The Entropy of a Correlated System of Nucleons

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Outline

1. Introduction
2. Self Consistent Green's Function Method at Finite Temperature
3. The Entropy of a Correlated System of Nucleons

Outline

Introduction

A Correlated System of Nucleons

Nuclear Matter

- Infinite system of nucleons
- High densities $\rho \sim 10^{14}$ g cm^{-3} $\Rightarrow$ strong interaction
- Model heavy nuclei cores and neutron stars

Finite Temperature

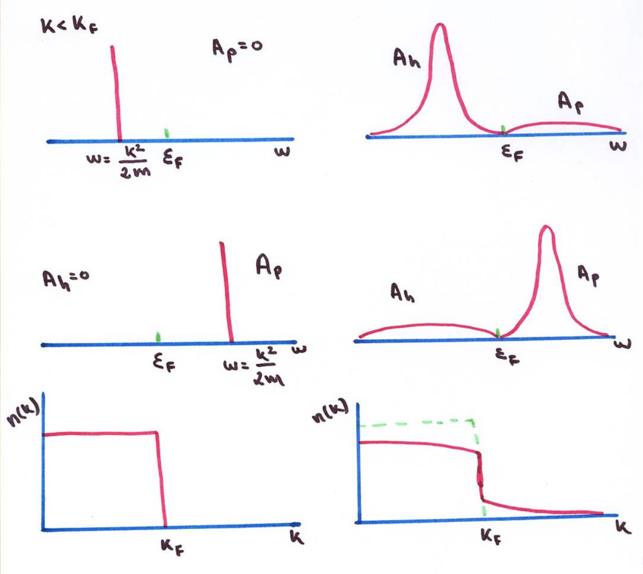
- Study hot nuclear systems:
  - Proto-neutron stars
  - AA collisions
- $T \sim 10 - 20$ MeV $\Rightarrow$ T/ϵ_F small...
- but $T > \Delta!$ $\Rightarrow$ Avoid pairing instability.

Introduction

A Correlated System of Nucleons

- Final aim: complete many-body treatment of in-medium nucleon properties from realistic NN potentials (CDBONN).
- NN potentials fit scattering data:
  - Strong short range repulsion
  - Tensor components
- Evidencies from $(e, e'p)$ experiments:
  - Partial occupation of single-particle states
  - Fragmentation in energy of single-particle strength
- Correlations=beyond mean-field + beyond quasi-particle

Nucleon depletion



Outline

Finite temperature spectral function

Lehmann's representation:

$$A(k,\omega) = -2 \, \mathrm{Im} \, G(k,\omega+i\eta) \leftrightarrow G(k,\omega+i\eta) = \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega}{2\pi} \frac{A(k,\omega')}{\omega-\omega'+i\eta}$$

It gives information about:

- Momentum distribution:

$$n(k) = \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega}{2\pi} A(k,\omega) f(\omega)$$

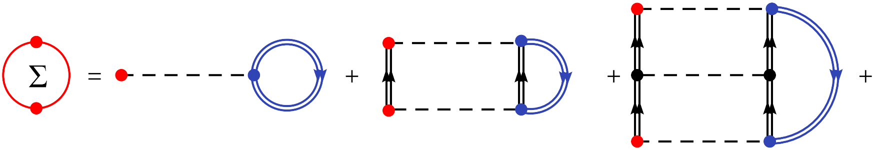
- Energy (Koltun's sum-rule):

$$\frac{E}{A} = \frac{\nu}{\rho} \int \frac{\mathrm{d}^3k}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega}{2\pi} \frac{1}{2} \left(\frac{k^2}{2m} + \omega \right) A(k,\omega) f(\omega)$$

with $f(\omega)$ the Fermi-Dirac distribution: $f(\omega) = \frac{1}{1+e^{\beta(\omega-\mu)}}$.

Ladder approximation to Σ

Dyson's Equation $\Rightarrow G(k,\omega) = \frac{1}{G_0^{-1}(k,\omega) - \Sigma(k,\omega)}$

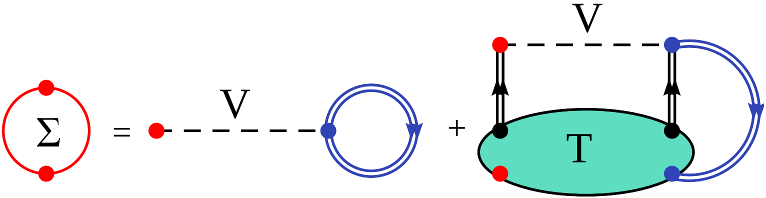


The complex self-energy Σ is a sum of two terms:

- A generalized Hartree-Fock contribution
- A T-matrix term

Amounts for a truncation: $G_{II} \sim G \times G$

Ladder approximation to Σ



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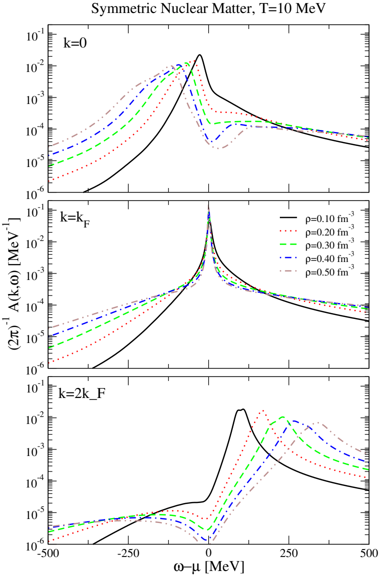
Spectral functions: symmetric nuclear matter



- Quasi-particle peak shifting with k.
- Peaks broaden with ρ at $k = 0$ and $k = 2k_F$.
- At $k = k_F$ the peaks become narrower with ρ.
- Negative energy tails more important as ρ increases.

Outline

TD properties of correlated systems

- How do we compute $F = E - TS$?
- Does the width of particles affect the entropy of the system?
- Can we compute the entropy with the single knowledge of the one-particle propagator G?

Partition function from G

- Expression for the partition function:

$$\ln Z = \mathrm{Tr}\,\Sigma(k,z_\nu)G(k,z_\nu) + \mathrm{Tr}\,\ln\left[-G^{-1}(k,z_\nu)\right] - \Phi\left[G\right]$$

1. Stationary under variations of the propagator:

$$\left.\frac{\delta \ln Z}{\delta G}\right|_{G_0} = 0\,.$$

2. Equivalent to the linked cluster expansion:

$$\ln Z = \ln Z_0 - \int_0^1 \frac{\mathrm{d}\lambda}{2\lambda}\,\mathrm{Tr}\,\Sigma_\lambda(k,\omega_+)G_\lambda(k,\omega_+)\,.$$

- The trace Tr is a sum over k and z_ν and can be performed:

$$\Omega = \int \frac{\mathrm{d}^3k}{(2\pi)^3}\int_{-\infty}^{\infty}\frac{\mathrm{d}\omega}{2\pi}f(\omega)\,\mathrm{Im}\left\{\ln\left[-G^{-1}(k,\omega_+)\right] + \Sigma(k,\omega_+)G(k,\omega_+)\right\} + T\Phi$$

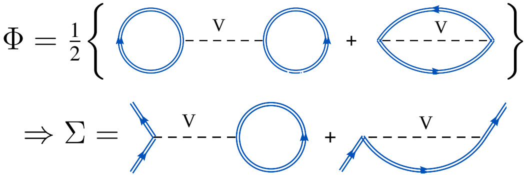
Φ functional

Sum of all the 2PI irreducible skeleton diagrams.

1. If microscopic conservation laws are to fullfilled:

$$\Sigma = \frac{\delta \Phi}{\delta G} .$$

2. Diagramatically:



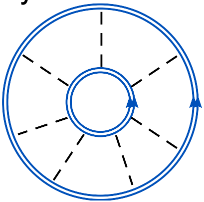
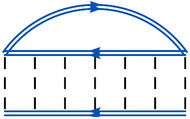
Φ functional

Sum of all the 2PI irreducible skeleton diagrams.

1. If microscopic conservation laws are to fullfilled:

$$\Sigma = \frac{\delta \Phi}{\delta G} .$$

2. Diagramatically:

$\Phi = \sum_n \frac{1}{n} \Big\{$  $\Big\} \Rightarrow \Sigma = \sum_n$ 

Entropy

The entropy can be decomposed in two contributions:

1. A non-analytical contribution:

$$S' = \int \frac{d^3k}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \sigma(\omega) \frac{\partial}{\partial \omega} \left\{ A(k,\omega) \, \mathrm{Re}\, \Sigma(k,\omega) \right\} - \frac{\partial T \Phi}{\partial T}$$

that will be neglected $\Rightarrow$ Avoid computing Φ.

2. A statistical factor times a weighting function:

$$S_{DQ} = \int \frac{d^3k}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \sigma(\omega) \, \mathcal{B}(k,\omega)$$

with $\sigma(\omega) = -\{ f(\omega) \ln f(\omega) + [1 - f(\omega)] \ln[1 - f(\omega)] \}$

Pethick and Carneiro: PRB,**11**,1107 (1975)

$\mathcal{B}$ spectral function

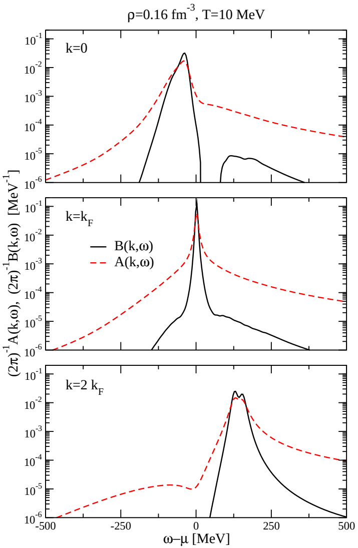
Can be expressed in different terms:

$$\mathcal{B}(k,\omega) = \frac{1}{2}\frac{\Gamma^3(k,\omega)}{\left[\left[\mathrm{Re}G^{-1}(k,\omega)\right]^2 + \left[\Gamma(k,\omega)/2\right]^2\right]^2}\left\{1 - \frac{\partial\,\mathrm{Re}\Sigma(k,\omega)}{\partial\omega}\right\} - \frac{1}{2}\frac{\Gamma^2(k,\omega)}{\left[\left[\mathrm{Re}G^{-1}(k,\omega)\right]^2 + \left[\Gamma(k,\omega)/2\right]^2\right]^2}\mathrm{Re}G^{-1}(k,\omega)\frac{\partial\Gamma(k,\omega)}{\partial\omega}$$

or:

$$\mathcal{B}(k,\omega) = A(k,\omega)\left\{1 - \frac{\partial\,\mathrm{Re}\Sigma(k,\omega)}{\partial\omega}\right\} + \Gamma(k,\omega)\frac{\partial\,\mathrm{Re}G(k,\omega)}{\partial\omega}$$

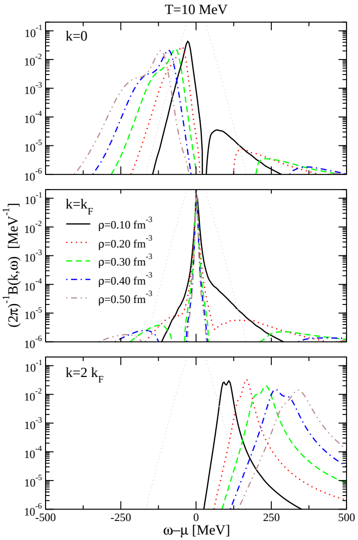
A vs. $\mathcal{B}$-spectral function



- $\mathcal{B}$ peaks narrower than A.
- $\mathcal{B}$ peaks higher than A.
- $\mathcal{B}$ has less high energy tails.
- Sum-rule for $\mathcal{B}$:

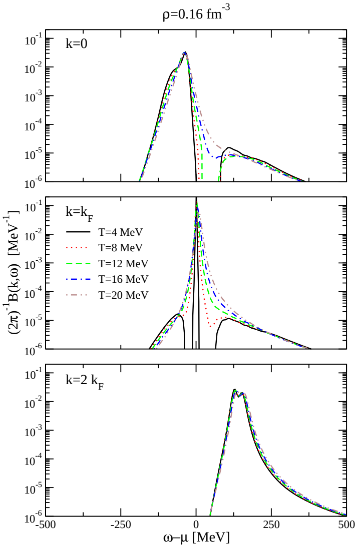
$$\int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \mathcal{B}(k, \omega) = 1$$

$\mathcal{B}$-spectral function - ρ dependence



- Peaks broaden with density at $k \neq k_F$.
- Very narrow peaks at $k = k_F$.
- High energy tails decrease with ρ.

$\mathcal{B}$-spectral function - T dependence



- Peaks do not change relative to μ.
- At low T clear separation btw peak and background.
- With increasing T, more energy tails.

Entropies

- Dynamical quasi-particle entropy:

$$S_{DQ} = \int \frac{\mathrm{d}^3k}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega}{2\pi} \sigma(\omega)\, \mathcal{B}(k,\omega)$$

VS...

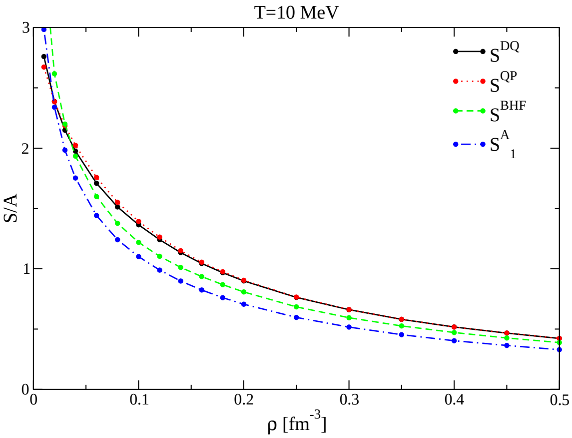
- Quasi-particle entropy:

$$S_{SPE} = -\int \frac{\mathrm{d}^3k}{(2\pi)^3} \left\{ f[\epsilon_{QP}(k)] \ln f[\epsilon_{QP}(k)] + \left[1 - f[\epsilon_{QP}(k)]\right] \ln \left[1 - f[\epsilon_{QP}(k)]\right] \right\}$$

- $A(k,\omega)$-entropy:

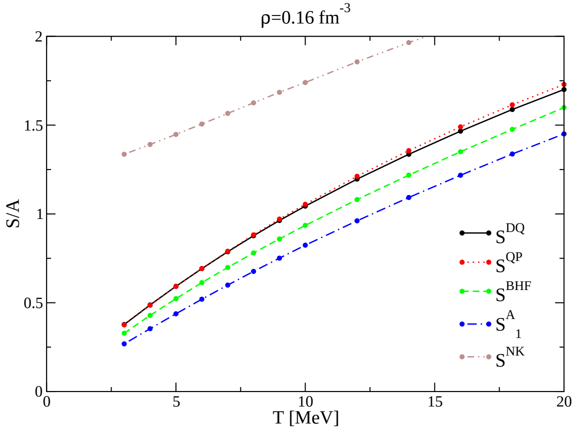
$$S_A = \int \frac{\mathrm{d}^3k}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega}{2\pi} \sigma(\omega)\, A(k,\omega)$$

Entropy - ρ dependence



- Width effects are small and tend to order the system.
- BHF entropy lower.
- $A(k,\omega)$ entropy lower.

Entropy - T dependence



- Linear law:

$$\frac{S}{A} = \frac{\pi^2}{3\rho} N(0) T$$

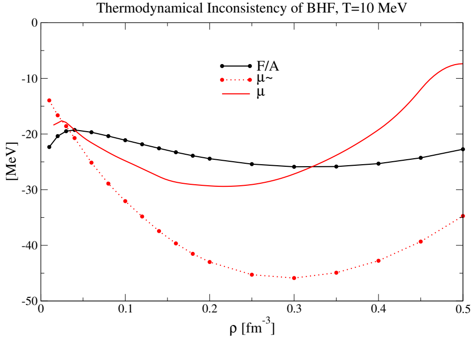
- Different densities of states $N(0)$.
- Width effects grow with T.

TD consistency

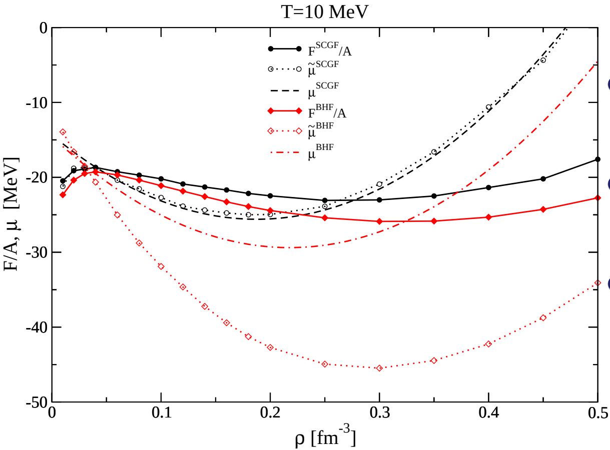
- Thermodynamical inconsistency of many-body approaches:

$$\rho = 2 \int \frac{d^3k}{(2\pi)^3} n(k, \tilde{\mu}) \quad \text{vs.} \quad \mu = \left. \frac{\partial F}{\partial N} \right|_T$$

- Hugenholtz-Van Hove theorem: $\tilde{\mu}(\rho = \rho_0) = \frac{F}{A}(\rho = \rho_0)$



Thermodynamical consistency of SCGF



- HvH theorem well fullfilled!
- In BHF badly violated by 20 MeV.
- Neglecting S' is good approx.

Summary

- In-medium nucleon spectral functions at a wide range of densities and temperatures (and asymmetries!).
- The entropy can be calculated from a statistical factor times a narrow and high $\mathcal{B}$-spectral function.
- For the ρ-T range explored, the dynamical quasi-particle approximation S_{DQ} works very well.
- First step towards using these data in astrophysical or heavy ion phenomena.

For Further Reading I

- T. Frick and H. Müther,
  Self-consistent solution to the nuclear many-body problem at finite temperature,
  Physical Review C **68**, 034310 (2003).
- T. Frick, H. Müther, A. Rios, A. Polls and A. Ramos,
  Correlations in hot asymmetric nuclear matter,
  Physical Review C **71**, 014313 (2005).
- A. Rios, A. Polls and H. Müther,
  Sum rules of single-particle spectral functions in hot asymmetric nuclear matter,
  Physical Review C **73**, 024305 (2006).

For Further Reading II

A. Rios, A. Polls, A. Ramos and H. Müther,
The entropy of a correlated system of nucleons,
In preparation.