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QCD Resummation for Heavy Quarkonium Production in High Energy Collisions

- Brief introduction
- Success and failure of existing models
- Huge high order corrections?
- QCD resummation and new factorization
- Summary and conclusions

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Based on work with Z.-B. Kang, G. Nayak, and G. Sterman

Heavy quarkonium production

QWG Report, N. Brambilla et al, hep-ph/0412158

- Offers a unique perspective to the hadronization

  Production of heavy quarks is effectively perturbative:

$$\Delta r \sim \frac{1}{2m_Q} \leq 0.1 \text{ fm} \quad \text{(for a charm-quark pair)}$$

$$\leq 0.025 \text{ fm} \quad \text{(for a b-quark pair)}$$

Heavy quark pairs are produced at a distance scale much less than fm

- Heavy quarkonium provides a non-relativistic system, potentially, very similar to a QED bound state:

Charm: $\frac{v^2}{c^2} \sim \frac{k_Q^2}{m_Q^2} \sim \frac{|M^2 - 4m_c^2|}{4m_c^2} \sim 0.3$ **Bottom:** $\frac{v^2}{c^2} \sim 0.1$

➡ Heavy quark potential: $V_{Q\bar{Q}}(r)$

Large mass and small binding energy

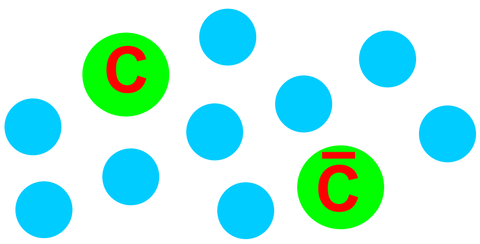
Heavy quarkonium in medium

- Could be a good probe for Quark Gluon Plasma (QGP)

  - The transition from a heavy quark pair to a quarkonium should be sensitive to the soft physics or medium properties

    Quarkonium binding energy: $\frac{\left|M^2 - 4m_Q^2\right|}{4m_Q^2} \ll 1$

  - Color screening in QGP suppresses the formation of J/ψ



Potential: $V_{Q\bar{Q}}(r) \Rightarrow V_{Q\bar{Q}}(r,T)$

Wave function: $\Phi_{Q\bar{Q}}(r) \Rightarrow \Phi_{Q\bar{Q}}(r,T)$

J/ψ formation rate $\propto \left|\Phi_{Q\bar{Q}}(r,T)\right|^2$

Matsui & Satz (1986)

J/ψ suppression ⇔ medium properties

Question

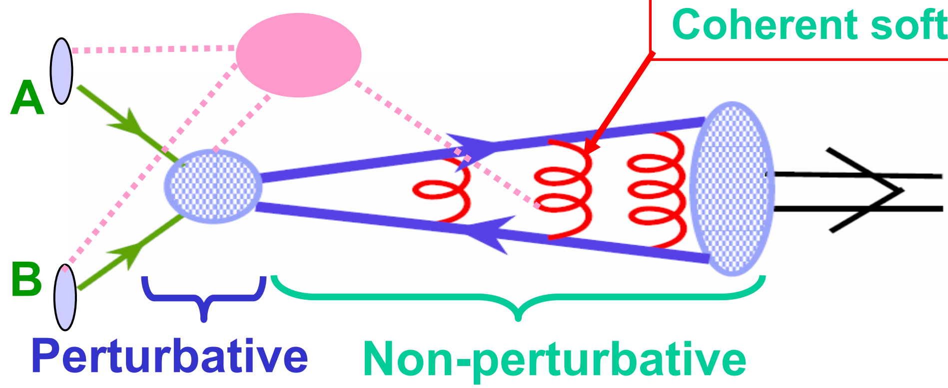
Do we understand the production mechanism of J/ψ well enough to calibrate the production rate and to extract the information on QGP?

May be, may be not

Note: it has been more than 30 years since the discovery of J/ψ!

The basic production mechanism

❑ Production of a heavy quark pair:



Nayak, Qiu, Sterman

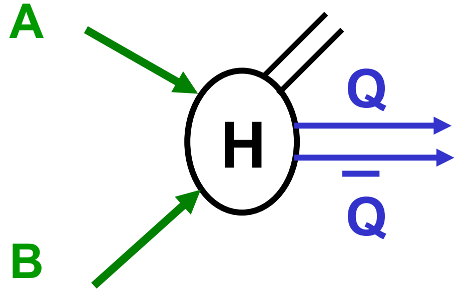
$$\Delta r \leq \frac{1}{2m_Q}$$

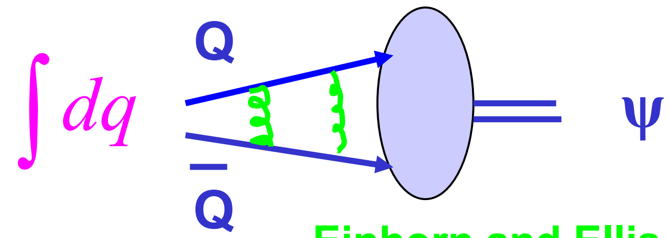
❑ Hadronization of the pair ⟹ models:

$$\sigma_{AB\to h} = \sum_{states} \int d\Gamma_{Q\bar{Q}} \frac{d\sigma_{AB\to states(Q\bar{Q})}}{d\Gamma_{Q\bar{Q}}} F_{states(Q\bar{Q})\to h}\left(p_Q, p_{\bar{Q}}, p_h\right)$$

Different models ⇔ Different assumptions/treatments on how the heavy quark pair becomes a quarkonium?

Color singlet model





Einhorn and Ellis (1975), …

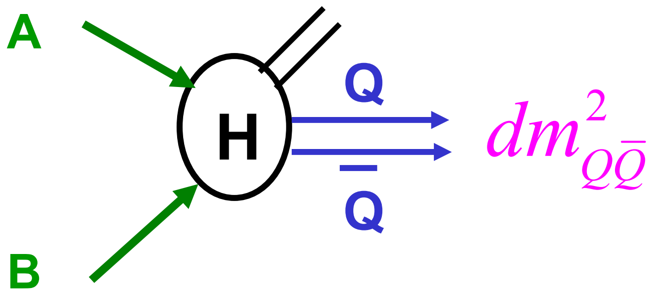
- color singlet pair
- right quantum numbers for the quarkonium
- same wave function for production and decay

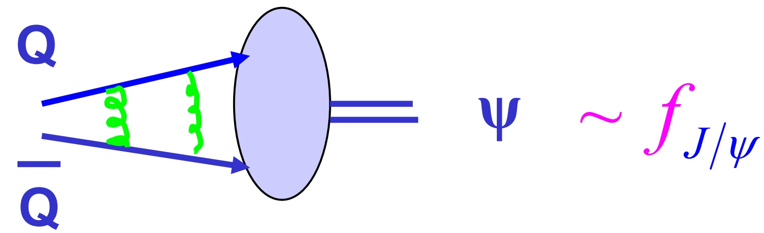
$$\sigma_{AB\to\psi} \propto \sigma_{AB\to(Q\bar{Q})} \left|R_\psi(0)\right|^2$$

- absolutely normalized predictions
- predictions on polarization
- quantum interference between production and formation suppressed

Works well for J/ψ production in photo-production and others
But, one order of magnitude too small for CDF data, …

Color Evaporation Model





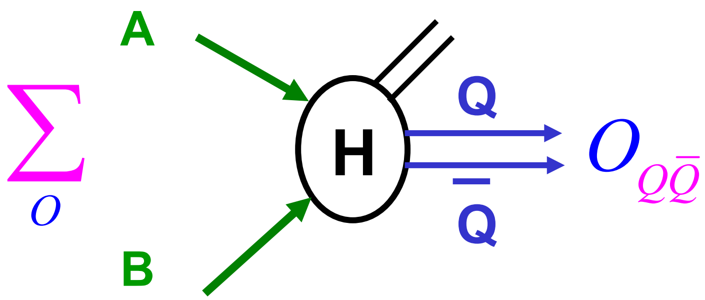
Fritsch (1978); Halzen; …

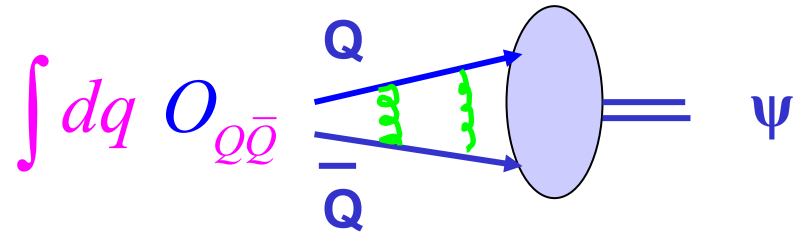
- all pairs with invariant mass less than open flavor threshold
- color and spin average
- a single constant for non-perturbative formation
- one constant for one quarkonium state

$$\sigma_{AB\to J/\psi} = f_{J/\psi} \int dm^2_{Q\bar{Q}} \frac{d\sigma_{AB\to(Q\bar{Q})}}{dm^2_{Q\bar{Q}}}$$

Works well for total cross sections, not perfect for distributions,
Predicts zero polarization for quarkonium production

Non-relativistic QCD (NRQCD) model





Bodwin, Braaten, Lapage (1994); …

- A generalization of color singlet model
- All color and spin states of HQ pairs -> quarkonium
- Expansion of HQ velocity

- "Integrate out" HQ dynamics: $(\mathcal{O}(\alpha_s^n(m_Q)))$
- Factorize hadronization: $(\mathcal{O}(v_{\rm rel}^n))$
  universal matrix elements

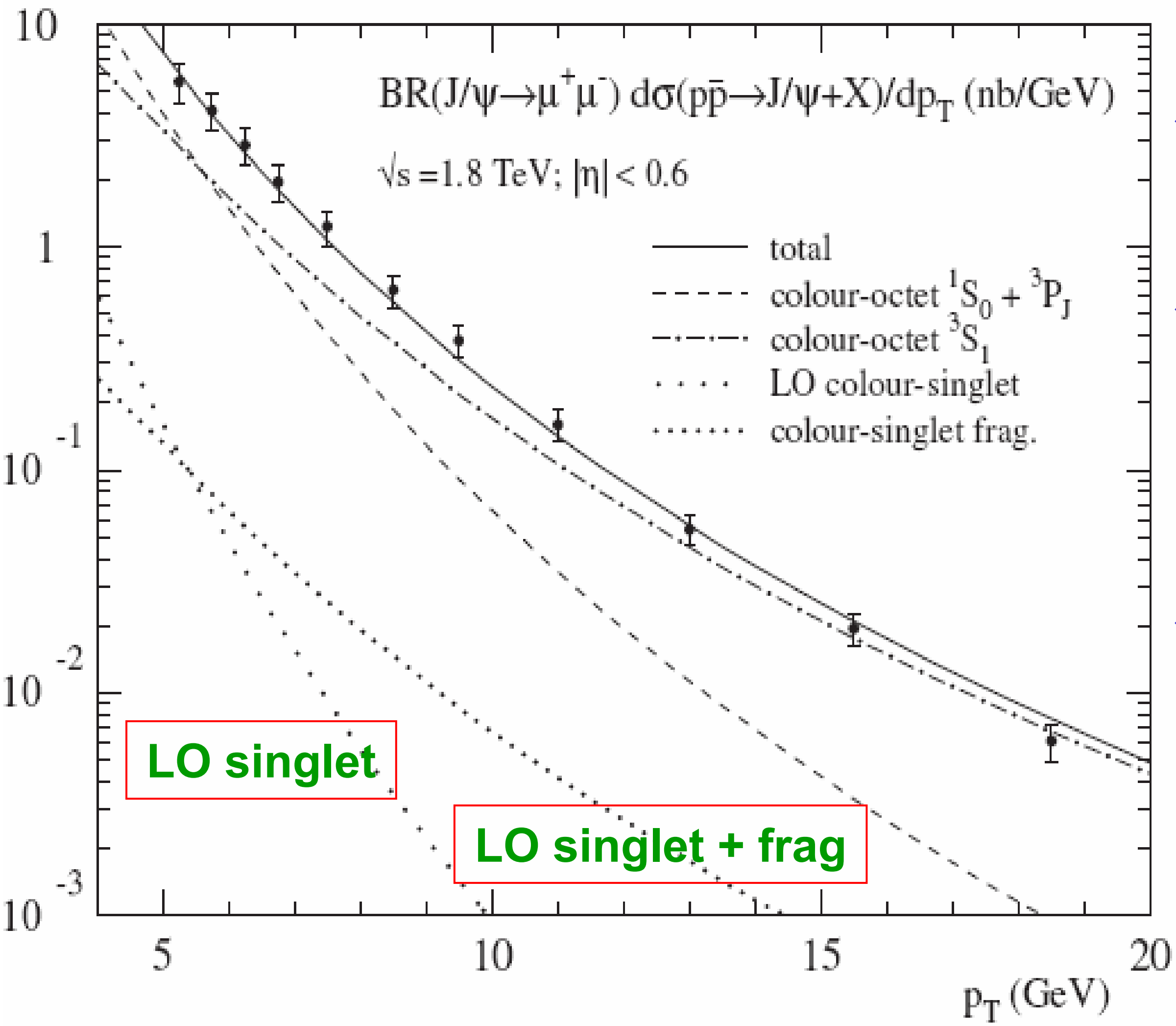
$$\sigma_{AB\to J/\psi}\left(M_{J/\psi}\right)\approx\sum_{[O]}\sigma_{AB\to[O]}\left(2m_{c\bar{c}}=M_{J/\psi}\right)\left\langle O_{J/\psi}(0)\right\rangle$$

It has been the most successful model …

Successes of the production models

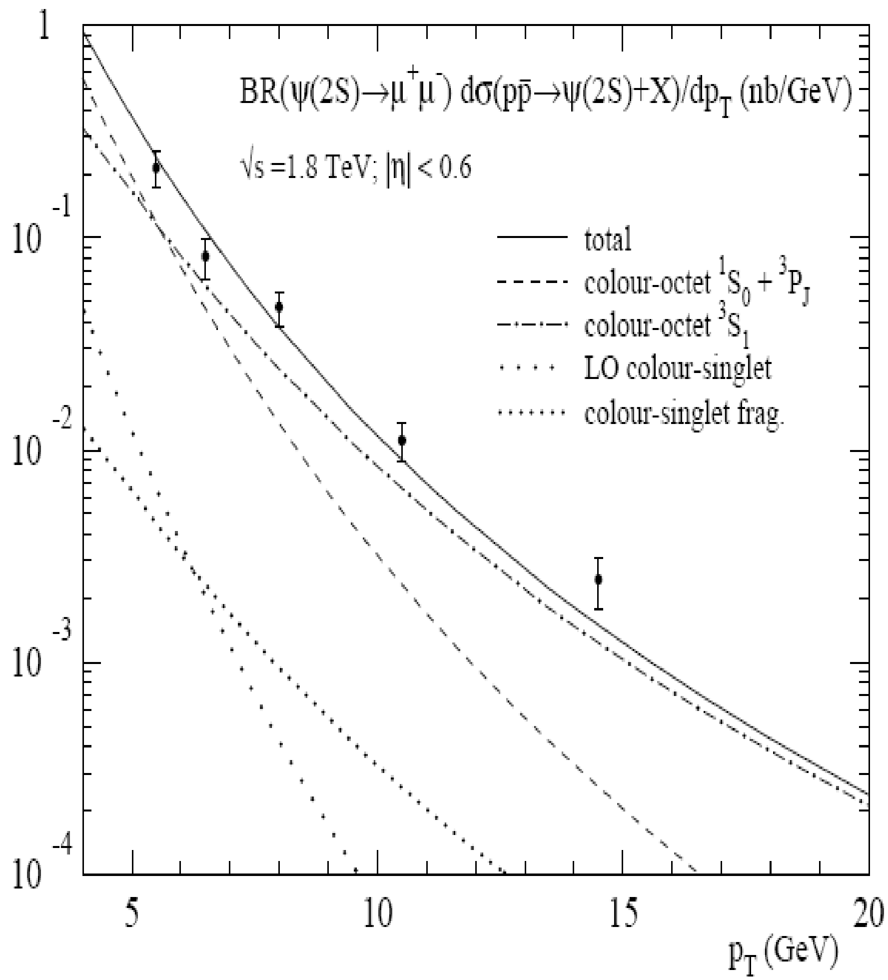
❑ Unpolarized J/ψ at the Tevatron:

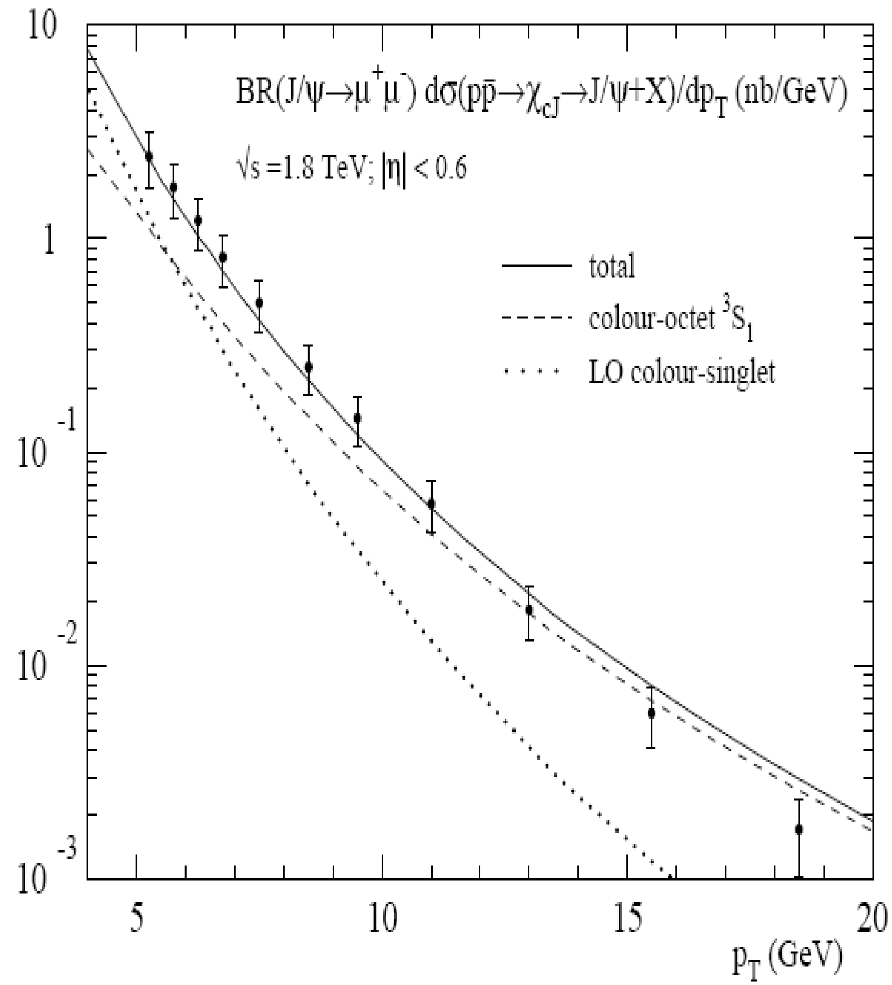
M. Kramer, 2001



- ❖ Data is not consistent with color singlet model
- ❖ Data is consistent with NRQCD, with matrix elements fixed by the data
- ❖ CEM predicts a similar p_T distribution

❑ Works for other states too:





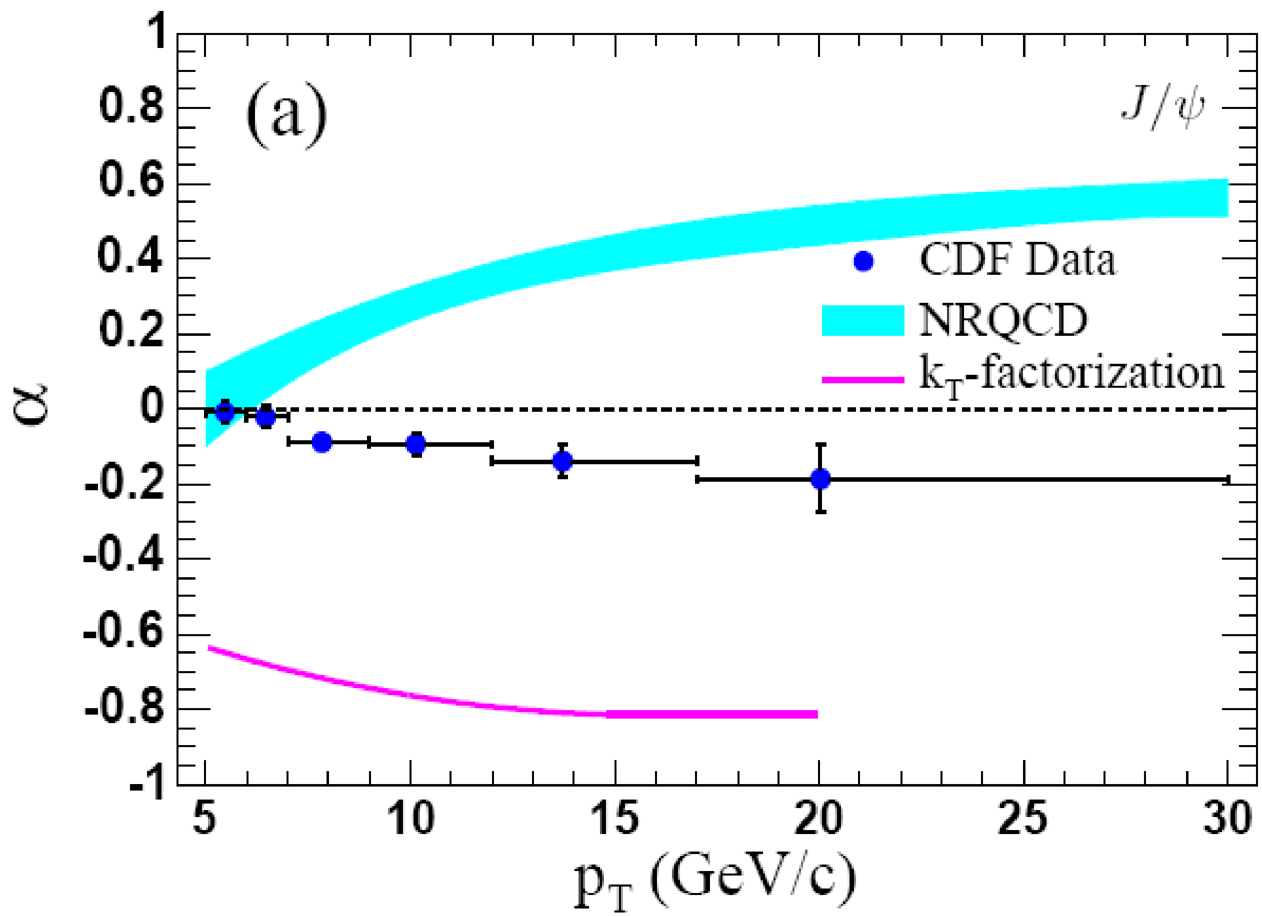
E. Braaten et al. Annu. Rev. Nucl. Part. Sci. 46, 197 (1996)

Difficulties

- Transverse polarization at high p_T?

NRQCD: Cho & Wise, Beneke & Rothstein, 1995, …

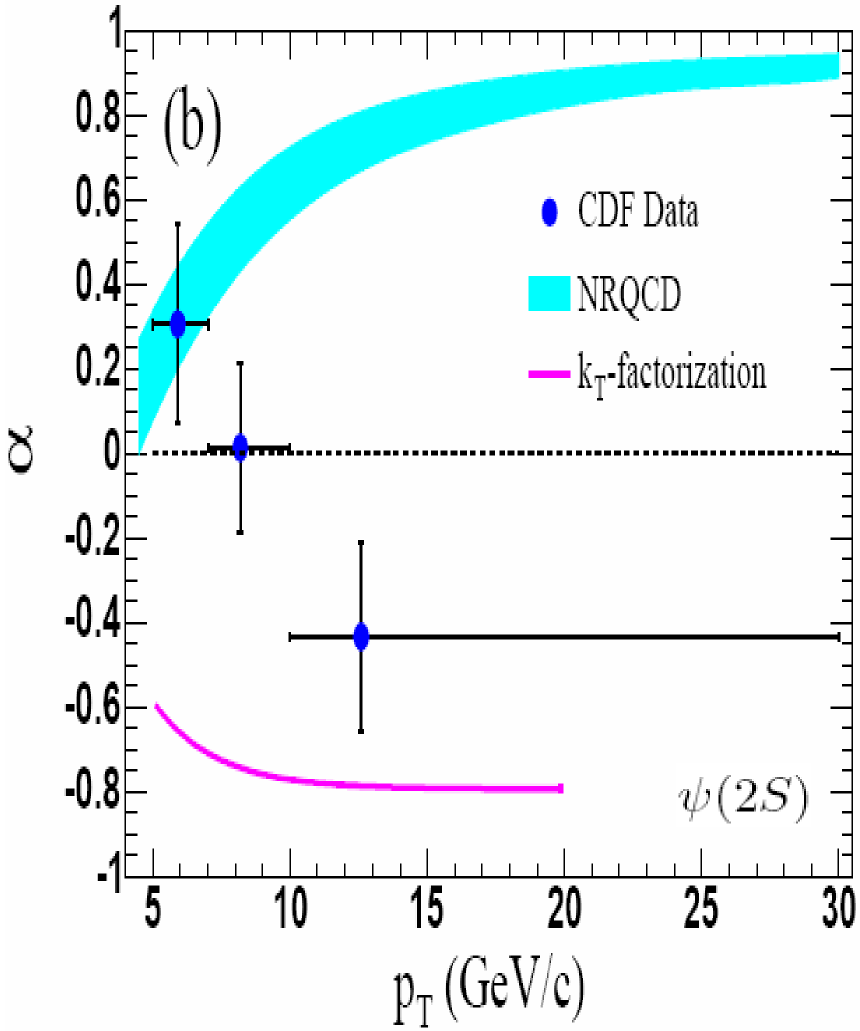
KT-fact: Baranov, 2002



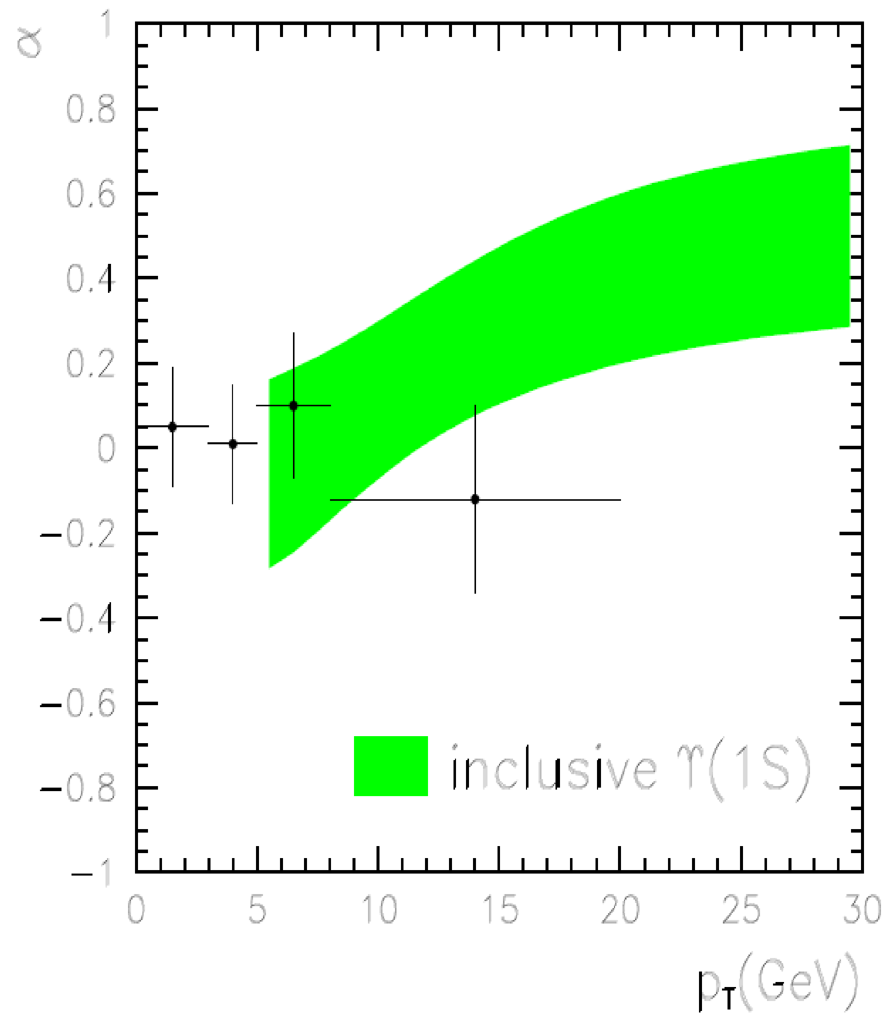
$$\alpha = \frac{\sigma_T - \sigma_L}{\sigma_T + \sigma_L}$$

CDF Collab. arXiv:0704.0638 [hep-ex]

❑ Same problem for other states:



CDF Collab. arXiv:0704.0638 [hep-ex]



Braaton & Lee, PRD63, 071501 (2001)

Double $c\bar{c}$ production in e^+e^-

- Inclusive production:

$$\sigma(e^+e^- \to J/\psi c\bar{c})$$

Belle: $(0.87^{+0.21}_{-0.19} \pm 0.17)$ pb

NRQCD: ~ 0.07 pb

Kiselev, et al 1994,
Cho, Leibovich, 1996
Yuan, Qiao, Chao, 1997

- Ratio to light flavors:

$$\sigma(e^+e^- \to J/\psi c\bar{c})/\sigma(e^+e^- \to J/\psi X)$$

Belle: $0.59^{+0.15}_{-0.13} \pm 0.12$

Message:

Production rate of $e^+e^- \to J/\psi c\bar{c}$ is larger than all these channels: $e^+e^- \to J/\psi gg\,, e^+e^- \to J/\psi q\bar{q}, \ldots$ combined ?

Question and facts

Question:

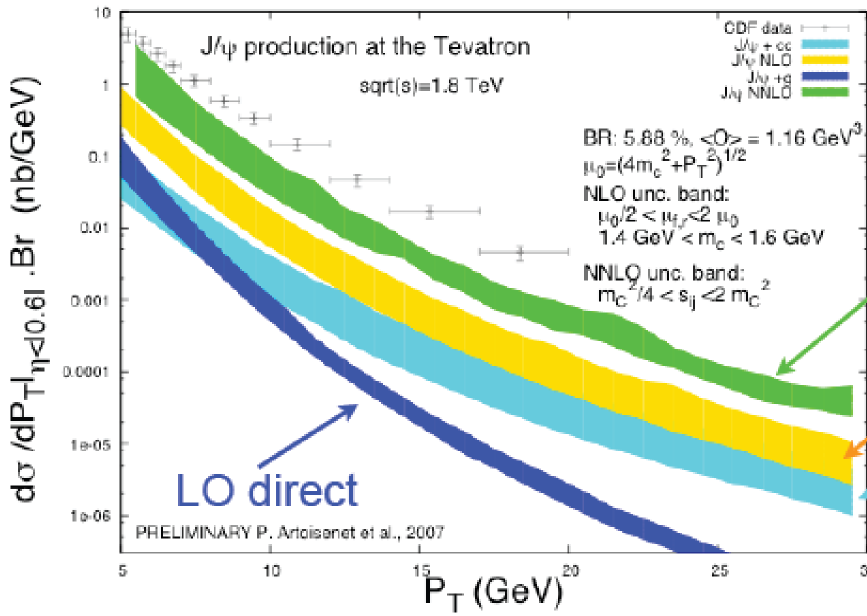
Is the factorization valid for these observables?

- None of these production models have been proved!
  - NRQCD formalism for heavy quarkonium decay was proved, but, not for production
    Bodwin, Braaten, Lapage (1994); …
    Nayak, Qiu, Sterman, 2005, 2006
  - Not work when additional heavy quark velocity is involved, such as, the associated production
    Nayak, Qiu, Sterman, 2007
- Huge high order corrections to the singlet channel

  Large scale dependence, even with high order terms

J. Campbell et al PRL 2007
P. Artoisnet, F. Maltoni, et al. 2008

Huge high order corrections



Color-singlet contribution for J/ψ and Upsilon production at Tevatron

NNLO

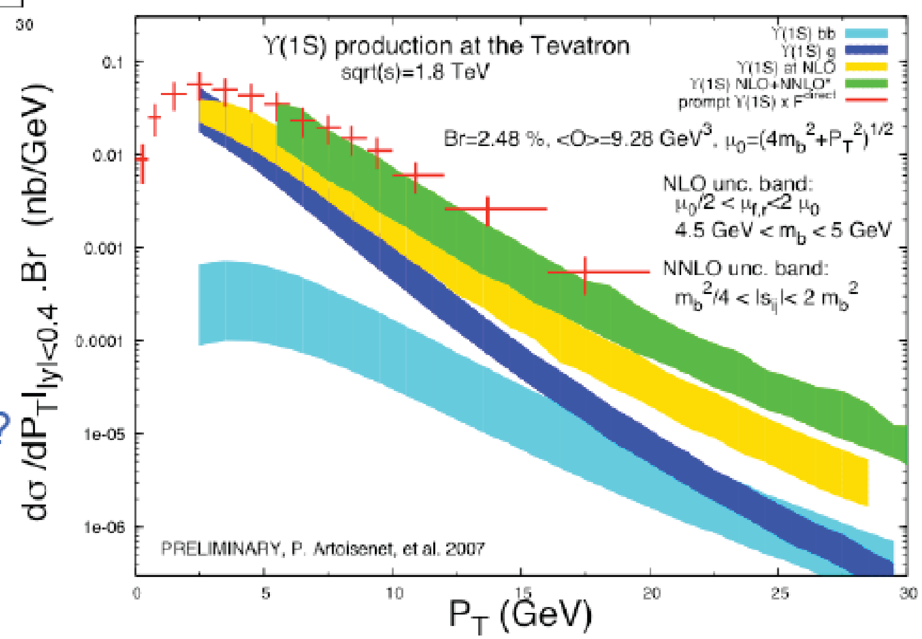
NLO

LO associate

P. Artoisenet, F. Maltoni, et.al. 2007

Large uncertainty band
⇒strong scale dependence

Large NLO, NNLO contribution
⇒ how perturbative series converge?

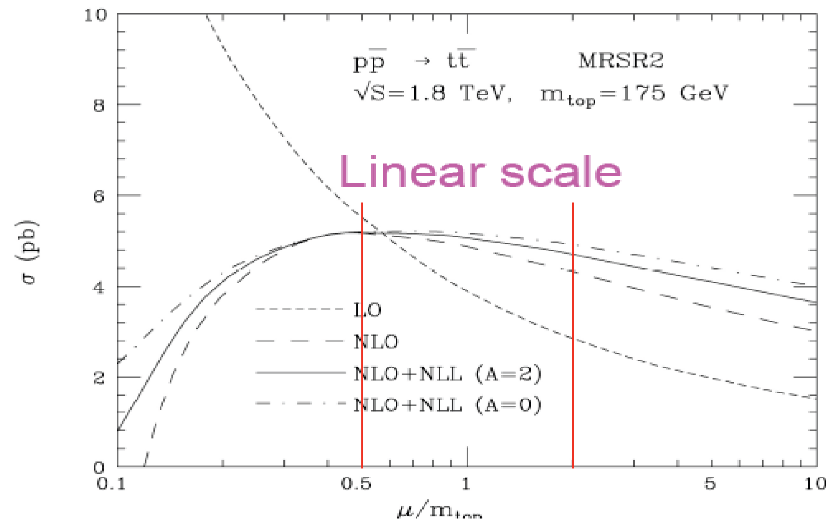


Scale dependence of cross sections

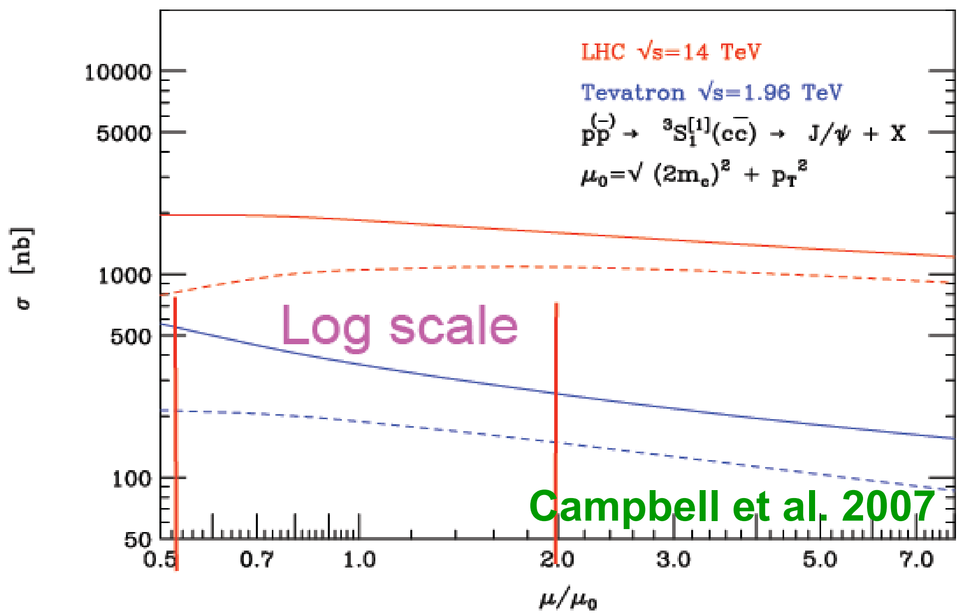
- $t\bar{t}$ - cross section NLO:

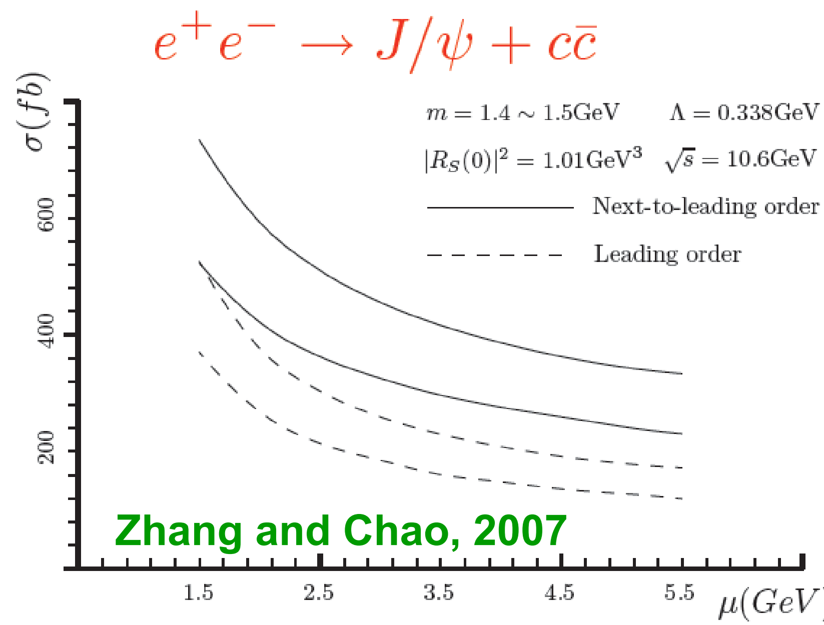
  With NLO correction, the scale dependence is much reduced!

  Bonciani et al. Nucl. Phys. B, 1998



- Large scale dependence for J/ψ even at NLO:





Goal of the rest discussion

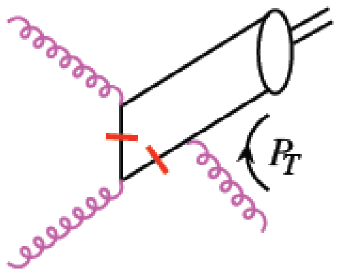
- Not to prove or disprove these production models

Nayak, Qiu, and Sterman, 2005-2007

- Re-organization of the perturbative part:
  - Multiple hard scales
  - Resummation of large logarithms
  - More stable predictions
  - The method is good for both NRQCD and Color Evaporation

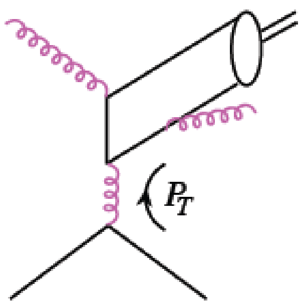
Why high order terms are so large?

❑ LO:



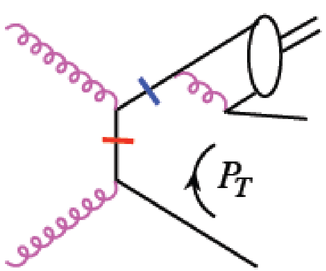
$\alpha_s^3 \frac{(2m)^4}{P_T^8}$

❑ NLO – new subprocess:



$\alpha_s^4 \frac{(2m)^2}{P_T^6}$

❑ NLO to existing LO:



$\alpha_s^4 \frac{1}{P_T^4}$

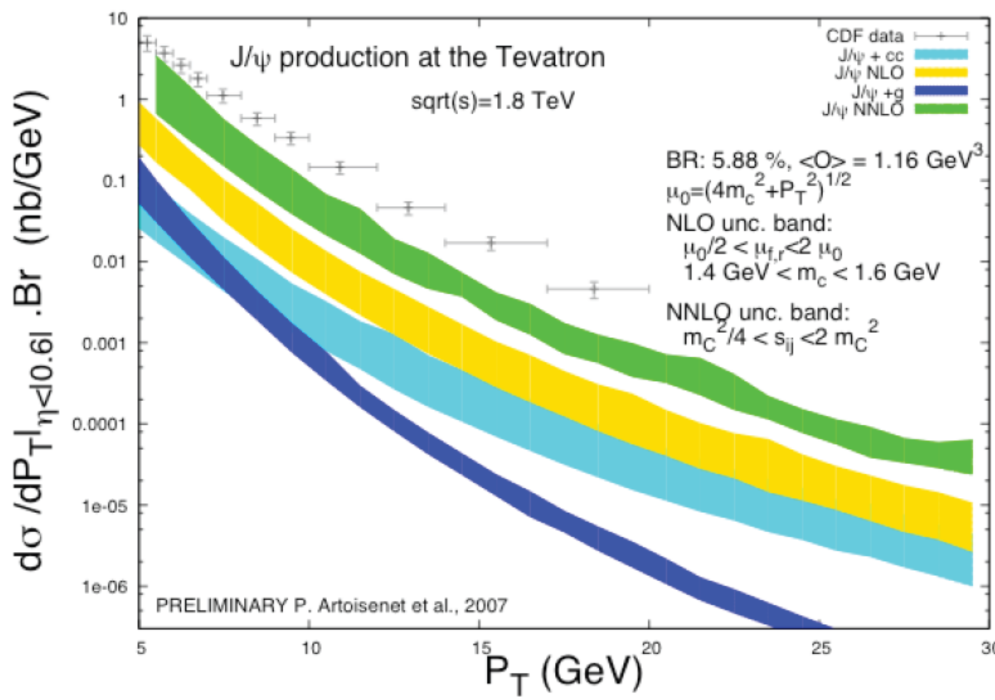
Associated production

❖ Scale dependence $\phi(x,\mu)^2\alpha_s^3(\mu)$

❖ P_T dependence: $\frac{1}{P_T^8}$

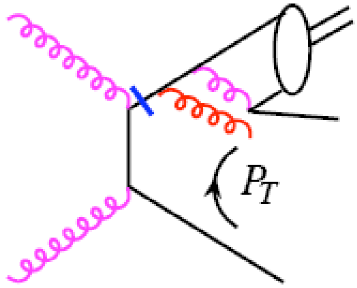
❖ High power in $\alpha_s(\mu)$

❖ Low power in P_T



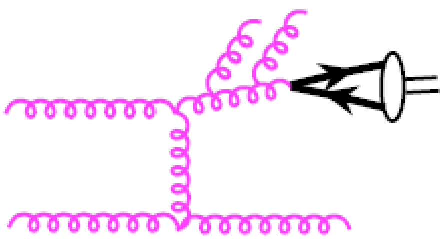
Large logarithmic contribution

- Fragmentation logarithms - NNLO:



$$\alpha_s^4 \frac{1}{P_T^4} \cdot \left(\alpha_s \ln \left[\frac{P_T^2}{m^2} \right] \right)$$

- Same logarithms in the octet channels:



As well as in the calculation of color evaporation model – two scales

- Large mass – the logarithms are perturbative

  - Resummation of such perturbative logarithms is necessary
  - Re-organization of the perturbative part

Resummation of fragmentation logarithms

- Fragmentation contribution:

Braaten et al, 1993

$$\sigma^F(pp \to H + X) = \sum_{i,j,k} \int dx_1 dx_2 dz \phi_{i/p}(x_1)\phi_{j/p}(x_2)\hat{\sigma}[ij \to k] D_{k \to H}(z)$$

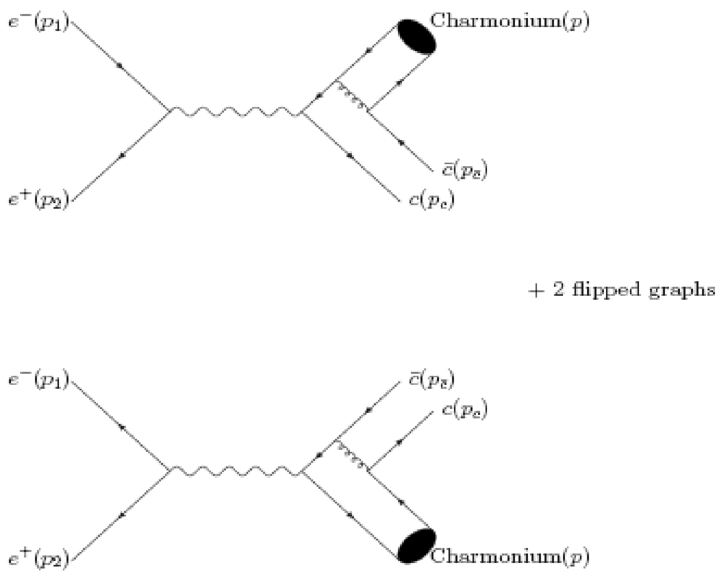
Resum all logs

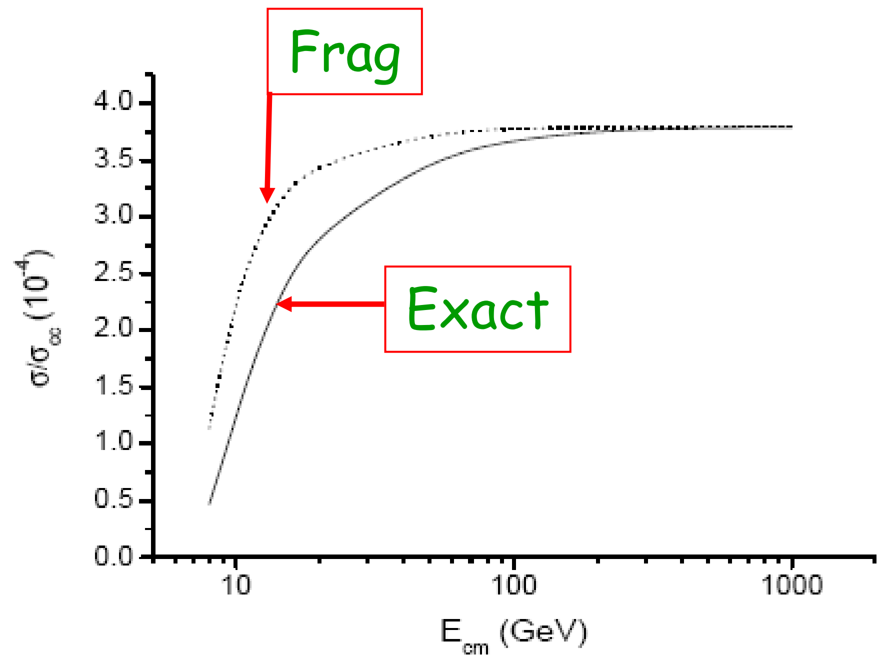
Dominant contribution when $P_T >> m$

- Interplay between the fixed order and fragmentation:

Ex: $e^+e^- \to J/\psi + c\bar{c}$

Liu et al, 2004





Factorization with QCD resummation

- Relation between the fixed order and fragmentation:

$P_T^2 \sim m^2 \qquad \sigma \approx \sigma^{\text{Pert}}$ Calculated by fixed order perturbation

$P_T^2 \gg m^2 \qquad \sigma \approx \sigma^{\text{Frag}}$ Logs dominate and need to be resummed

- Questions:
  - ❖ How to transform smoothly between these two regimes?
  - ❖ How to avoid double counting beyond the LO?

- New factorization formula:

Qiu et al 2001, Berger et al 2002
for low mass Drell-Yan

$\sigma = \sigma^{\text{Dir}} + \sigma^{\text{Frag}}$ ⟵ Resum all logarithms between P_T and m

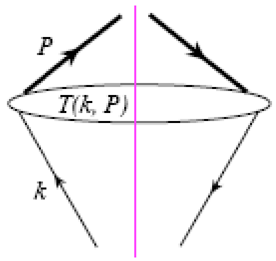
$\sigma^{\text{Dir}} \equiv \sigma^{\text{Pert}} - \sigma^{\text{Asym}}$ ⟵ No large logarithms, evaluated at $1/P_T$

The separation between the "Direct" and the "Frag" contribution depends on the definition of the fragmentation function

Quarkonium fragmentation functions

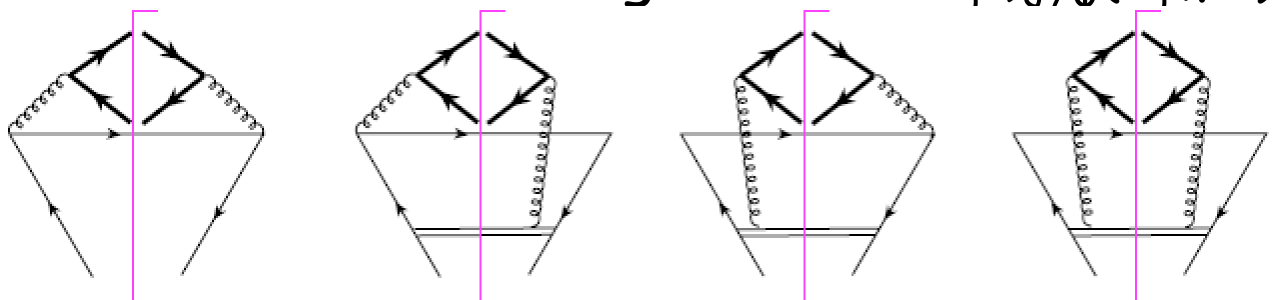
- Operator definition of $D_{f\to J/\psi}(z_f,\mu^2)$:

$$D_{k\to H}(z_f,\mu^2) =$$



$$= \int_{k^2\le\mu^2} \frac{d^4k}{(2\pi)^4}\frac{z_f^2}{4k^+}\delta\left(z_f - \frac{P^+}{k^+}\right)\mathrm{Tr}\left[\gamma^+ T(k,P)\right]$$

- Calculation of the leading order $D^{(0)}_{f\to J/\psi}(z_f,\mu^2)$:



$$D^{(0)}_{q\to J/\psi}(z_f,\mu) = \frac{\alpha_s^2}{36m^3}\langle O_8(^3S_1)\rangle \cdot \left[\frac{(z_f-1)^2+1}{z_f}\ln\left(\frac{z_f\mu^2}{4m^2}\right) - z_f\left(1-\frac{4m^2}{z_f\mu^2}\right)\right]$$

- HQ mass is a perturbative scale, and cuts off the collinear region
- Scheme dependence from the $k^2 < \mu^2$
- "Perturbative" – different from the light-hadron fragmentation func

Resummation and inhomogeneous evolution

- Resummation: Qiu and Zhang, 2001
  - Identify the logarithms
  - Derive the renormalization group or evolution equation for the log
  - Resummation – solve the evolution equation

$$\mu^2 \frac{d}{d\mu^2} D_{q\to J/\psi}(z_f,\mu) = \gamma_{q\to J/\psi}(z_f,\mu) + \frac{\alpha_s}{2\pi}\int_{z_f}^1 \frac{d\xi}{\xi} P_{q\to q}\left(\frac{z_f}{\xi}\right) D_{q\to J/\psi}(\xi,\mu) + \cdots$$

- "Perturbative" contribution ⟶ inhomogeneous term:

$$\mu^2 \frac{d}{d\mu^2} D^{(0)}_{q\to J/\psi}(z_f,\mu) = \gamma_{q\to J/\psi}(z_f,\mu)$$

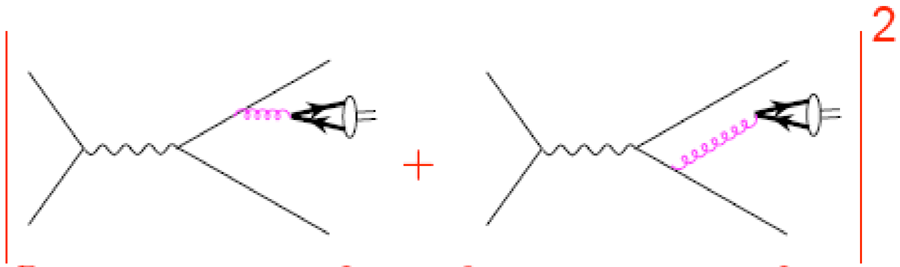
$$\gamma_{q\to J/\psi}(z_f,\mu) = \frac{\alpha_s^2}{36m^3}\langle O_8(^3S_1)\rangle \left[\frac{(z_f-1)^2+1}{z_f} - \frac{4m^2}{\mu^2}\right] \theta\left(\mu^2 - \frac{4m^2}{z_f}\right)$$

- "Input" distribution – boundary condition:

$$D_{q\to J/\psi}\left(z_f, \mu_0^2 = 4m^2/z_f\right) = 0$$

Case study: $e^+e^- \to J/\psi + X$

- NRQCD LO perturbative:



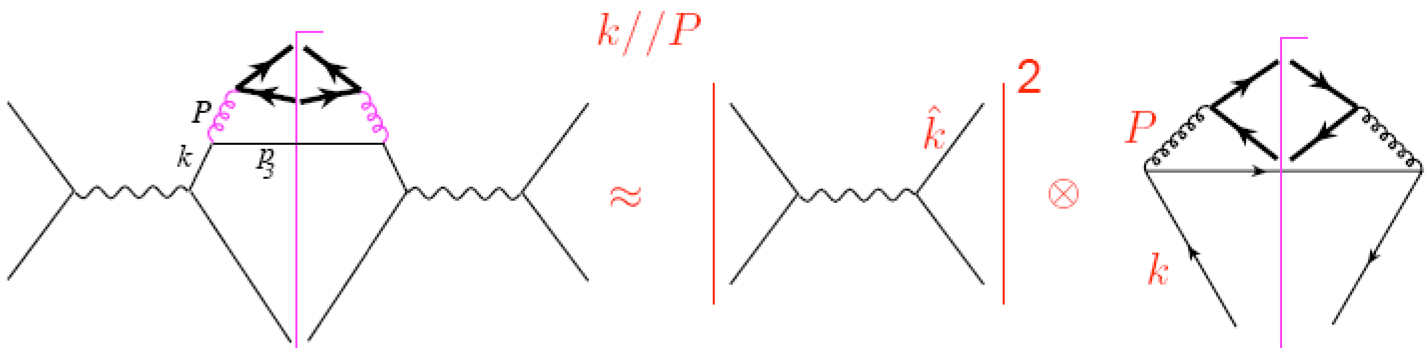
$$z = \frac{2E_{J/\psi}}{\sqrt{s}} \qquad \xi = \frac{4m^2}{s}$$

$$z_L = \sqrt{z^2 - 4\xi}$$

$$\ln\left(\frac{E^2_{J/\psi}}{zm^2}\right)$$

$$\frac{d\sigma^{Pert}}{dE_{J/\psi}} = \sigma_0 \cdot \frac{2}{\sqrt{s}} \frac{\alpha_s^2}{18} \frac{\langle O_8(^3S_1)\rangle}{m^3} \left[\left(\frac{(z-1)^2+1}{z} + 2\xi\frac{2-z}{z} + \xi^2\frac{2}{z}\right) \ln\frac{z+z_L}{z-z_L} - 2z_L\right]$$

- Identify the logarithm without full calculation - $\sigma^{\rm Asym}$



$$\frac{d\sigma^{Asym}}{dE_{J/\psi}} \approx \sigma_0 \cdot D^{(0)}_{q\to J/\psi}(z_f, \mu^2, 4m^2) \frac{dz_f}{dE_{J/\psi}} \qquad z_f = \frac{P^+}{k^+} = \frac{1}{2}[z + z_L]$$

Smooth transition

- **Direct contribution:** $\sigma^{\text{Dir}} = \sigma^{\text{Pert}} - \sigma^{\text{Asym}} = \sigma^{\text{Pert}} - 2\sigma_0 \cdot D^{(0)}_{q\to J/\psi}\left(z,\mu^2\right)$

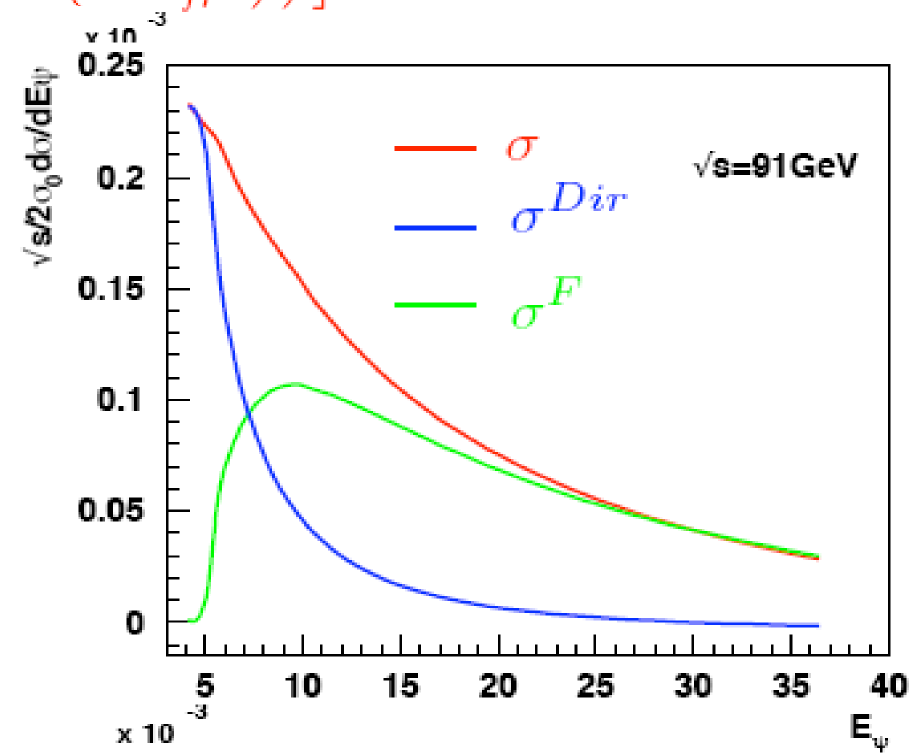
$$\frac{d\sigma^{Dir}}{dE_{J/\psi}} = \sigma_0 \cdot \frac{2}{\sqrt{s}}\frac{\alpha_s^2}{18}\frac{\langle O_8(^3S_1)\rangle}{m^3}$$
$$\times\left[\left(\frac{(z-1)^2+1}{z} + 2\xi\frac{2-z}{z} + \xi^2\frac{2}{z}\right)\ln\frac{z+z_L}{z-z_L} - 2z_L - \frac{z_f}{z_L}\left(\frac{(z_f-1)^2+1}{z_f}\ln\left(\frac{z_f\mu^2}{4m^2}\right) - z_f\left(1-\frac{4m^2}{z_f\mu^2}\right)\right)\right]$$
$$\mu = 2E_{J/\psi}$$

Cancellation of the logs

- **Full cross section:**

$$\sigma = \sigma^{\text{Dir}} + \sigma^{\text{Frag}}$$

  - smooth transition:
    - when $E_{J/\psi}\sim m$: $\quad \frac{d\sigma}{dE_{J/\psi}} \sim \frac{d\sigma^{Dir}}{dE_{J/\psi}}$
    - when $E_{J/\psi} >> m$: $\quad \frac{d\sigma}{dE_{J/\psi}} \sim \frac{d\sigma^{F}}{dE_{J/\psi}}$

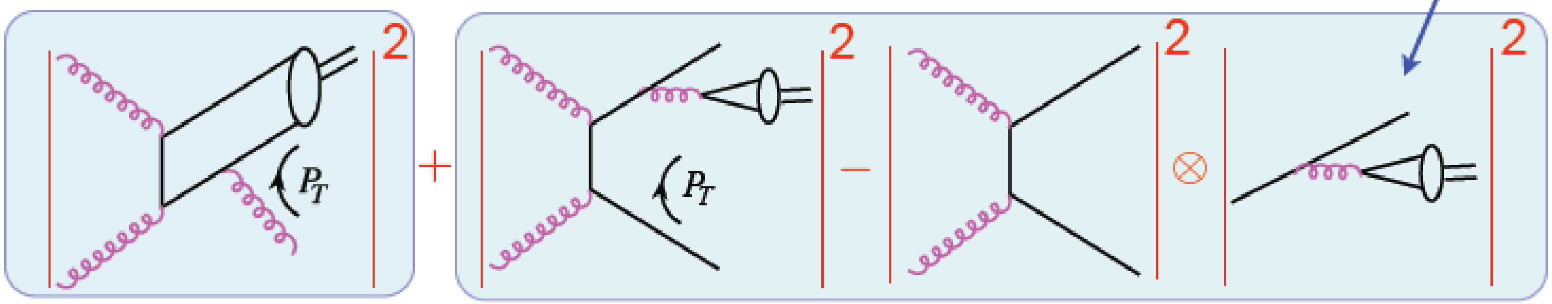


Hadronic collision – in progress

$$\sigma = \sigma^{Dir} + \sigma^{F}$$

- Direct contribution: $\sigma^{Dir} = \sigma^{Pert} - \sigma^{Asym}$

$D^{(0)}_{q\to H}(z,\mu^2)$

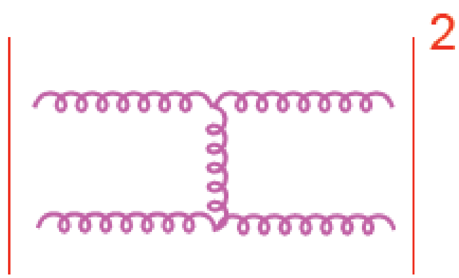


$+\cdots$

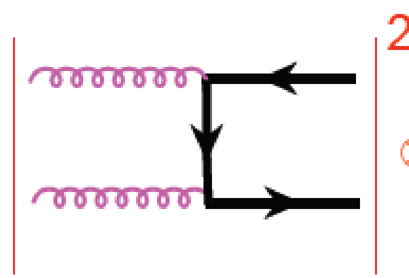
LO NLO

- Fragmentation contribution:

$\sigma^F =$



$\otimes D_{g\to H} +$



$\otimes D_{Q\to H} + \ldots$

Stay tuned!

Summary

- We proposed a QCD resummed factorization formula for heavy quarkonium production
- We re-organize the perturbative series of any production models (NRQCD, Color Evaporation, …)
- The new formula is perturbatively more stable and reliable for a wide range of collision energies