Electromagnetic Form Factors of Hadrons and Higher Fock Components

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Pion ff in the space-like region $\rightarrow$ J. P. B. C. de Melo, T. Frederico, E. Pace, G. Salme', Nucl. Phys. **A 707** (2002) 399

Pion ff in the space-like and time-like regions $\rightarrow$ J. P. B. C. de Melo, T. Frederico, E. Pace, G. Salme', Phys. Lett. **B 581** (2004) 75

Space-like and time-like pion electromagnetic form factor and Fock state components within the Light-Front dynamics $\rightarrow$ J. P. B. C. de Melo, T. Frederico, E. Pace, G. Salme', Phys. Rev. D **73**, 074013 (2006)

Outline

- Motivations

- A covariant expression for the EM current: the Mandelstam Formula

- Pion EM Form Factor in the space- and time-like regions
- Nucleon EM Form Factors in the space-like region, PRELIMINARY

- Conclusion & Perspectives

Motivations

The investigation of hadron EM form factors in the space- and time-like regions, within the light-front dynamics,

- opens a unique possibility to study the hadronic state, in both the valence and the nonvalence sector (Brodsky, Pauli & Pinsky, Phys. Rep. **301** (1998) 299)

$$|meson\rangle = \ |q\bar{q}\rangle + |q\bar{q}q\bar{q}\rangle + |q\bar{q}\ g\rangle.....$$

$$|baryon\rangle = \underbrace{|qqq\rangle}_{\text{valence}} + \underbrace{|qqq\ q\bar{q}\rangle + |qqq\ g\rangle.....}_{\text{nonvalence}}$$

★ A meaningful Fock expansion within LF framework

★★ Zero modes $\rightarrow \chi$SB for fermions

- yields the possibility to address the vast phenomenology of hadronic resonances (Vector Meson propagation...) in the time-like region, and then to impose strong constraints on dynamical models pointing to a microscopical description of hadrons

- allows one to obtain insights into the two-body currents associated to the $q\bar{q}$ pair production, (very important in reference frames where $q^+ \neq 0$).

The Mandelstam Formula for the EM current

Our guidance ⇒ the Mandelstam formula, that yields a covariant expression for the em current of the hadron.

For a global investigation of SL and TL regions we need to change frame, from the $q^+ = 0$ frame (a standard choice within LF) to a $q^+ \neq 0$ frame (F.M. Lev, Pace and G.S. NPA 641 (1998) 229).

In the TL region

$$j^\mu = -\imath e \int \frac{d^4k}{(2\pi)^4} \, Tr \left[S_Q(k - P_h) \lambda_{\overline{h}}(k - P_h, P_{\overline{h}}) \, S(k - q) \times \right.$$
$$\left. \Gamma^\mu(k, q) \, S(k) \overline{\lambda}_h(k, P_h) \right]$$

- $S(p) = \dfrac{1}{\not p - m + \imath\epsilon}$, with m the mass of the constituent quark struck by the virtual photon
- $S_Q(p)$ is the propagator of the spectator constituent quark or diquark (in a simple picture of baryons)
- $\lambda_h(k, P_h)$ is the hadron vertex function; P_h^μ and $P_{h'}^\mu$ are the hadron momenta. Very important: it contains a Dirac structure, i.e. a proper combination of Dirac matrices.
- $\Gamma^\mu(k, q)$ is the quark-photon vertex (q^μ the virtual photon momentum)

TL → SL $\qquad P_h^\mu \;\rightarrow\; -P_h^\mu$

(The initial hadron vertex becomes $\lambda_h(-k, P_h)$).

A first application $\Rightarrow$ Pion.

In the TL region one has

$$j^\mu = -\imath 2e\frac{m^2}{f_\pi^2}N_c\int\frac{d^4k}{(2\pi)^4}\Lambda_{\bar\pi}(k-P_\pi,P_{\bar\pi})\overline{\Lambda}_\pi(k,P_\pi)\times$$
$$Tr[S(k-P_\pi)\,\gamma^5 S(k-q)\,\Gamma^\mu(k,q)\,S(k)\,\gamma^5]$$

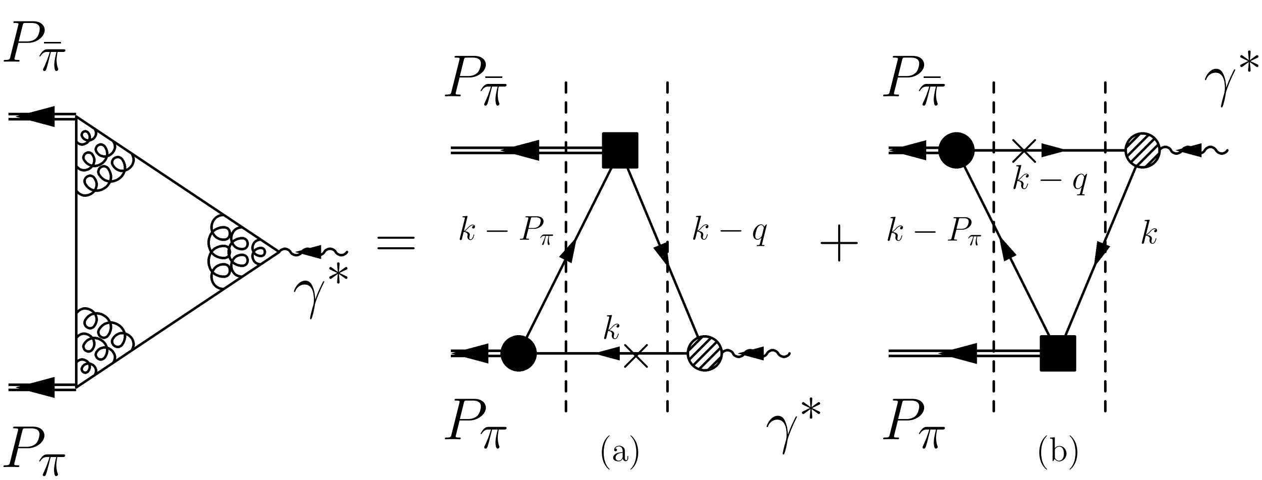
γ_5 is the Dirac structure in $\lambda_\pi(k,P_\pi)$, from a simple effective quark-pion Lagrangian

Projecting out the Mandelstam Formula on the Light Front

...through a k^- integration, (only the poles of the Dirac propagators taken into account), in a reference frame where

$$q^+ > 0\,,\quad \mathbf{q}_\perp = 0$$

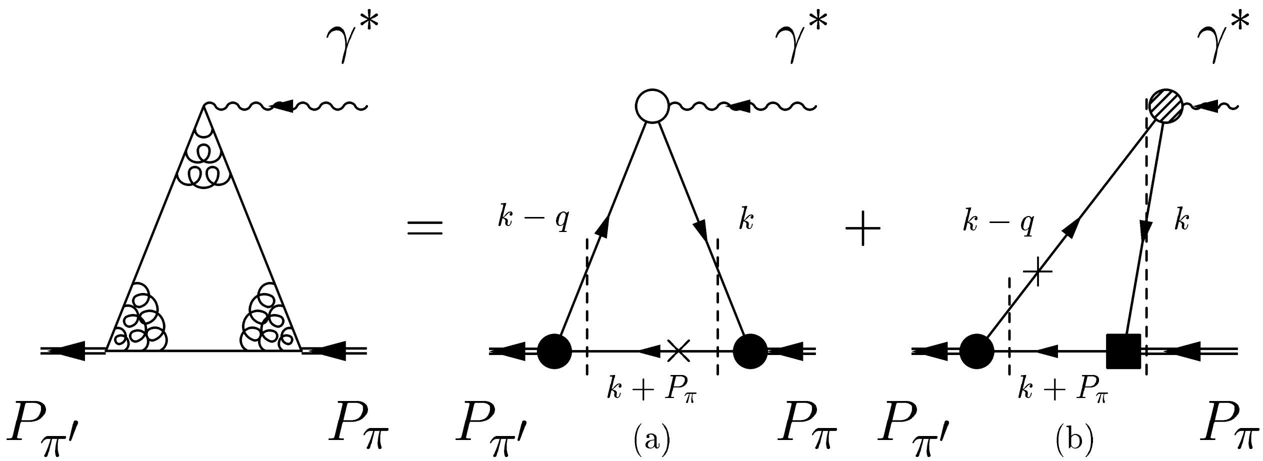
Time-like region



(val.) $0 < k^+ < P_\pi^+$ $\quad$ $P_\pi^+ < k^+ < q^+$ (non-val.)

$\times \;\Rightarrow\;$ k on its mass shell : $k^-_{on} = (m^2 + k_\perp^2)/k^+$

Space-like region



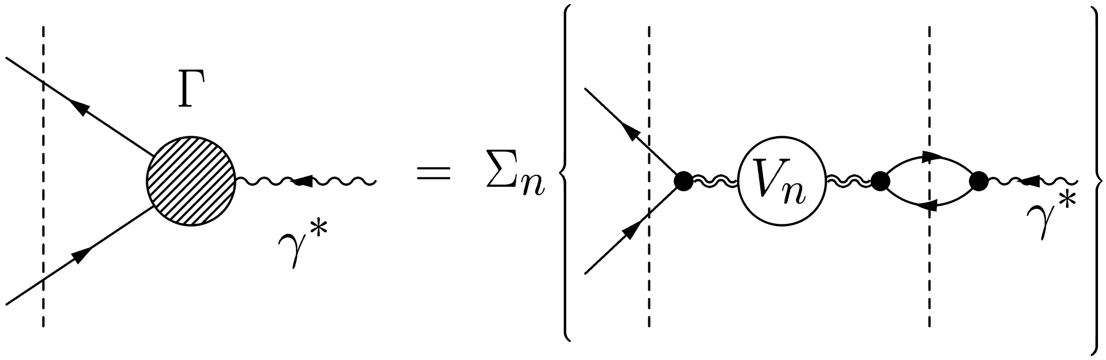
$0 < k^+ + P_\pi^+ < P_\pi^+$ $\quad\quad$ $0 < k^+ < q^+$

In (b), a $q\bar{q}$ pair is produced by the virtual photon.

⋆ First Problem: How to model the quark-photon vertex ? ⋆

⋆ ⋆ Second Problem: How to connect the Fock language with the Bethe-Salpeter one, e.g. how to describe the amplitude for the emission or absorption of a pion by a quark, ■(non valence component) , and the $q\bar{q}$-pion vertex,● (valence component)? ⋆ ⋆

In the limit $m_\pi \to 0$, only one diagram contributes both in TL and SL regions: the one where where the non valence component (higher Fock components) is acting. Then, the quark-photon vertex is dominated by the $q\bar{q}$ production.



⋆ A Vector Meson Dominance approximation has been applied to the quark-photon vertex, when a $q\bar{q}$ pair is produced

$$\Gamma^{+}(k,q) = \sqrt{2}\sum_{n,\lambda}\left[\epsilon_\lambda \cdot \widehat{V}_n(k,k-q)\right]\Lambda_n(k,P_n) \times \frac{[\epsilon^{+}_\lambda]^* f_{Vn}}{(q^2 - M_n^2 + \imath M_n \Gamma_n(q^2))}$$

- f_{Vn} is the decay constant of the n-th vector meson into a virtual photon (to be calculated in our model **!**), M_n the mass, $\Gamma_n(q^2) = \Gamma_n q^2/M_n^2$ (for $q^2 > 0$) the corresponding total decay width and ϵ_λ is the VM polarization
- $\left[\epsilon_\lambda \cdot \widehat{V}_n(k,k-q)\right]\Lambda_n(k,q) \equiv$ VM vertex function.

$$\widehat{V}_n^{\mu}(k,k-q) = \gamma^\mu - \frac{k^\mu_{on} - (q-k)^\mu_{on}}{M_0(k^+,\mathbf{k}_\perp;q^+,\mathbf{q}_\perp) + 2m} \quad ,$$

generates the proper Melosh rotations for $^3S_1$ states. M_0 is the standard light-front free mass. [Jaus PRD 41 (1990) 3394)]

$\Lambda_n(k,q)$ is the momentum-dependent part of the Bethe-Salpeter amplitude.

The decay constant, f_{Vn}, is evaluated assuming that: i) $\Lambda_n(k, P_n)$ does not diverge in the k^- complex-plane for $|k^-| \to \infty$, and ii) the contributions of its singularities in the k^- integration are negligible.

$$f_{Vn} = -\frac{N_c \; P_n^+}{4(2\pi)^3} \int_0^{P_n^+} \frac{dk^+ \; d\mathbf{k}_\perp}{k^+ \; (P_n^+ - k^+)} \frac{\Lambda_n(k, P_n)|_{[k^- = k_{on}^-]}}{[M_n^2 - M_0^2(k^+, \mathbf{k}_\perp; P_n^+, \mathbf{P}_{n\perp})]}$$
$$Tr\left[(\not k - \not P_n + m)\gamma^+(\not k + m)\widehat{V}_{nz}(k, k - P_n)\right] .$$

⋆⋆ The k^-−shell amplitude of the VM in the valence sector, $0 < k^+ < P_n^+$, has been related to the light-front VM wave function

$$\Lambda_n(k, P_n)|_{[k^- = k_{on}^-]} = \psi_n(k^+, \mathbf{k}_\perp; P_n^+, \mathbf{P}_{n\perp}) \; \frac{[M_n^2 - M_0^2(k^+, \mathbf{k}_\perp; P_n^+, \mathbf{P}_{n\perp})]}{P_n^+}$$

$\psi_n(k^+, \mathbf{k}_\perp; P_n^+, \mathbf{P}_{n\perp})$ is

- an eigenfunction of a relativistic CQ square mass operator (Frederico, Pauli & Zhou, PRD 66 (2002) 116011), with confinement (harmonic oscillator potential) and $\pi - \rho$ splitting (Dirac-delta interaction in the pseudoscalar channel). A natural explanation of the "Iachello-Anisovitch law" ($M_n^2 \sim M_{gr}^2 + \omega \; (n-1)$; n is the radial quantum number) is obtained. No isospin breaking ($\rho \equiv \omega$).

- normalized to the probability of the lowest $(q\bar{q})$ Fock state (i.e. the valence component). The $q\bar{q}$ probability can be roughly estimated in a simple model (de Melo et al. PRD 73, (2006) 074013) that reproduces the "Iachello-Anisovitch law", for the meson mass spectra.

$\star\star$ In order to describe the emission (absorption) of a pion by a quark, we assumed a point-like interaction [Choi & Ji (PLB 513 (2001) 330)]. In the present approach, the coupling constant is fixed by the normalization of the pion form factor.

$\star\star$ A new issue: the instantaneous contributions.

What does "instantaneous" mean ?

Let us consider the free Dirac propagator

$$\frac{\not k + m}{k^2 - m^2 + \imath\epsilon} = \frac{\not k_{on} + m}{k^+(k^- - k^-_{on} + \frac{\imath\epsilon}{k^+})} + \frac{\gamma^+}{2k^+}$$

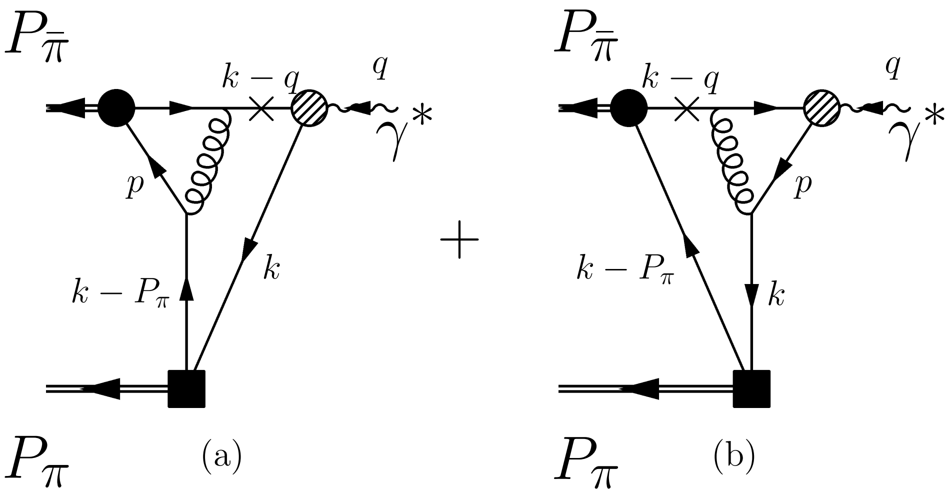
Instantaneous term in the free propagator $\uparrow$

with $k^-_{on} = (m^2 + |\vec{k}_\perp|^2)/k^+$

Instantaneous in the light-front time: $x^+ = t + z$ **!** The Fourier transform on k^- of the second term contains: $\delta(x^+)$.

For $m_\pi \to 0$, only instantaneous contributions survive, as a simple consequence from the trace in the j^μ.

See what follows $\Rightarrow$



Instantaneous contributions to the time-like em form factor of a massless pion. The instantaneous quark contribution (vertical line) is attached to the pion vertex in (a) and to VM vertex in (b). The shaded circle represents the dressed photon vertex.

In the vertex functions with an instantaneous quark contribution, $\Lambda^{ist}_{\pi(n)}$, we assume that the very short-range part of the one-gluon-exchange interaction, which includes spin-spin terms, is the dominant one

$$\Lambda^{ist} = \mathcal{K}^{ist}\, G_0\, \Lambda^{full} \sim \mathcal{C}\, \Lambda^{full}$$

$\mathcal{K}^{ist}$ is the Bethe-Salpeter kernel for the instantaneous vertex function Λ^{ist}, G_0 the propagator of two free quarks and Λ^{full} the full vertex function (we assume still related to the LF meson wave function as in the case of the valence sector).

The constant $\mathcal{C}$ is thought to roughly describe the effects of the short-range interaction.

We use the relative weight, $w_{VM} = \mathcal{C}_{VM}/\mathcal{C}_\pi$, as a free parameter.

Pion EM Form Factor in the space- and time-like regions

The pion EM form factor can be extracted using the definitions

$$j^{\mu}_{TL} = \langle \pi \bar{\pi} | \bar{q}(0) \gamma^{\mu} q(0) | 0 \rangle = e \, (P^{\mu}_{\pi} - P^{\mu}_{\bar{\pi}}) \, F_{\pi}(q^2) \quad ,$$

$$j^{\mu}_{SL} = \langle \pi | \bar{q}(0) \gamma^{\mu} q(0) | \pi' \rangle = e \, \left(P^{\mu}_{\pi} + P^{\mu}_{\pi'}\right) \, F_{\pi}(q^2)$$

From i) the Mandelstam formula, ii) integrating over k^- taking into account only the poles of Dirac propagators and iii) putting $m_{\pi} \to 0$ one obtains the following expression of the EM pion form factor

calculated $\downarrow$

$$F_{\pi}(q^2) = \sum_n \frac{f_{Vn}}{q^2 - M_n^2 + \imath M_n \Gamma_n(q^2)} \, g^{+}_{Vn}(q^2)$$

calculated $\uparrow$

Note, for $q^2 = M_n^2$, $g^{+}_{Vn}(M_n^2)$ yields the decay constant $VM \to \pi \bar{\pi}$.

Fixed parameters

Vector-meson masses, M_n, and widths, Γ_n, used in the model.
From PDG 04, *Akhmetshin et al., PLB **509**, 217 (2001) and **Anisovich et al., PLB **542**, 8 (2002).

Meson	M_n (MeV)	$M_n^{\rm exp}$ (MeV)	Γ_n (MeV)	$\Gamma_n^{\rm exp}$ (MeV)
$\rho(770)$	770	775.8 ± 0.5	146.4	146.4 ± 1.5
$\rho(1450)$	1497^*	1465.0 ± 25.0	226^*	400 ± 60
$\rho(1700)$	1720	1720.0 ± 20.0	220	250 ± 100
$\rho(2150)$	2149	2149.0 ± 17	230^{**}	363 ± 50

In the calculation, 20 vector mesons are taken into account to reach a convergence up to $q^2 = 10\ (GeV/c)^2$. The VM masses for $M_n > 2150\ MeV$ are from the FPZ model.

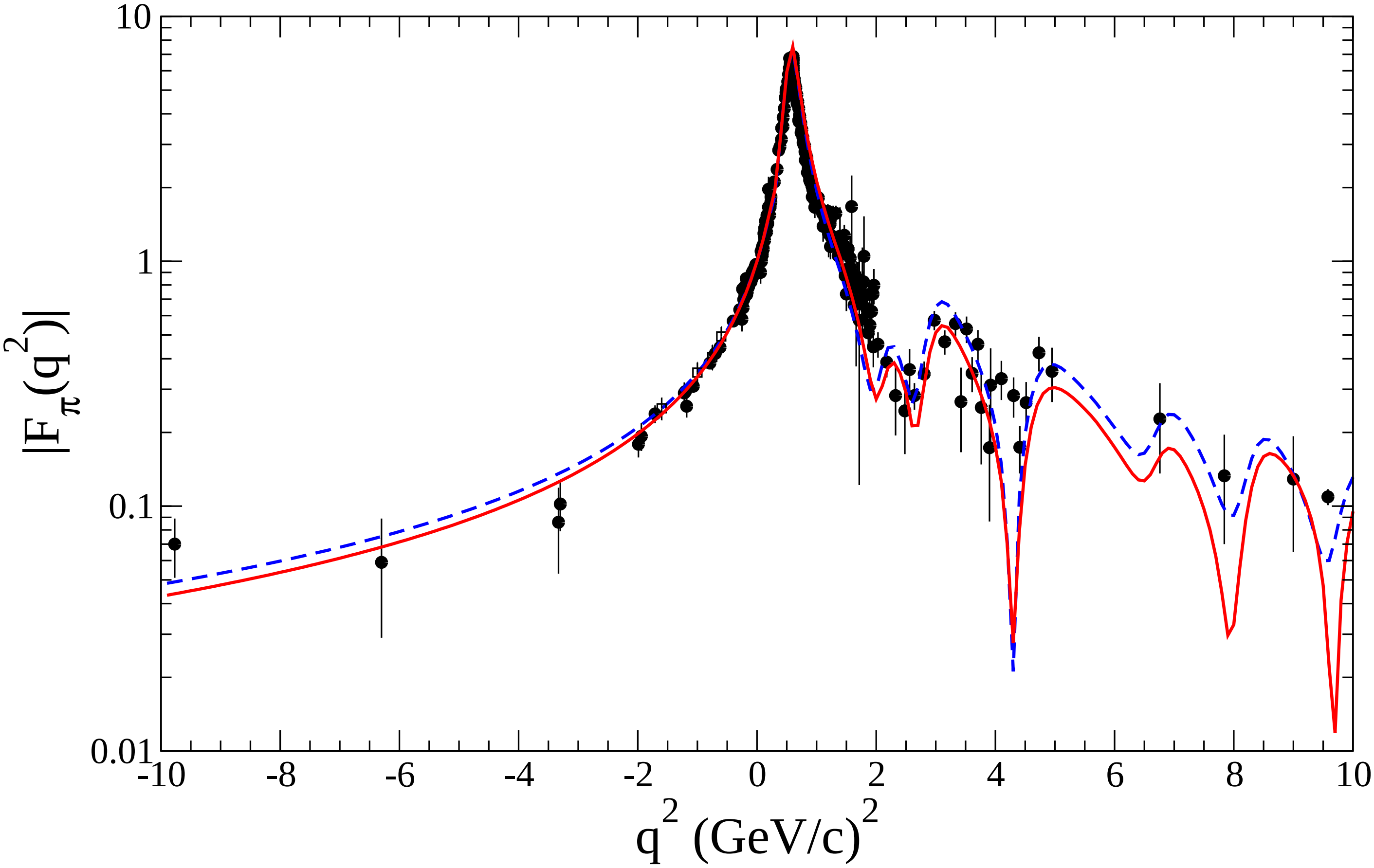
Adjusted parameters

1) The width, Γ_n, of the vector mesons with mass $> \ 2.150\ GeV$. The chosen value $\Gamma_n = 0.15\ GeV$ is similar to the width of the first four VM's

2) w_{VM}, that weights the instantaneous contributions. We obtained $w_{VM} = -0.7$ for a global fit, and $w_{VM} = -1.5$ for an improved description of the ρ peak region.

Pion EM Form Factor in the SL and TL regions

Comparison with Exp. data



●: Data, R. Baldini et al. (EPJ. C11 (1999) 709, and Refs. therein.)

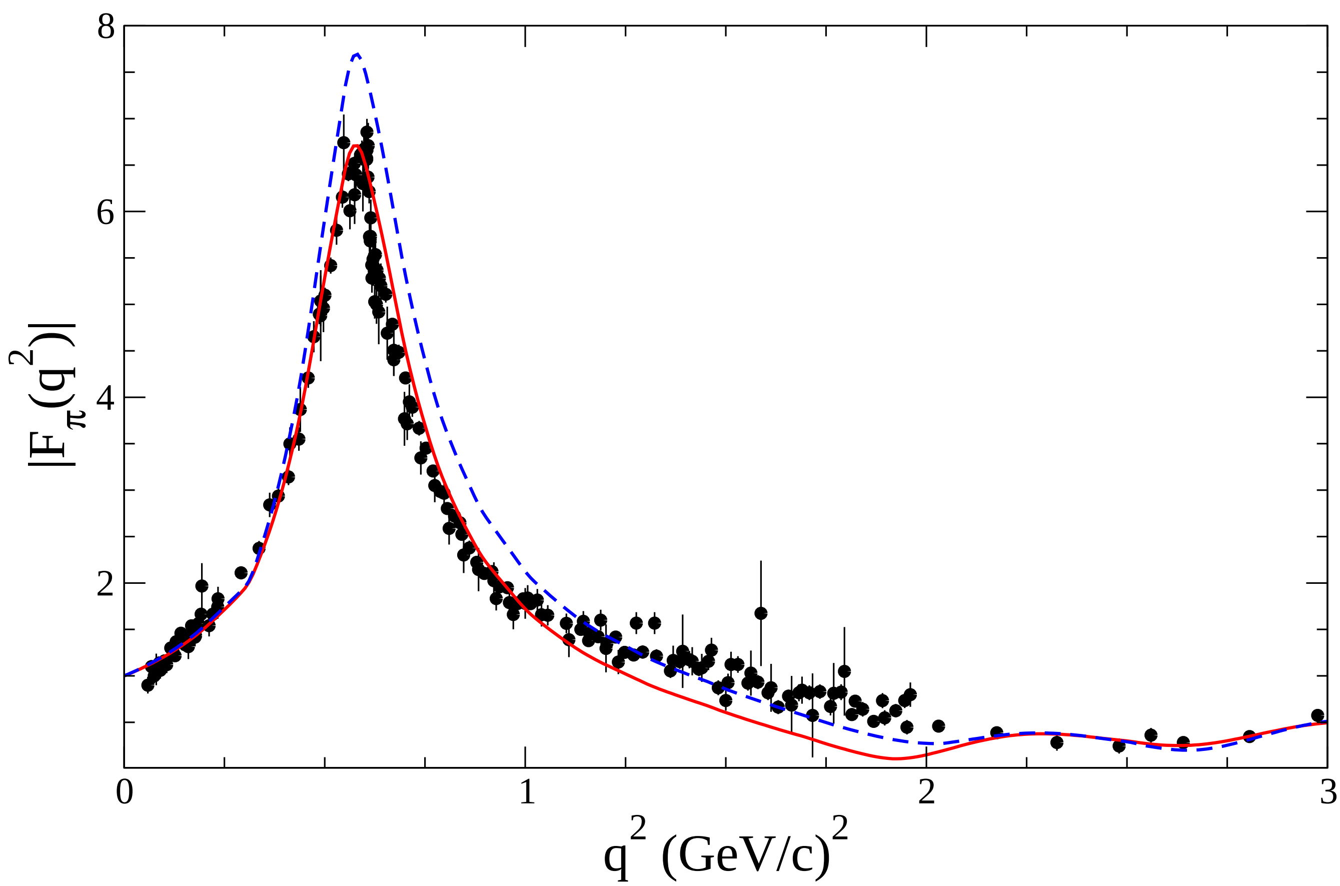
Solid line: calculation with the pion w.f. from the FPZ model for the Bethe-Salpeter amplitude in the valence region ($w_{VM} = -0.7$).

Dashed line: the same as the solid line, but with the asymptotic pion w.f. $(\Lambda_\pi(k; P_\pi) = 1)$

$$\psi_\pi(k^+, \mathbf{k}_\perp; P_\pi^+, \mathbf{P}_{\pi\perp}) = \frac{m}{f_\pi} \frac{P_\pi^+}{[M_\pi^2 - M_0^2(k^+, \mathbf{k}_\perp; P_\pi^+, \mathbf{P}_{\pi\perp})]}$$

To show the sensitivity to the choice of different weights for the instantaneous Bethe-Salpeter amplitudes of pion and VM's

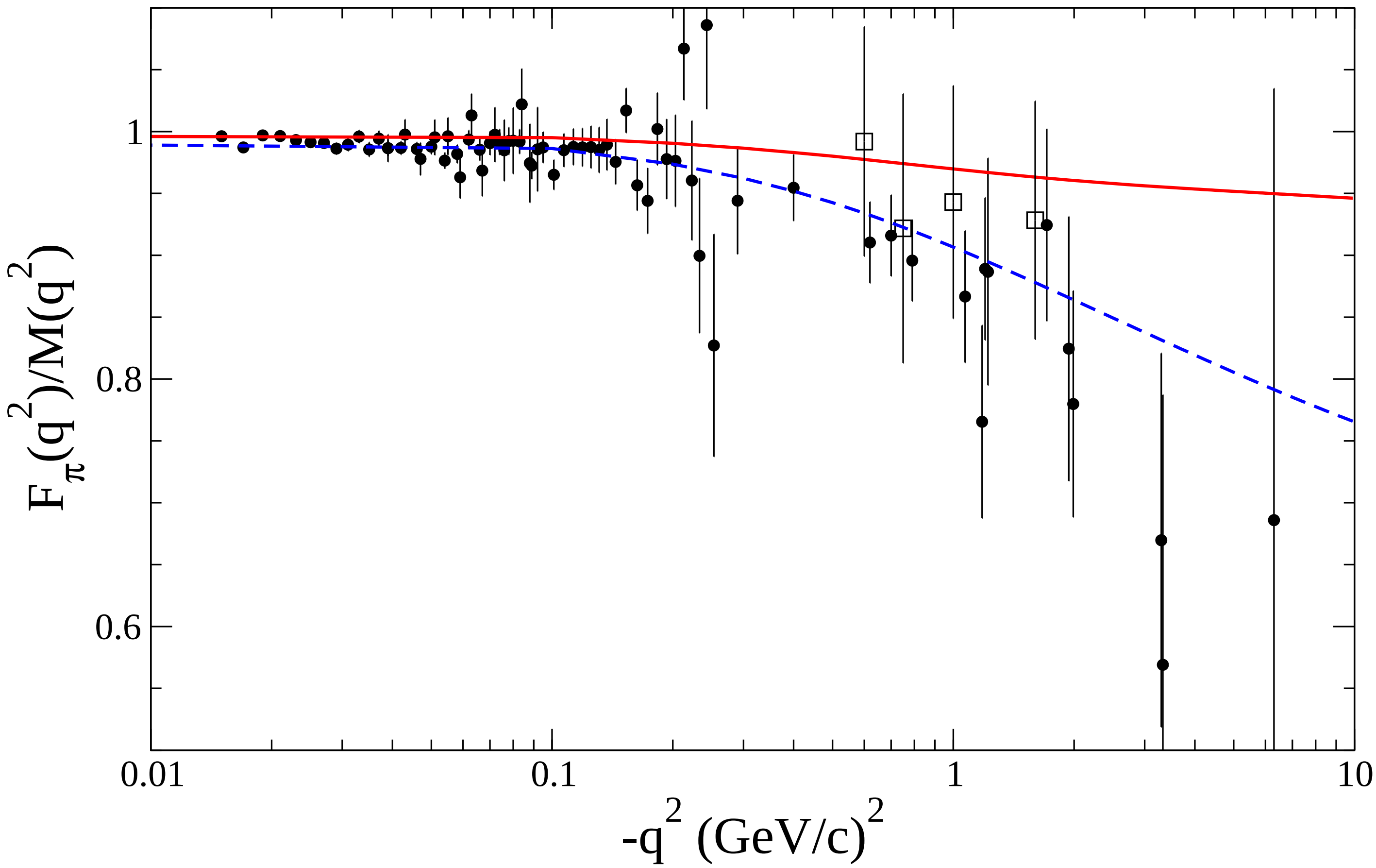
Pion form factor in the ρ-peak region



Solid line: calculation with the pion w.f. from the FPZ model for the Bethe-Salpeter amplitude in the valence region and $w_{VM} = -1.5$.

Dashed line: the same as the solid line but with $w_{VM} = -0.7$.

Pion form factor in the space-like region.



The ratio $R_\pi(q^2) = F_\pi(q^2)/\left[1/(1-q^2/m_\rho^2)\right]$ vs q^2, in the SL region. ●: data from Baldini et al.; □: TJLAB data, Volmer et al, PRL **86**, 1713 (2001).

Solid line: calculation with the pion w.f. from the FPZ model and $w_{VM} = -1.5$.

Dashed line: the same as the solid line but with $w_{VM} = -0.7$).

★ The good agreement with the experimental form factor at low momentum transfers is expected, since we have built-in the generalized ρ-meson dominance.

- The heights of the bumps in the TL region are well reproduced in our model. This feature is related to the calculated value, $g^{+}_{Vn}(q^2)$, of the decay constant $VM \to \pi\bar{\pi}$.
- A more refined description of the instantaneous contribution should help to fill the deep near 2 $(GeV/c)^2$.
- The introduction of ω-like and ϕ-like mesons could obviously improve the description of the data in the TL region.
- In the TL region, at high values of the momentum transfer, the agreement with the data is quite reasonable.

More Results

$$\Gamma_{e^+e^-} = \frac{8\pi\alpha^2 \; f^2_{Vn}}{(3M^3_n)}$$

Meson	$\Gamma_{e^+e^-}$ (KeV)	$\Gamma^{exp}_{e^+e^-}$ (KeV)
ρ(770)	6.98	7.02 ± 0.11
ρ(1450)	1.04	1.47 ± 0.4
ρ(1700)	0.98	$> 0.23 \pm 0.1$
ρ(2150)	0.65	-

The EM Nucleon Form Factors

The Dirac structure of the quark-nucleon vertex is suggested, as in the case of the quark-pion vertex, by an effective Lagrangian (de Araujo et al PLB B478 (2001) 86)

$$\mathcal{L}_{eff}(x) = \frac{\epsilon_{abc}}{\sqrt{2}} \int d^4x_1\, d^4x_2\, d^4x_3 \mathcal{F}(x_1, x_2, x_3, x) \sum_{\tau_1,\tau_2,\tau_3} \times$$

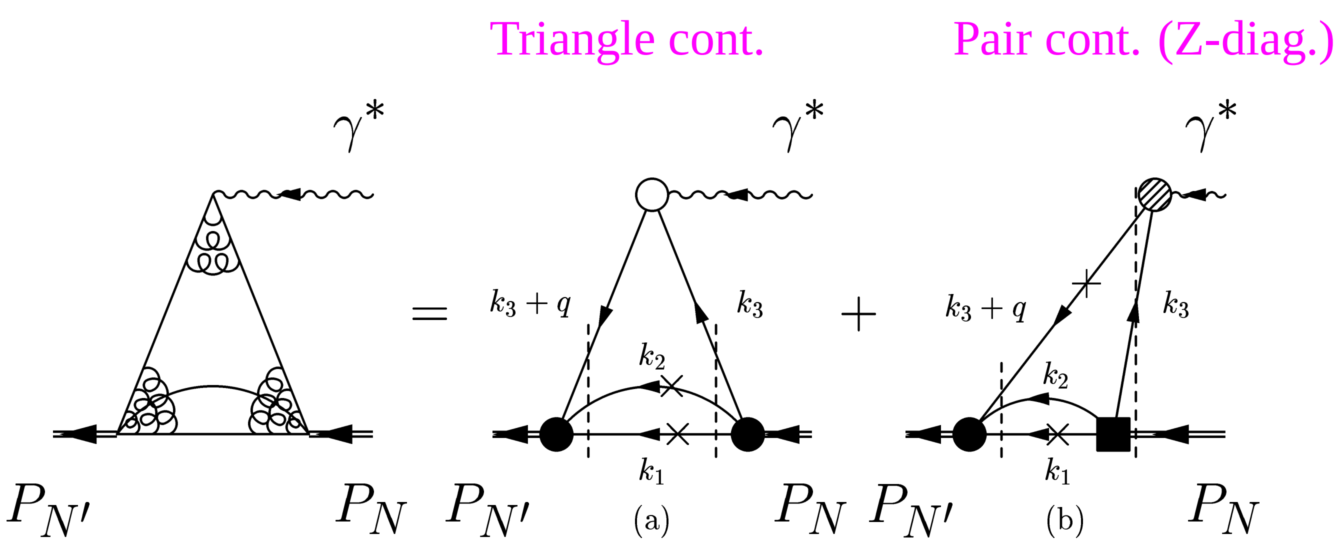
$$\Big[m_N\, \alpha\;\, \bar{q}^a(x_1,\tau_1) \imath\tau_y\, \gamma^5\, q^b_C(x_2,\tau_2)\, \bar{q}^c(x_3,\tau_3)\; - \;\frac{(1-\alpha)}{\sqrt{3}} \times$$

$$\bar{q}^a(x_1,\tau_1)\, \vec{\tau}\, \imath\tau_y\, \gamma^5\, \gamma_\mu\, q^b_C(x_2,\tau_2) \cdot \bar{q}^c(x_3,\tau_3) \vec{\tau}\, (-\imath\, \partial^\mu) \Big]\, \psi_N(x,\tau_N)$$

$$+ \ldots$$

for the present time $\alpha = 1$ (i.e. T=0, S=0 quark pair, only)

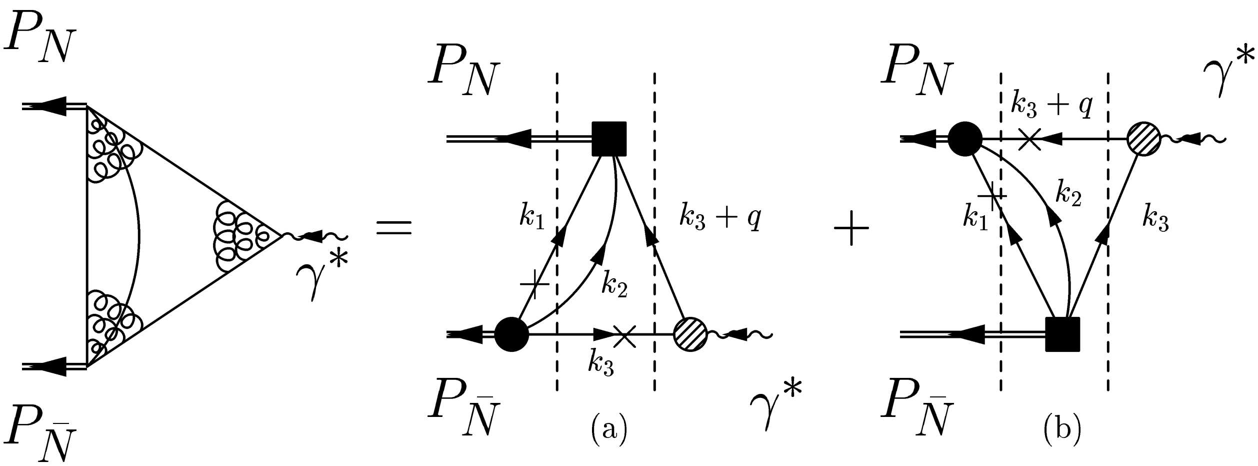
Space-like



(val.) $0 < k^+_{1(2)} < P^+_N$ $\quad$ $P^+_N < k^+_1 < q^+$ (non-val.)

$\times \;\Rightarrow\;$ k_1 on the mass shell : $k^-_{1on} = (m^2 + k^2_{1\perp})/k^+_1$

Time-like



The non valence contribution of the photon is involved: $|q\bar{q}, q\bar{q}, q\bar{q}\rangle$

Definition

The nucleon em form factors (Dirac and Pauli ff's) are introduced as usual from the matrix elements of the macroscopic em current

$$\langle N;\sigma',p'|j^\mu\,|p,\sigma;N\rangle = \bar{U}_N(p',\sigma')\,\left[-F_2(Q^2)\frac{p'^\mu+p^\mu}{2M_N}+\right.$$

$$\left.\left(F_1(Q^2)+F_2(Q^2)\right)\,\gamma^\mu\right]\,U_N(p,\sigma) =$$

$$= \int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}Tr_{\tau_{(1,2)}}\,Tr_{\tau_{(3,N)}}\,Tr_{\Gamma_{(1,2)}}\,\left\{S_C^{-1}(k_2)\times\right.$$

$$\left.\bar{\Phi}_N^{\sigma'}(k_1,k_2,k_3',p')\,S^{-1}(k_1)\,\mathcal{I}^\mu(k_1,k_2,k_3,q)\,\,\Phi_N^\sigma(k_1,k_2,k_3,p)\right\}$$

where $\mathcal{I}^\mu(k_1,k_2,k_3,q)$ is the quark-photon vertex, $\Phi_N^\sigma(k_1,k_2,k_3,p)$ the Bethe-Salpeter amplitude that contains a Dirac structure (highly non trivial...) and a dependence upon the four-momenta of the quarks.

More details on the ingredients...

Quark-Photon Vertex

$$\mathcal{I}^{\mu} = \mathcal{I}^{\mu}_{IS} + \tau_z \mathcal{I}^{\mu}_{IV}$$

and each term contains a contribution corresponding to a purely valence sector (Space-like only) and a contribution corresponding to the pair production (or Z-diagram).

In turn, the Z-diagram contribution can be decomposed in a bare term + a Vector Meson Dominance term (according to the decomposition of the photon state in bare, hadronic [and leptonic] contributions), viz

$$\mathcal{I}^{\mu}_{IS(IV)}(k,q) = \mathcal{N}_{IS(IV)}\theta(p^+ - k^+)\,\theta(k^+)\gamma^{\mu} \; + \\ +\theta(q^+ + k^+)\,\theta(-k^+)\;\{Z_b\,\mathcal{N}_{IS(IV)}\gamma^{\mu} + Z_V\;\Gamma^{\mu}_V[k,q,IS(IV)]\}$$

with $\mathcal{N}_{IS} = 1/6$ and $\mathcal{N}_{IS} = 1/2$. The constant Z_b (bare term) and Z_V (VMD term) are unknown renormalization constants to be extracted from the phenomenological analysis of the data.

Momentum Dependence of the Bethe-Salpeter Amplitude

In the valence sector, namely the spectator quarks are on their-own k^--shell, the momentum dependence, reduced to a 3-momentum dependence by the LF projection, is approximated through a Nucleon Wave Function a la Brodsky (PQCD inspired), namely

$$\mathcal{W}_N \sim \frac{1}{[\beta^2 + M_0^2(1,2,3)]^3}$$

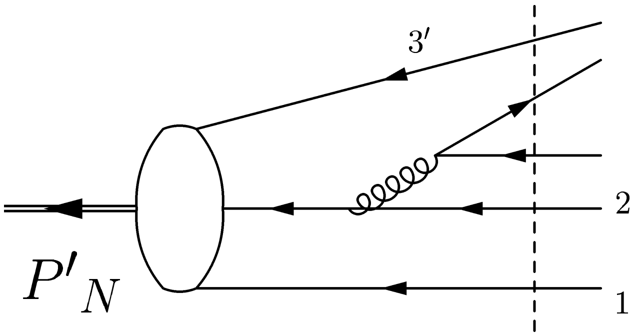
β fixed through anomalous magnetic moments

Proton: 2.878 (Exp. 2.793)

Neutron : -1.859 (Exp. -1.913)

In the non-valence sector, relevant for evaluating the Z-diagram contribution, the momentum dependence is approximated by

$$G_N \sim \frac{1}{[\beta^2 + M_0^2(1,2)]^2} \left\{ \frac{1}{[\beta^2 + M_0^2(3',2)]} + \right.$$
$$\left. + \frac{1}{[\beta^2 + M_0^2(3',1)]} \right\}$$

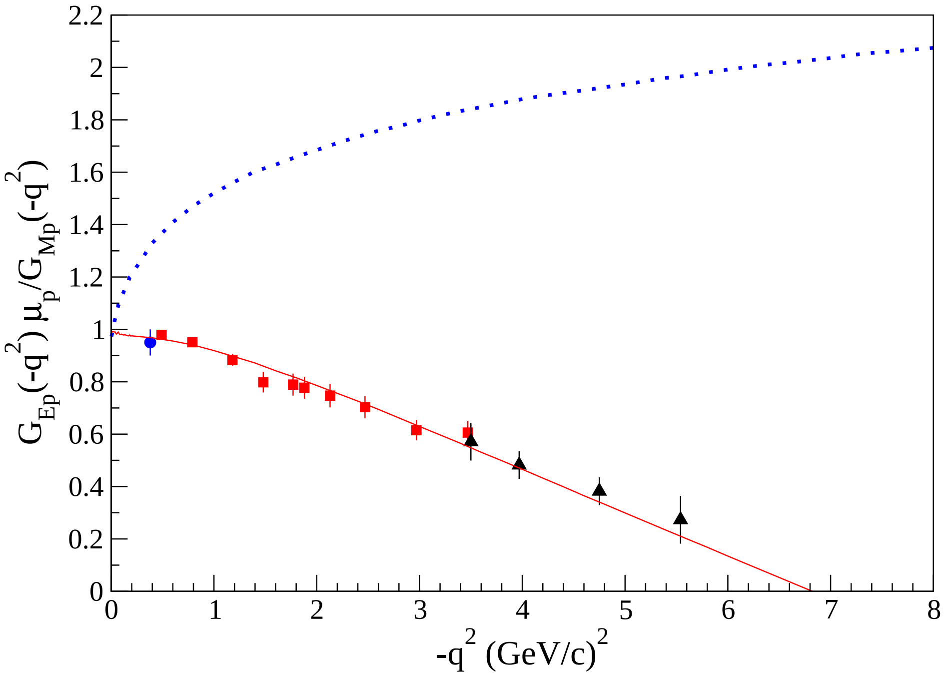


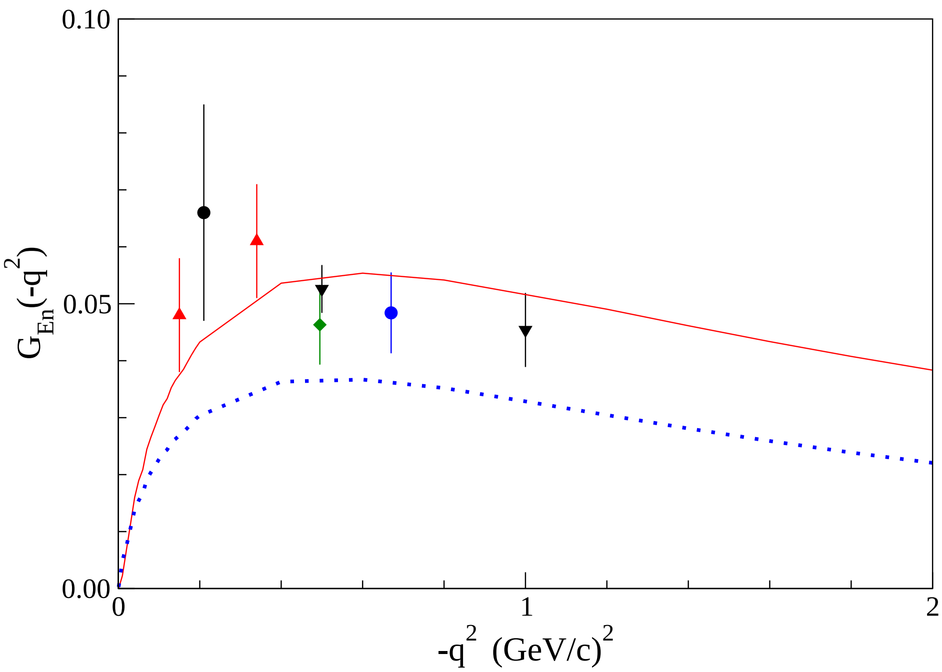
Adjusted parms

- Z_b and Z_V: renormalization constants for the pair production terms
- For this preliminary calculation: two parameters for a running mass in the valence sector (quark mass adopted: 220 MeV), range of variation from 1 to 0.9 for $-10\ GeV^2 < q^2 < 0$.
- For this preliminary calculation: two parameters for a running mass in the non-valence sector (quark mass adopted: 220 MeV), range of variation from 1 to 0.8, for $-10\ GeV^2 < q^2 < 0$.

Preliminary Results in the Space-like Region:

$G_E^p \mu_p / G_M^p$, G_E^n, G_M^p / G_D and G_M^n / G_D, with $G_{E(M)}^{p(n)}$ the Sachs form factors, and $G_D = 1/(1 - q^2/0.71)^2$



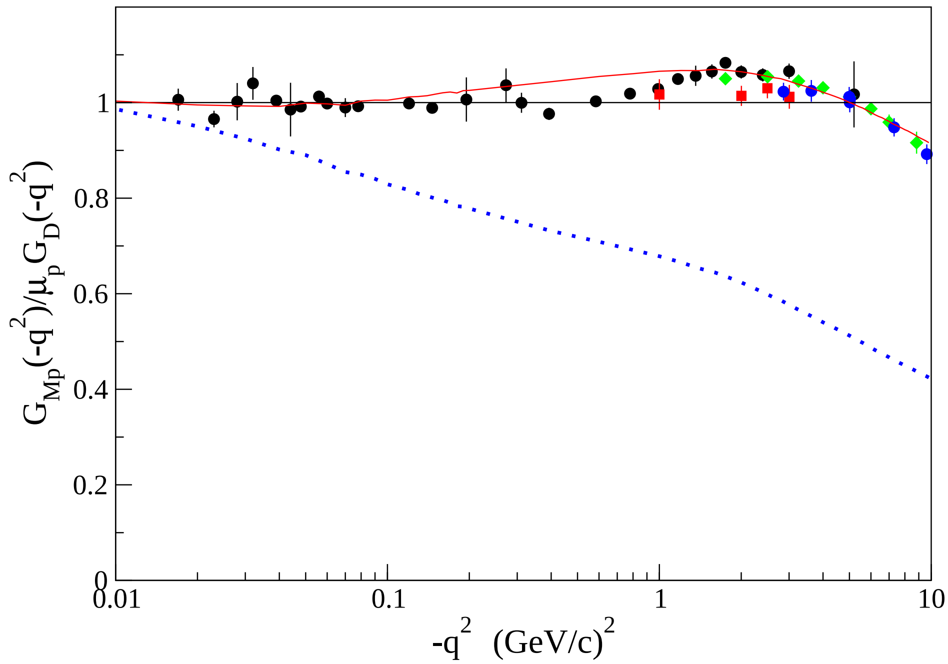


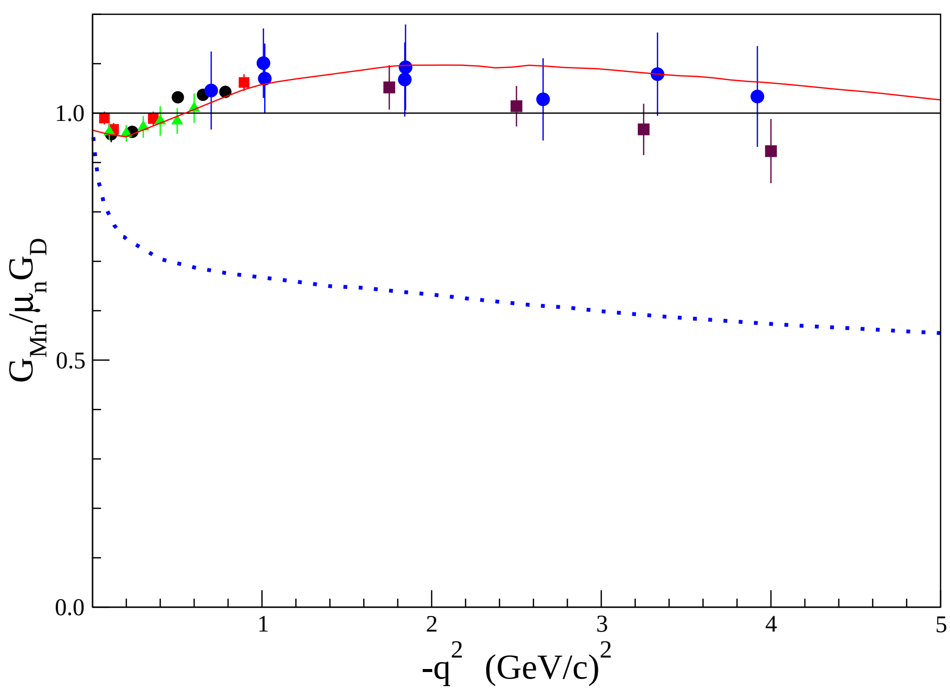
Solid line: full calculation $\equiv \mathcal{F}_\triangle + Z_b \mathcal{F}_{bare} + Z_V \mathcal{F}_{VMD}$

Dotted line: $\mathcal{F}_\triangle$ (elastic contribution only)

Data: www.jlab.org/ cseely/nucleons.html and Refs. therein.

The possible zero in $G_E^p \mu_p / G_M^p$ seems in strong relation to the Z-diagram contribution, i.e. higher Fock components.





Solid line: full calculation $\equiv \mathcal{F}_{\triangle} + Z_b \mathcal{F}_{bare} + Z_V \mathcal{F}_{VMD}$

Dotted line: $\mathcal{F}_{\triangle}$ (elastic contribution only)

Data: www.jlab.org/ cseely/nucleons.html and Refs. therein.

Conclusions & Perspectives

- Constructing a microscopical model for calculating the hadron em form factors in both SL and TL region
- Quark-photon vertex approximated by a VMD model for the process where a virtual photon materializes in a $q\bar{q}$ pair plus a bare term
- The Z-diagram (higher Fock components) essential for both Pion and Nucleon, in the reference frame adopted ($q^+ \neq 0$)
- Pion: Results present a very reasonable agreement with the TL data, while in the SL region the model works very well.
- Nucleon: Preliminary results in the SL region. The Z-diagram is necessary for a reasonable description. The possible zero in $G_E^p \mu_p / G_M^p$ seems in strong relation to the Z-diagram contribution.

Next step, a calculation of the nucleon EM form factors in the TL