Unified description of equation of state and transport properties of neutron star matter

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Outline

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- ⋆ Correlated basis function CBF & cluster expansion formalism

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- ⋆ Viscosity and thermal conductivity of pure neutron matter
- ⋆ Summary and prospects

Motivation

- ⋆ The description of neutron star properties (mass, radius, moment of inertia, oscillation modes leading to gravitational wave emission, cooling rate) requires the knowledge of a number quantities, including the *equation of state* (EOS) and the *transport coefficients* (viscosity, thermal conductivity, . . .) of neutron star matter

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- ⋆ While the EOS is often obtained from *realistic dynamical models*, most available estimates of the transport coefficients are based on rather *crude assumptions*
- ⋆ Using the formalism of Correlated Basis Function (CBF) theory the EOS and the transport coefficients can be *consistently* obtained using an *effective interaction* derived from a state-of-the-art nucleon-nucleon (NN) potential model

Non relativistic nuclear many-body theory

⋆ Non relativistic pointlike protons and neutrons interacting through the hamiltonian

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$$H = T + V = \sum_i \frac{p^2}{2m} + \sum_{j>i} v_{ij} + \dots$$

- v_{ij} strongly constrained by deuteron properties and nucleon-nucleon (NN) scattering data: ANL v_{18} as an example

$$v_{ij} = \sum_{p=1,18} v_p(r_{ij}) O^p_{ij}$$

$$O^p_{ij} = [1, \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j, S_{ij}, \mathbf{L}\cdot\mathbf{S}, \mathbf{L}^2, \mathbf{L}^2(\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j), (\mathbf{L}\cdot\mathbf{S})^2] \otimes [1, \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j] \ ,$$

$$[1, \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j, S_{ij}] \otimes T_{ij} \ , \ \text{and} \ (\tau_{zi} + \tau_{zj})$$

$$S_{ij} = \sigma^\alpha_i \sigma^\beta_j \left(3 r^\alpha_{ij} r^\beta_{ij} - \delta^{\alpha\beta} \right) \ , \ T_{ij} = \tau^\alpha_i \tau^\beta_j \left(3 r^\alpha_{ij} r^\beta_{ij} - \delta^{\alpha\beta} \right)$$

Nucleon-nucleon scattering in free space

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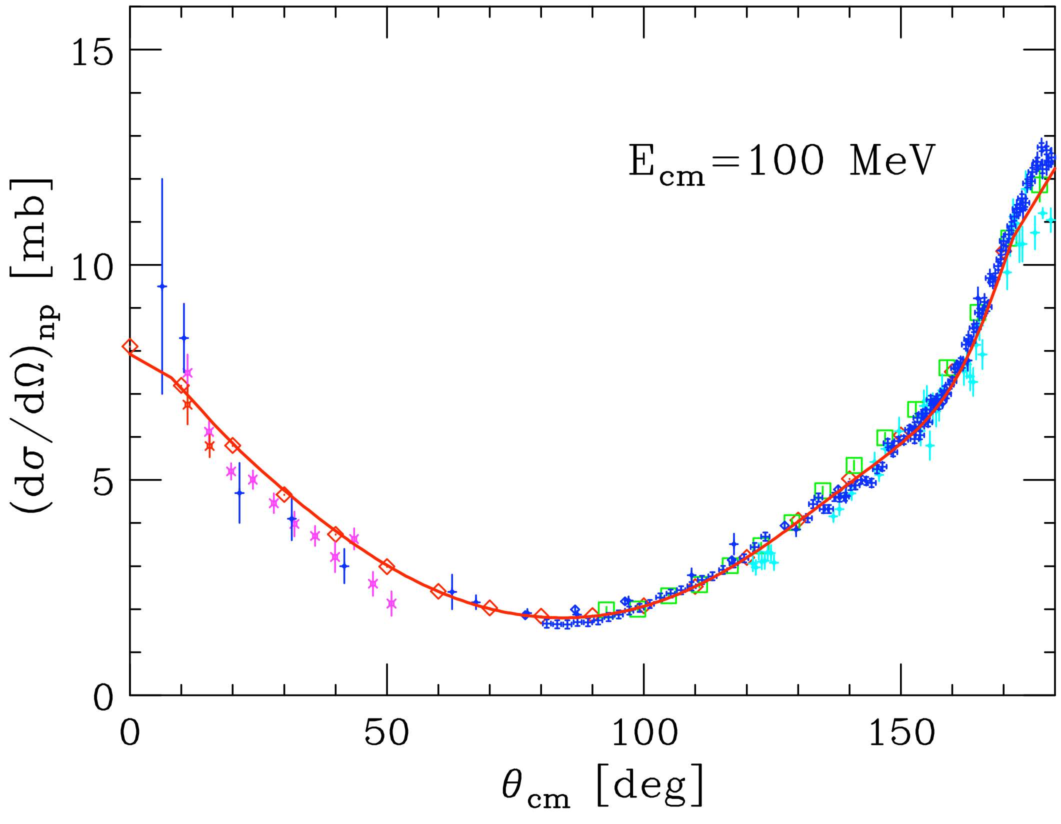
- ⋆ *np* differential x-section



Correlated Basis Function (CBF) formalism

- ⋆ Bottom line: due to the short range repulsive core of the NN potential, standard perturbation theory is not applicable
- ⋆ The *correlated* states are obtained from the Fermi gas (FG) states through the transformation

$$|n\rangle = F\,|n_{FG}\rangle$$

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- ⋆ The structure of F reflects the properties of the NN potential

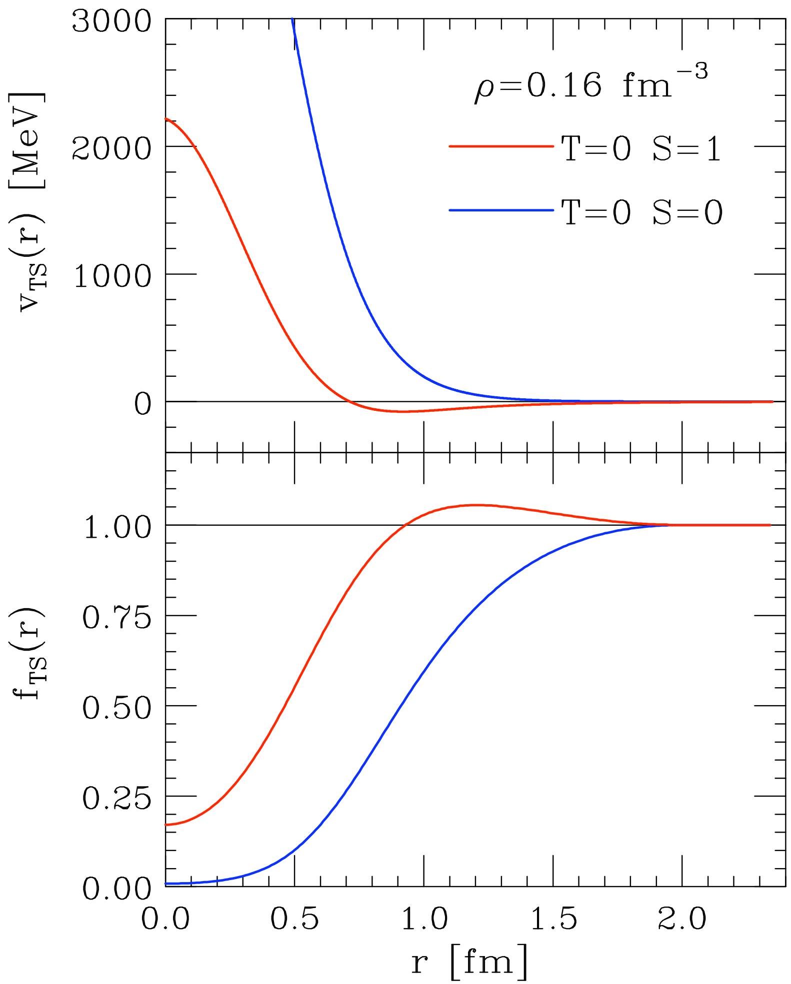
$$F = \mathcal{S}\prod_{j>i} f_{ij} \quad , \quad f_{ij} = \sum_p f_p(r_{ij}) O^p_{ij}$$

- ⋆ f_p determined from functional minimization of

$$\langle H\rangle = \frac{\langle 0|H|0\rangle}{\langle 0|0\rangle}$$

Shape of central correlation functions

- ANL v_8' potential $\rho = 0.16\ \text{fm}^{-3}$



Cluster expansion formalism

- ⋆ The expectation value of H in the correlated ground state can be expanded according to

$$\langle H \rangle = E_{FG} + \sum_{n \geq 2} (\Delta E)_n$$

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- ⋆ At two-body cluster level

$$\langle H \rangle \approx E_{FG} + \sum_{j>i} \langle ij | f_{12}^{\dagger} \left[-\frac{1}{m}(\nabla^2 f_{12}) - \frac{2}{m}(\boldsymbol{\nabla} f_{12}) \cdot \boldsymbol{\nabla} + v_{12} f_{12} \right] | ij - ji \rangle$$

$$|ij\rangle = \frac{1}{V} \, \mathrm{e}^{i(\mathbf{k}_i \cdot \mathbf{r}_1 + \mathbf{k}_j \cdot \mathbf{r}_2)} \, |ST\rangle$$

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- ⋆ Accurate calculations of $\langle H \rangle$ can be carried out for uniform matter using FHNC/SOC summation techniques and *CBF perturbation theory*

The CBF effective interaction

- The effective interaction is defined through

$$\langle H \rangle = \frac{\langle 0|T+V|0\rangle}{\langle 0|0\rangle} = \langle 0_{FG}|T+V_{\rm eff}|0_{FG}\rangle$$

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- Problem: inclusion of three-nucleon interactions, which are known to be needed to reproduce the binding energy of the few-nucleon systems and the equiliblrium propoerties of nuclear matter

The TNI model

- Within the TNI model of Lagaris & Pandharipande the effects of three-nucleon interactions are taken into account through
  - A density dependent modification of the NN potential at intermediate range, resulting in a repulsive contribution to $\langle H \rangle$

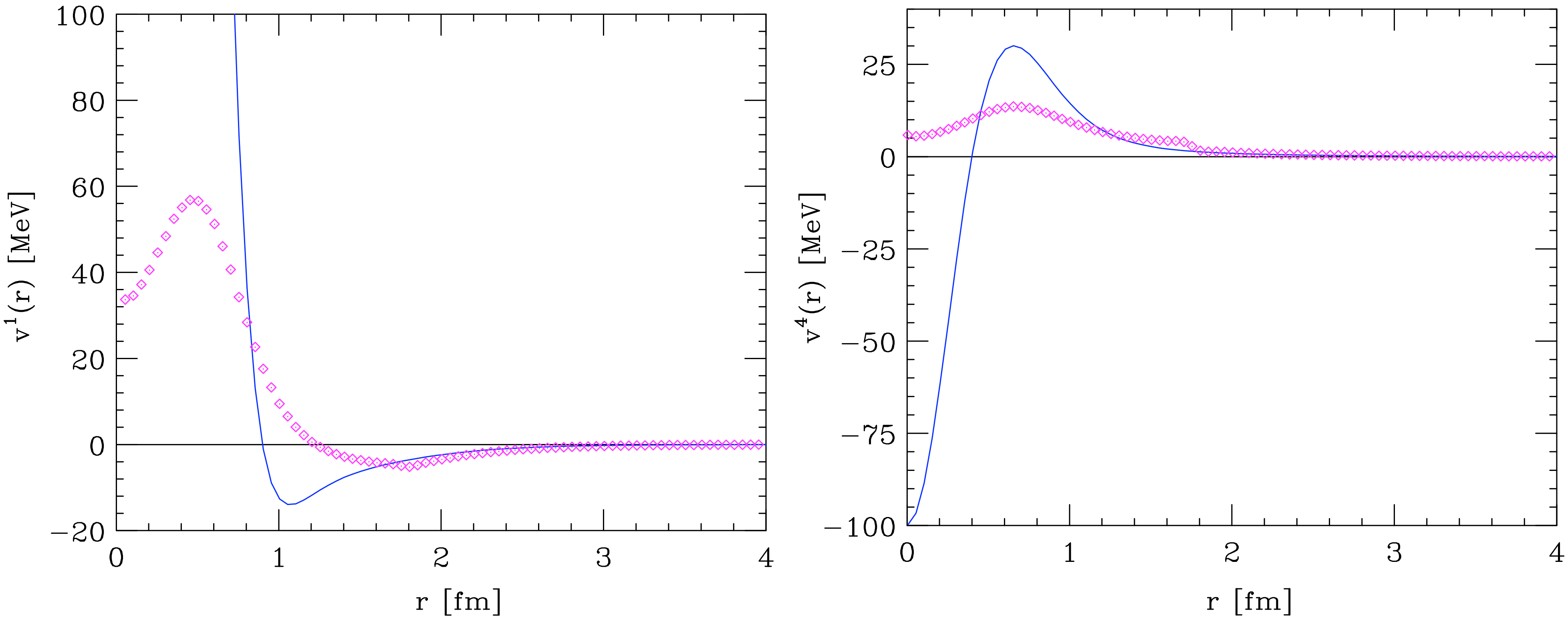
$$v_{ij} + TNR = \sum_{p=1,14} \left[v_\pi^p(r_{ij}) + v_I^p(r_{ij}) \mathrm{e}^{-\gamma_1 \rho} + v_S^p(r_{ij}) \right] O_{ij}^p$$

  - A purely phenomenological attractive correction to $\langle H \rangle$

$$TNA = \gamma_2 \rho^2 (3 - 2\beta) \mathrm{e}^{-\gamma_3 \rho} \ , \ \ \beta = (\rho_p - \rho_n)/\rho$$

- the model parameters γ_1, γ_2 and γ_3 are fixed fitting the equilibrium propertes of symmetric nuclear matter

Effective vs bare potential @ $\rho = 0.16$ fm^{-3}



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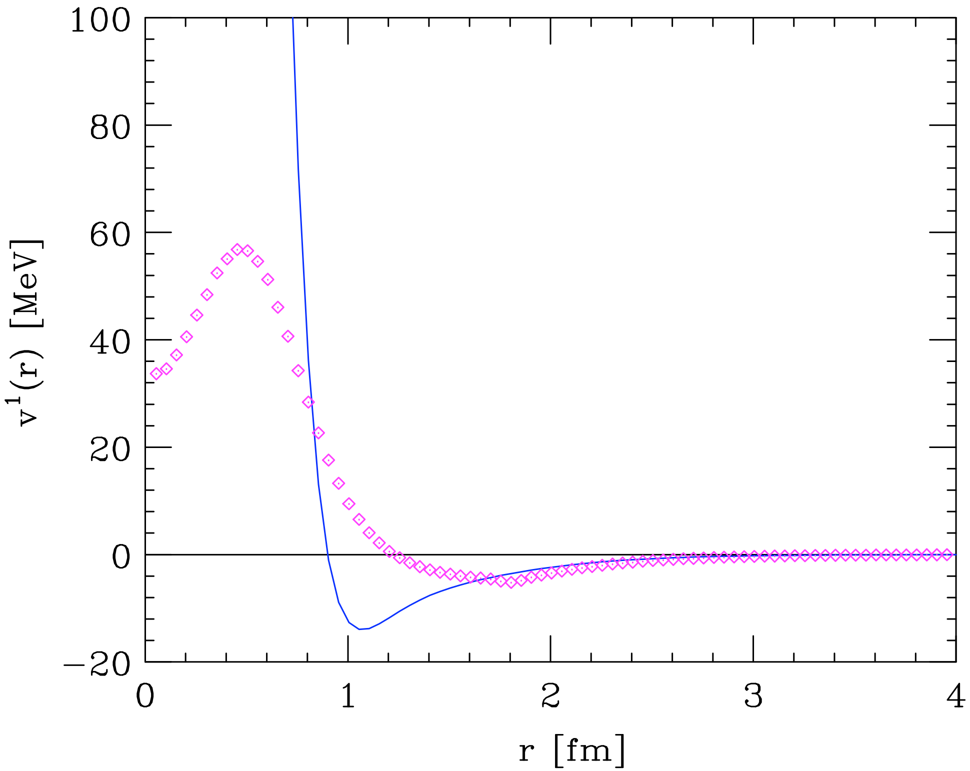
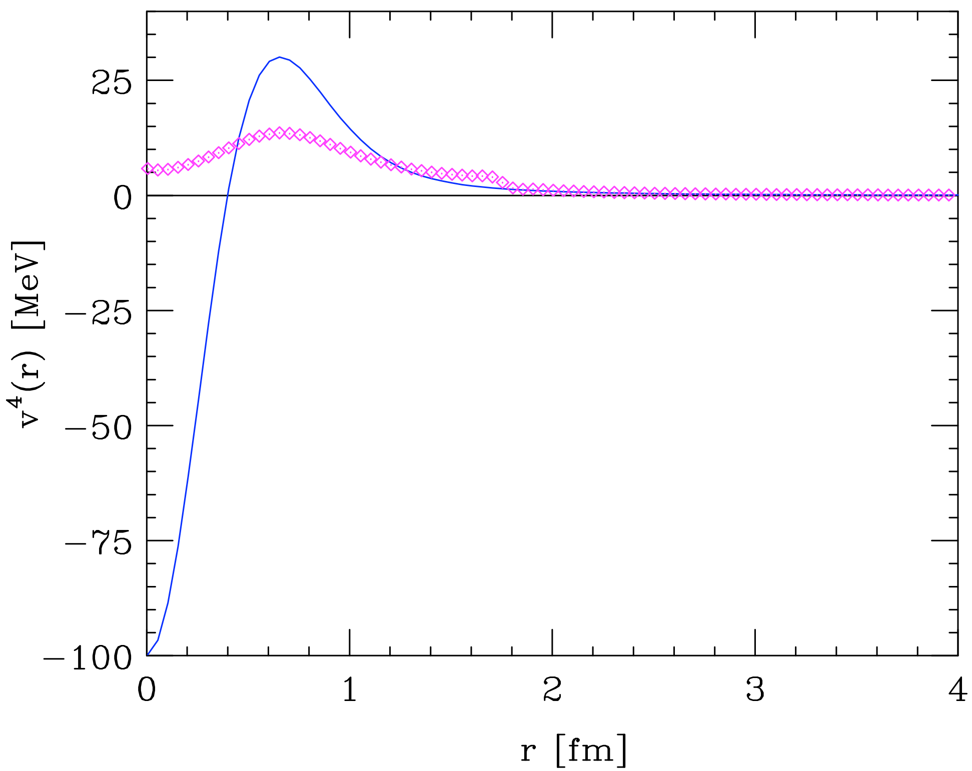
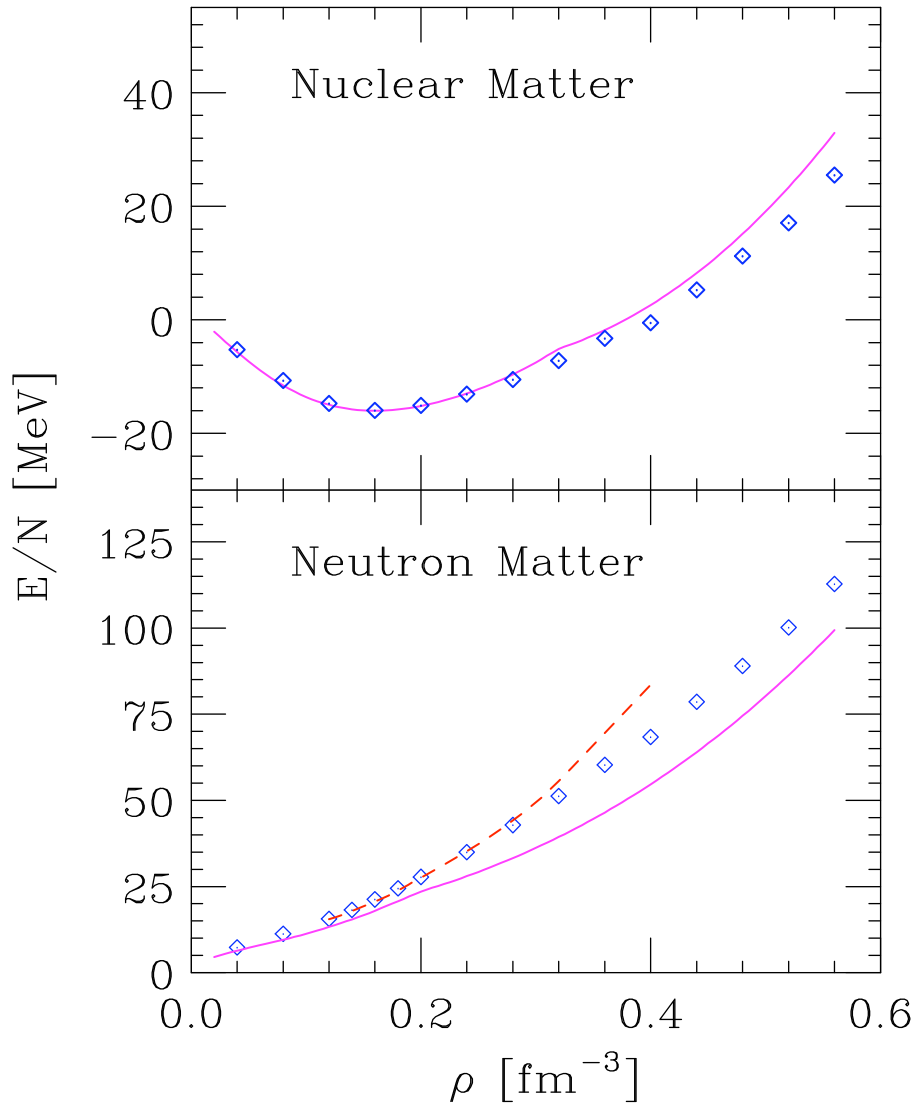




⋆ Using the effective interaction a variety of nuclear matter properties can be *consistently* calculated in *standard perturbation theory*

Energy per nucleon in matter

- ⋆ Diamonds: V_{eff}
- ⋆ Solid lines: Akmal, Pandharipande & Ravenhall (FHNC-SOC, ANL v_{18} + UIX)
- ⋆ Dashed line: Sarsa, Fantoni, Schmidt & Pederiva (AFDMC, ANL v_8' + UIX)



Transport in Fermi liquids: Abrikosov & Khalatnikov (AD 1957)

⋆ Starting point: Boltzman equation

$$\frac{\partial n}{\partial t} + \frac{\partial n}{\partial \mathbf{r}}\frac{\partial \epsilon}{\partial \mathbf{p}} - \frac{\partial n}{\partial \mathbf{p}}\frac{\partial \epsilon}{\partial \mathbf{r}} = I(n)$$

$$n = n_0 + \delta n \quad , \quad n_0 = \{1 + \exp[\beta(\epsilon - \mu)]\}^{-1}$$

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- Consider *shear viscosity* as an example. Using Landau theory of Fermi liquids AK obtain the *approximate* result

$$\eta_{AK} = \frac{1}{5}\rho m^* v_F^2 \tau \, \frac{2}{\pi^2(1-\lambda_\eta)}$$

⋆ quasiparticle lifetime and angle-averaged scattering probability (in the low temperature limit collisions only occur on the Fermi surface)

$$\tau T^2 = \frac{8\pi^4}{m^{*3}}\,\frac{1}{\langle W\rangle} \qquad \langle W\rangle = \int \frac{d\Omega}{2\pi}\,\frac{W(\theta,\phi)}{\cos\theta/2}$$

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⋆ exact solution by Brooker & Sykes (1968)

$$\eta = \eta_{AK} C(\lambda_\eta)$$

$$C(\lambda_\eta) = \frac{1-\lambda_\eta}{4} \sum_{k=0}^{\infty} \frac{4k+3}{(k+1)(2k+1)[(k+1)(2k+1) - \lambda_\eta]}$$

$$-2 < \lambda_\eta < 1 \quad , \quad 0.750 < C(\lambda_\eta) < 0.925$$

⋆ In medium scattering x-section

$$\frac{d\sigma}{d\Omega_{\mathbf{p}'}} = \frac{1}{16\pi^2} \frac{m^\star}{|\mathbf{p}_{rel}|} W(\theta,\phi) \;\; m^\star \, |\mathbf{p}'_{rel}|$$

⋆ Assuming that W is the same as in free space

$$W(\theta,\phi) = \frac{16\pi^2}{m^{\star 2}} \left(\frac{d\sigma_0}{d\Omega_{\mathbf{p}'}} \right)_{cm}$$

⋆ The transition probability can be obtained from the cross section measured in the center of mass frame at

$$E_{cm} = \frac{p_F^2}{2m}(1-\cos\theta) \;\; , \;\; \theta_{cm} = \phi$$

Effective mass from the effective interaction

⋆ Use

$$\frac{1}{m^\star} = \frac{1}{k}\frac{de_k}{dk}$$

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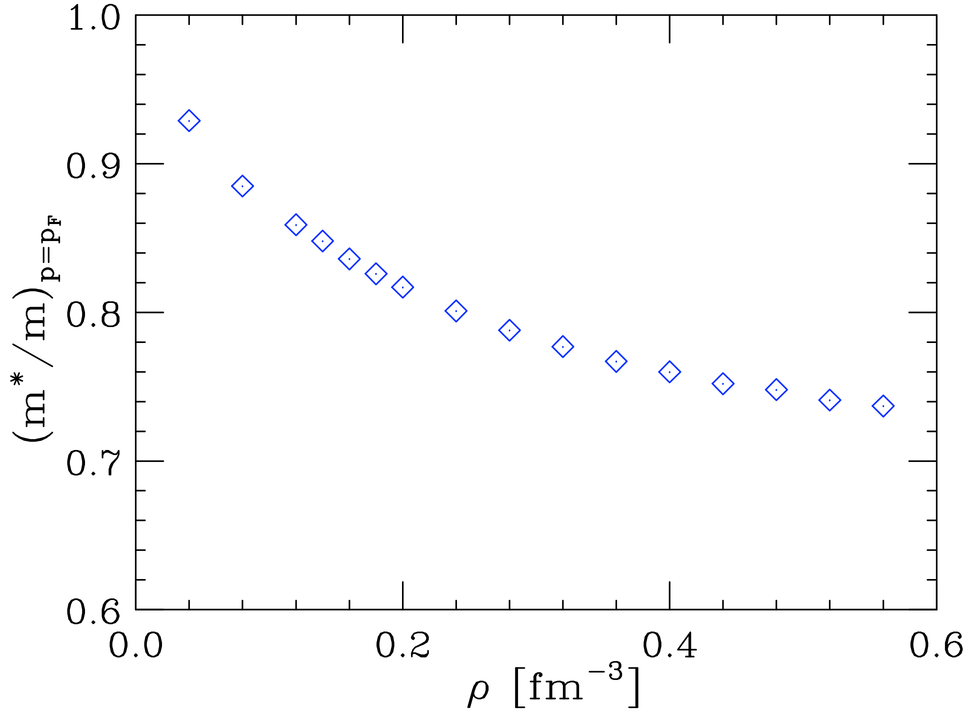

In medium scattering probability from the effective interaction

- From Fermi's golden rule

$$W(\mathbf{p},\mathbf{p}') = 2\pi \left|\hat{V}_{eff}(\mathbf{p}-\mathbf{p}')\right|^2 \rho(\mathbf{p}')$$

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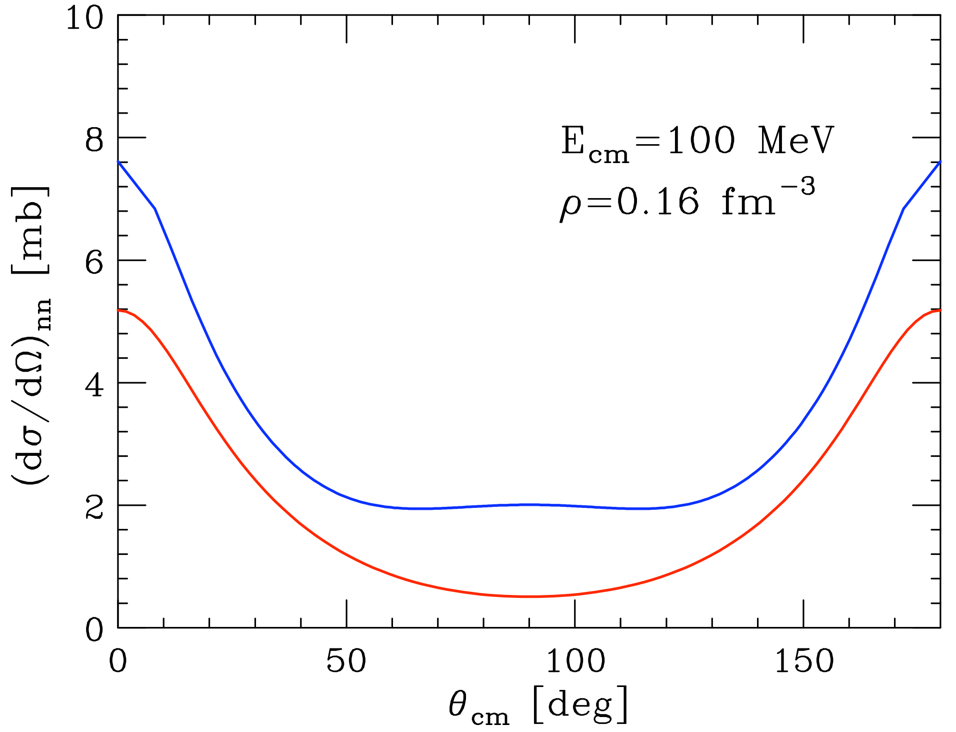
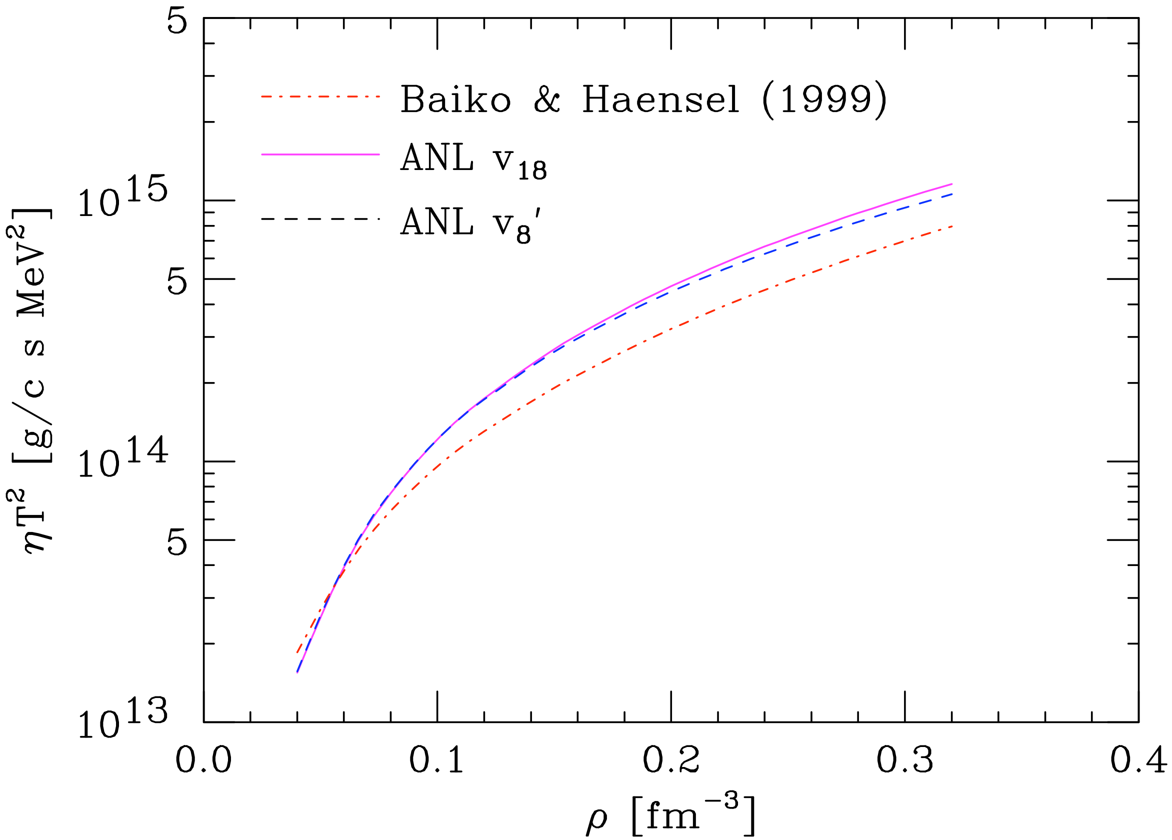
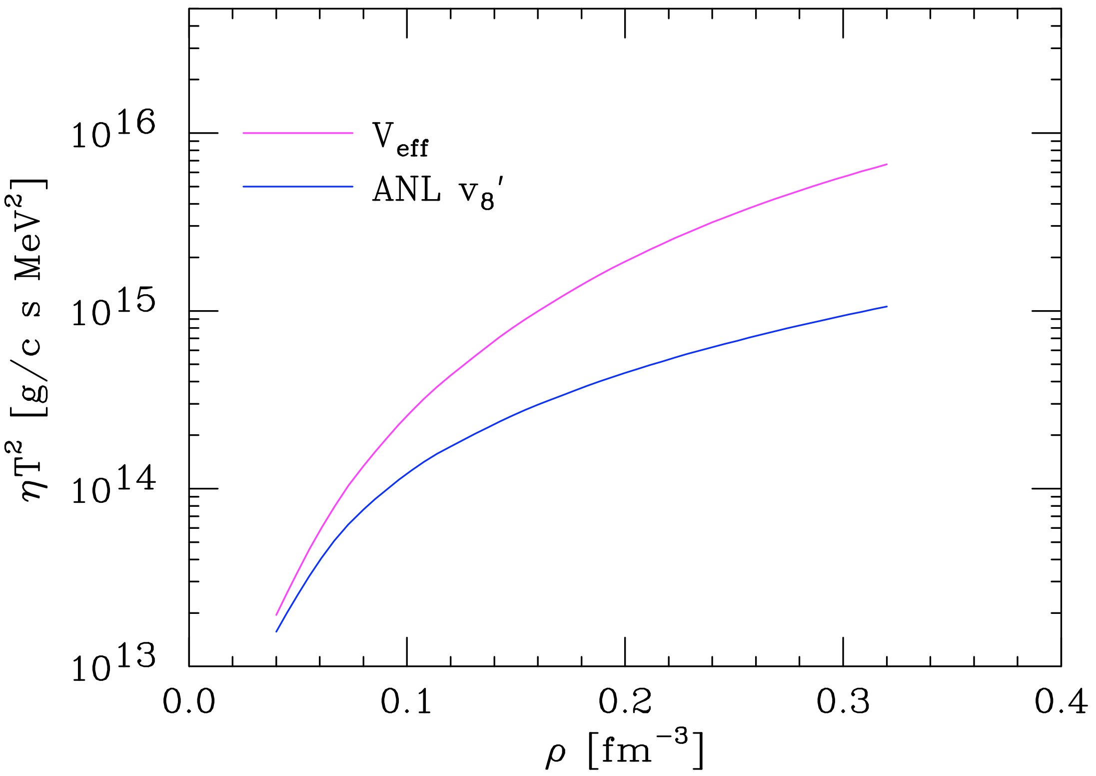


Viscosity from the free-space cross section

- ⋆ Results obtained from the free-space x-sections calculated using the ANL v_{18} and ANL v_8' (truncated at the spin-orbit level, $p = 8$) potentials
- ⋆ Baiko & Haensel calculation carried out using the measured scattering x-sections

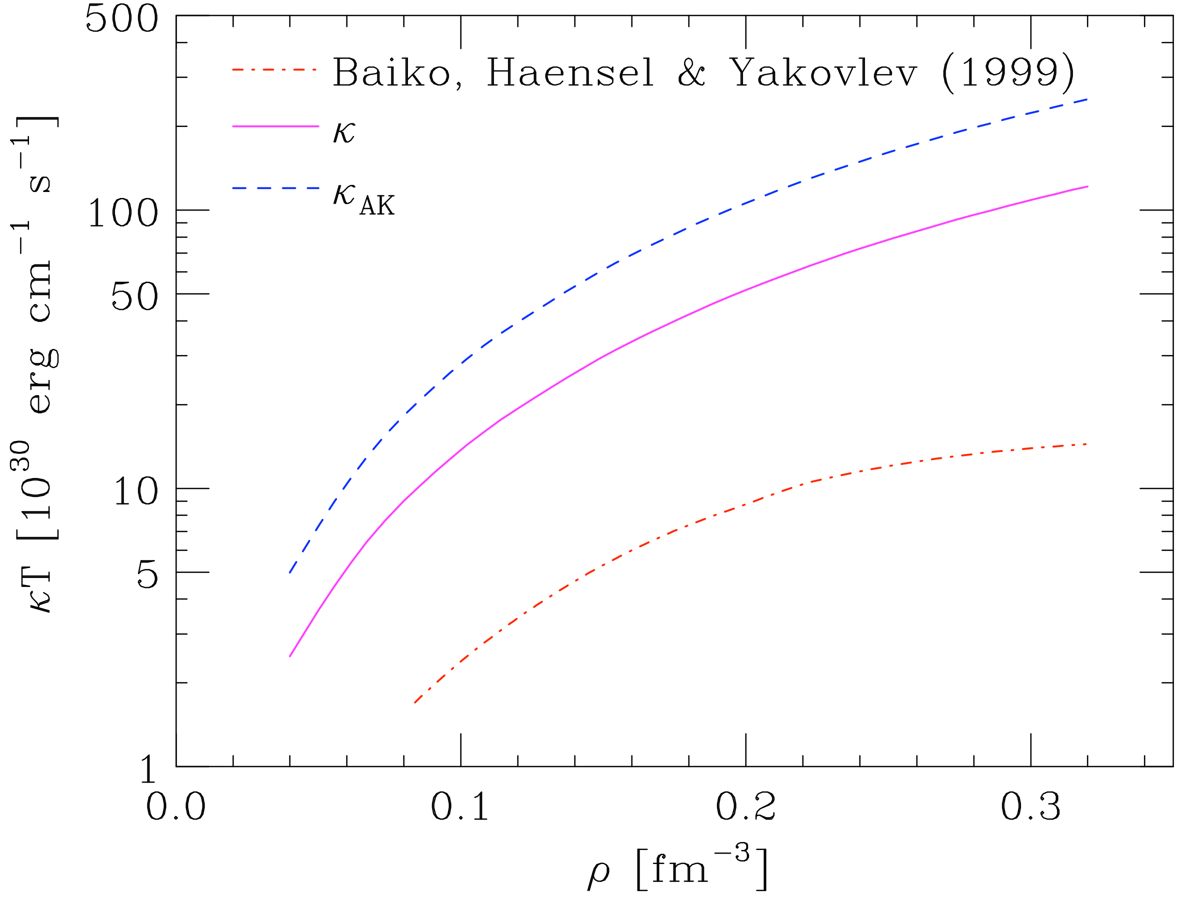


Viscosity from medium modified x-section



- ⋆ Medium effects increase ηT^2 by a factor $\sim 3 - 7$ @ $\rho/\rho_0 \sim 1 - 2$
- ⋆ Such a large increase likely to affect the onset of the instability of rotating neutron stars predicted by Chandrasekhar back in 1970

Thermal conductivity of pure neutron matter



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- The effective interaction, that naturally emerges from CBF theory and the cluster expansion formalism, provides a powerful tool to carry out calculations of a number of properties of neutron star matter - ranging from the EOS to single particle properties and transport coefficients - based on a highly realistic (i.e. strongly constrained by phenomenology) dynamical model.

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- Quantitative studied of the impact of the results obtained from the CBF effective interaction on neutron star cooling and the damping of neutron star oscillations are under way