Trinucleon Electromagnetic Form Factors and the Light-Front Hamiltonian Dynamics

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Outline

- Motivations
- Poincaré covariant EM Current in the Light-Front Hamiltonian Dynamics
- Bakamjian-Thomas-allowed Mass Operators
- Electromagnetic Observables: The Deuteron case (a resumé)
- Electromagnetic Observables: The Trinucleon case
- Conclusions & Perspectives

Some results already published + work in progress

F.M. Lev, E. Pace and G. Salmè, ”Electromagnetic and weak current operators for interacting Systems within the front-form dynamics”, Nucl. Phys. **A 641** (1998) 229.

F. M. Lev, E. Pace and G. Salmè, ”Poincaré covariant current operator and elastic electron-deuteron scattering in the front-form Hamiltonian dynamics”, Phys. Rev. **C 62** (2000) 0640004.

F. A. Baroncini, E. Pace and G. Salmè, ”Relativistic Hamiltonian Dynamics and Few-Nucleon Systems”, to be published in Few-Body Systems and arXiv:0712.0516 .

Motivations

- The *standard model of Few-Nucleon Systems, where nucleon and pion degrees of freedom are taken into account, is already at a very sophisticated stage, and many efforts are presently carried on in order to retain all the* general principles compatible with a theory where a fixed number of constituents is acting.

- On top of this, including relativity (as much as possible) represents an important goal, in view of the fact that i) the underlying theory is a local relativistic field theory, after all, and ii) the extraction of unambiguous signatures beyond the *standard model of Few-Nucleon Systems could be affected by relativistic effects.*

- *Field theoretical approaches based on the Bethe-Salpeter equations have been highly developed for two-nucleon System (Tjon, Gross,...), in particular within the so-called quasi-potential approximation (3D!), and for the trinucleon System many efforts are in progress.*

★ Aim : to construct a relativistic approach for Few-Nucleon System that i) retains the whole *successful phenomenology* already developed and ii) includes, *in a non perturbative way*, relativistic features, requested by Poincaré covariance.

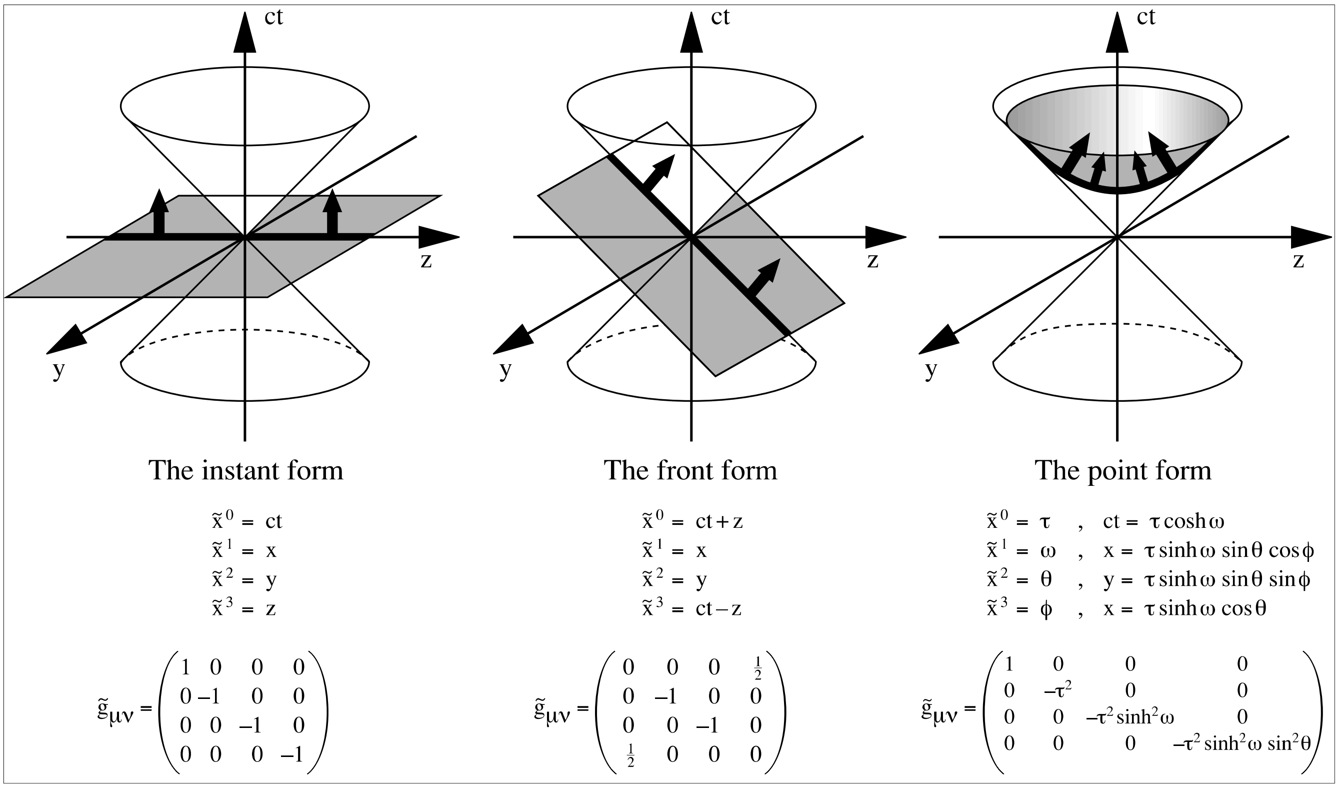
★★ A role for antinucleons ?

★ ★★ Electron scattering by Few-nucleon Systems yields a clean play ground for testing theoretical approaches.

Caveats: isobar configurations, MEC ....

Relativistic Hamiltonian Dynamics: a flash

- A reasonable compromise: i) fulfilling Poincaré covariance in a non perturbative way; ii) embedding the whole successful non relativistic phenomenology; iii) feasible numerical calculations; iv) fixed number of constituents; v) large class of allowed interactions.



After S.J. Brodsky, H.C. Pauli and S.S Pinsky, Phys. Rep. **301**, 299 (1998).

- Dirac proposed three forms for the the so-called Relativistic Hamiltonian Dynamics: Instant Form, Front Form or Light-Front Form (most widely adopted, a reduced impact by dynamics and $\longrightarrow$ light-cone DIS), Point Form (Dirac, Rev. Mod. Phys. 21 (1949) 392)

- Symmetries of the ”initial” hypersurface: the properties of invariance of the hypersurface, where the interacting System is ”sitting” at $\tau = 0$ (τ is the variable that labels the evolution of the System under the action of a Hamiltonian operator, containing the interaction) allows one to separate the 10 generators of the Poincaré group in two sets: the kinematical generators (that leave the initial hypersurface unchanged) and the dynamical ones (that move the System outside the initial hypersurface).

- Explicit construction of 10 generators, given the mass of the interacting System, (see the Bakamjian-Thomas approach PR 92 (1953) 1300)

- For A$\geq$ 3, cluster separability ($\rightarrow$ **Packing Operators**) must be implemented: macroscopic locality instead of the microscopic one, namely observables separated by space-like distances must commute (see. N.N. Sokolov Dokl. Akad. Nauk. 233 (1977) 575)

Summarizing : RHD rigorously fulfills the Poincaré covariance and, in some sense, falls between non-relativistic quantum mechanics and local relativistic field theory

Advantages and Drawbacks of choosing the Light front

Good news

- Maximal number of kinematical generators: 7.
- LF boosts form a subgroup, then the corresponding ”Wigner rotations” shrink to unity. This property allows one to separate intrinsic features of the System from the CM ones, in strict analogy with non relativistic theories.
- $P^+ \geq 0$. This produces a trivial vacuum in LF field theory with massive constituents. The physical vacuum coincides with the mathematical one, and therefore one can construct a meaningful Fock expansion from the LF vacuum.

Bad news

- Rotations around $\perp$ axes are dynamical.
- The case $P^+ = 0$ has to be carefully considered in theories with massless particles (zero-mode problem).

A possible approach for Nuclei: a Bakamjian-Thomas construction with a fixed number of constituents.

N.B. All the constituents are on their own mass shell (sharp difference from the explicitly covariant theory, more familiar...), then: $P^\mu \neq \sum_i p^\mu_{on}$. But, this on-mass-shell constraint allows one to define intrinsic variables as in the non relativistic case.

Electromagnetic Current Operator for Interacting Systems in the Light-Front Hamiltonian Dynamics

★To construct a current operator J^μ for an Interacting System that fulfills i) Poincaré, parity and time reversal covariance, ii) Hermiticity (F. Lev, E. Pace, G.S. Nucl. Phys. **A 641**, 229 (1998)),

★★one can translate those general properties into constraints on a suitable auxiliary operator, j^μ, acting on intrinsic variables.

★★★Extended Poincaré covariance of the EM current operator J^μ leads to the rotational covariance, around a given axis, of the auxiliary operator j^μ. The following analysis is performed in a Breit frame with the z-axis along the momentum transfer.

In a Breit frame where

$\boxed{\mathbf{q}_\perp = 0} \Rightarrow \boxed{q^+ \neq 0}$ and one can also choose

$\mathbf{P}_{f\perp} = \mathbf{P}_{i\perp} = 0$

The auxiliary operator acting on the intrinsic variables is

$$j^\mu(\mathbf{q}, M_f, M_i) \equiv \langle \mathbf{P}_{f\perp}, P_f^+ | \Pi_f \; J^\mu(0) \; \Pi_i | \mathbf{P}_{i\perp}, P_i^+ \rangle$$

where $\Pi \equiv$ projector onto the states of the (initial or final) System

If $J^\mu(0)$ is Poincaré covariant, then $j^\mu(\mathbf{q}, M_f, M_i)$ is rotationally covariant around $\widehat{q}$ and

$\boxed{\text{viceversa}}$

For an interacting System, a sensible approximation to $J^\mu(0)$ can be constructed through a suitable model for the auxiliary operator j^μ, starting with operators rotationally covariant.

In general

$$j^\mu_{if}(q\hat{e}_z) = \frac{\mathcal{J}^\mu_{if}(q\hat{e}_z)}{2} + \overbrace{L^\mu_\nu[r_x(-\pi)]\; e^{\imath\pi S_x} \frac{\mathcal{J}^\nu_{if}(q\hat{e}_z)^*}{2} e^{-\imath\pi S_x}}^{\text{from Hermiticity}}$$

i.e. $\left[j^\mu_{if}(q\hat{e}_z) \right]^* = j^\mu_{fi}(-q\hat{e}_z)$

$\mathcal{J}^\mu_{if}$ is a suitable approximation...

$\vec{S} \equiv$ the LF-spin operator of the System as whole, it acts on the ”internal” space and is unitarily related to the standard angular momentum operator through the Melosh operators

★ The current j^μ contains a many-body contribution produced by the presence of the dynamical operator S_x

★★ What about Current Conservation and Charge Normalization?

In the chosen Breit frame, CC and CN impose

$$CC: \quad j^+_{if}(q\hat{e}_z) = j^-_{if}(q\hat{e}_z)$$
$$CN: \quad j^+_{ii}(0) = e\,\Pi_i$$

Are those constrains satisfied by the simple approx.
$\mathcal{J}^\mu_{if}(q\hat{e}_z) \;=\; \Pi_f\; J^\mu_{free}(0)\,\Pi_i$?

In elastic processes, the Current Conservation follows from Poincaré covariance and Hermiticity (!) only.

Then

$$\mathcal{J}^\mu_{ii}(q\hat{e}_z) = \Pi_i\; J^\mu_{free}(0)\,\Pi_i$$

is acceptable ,

trivially one can show that $j^+_{if}(q\hat{e}_z) = j^-_{if}(q\hat{e}_z)$, within such an approximation

For fulfilling the Charge Normalization, one must have:

$$\mathcal{J}^{-}_{ii}(q\hat{e}_z) = \mathcal{J}^{+}_{ii}(q\hat{e}_z)$$

$$\text{but} \qquad \Pi_i \; J^{+}_{free}(0) \; \Pi_i \neq \Pi_i \; J^{-}_{free}(0) \; \Pi_i$$

N.B.: for evaluating of the EM form factors only $j^{+}_{ii}(q\hat{e}_z)$ and $j^{1(2)}_{ii}(q\hat{e}_z)$ are relevant.

To satisfy Poincaré, Hermiticity, CC and CN, one can choose, as a first step,

$$j^{\mu}(q\hat{e}_z) = \frac{\mathcal{J}^{\mu}(q\hat{e}_z)}{2} + L^{\mu}_{\nu}[r_x(-\pi)] \; e^{\imath\pi S_x} \frac{\mathcal{J}^{\nu}(q\hat{e}_z)^{*}}{2} e^{-\imath\pi S_x}$$

with

$$\mathcal{J}^{+}(q\hat{e}_z) = \mathcal{J}^{-}(q\hat{e}_z) = \Pi J^{+}_{free}(0)\Pi$$

$$\mathcal{J}^{1(2)}(q\hat{e}_z) = \Pi J^{1(2)}_{free}(0)\Pi$$

For a nucleus

$$J^{\mu}_{free}(0) = \sum_i J^{\mu}_{pi}(0)(1+\tau_3)/2 + J^{\mu}_{ni}(0)(1+\tau_3)/2$$

$$J^{\mu}_{N} = -F_{2N}(p^{\mu} + p'^{\mu})/2M + \gamma^{\mu}(F_{1N} + F_{2N})$$

N.B.: the presence of S_x introduces a class of two-body currents. More two-body currents in what follows.

Bakamjian-Thomas-allowed Mass Operators

For evaluating matrix elements of the EM Current, it is necessary to have eigenstates of the interacting System, that properly transform under the Poincaré group action.

The Bakamjian-Thomas construction suggests a necessary (not sufficient) condition (cf , e.g.,B.D. Keister and W. Polyzou Adv. Nucl. Phys. **20**, (1991)) for modeling a suitable mass operator, M: *the interaction term (e.g. the difference between M and the free mass, i.e.,$V = M - M_0$) must depend upon intrinsic variables combined in scalar products.*

More formally $[\vec{\mathcal{B}}_{LF}, V] = [\vec{S}_{free}, V] = [P_\perp, V] = [P^+, V] = 0$

where $\vec{\mathcal{B}}_{LF}$ are the LF-boosts, $\vec{S}_{free} \equiv$ the angular momentum operator for the non interacting case (note that $S^2_{free} = S^2_{int}$ and $S_{free,z} = S_{int,z}$, i.e. the eigenvalues of S^2_{free} and $S_{free,z}$ can be used for labeling the eigenstates of the interacting System)

Then, any *non relativistic* Mass operator, fulfilling the above constraints, can be adopted as an effective approximation of the *true* (?!) Mass operator, and embedded in a Bakamjian-Thomas construction.

Then the eigenfunctions to be used in a LF calculations, can be ”non relativistic solutions”, adopting standard Clebsch-Gordan machinery, $\boxed{\text{but}}$ introducing Melosh Rotations, that relate LF angular momenta to the standard ones.

Deuteron EM observables: a resumé

A first application: the Deuteron

Magnetic moment (in nuclear magnetons) and quadrupole moment (in fm^2) ; P_D is the D-state percentage. (LPS PRL **83** (1999) 5250)

$$\mu_d = \frac{m_p}{(\sqrt{2}m_d)} \lim_{Q\to 0} \frac{1}{Q}[\mathcal{J}^1_{1,0} - \mathcal{J}^1_{0,1}]$$

$$Q_d = \frac{\sqrt{2}}{m_d} \lim_{Q\to 0} \frac{1}{Q^2}[\mathcal{J}^+_{0,0} - \mathcal{J}^+_{1,1}]$$

where $\mathcal{J}^{1(+)}_{fi} = \langle m_d, S_z = f|\mathcal{J}^{1(+)}|m_d, S_z = i\rangle$

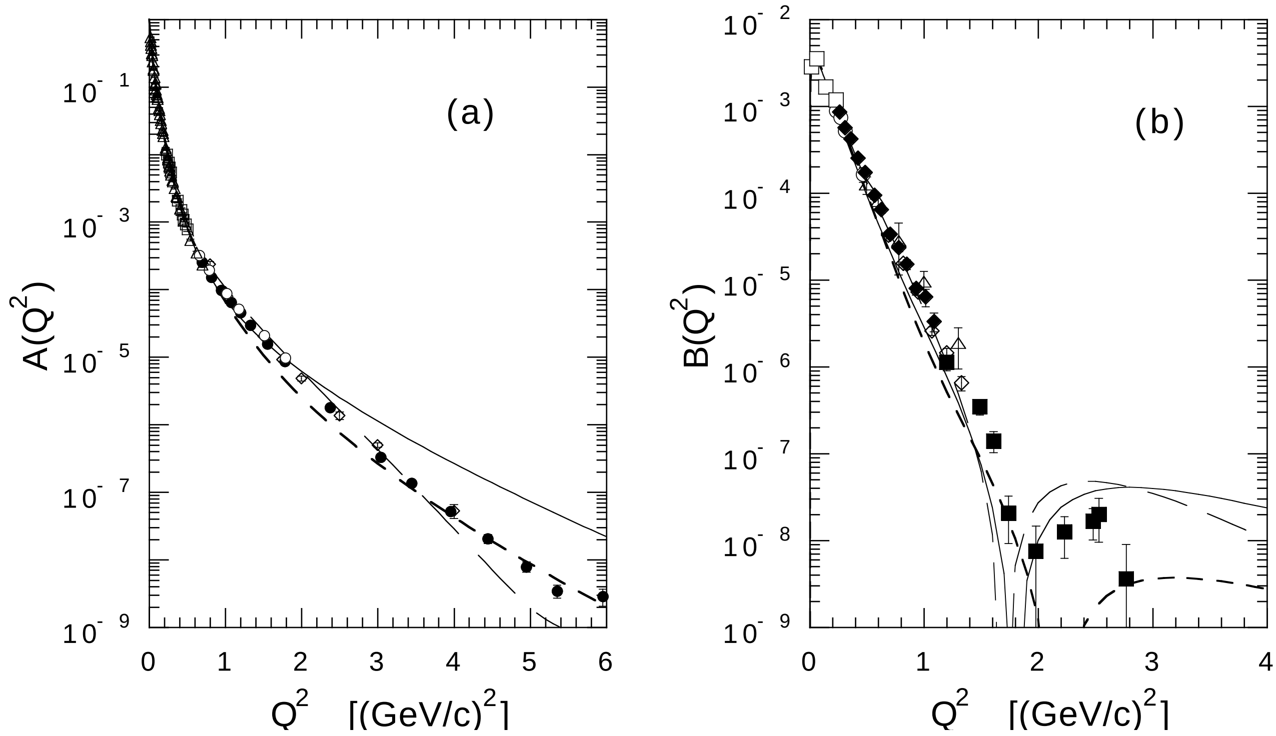
Interaction	P_D	μ_D^{NR}	μ_D^{LFD}	Q_D^{NR}	Q_D^{LFD}
CD-Bonn	4.83	0.8523	0.8670	0.2696	0.2729
Nijm1	5.66	0.8475	0.8622	0.2719	0.2758
RSC93	5.70	0.8473	0.8637	0.2703	0.2750
Av18	5.76	0.8470	0.8635	0.2696	0.2744

Exp.	0.857406(1)	0.2859(3)

Does the role of MEC shrink? Could the pair diagram (a well known relativistic effect) remove the remaining differences? In LF, instantaneous term is the relevant one for μ_D

Deuteron EM ff's (LPS, PRC **C 62** (2000) 0640004)

$$A(Q^2) = G_C^2 + \frac{8}{9}\tau^2 G_Q^2 + \frac{2}{3}\tau G_M^2$$
$$B(Q^2) = \frac{4}{3}\tau(1+\tau)G_M^2$$



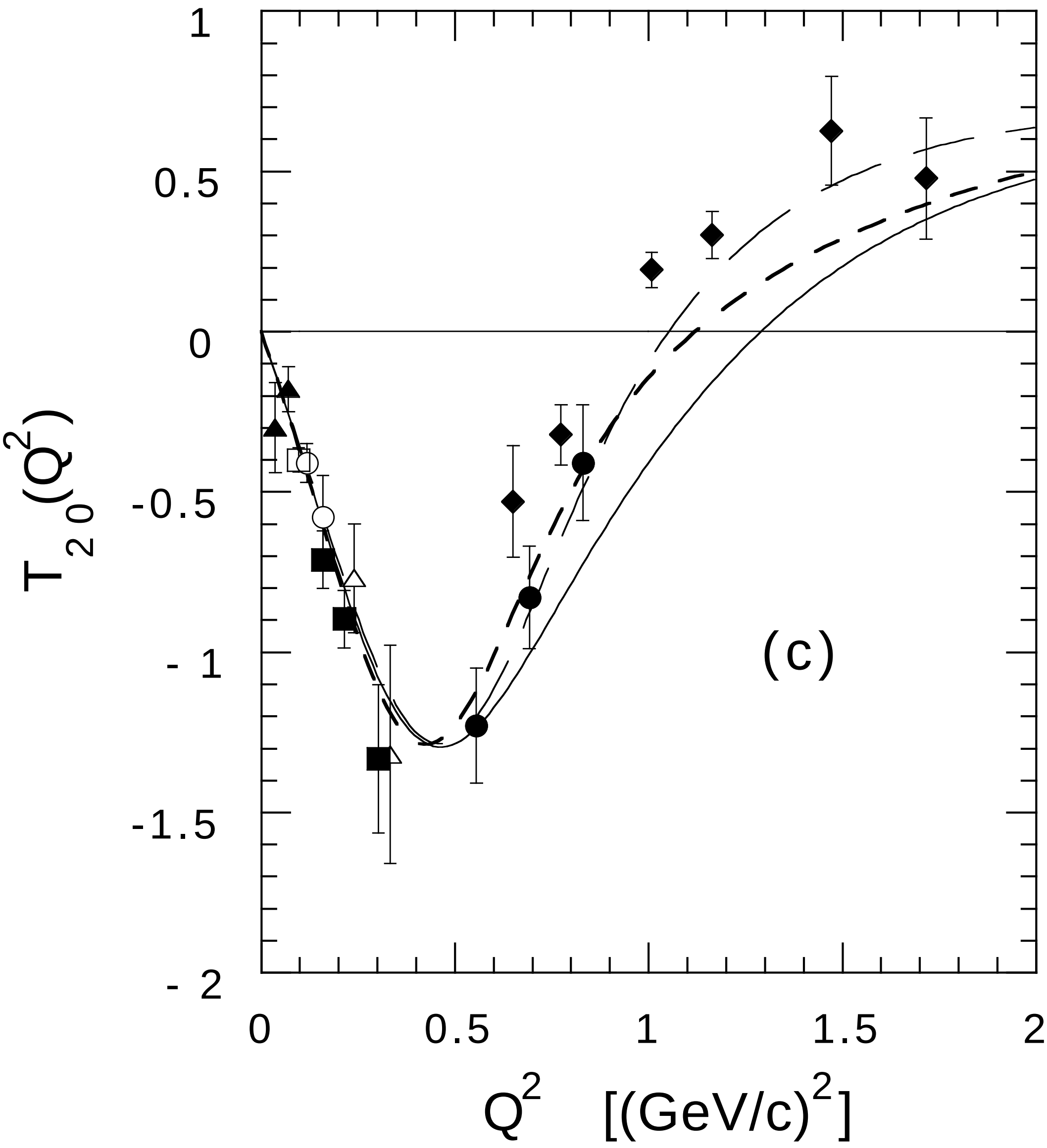
RSC $N-N$ interaction + Gari-Krümpelmann Nucleon ff's (ZPA **322** (1985) 689)

Solid line: full result with the Poincaré covariant current operator, in the Breit frame where $\boxed{\mathbf{q}_\perp = 0}$

Long-dashed line: non relativistic result in the same Breit frame.

Dashed line: the argument of the Nucleon ff's, $(p'_1 - p_1)^2 \rightarrow -Q^2$.

$$T_{20}(Q^2) = -\tau \frac{\sqrt{2}}{3} \frac{[\frac{4}{3}\tau G_Q^2 + 4 G_Q G_C + f G_M^2]}{A + B \tan^2(\theta/2)}$$



$RSC\ N-N$ interaction + Gari-Krümpelmann nucleon f. f.

Solid line: full result with the Poincaré covariant current operator in the Breit frame where $\boxed{\mathbf{q}_\perp = 0}$.

Long-dashed line: non relativistic result in the same Breit frame.

Dashed line: the argument of the Nucleon ff's, $(p_1' - p_1)^2 \to -Q^2$.

Conclusions: Dependence upon two-nucleon interactions and upon nucleon form factors. EM current operator has to be improved!

Trinucleon EM form factors

Ingredients

★ A Breit frame where

$$\boxed{\mathbf{q}_\perp = 0} \Rightarrow \boxed{q^+ \neq 0}$$

★★ A Poincaré covariant current

$$j^\mu(q\hat{e}_z) = \frac{\mathcal{J}^\mu(q\hat{e}_z)}{2} + L^\mu_\nu[r_x(-\pi)]\, e^{\imath\pi S_x} \frac{\mathcal{J}^\nu(q\hat{e}_z)^*}{2} e^{-\imath\pi S_x}$$

with

$$\mathcal{J}^+(q\hat{e}_z) = \mathcal{J}^-(q\hat{e}_z) = \Pi J^+_{free}(0)\Pi$$
$$\mathcal{J}^{1(2)}(q\hat{e}_z) = \Pi J^{1(2)}_{free}(0)\Pi$$

$\Pi \equiv$ projector onto the subspace of a trinucleon bound state $|\chi_{\frac{1}{2}}\rangle$ of mass M_T and spin $1/2$,
$J^\mu_{free}(0) = \sum_i J^\mu_{pi}(0)(1+\tau_3)/2 + J^\mu_{ni}(0)(1+\tau_3)/2$ with
$J^\mu_N = -F_{2N}(p^\mu + p'^\mu)/2M + \gamma^\mu(F_{1N} + F_{2N})$.

$$F^{T_z}_{ch}(Q^2) = \frac{1}{2}\, Tr[\mathcal{I}^+(T_z)] \qquad F^{T_z}_{mag}(Q^2) = -i\frac{M}{Q}\, Tr[\hat{\sigma}_y\, \mathcal{I}_x(T_z)]$$

with $\mathcal{I}^r_{\sigma'\sigma}(T_z) \equiv \langle\Psi^{\frac{1}{2}T_z}_{\frac{1}{2}\sigma'}, P'|\, \mathcal{J}^r\, |\Psi^{\frac{1}{2}T_z}_{\frac{1}{2}\sigma}, P\rangle$

★★★ A trinucleon bound state, obtained through a variational technique by Kievsky, Rosati, Viviani (NPA 577 (1994) 511) with two-body forces, AV18, and three-body ones, UIX. ^{3}He and ^{3}H are distinct, since the Coulomb forces are taken into account.

- S+S', P and D waves included
- Melosh Rotations fully considered
- 6D Montecarlo integrations

Table 1: Magnetic moments and charge radii of ^{3}He and ^{3}H. The two-body force, $Av18$, is included. $\mathcal{P}_{S+S'}(Av18) \sim 91.4\%$, $\mathcal{P}_P(Av18) \sim 0.07\%$, $\mathcal{P}_D(Av18) \sim 8.5\%$

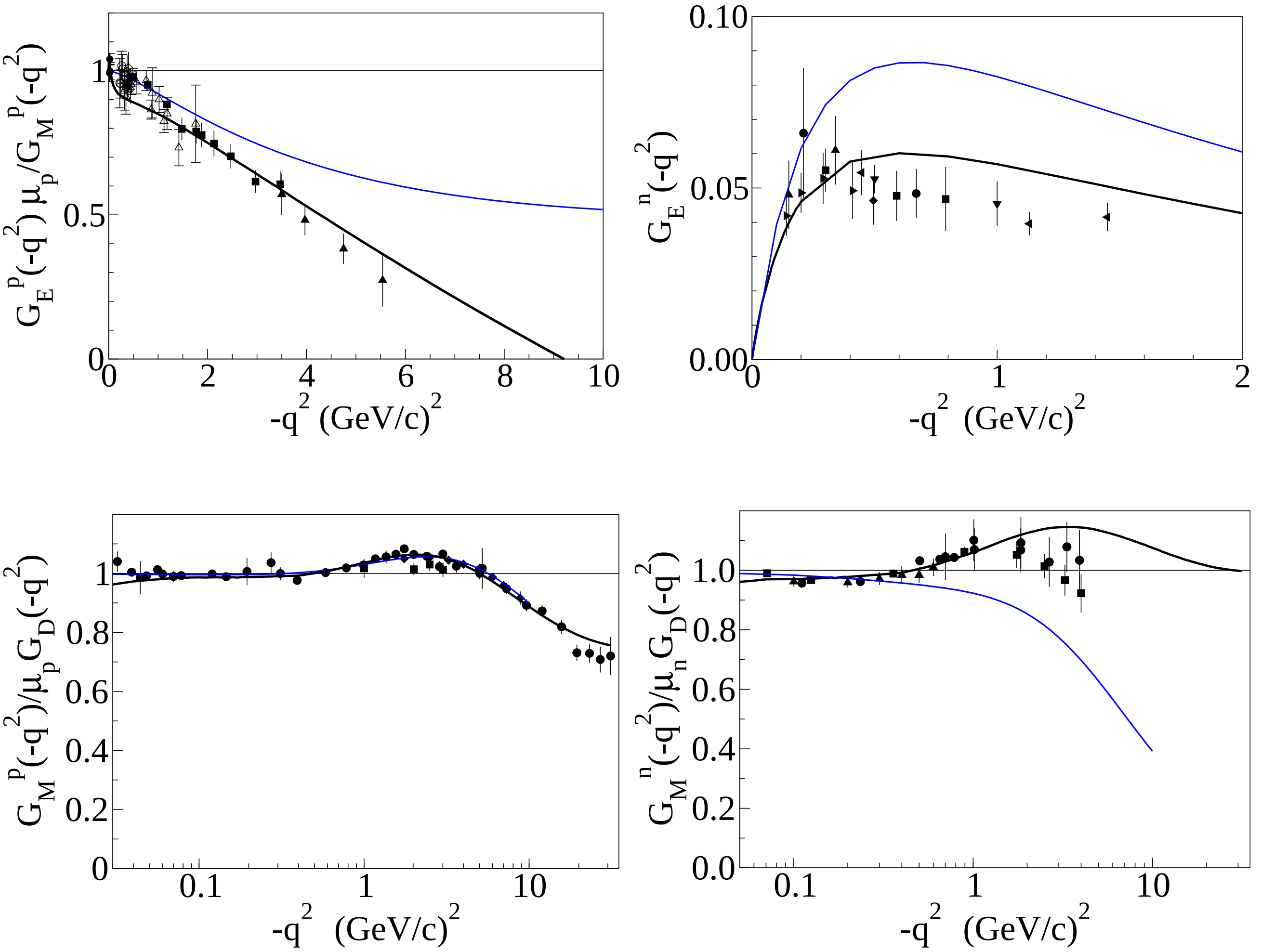
Theory	$\mu(^3\mathrm{He})$	$\mu(^3\mathrm{H})$	$r_{ch}(^3\mathrm{He})$fm	$r_{ch}(^3\mathrm{H})$fm
NR(S+S')	-1.700(1)	2.515(3)	1.926(3)	1.726(3)
LF(S+S')	-1.758(1)	2.600(3)	1.949(3)	1.771(3)
NR(S+S'+P+D)	-1.762(1)	2.579(2)	1.916(4)	1.718(4)
LF(S+S'+P+D)	-1.834(2)	2.674(2)	1.941(4)	1.759(4)
Exp.	-2.1276	2.9789	1.959(30)	1.755(86)

Table 2: Magnetic moments and charge radii of ^{3}He and ^{3}H Two- and three-body forces, $Av18 + UIX$, are included. $\mathcal{P}_{S+S'}(Av18 + UIX) \sim 90.5\%$ $\mathcal{P}_P(Av18 + UIX) \sim 0.01\%$ $\mathcal{P}_D(Av18 + UIX) \sim 9.3\%$.

Theory	$\mu(^3\mathrm{He})$	$\mu(^3\mathrm{H})$	$r_{ch}(^3\mathrm{He})$fm	$r_{ch}(^3\mathrm{H})$fm
NR(S+S')	-1.697(1)	2.494(2)	1.848(3)	1.695(3)
LF (S+S')	-1.759(2)	2.588(2)	1.870(3)	1.712(3)
NR(S+S'+P+D)	-1.760(1)	2.569(2)	1.841(4)	1.666(4)
LF (S+S'+P+D)	-1.837(2)	2.669(2)	1.867(4)	1.690(4)
Exp.	-2.1276	2.9789	1.959(30)	1.755(86)

Nucleon Electromagnetic Form Factors

In the calculation we used the Gari-Krümpelman Nucleon ff's and the ones recently obtained within a Light-front approach by de Melo, Frederico, Pace Pisano and G.S (arXiv:0804.1511, for the π see PRD 73 (2006))

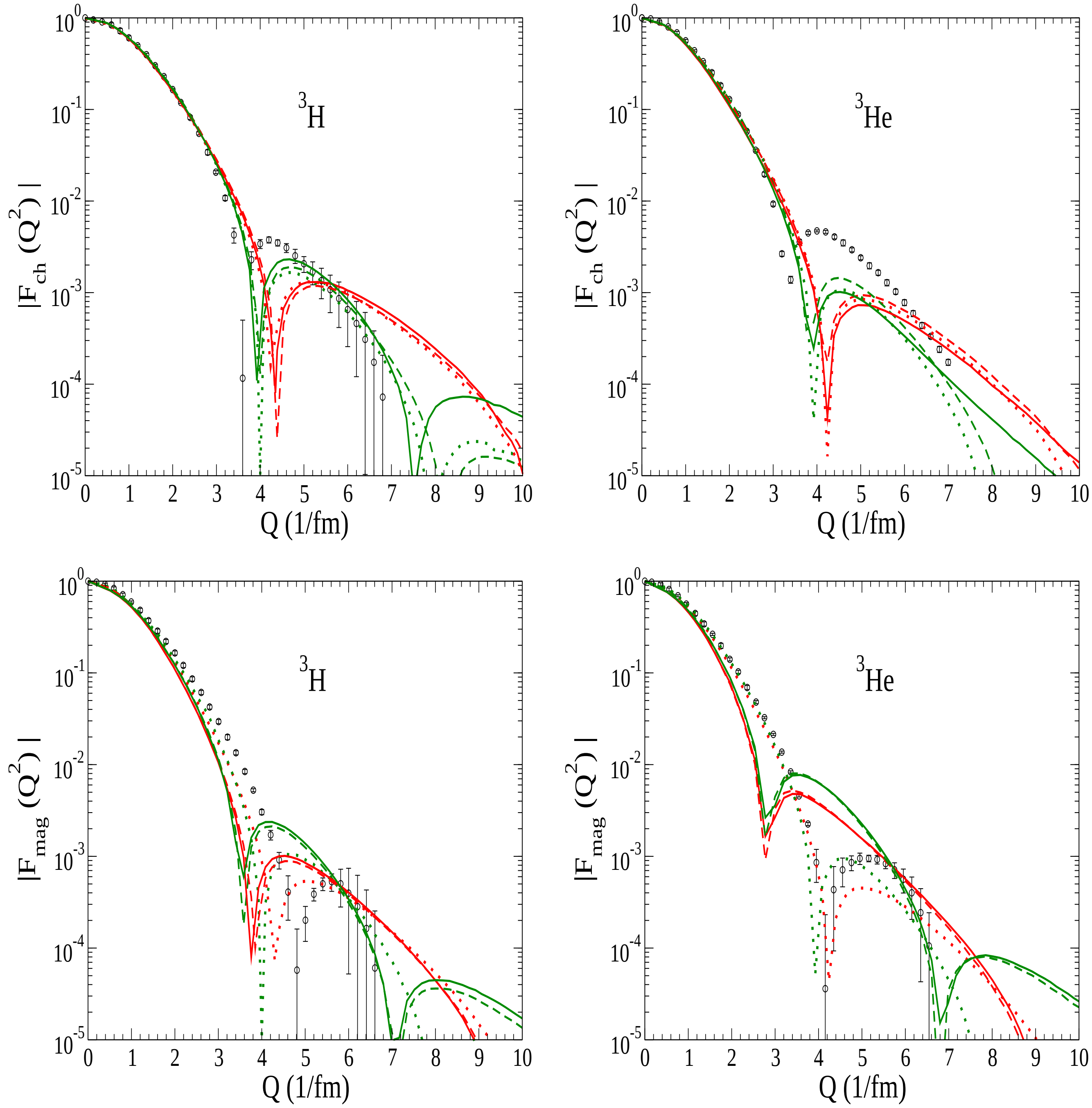


Solid line: LF Nucleon ff's

Solid line: Gari-Krümpelman

Charge and Magnetic form factors of ^{3}H and ^{3}He in a frame where $\boxed{\mathbf{q}_\perp = 0}$, and AV18 Two-body forces + Coulomb

For the first time in LFD **!**, but without two-body dynamical currents

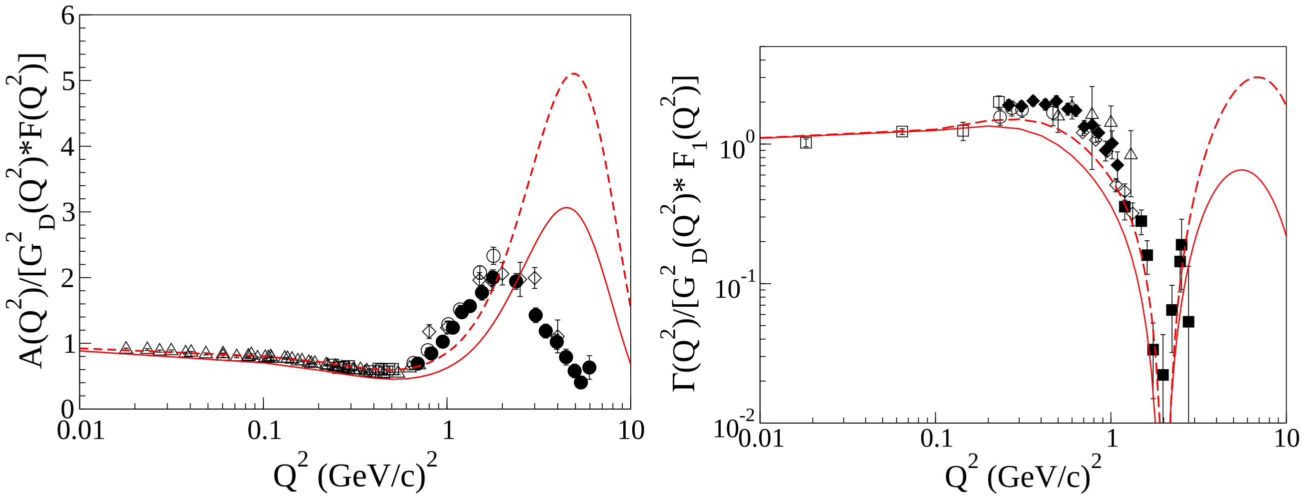


Solid line: LF full calculation ($S + S' + P + D$) and LF Nucleon ff's; dashed line: LF full calculation and Gari-Krümpelman N ff's; dotted line: $S + S'$ waves + GK.

In Green color, NR calculations

- Standard drawbacks point to an improving of the LF current operator
- Sizable relativistic effects on the ff's tails. Interesting region: $Q > 7\ (1/fm)$.
- Low dependence upon Nucleon ff's (different from the Deuteron).

Deuteron form factors with AV18 & Gari-Krümpelmann vs. LF Nucleon ff's (isoscalar combination)

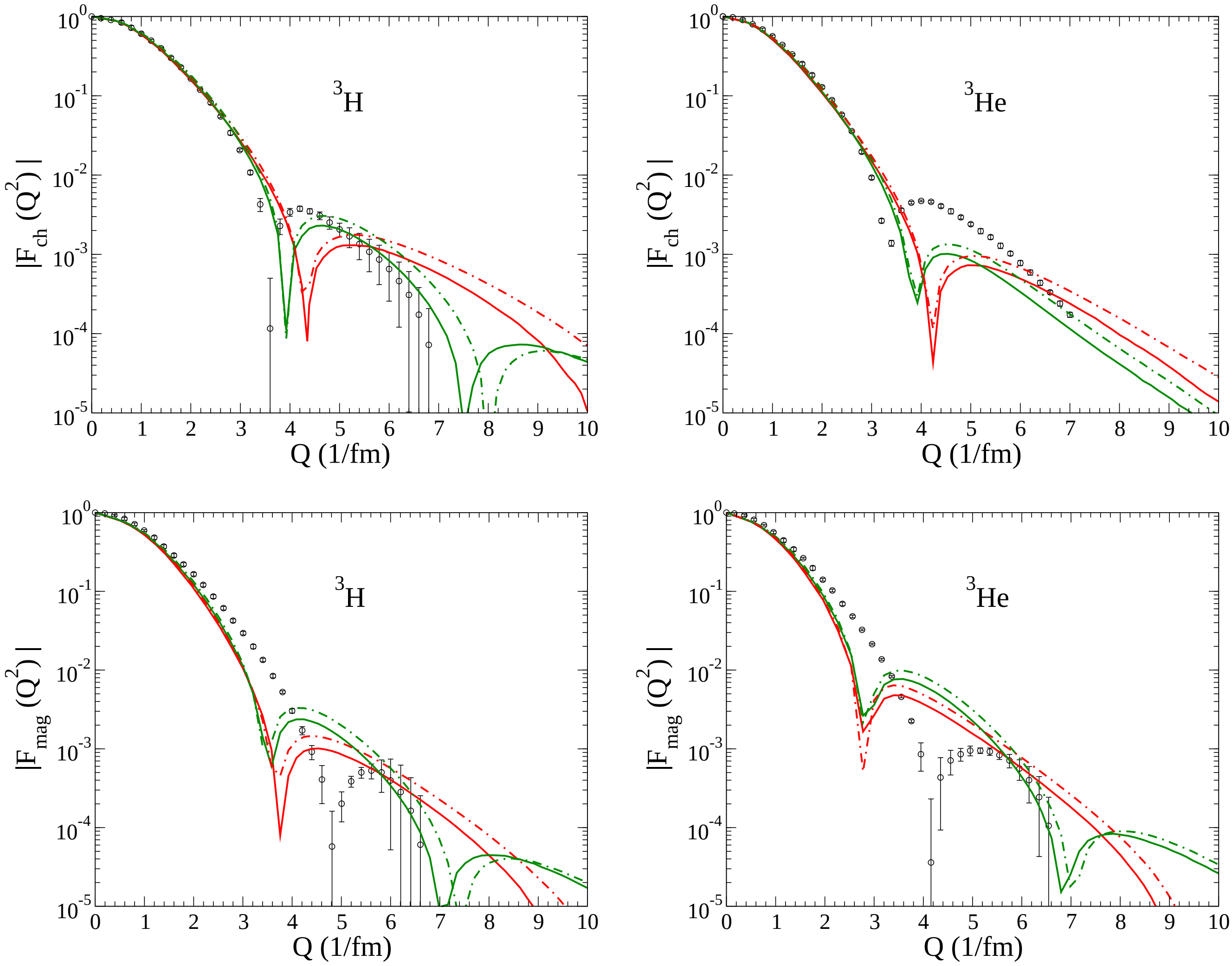


Solid line: AV18 + LF Nucleon ff's

Dashed line: AV18 + GK Nucleon ff's

Charge and Magnetic form factors of ^{3}H and ^{3}He in a frame where $\boxed{\mathbf{q}_\perp = 0}$, and AV18(2BF) + UIX (3BF)

For the first time in LFD !, but without two-body dynamical currents



Solid line: LF calculation with 2BF (AV18) & LF Nucleon ff's.

Dash-dotted line: LF calculation with 2BF+3BF (AV18+UIX) & LF Nucleon ff's.

In Green color NR calculations

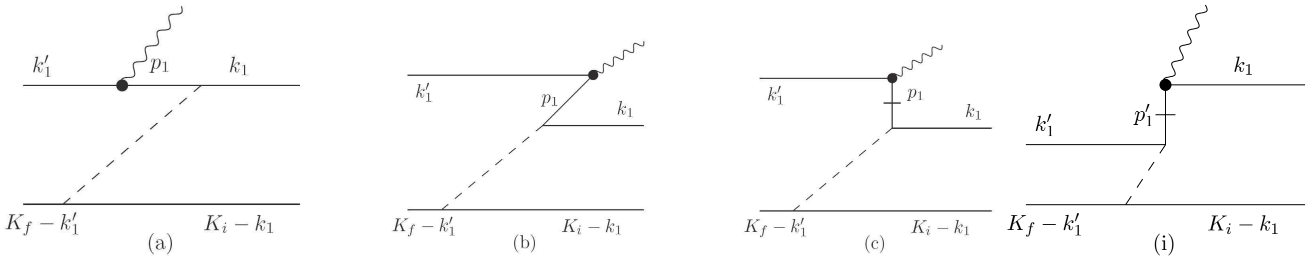
Tails increase under the effect of 3BF's, that produce more binding (smaller charge radii)!

Two-Body Current in the LF approach

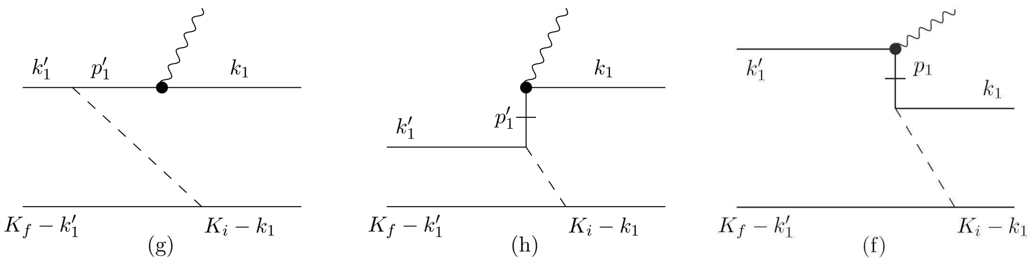
Main lesson: more two-body currents are necessary!

A simple model: from a 4D Yukawa model, in ladder approximation, one obtains a 3D current on the LF, that fulfills the Ward-Takashi Identity (also the general case in Frederico, Marinho, Pace, Sauer, G.S.: arXiv:0805.0707 and PRD in print).

The first-order current operator is (at least three particles in flight)



pair term instantaneous in LF time



instantaneous in LF time

Inspired by such a model we are evaluating the two-body current contribution for the Deuteron. Actual calculations in progress.

One can anticipate that i) the pair term affects all the three ff's, while the instantaneous term (present only for fermions) contributes to the magnetic one, ii) the pair term vanishes for $q^+ \to 0$, as it must do, while the instantaneous one survives, iii) the remaining, on-mass shell, term affect all the Deuteron ff's in the whole range of q^+, i) the pair term should be maximal at $q^+ \sim m_N$

Conclusions & Perspectives

In order to construct a *Standard Model for Few-Nucleon Systems it is necessary to take into account relativistic effects*

- We have extended our analysis of the elastic EM observables from Deuteron $\rightarrow$ ^{3}H and ^{3}He, within the Light-Front Hamiltonian Dynamics (Bakamjian-Thomas construction), taking profit of the successful phenomenology developed for Few-Nucleon Systems, and fulfilling both extended Poincaré covariance and Hermiticity
- Few % effects for observables at $Q^2 = 0$, but in the correct direction. Sizable effects for $Q > 1.5\ GeV/c$. An interesting dependence upon 3BF beyond the same limit.
- A Systematic analysis of a new class of two-body currents for the Deuteron is in progress.