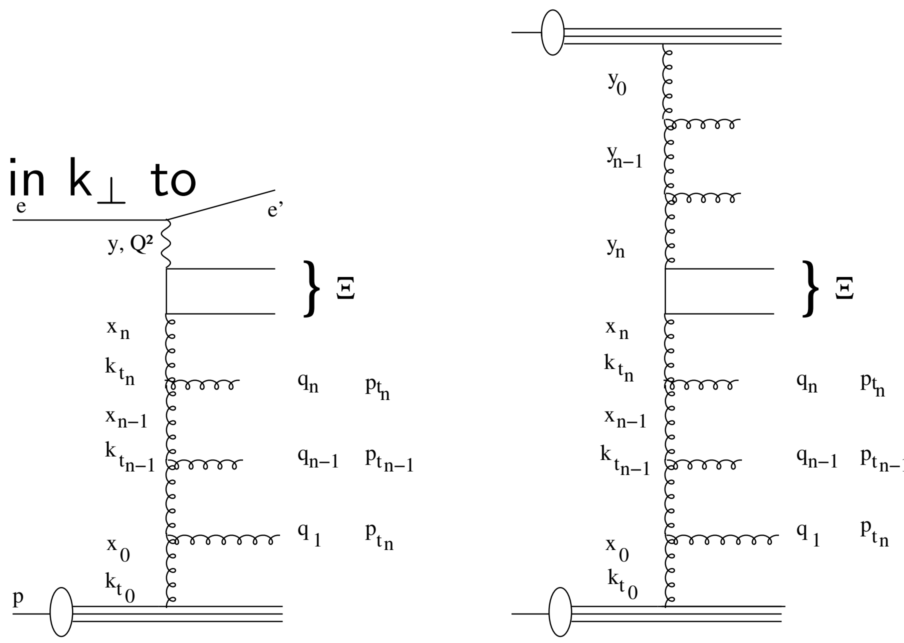
III. Jet correlations in small-x final states

◇ All MC's based on u-pdf's rely on factorization in $k_\perp$ to

a) generate hard-scattering event (hard ME)

b) couple it to initial-state shower

◇ differ by detailed model for initial state



(a) (b)

• all implement correct $\alpha_s^n x^{-1} \ln^{n-1} x$ behavior for spacelike evolution at $x \ll 1$ to all orders in α_s

$$\exp \int (dk^2/k^2) \gamma(\alpha_s(k^2))$$

• resum non-universal $\alpha_s^k \ln^k (s/p_\perp^2)$ (in certain cases)

• subleading contributions possibly important for final states

Implementations:

Höche, Krauss and Teubner, arXiv:0705.4577 (BFKL)

Golec, Jadach, Placzek, Stephens, Skrzypek, hep-ph/0703317 (CCFM)

LDCMC Lönnblad & Sjödahl, 2005; Gustafson, Lönnblad & Miu, 2002 (LDC)

CASCADE Jung, 2004, 2002; Jung and Salam, 2001 (CCFM)

SMALLX Marchesini & Webber, 1992 (CCFM)

Advantages over standard Monte-Carlo like PYTHIA or HERWIG:

- better treatment of high-energy logarithmic effects
- likely more suitable for simulating underlying event's $k_\perp$

Current limitations:

- radiative terms associated to $x \sim 1$ not automatically included
- procedure to correct for this not yet systematic

  $\hookrightarrow$ e.g.: LO-DGLAP in Höche et al

- quark contributions in initial state included partially

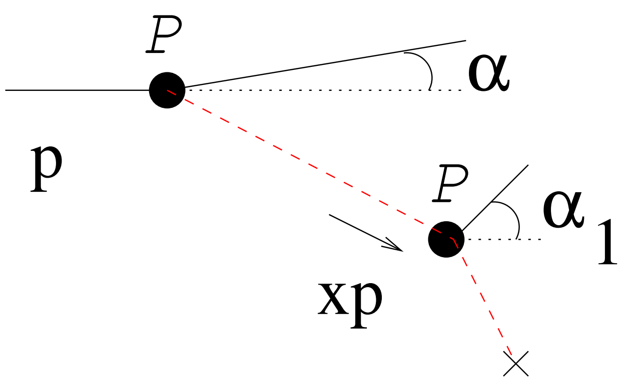
  $\hookrightarrow$ see also: $k_\perp$ kernel for sea-quark evolution [Catani & H]

- limited knowledge of u-pdf's [Jung et al., arXiv:0706.3793; J. R. Andersen et al., 2006]

Basic ingredients in the CASCADE Monte-Carlo

- ME by perturbative computation
- branching eq. : $\mathcal{A}(x,k_T,\mu) \; = \; \mathcal{A}_0(x,k_T,\mu) + \int \frac{dz}{z} \int \frac{dq^2}{q^2}\, \Theta(\mu - zq)$

$$\times \quad \Delta(\mu, zq)\; \mathcal{P}(z,q,k_T)\; \mathcal{A}(\frac{x}{z}, k_T + (1-z)q, q)$$



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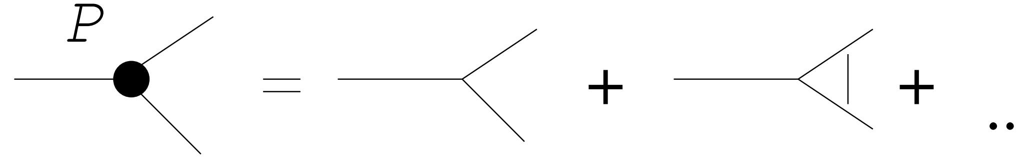


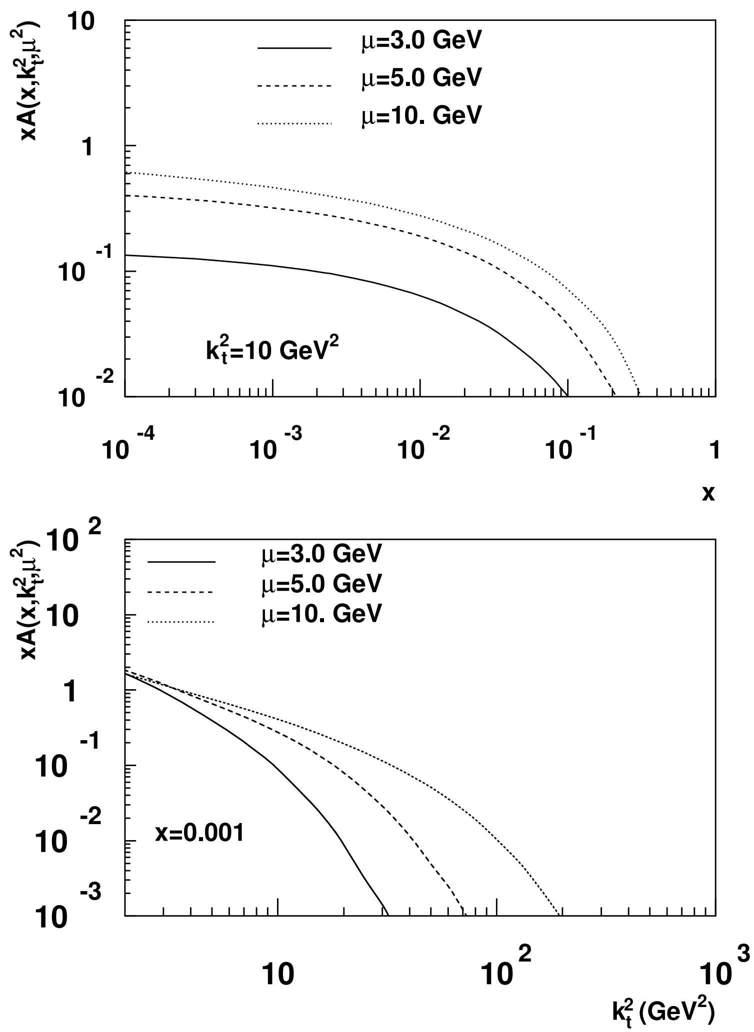
(left) Coherent radiation in the space-like parton shower for $x \ll 1$;
(right) the unintegrated splitting function $\mathcal{P}$, including small-x virtual corrections.

$$\alpha/x > \alpha_1 > \alpha \quad (\mathrm{small - x\;\; coherence\;\; region})$$

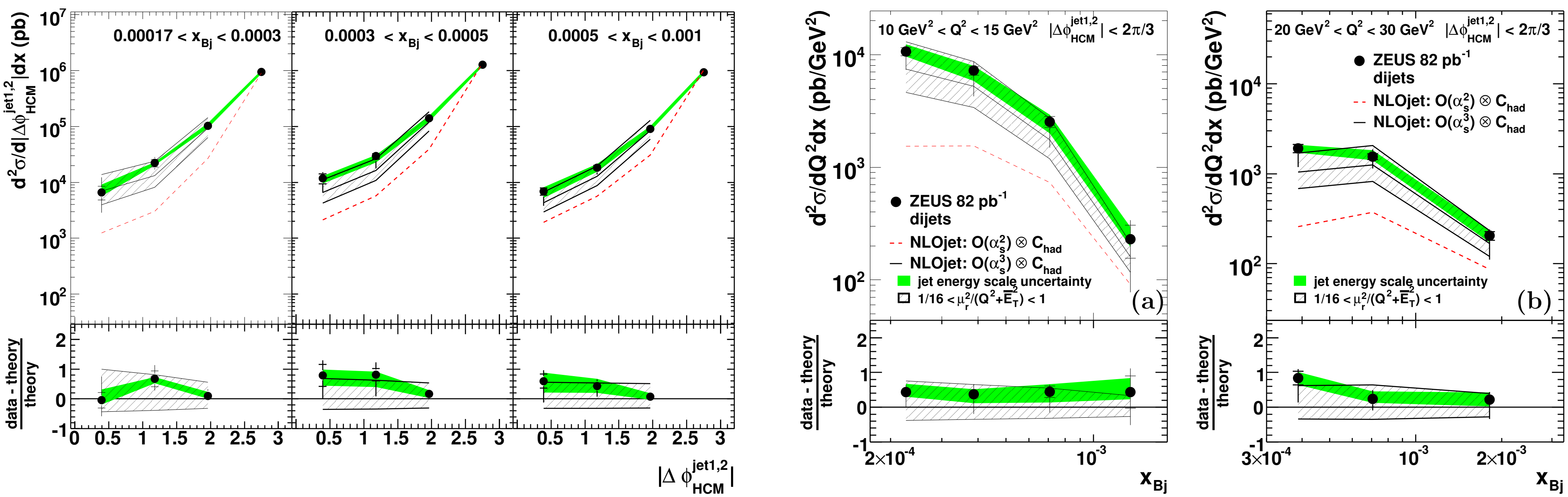
U-pdf fits $\oplus$ evolution

[$\hookrightarrow$ DIS, jets, heavy flavors]

unintegrated gluon distribution as a function of x , $k_\perp$, μ [Hansson & Jung, 2007]



MULTI-JET DIS PRODUCTION AND NLO RESULTS



(left) Azimuth dependence and (right) Bjorken-x dependence of di-jet distributions

- measured by ZEUS [arXiv:0705.1931]
- compared with NLO results [Z. Nagy and Z. Trocsanyi, PRL 87 (2001)]

◇ large variation from order-α_s^2 to order-α_s^3 prediction as $\Delta\phi$ and x decrease

$\Rightarrow$ sizeable theory uncertainty at NLO (underestimated by "μ error band")

- Jet clustering and hadronization:

  ▷ moderate hadronization corrections from jet algorithm used by Zeus and H1
  [arXiv:0705.1931 [hep-ex]; hep-ex/0310019]

  ▷ jet clustering free of non-global logarithms
  [Dasgupta et al., hep-ph/0610242]

  ▷ asymmetric jet cuts to avoid double logs in minimum p_T
  [Banfi and Dasgupta, hep-ph/0312108]

  ▷ nonperturbative corrections in inverse powers of Q moderate for $Q^2 > 10$GeV^2

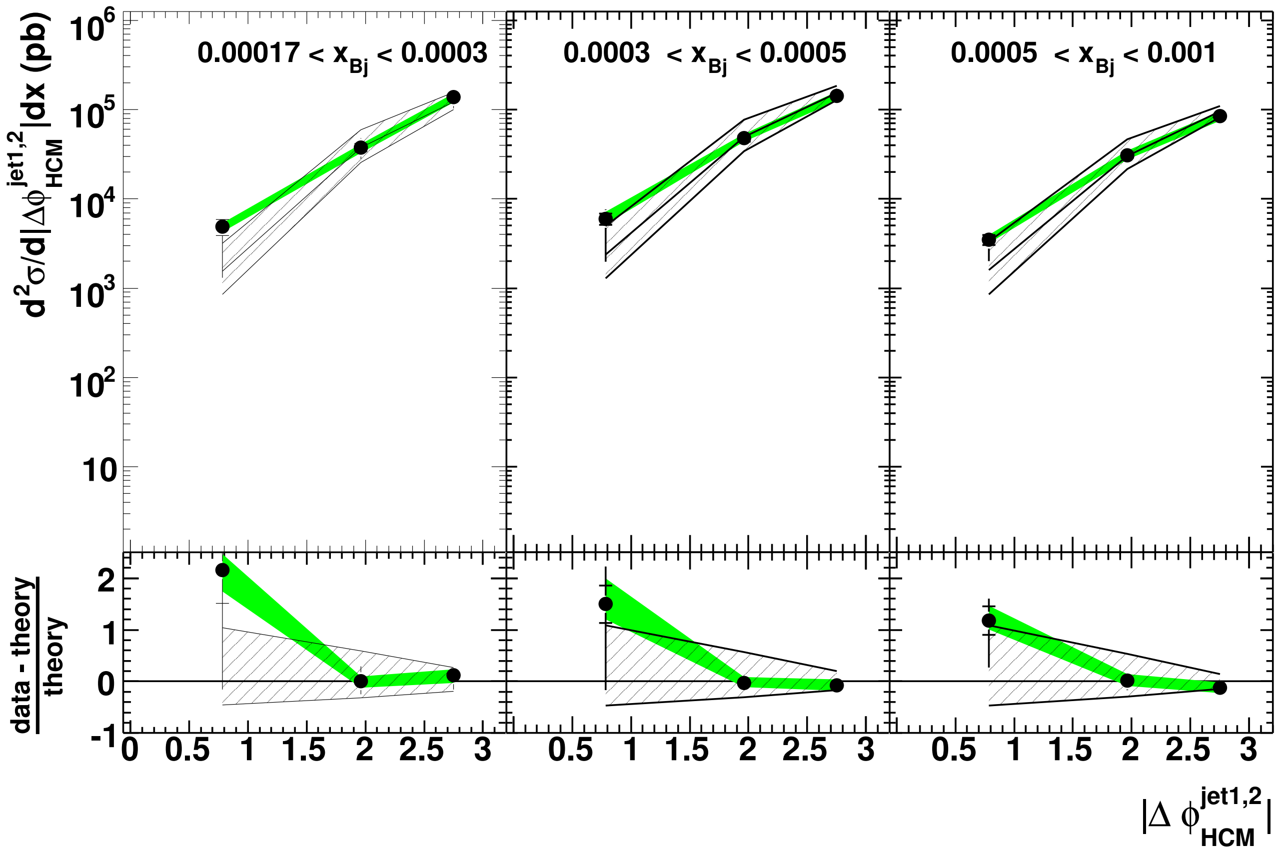
- Radiative effects at higher order:

  ♢ fixed-order beyond NLO is outside present reach for multi-jets in ep and pp

  ♢ enhanced (soft/collinear) higher orders from near back-to-back region
  Y.Delenda et al., arXiv:0706.2172; arXiv:0804.3786; HERWIG

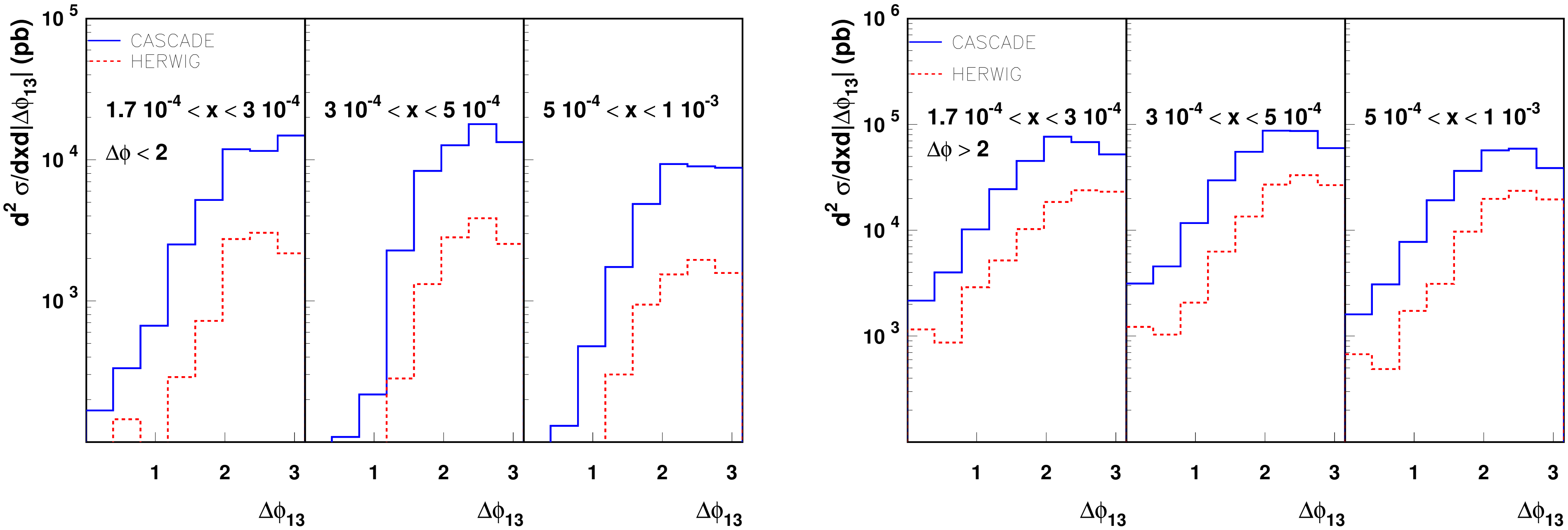
  ♢ largest effects seen at small $\Delta\phi$ (3 well-separated hard jets)

azimuthal distribution in 3-jet cross section [Zeus, 2007]



- besides angular correlations, sizeable NLO uncertainties in other associated distributions
  - NLO results much more stable for inclusive jet cross sections

AZIMUTHAL DISTRIBUTION OF THE THIRD JET



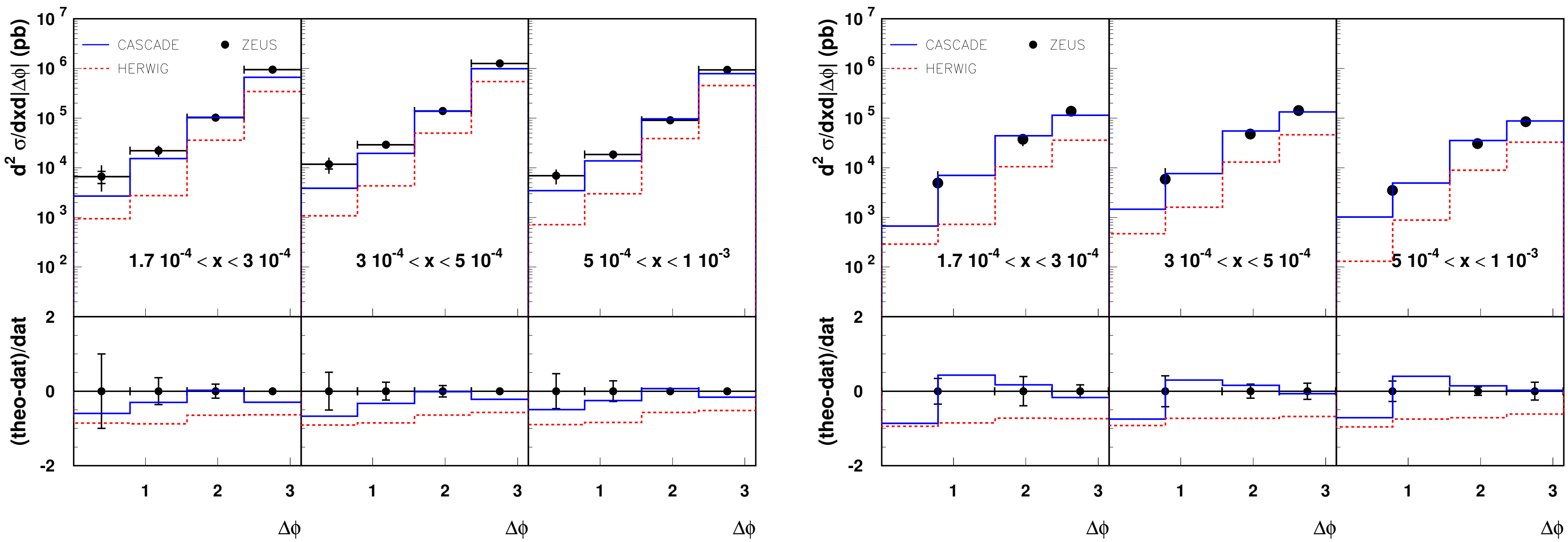
Cross section in the azimuthal angle between the hardest and the third jet

for small (left) and large (right) azimuthal separations between the leading jets

Jung & H, arXiv:0712.0568 [hep-ph]

- small $\Delta\phi \Rightarrow$ non-negligible initial $k_\perp \Rightarrow$ larger corrections to collinear ordering
- curves become closer at large $\Delta\phi$

Angular jet correlations from CASCADE and HERWIG compared with DIS data

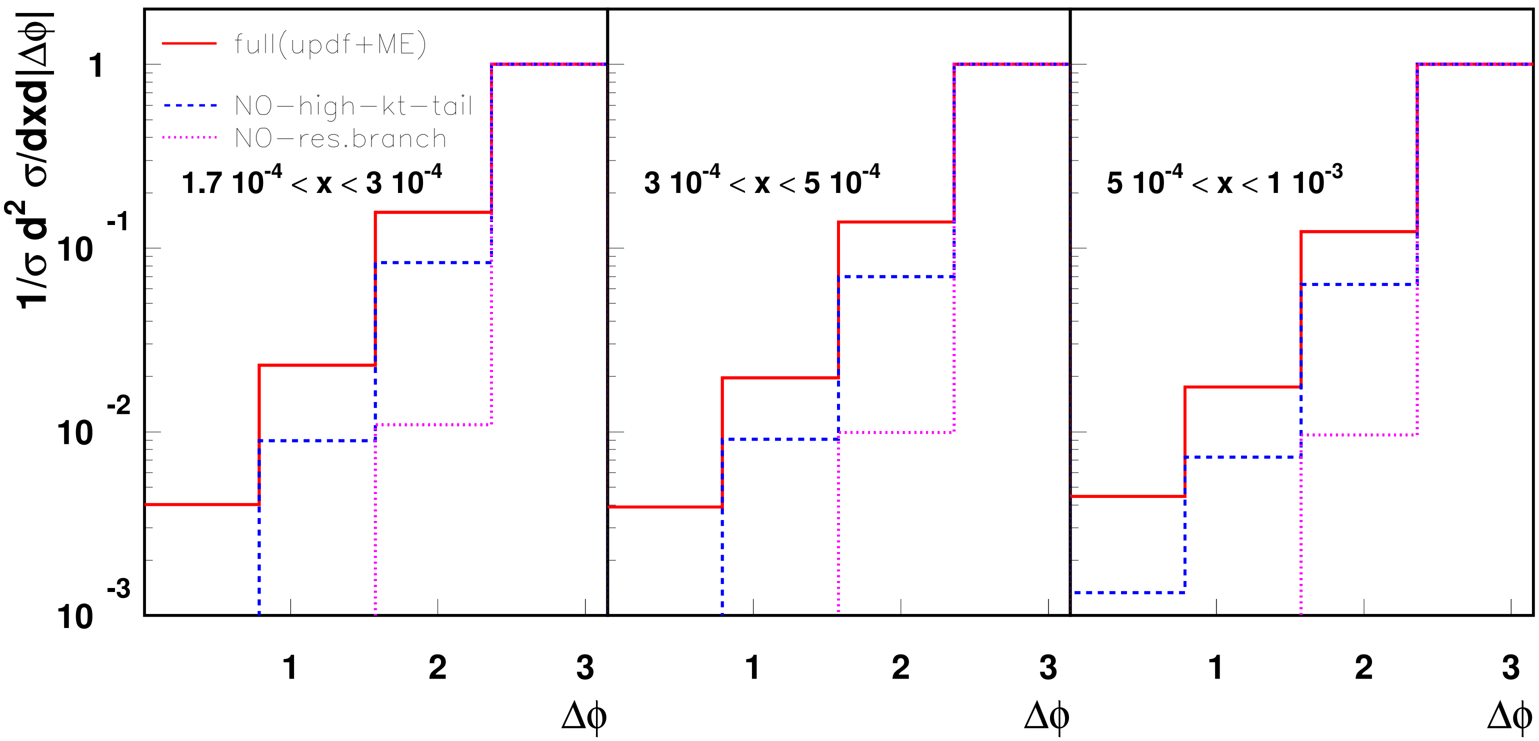


(left) di-jet cross section; (right) three-jet cross section

Jung & H, arXiv:0712.0568 [hep-ph]

- different shapes from the two MC
- largest differences at small $\Delta\phi$
- good description of measurement by CASCADE

Normalize to the back-to-back cross section:



—— updf $\oplus$ ME

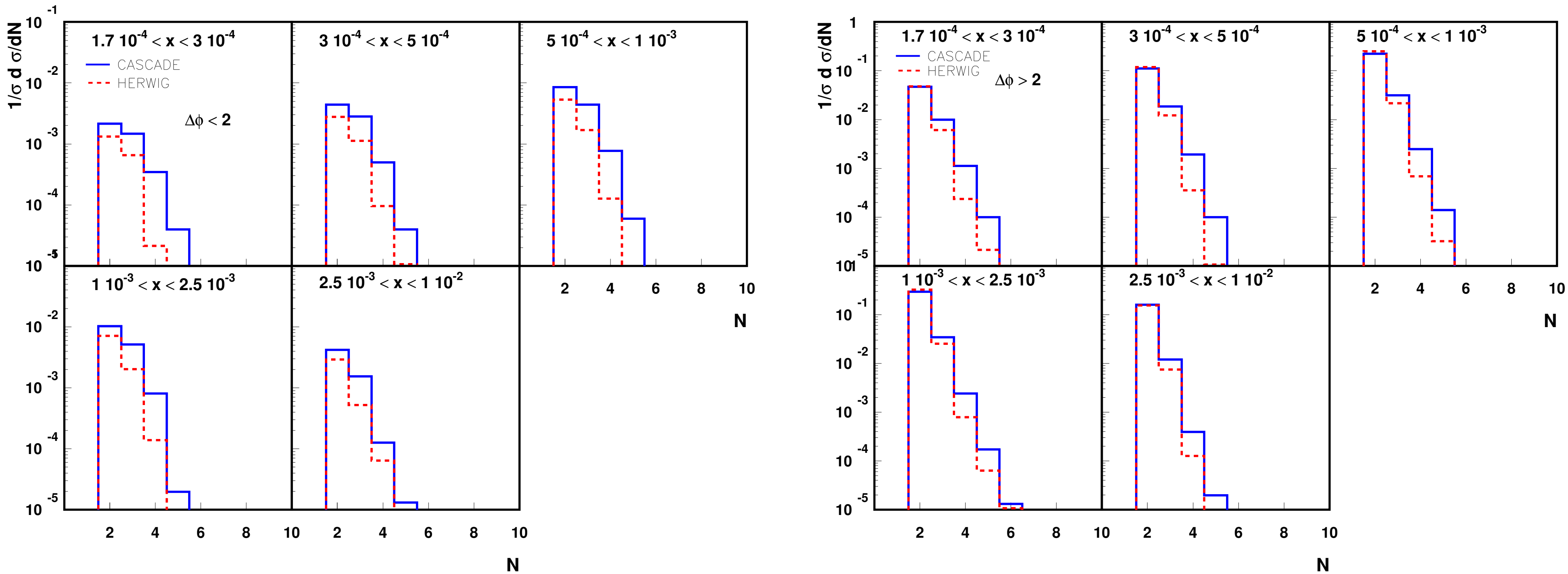
- - - updf $\oplus$ $\text{ME}_{collin.}$: $\mathcal{M} \rightarrow \mathcal{M}_{collin.}(k_T) = \mathcal{M}(0_\perp)\ \Theta(\mu - k_T)$

······ no resolved branching : $\mathcal{A} \rightarrow \mathcal{A}_{no-res.}(x, k_T, \mu) = \mathcal{A}_0(x, k_T, Q_0)\ \Delta(\mu, Q_0)$

▷ high-$k_\perp$ component in ME essential to describe correlation at small $\Delta\phi$

▷ $k_\perp$-dependence in u-pdf alone not sufficient

JET MULTIPLICITIES

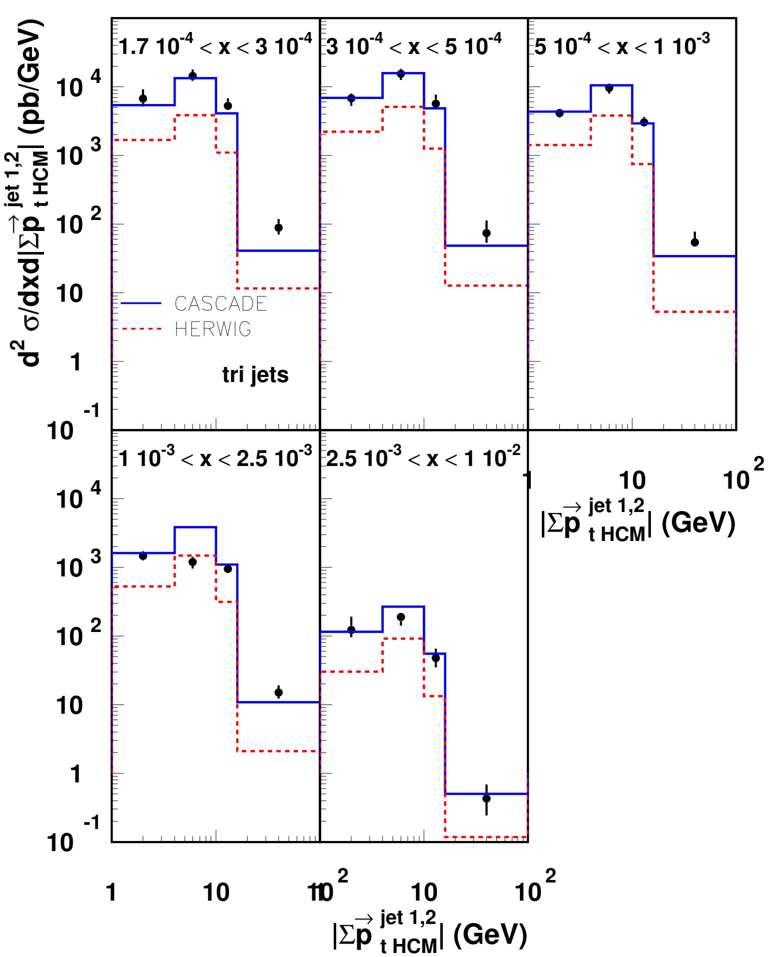


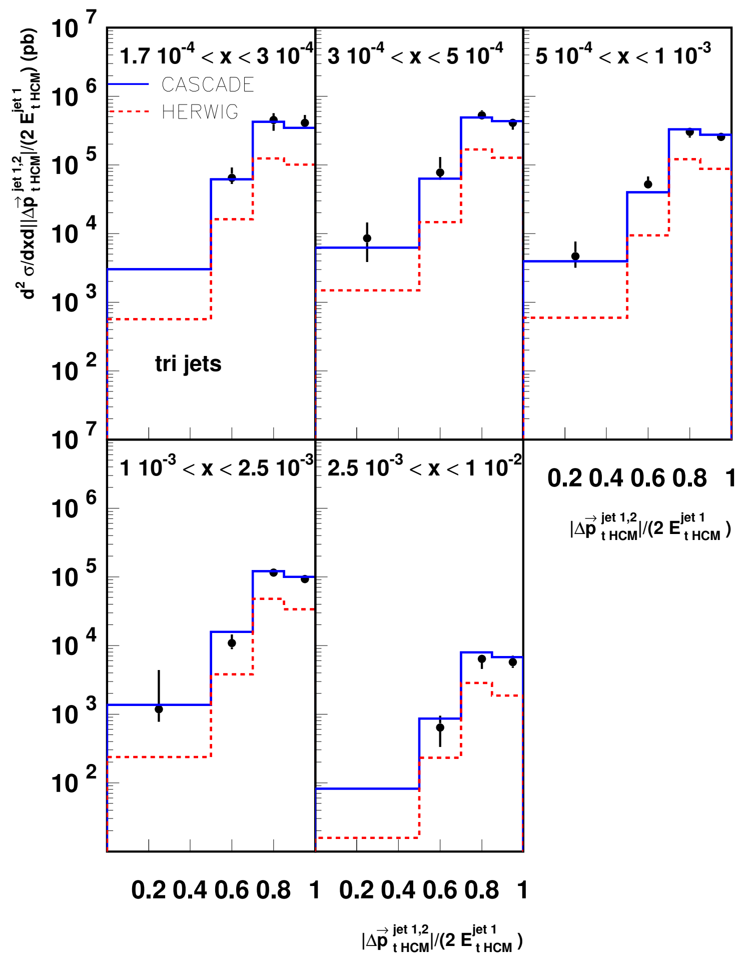
(left) $\Delta\phi < 2$; (right) $\Delta\phi > 2$

[Jung & H, arXiv:0805.1049]

- larger contribution from high multiplicity in the MC with u-pdf

MOMENTUM CORRELATIONS [Jung & H, arXiv:0805.1049]





- correlations in the transverse momentum imbalance between the leading jets

Summary on DIS 3-jet

▷ U-pdfs $\oplus$ $k_\perp$-dependent hard MEs describe

multi-jet measurements including correlations.

▷ Physical picture: • growth of $k_\perp$ along spacelike jet

• finite-angle emission corrections

▷ Furthermore:

- Results similar to HERWIG if reduced to $k_\perp$-ordered phase space

- Similar to fixed NLO where corrections are not large

▷ Non-forward jets $\Rightarrow$ results less dependent on details of u-pdf evolution models

IV. Further developments and conclusions

$\diamondsuit$ Forward-region observables

- More pronounced dependence on evolution model

- better understanding of u-pdf's needed in target fragmentation region

[Trentadue et al, 2007]

$\diamondsuit$ Gluon-gluon fusion processes:

- production of b, c

large NLO uncertainties at LHC energies

[Nason et al. 2004]

- final states with Higgs

10 $\div$ 20 $\%$ effects in p_t spectrum from $x \ll 1$ terms

[Kulesza, Sterman & Vogelsang, 2004]

ISSUES AT HIGHER ORDER

- u-pdf defined gauge-invariantly for small x by high-energy factorization
  - general definitions including x ~ 1?

Collins, Rogers and Stasto, arXiv:0708.2833

H, PLB 655 (2007) [hep-ph/0702196]

- soft gluon exchange with spectator partons in pp collisions

$\longrightarrow$ possibly factorization-breaking? (back-to-back dihadron production)

Mulders, Bomhof, Collins,

Vogelsang, Qiu, Yuan, Pijlman, ...

2006-2008

$\diamondsuit$ appears at N^{3}LO (2 soft, 1 collinear partons)

$\diamondsuit$ does it survive destructive interference from multiple emission?

- Note: Coulomb/radiative mixing terms also appear to break coherence in di-jet cross sections with gap in rapidity

Forshaw, Kyrieleis & Seymour, JHEP 08 (2006)

MORE ON U-PDF'S WITH SUBTRACTIVE REGULARIZATION

One loop expansion: $[\zeta = (p^{+2}/2) u^- / u^+]$

$$f_{(1)}^{(\text{subtr})}(x, k_\perp) = P_R(x, k_\perp) - \delta(1-x)\,\delta(k_\perp) \int dx'\, dk'_\perp\, P_R(x', k'_\perp) \quad (\leftarrow \text{from numerator})$$

$$- \; W_R(x, k_\perp, \zeta) + \delta(k_\perp) \int dk'_\perp\, W_R(x, k'_\perp, \zeta) \quad (\leftarrow \text{from vev's})$$

with $P_R = \alpha_s C_F / \pi^2 \left\{ 1/[(1-x)\ (k_\perp^2 + m^2 (1-x)^2)] + \ldots \right\}$ = real emission prob.

$W_R = \alpha_s C_F / \pi^2 \left\{ 1/[(1-x)\ (k_\perp^2 + 4\zeta(1-x)^2)] + \ldots \right\}$ = counterterm

- ζ-dependence cancels upon integration in $k_\perp$

$$\Rightarrow \mathcal{O} = \int dx\ dk_\perp\ f_{(1)}^{(\text{subtr})}(x, k_\perp)\ \varphi(x, k_\perp)$$

$$= \int dx\ dk_\perp\ \{ P_R\ [\varphi(x, 0_\perp) - \varphi(1, 0_\perp)] + (P_R - W_R)\ [\varphi(x, k_\perp) - \varphi(x, 0_\perp)] \}$$

- first term: usual $1/(1-x)_+$ distribution
- second term: singularity in P_R cancelled by W_R

Note: it works because terms in $\delta(1-x)$ cancel between the two vev's,

$$-W_R(x, k_\perp, \zeta) + \delta(1-x)\,\delta(k_\perp) \int dx' dk'_\perp W_R$$

and

$$+\delta(k_\perp) \int dk'_\perp W_R(x, k'_\perp, \zeta) - \delta(1-x)\,\delta(k_\perp) \int dx' dk'_\perp W_R \quad .$$

- virtual correction to gauge link does not depend on $y_\perp$

G. Korchemsky

V. Braun et al

▷ subtractions have (relatively) simple form in <u>coordinate space</u>

▷ operator representation valid to all orders

▷ one-loop counterterm gives extension for k$_\perp \neq 0$ of the plus-distribution regularization

Conclusions

- Branching methods based on $k_\perp$-dependent u-pdfs and MEs useful for simulation of high-energy parton showers

  ▷ $k_\perp$ shower Monte-Carlo gives good description of small-x multi-jet final states

- Extension of u-pdf's over whole phase space important to turn these Monte-Carlo's into general-purpose tools

  ▷ special issues at $x \sim 1$ (and matching with small x) relevant for showering algorithm