# Hard exclusive reactions and hadron structure : some new results

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.

Based on work in collaboration with

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# Factorization of Hard Exclusive processes

• DIS : INCLUSIVE / Large vs Short distance $\longrightarrow$
Stucture Function =Pert. Coef. Funct. $\times$ Parton dist.

• DVCS : EXCLUSIVE $\gamma^* \ N \ \longrightarrow \ \gamma \ N'$
$\longrightarrow$ Amplitude =Pert. Coef. Funct. $\times$ GPD
*Generalized Parton Distributions*

• Deep EXCLUSIVE meson production
Amplitude =Pert. Coef. Funct. $\times$ GPD $\times$ DA

• CROSSING $\rightarrow \gamma^*\gamma \rightarrow M_1 \ M_2$ near threshold
Amplitude =Pert. Coef. Funct. $\times$ GDA
*Generalized Distribution Amplitude*

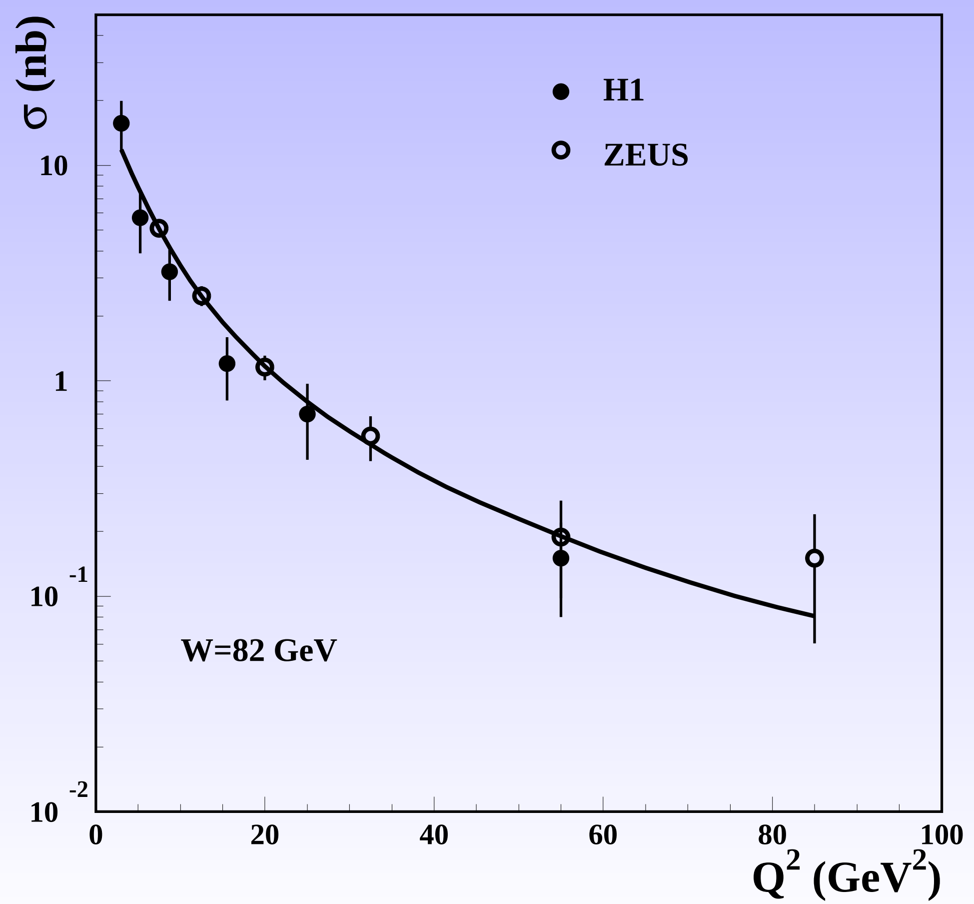
# Successes of Factorized framework

• Consistent picture in QCD
*Evolution Equations interpolate between DGLAP (e.g. for structure functions) and ERBL (e.g. for form-factors) equations*

• SCALING, e.g.
handbag dominance $\equiv$ (generalized) Bjorken scaling

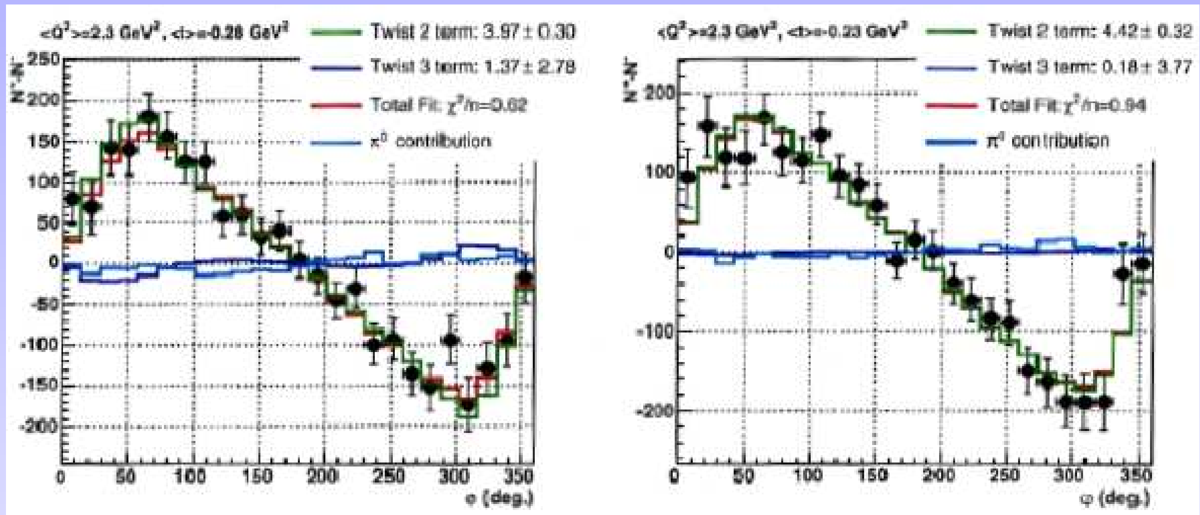
# Successes of Factorized framework

- Right order of magnitudes with experimental results, for DVCS (Guzey + Polyakov 2005)

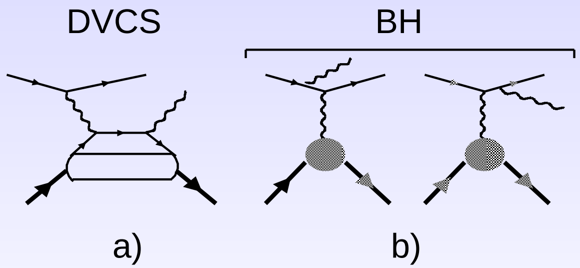


# Successes of Factorized framework

- Angular dependence of asymmetries coming from Bethe Heitler / DVCS interference




JLab data at $Q^2 = 2.3\,\mathrm{GeV}^2$,
$t = -0.28$ and $-0.23\,\mathrm{GeV}^2$



# Generalized Parton Distributions

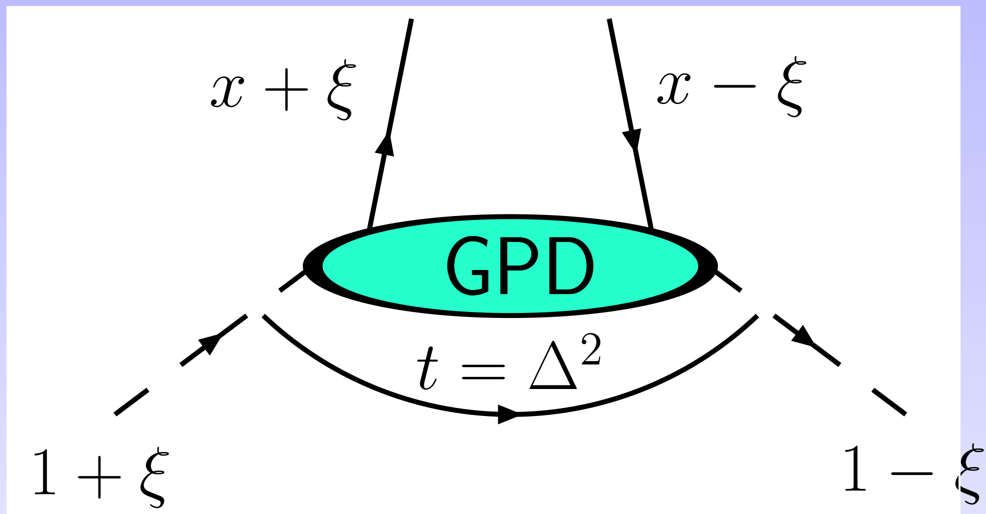
• *Non forward* Matrix elements of non-local light-cone operators, e.g. for a nucleon

$$\langle N(p,\lambda)|\bar{\psi}(-z/2)\Gamma[-z/2;z/2]\psi(z/2)|N'(p',\lambda')\rangle$$
$$\Gamma = \gamma_\mu \;, \quad \gamma_\mu\gamma^5 \;, \quad \sigma_{\mu\,\nu}$$

• Fourier Transform + Decomposition $\longrightarrow$ 8 GPDs :
chiral even:
$H(x,\xi,t), E(x,\xi,t), \tilde{H}(x,\xi,t), \tilde{E}(x,\xi,t)$
chiral odd:
$H_{Ti}(x,\xi,t), i = 1,..,4$ (transversity)

# Kinematics:

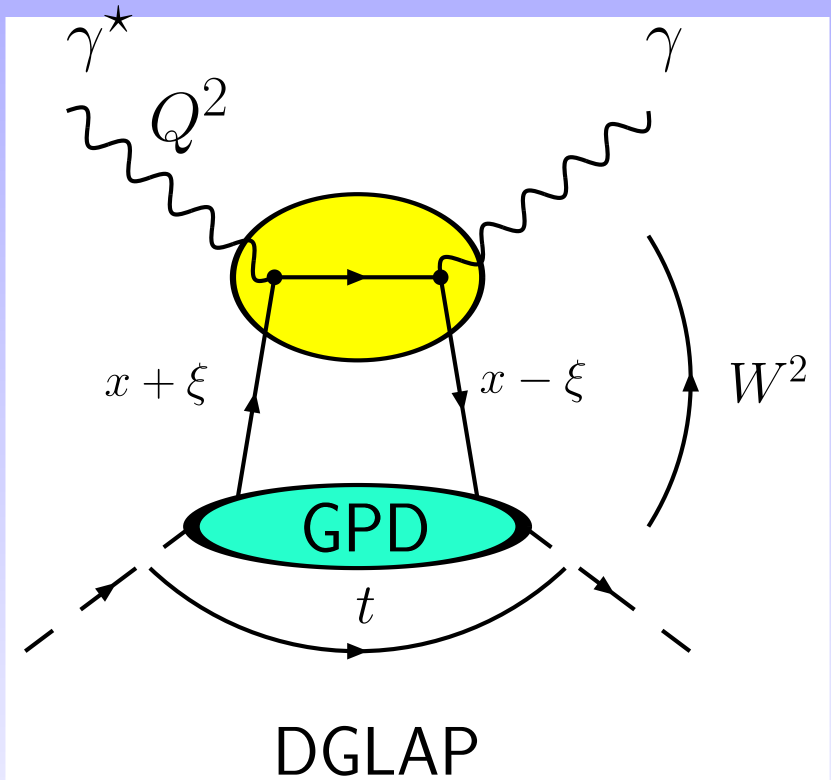
- Notation: $\rightarrow \Delta^+ = -2\xi P^+$
$(2P = p + p', \Delta = p' - p)$ $\xi$ = skewedness

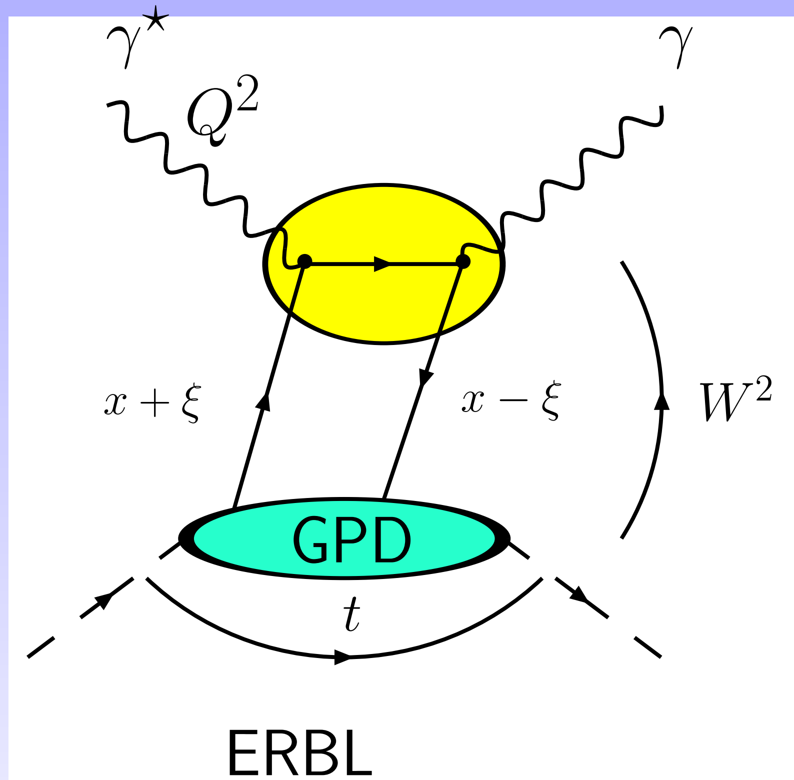



- $\Delta^2 = t << Q^2$ $\quad t-$dependence parametrized as in Form Factors

# Properties of Generalized Parton Distributions

- Two quite distinct regions : $x > \xi$ : DGLAP
$x < \xi$ : ERBL




- Limits at zero skewedness $\longrightarrow$ Usual parton dist.

# Properties of Generalized Parton Distributions

• First $x-$moment $\longrightarrow$ Form Factors ($\xi$ independent), e.g.

$$F_1^q(t) = \int_{-1}^{1} dx\ H_q(x,\xi,t)$$

• Second $x-$moment $\longrightarrow$ Spin Sum Rule (through energy-momentum tensor), e.g.

$$2\langle J_q^3\rangle = \int_{-1}^{1} dx\ x\ \left[H_q(x,\xi,t=0) + E_q(x,\xi,t=0)\right]$$

# Properties of Generalized Parton Distributions

- Lorentz invariance $\longrightarrow$ Polynomiality ($\longrightarrow$ Double distributions), e.g.

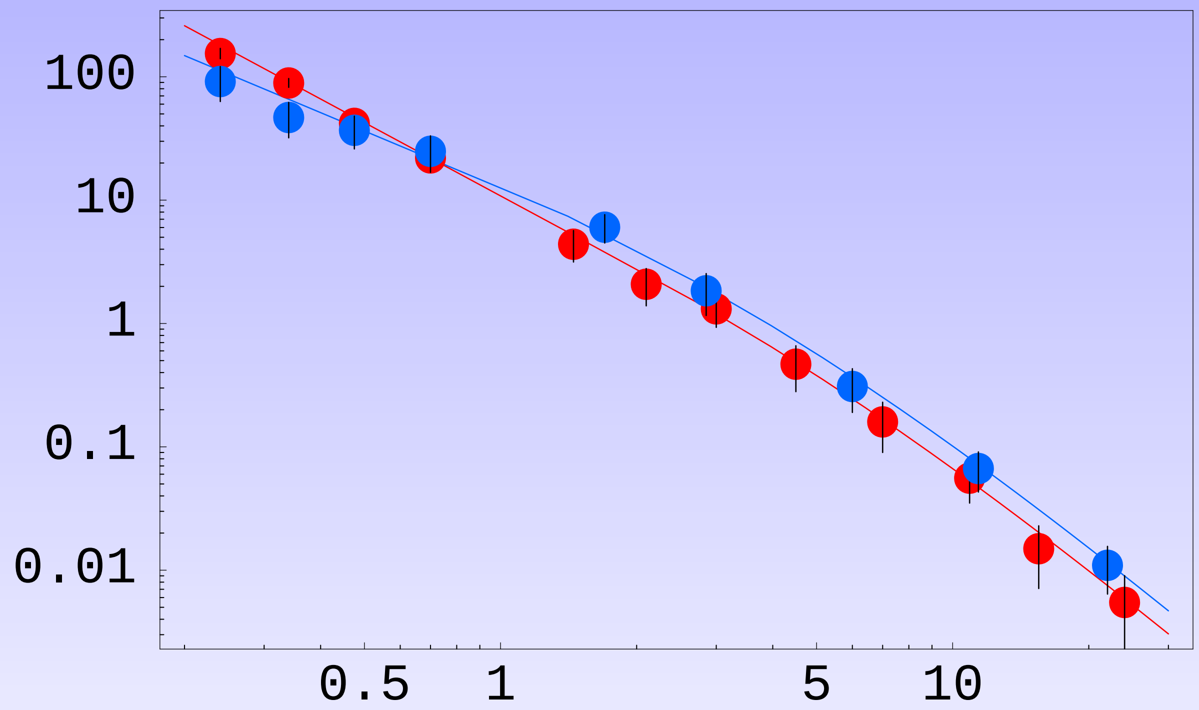
$$\int_{-1}^{1} dx\, x^n\, H^q(x,\xi,t) = \sum_{i=0}^{n} (2\xi)^i A^q_{n+1,i}(t) + \text{'D-term'}$$

- Positivity constraints in DGLAP region, e.g.

$$|H^q_\pi(x,\xi,t)| \leq \sqrt{q_\pi(\frac{x+\xi}{1+\xi}) . q_\pi(\frac{x-\xi}{1-\xi})}$$

# When do we access the factorization regime ?

- dVCS $\longrightarrow$ wait for experimental talks today ...
- crossed process $\longrightarrow$ LEP2 data : EARLY SCALING

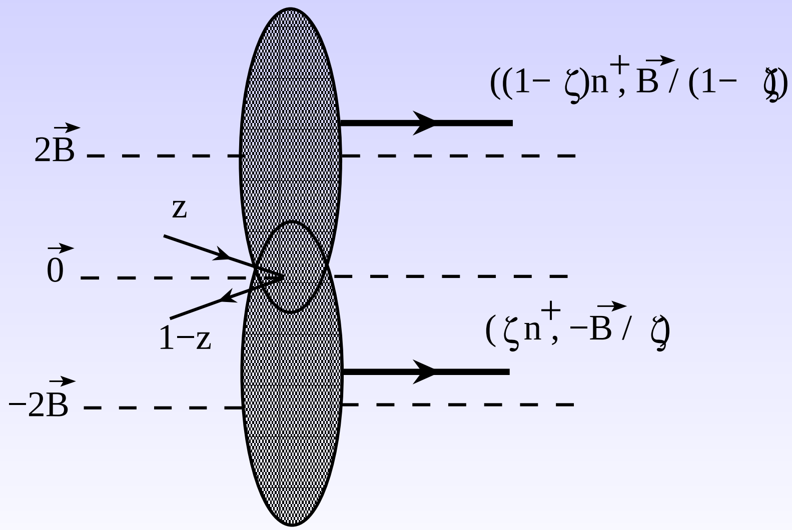



$Q^2$ dependence of $\gamma^*\gamma \longrightarrow \rho^+\rho^-$ and $\gamma^*\gamma \longrightarrow \rho^0\rho^0$

blue red

# Impact picture Representation

- $t$ dependence of GPDs maps transverse position of quarks in proton.
Fourier transform GPD at zero skewedness
$q(x, b_T) = (2\pi)^{-2} \int d^2\Delta e^{i\Delta . b} H(x, \xi = 0, t)$
Generalize at $\xi \neq 0 \rightarrow$ *Quantum femtophotography*.
- $W^2$ dependence of $\gamma^* \gamma \rightarrow M_1 \; M_2$ maps impact representation of hadronization.



# Some new results

- Transversity GPDs

- Searching for EXOTIC HADRONS

- Describing other processes through TDAs
$\bar{N}N \to \gamma^*\gamma$ and $\bar{N}N \to \gamma^*\pi$

# Transversity GPDs

Transversity dependent quark distribution $h_1(x) \rightarrow$
4 transversity GPDs

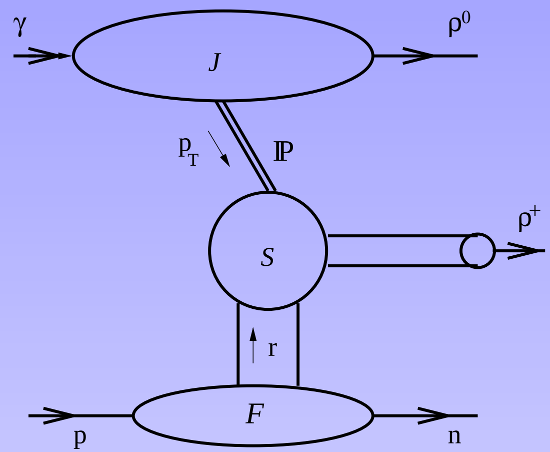
• How to access them ?

Chiral odd functions come in pairs ->
try electroproduction of $\rho_T$

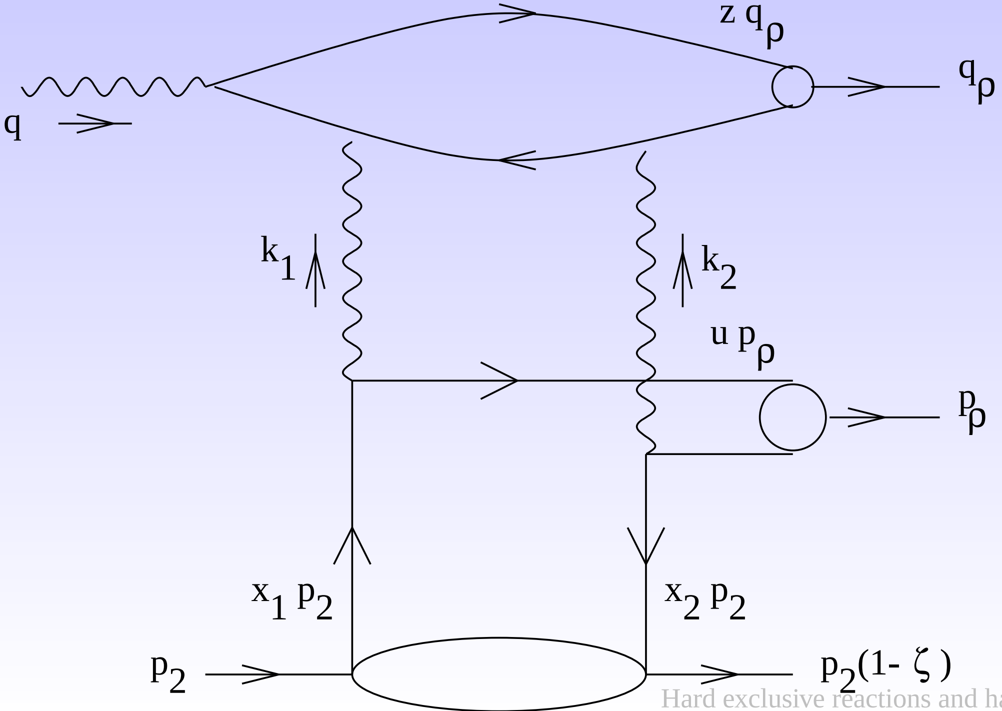
•BUT *zero* amplitude for $\gamma^* N \rightarrow \rho_T N'$ :
use Pomeron analog

$\mathcal{P} N \rightarrow \rho_T N'$ *i.e.* $\gamma^* N \rightarrow \rho_L \rho_T N'$

# Transversity GPDs




P =2 gluons, at Born order 6 diagrams, e.g.

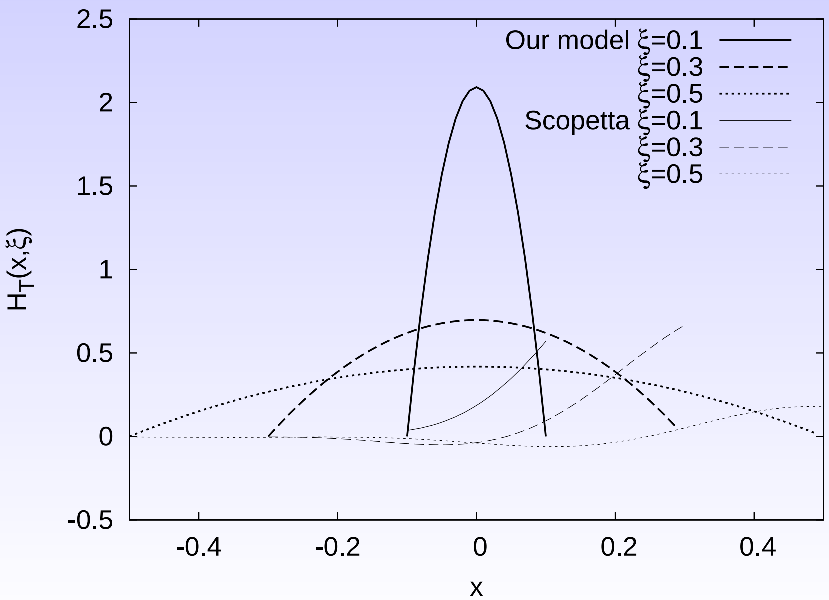


# Models for transversity TDA, $H_T$:

(i) axial meson $A = b_1(1235)$ exchange dominance

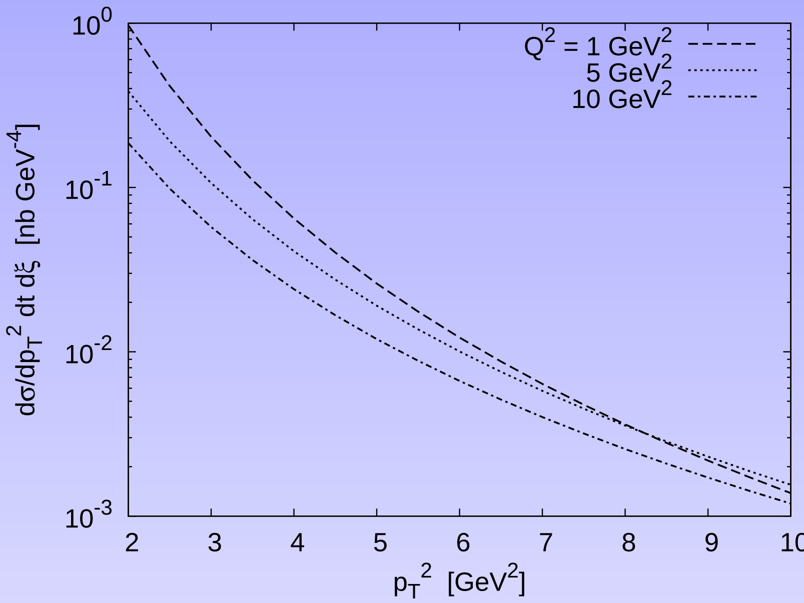
$$H_T^a(x,\xi) = \frac{g_{ANN} f_A^{a\perp} (\Delta \cdot S_T)^2}{2M_N\, m_A^2} \frac{\phi_\perp(\frac{x+\xi}{2\xi})}{2\xi} \,,$$

with $b_1$ distribution amplitude $\phi_\perp^A(u)$ (only ERBL)
(ii) the bag model of transversity (Scopetta 2005)

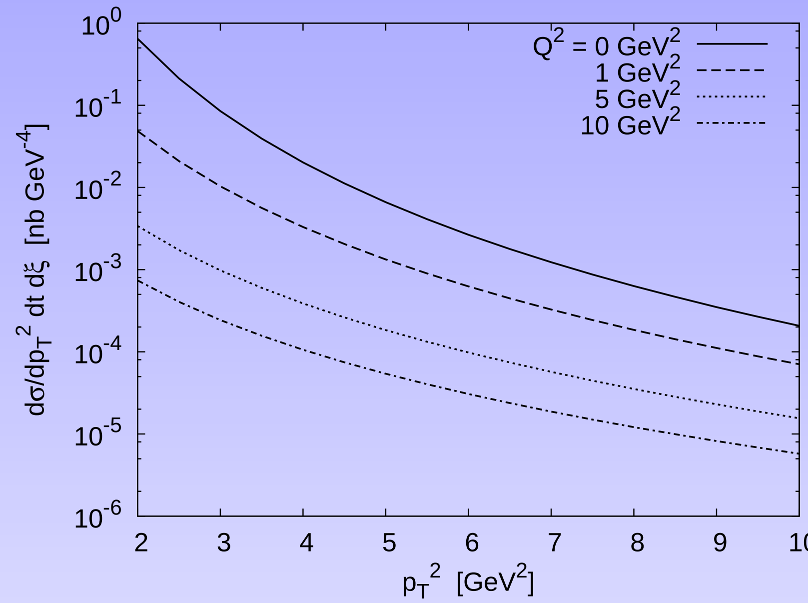


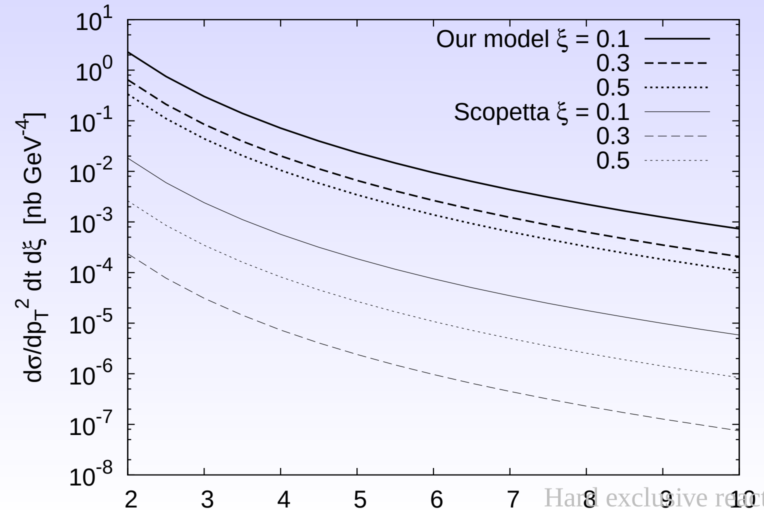
# Diff. cross sec. for $\gamma^{(*)}(Q)\, p \to \rho^0_L\, \rho^+_{L,T}\, n$

$\rho^+_L$




$\rho^+_T$



# Exotic meson exclusive production

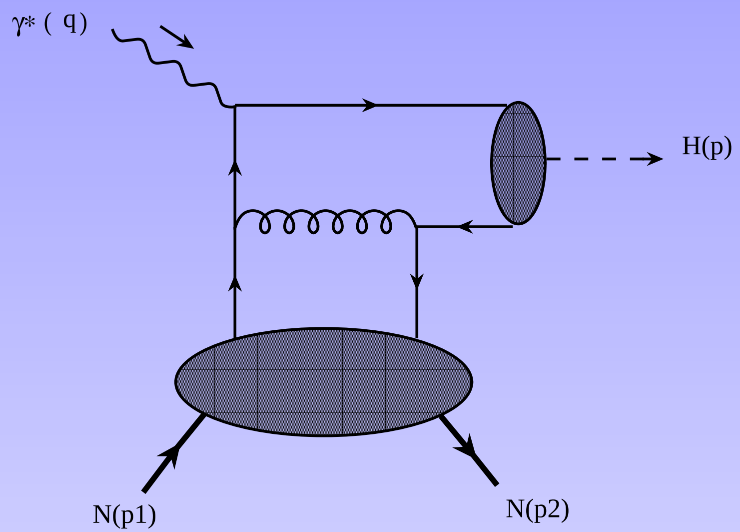
Exotic Hybrid Meson $\pi_1$ with $J^{PC} = 1^{-+}$
Define $\pi_1$ Distribution Amplitude as usual :

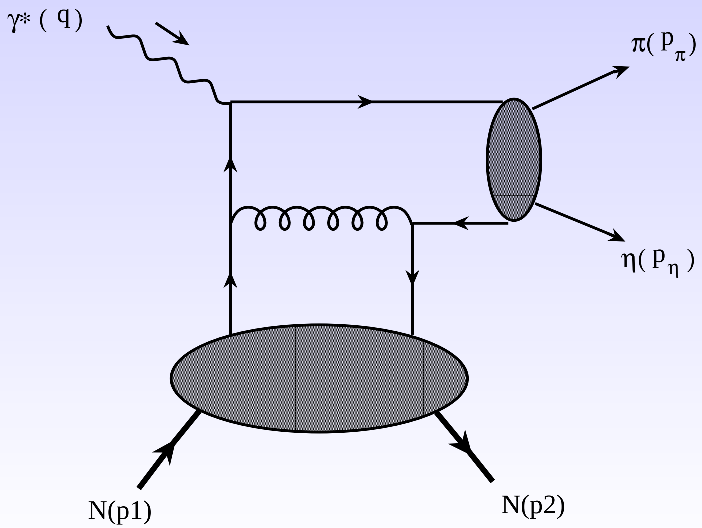
$$\langle \pi_1(p,\lambda)|\bar{\psi}(-z/2)\gamma_\mu[-z/2;z/2]\psi(z/2)|0\rangle =$$

$$i f_{\pi_1} M_{\pi_1} \left[ p_\mu \frac{e^{(\lambda)} \cdot z}{p \cdot z} \int_0^1 dy e^{i(\bar{y}-y)p\cdot z/2} \phi_L^H(y) \right]$$

- same twist as $\rho$ Distribution Amplitude
- QCD sum rules $\longrightarrow f_{\pi_1} \sim 50$ MeV
- Similar electroproduction cross sections in $ep$ collisions.
- Also possible in $e\gamma$ collisions

# Exotic meson exclusive production



# Comparison of $\rho^0$ and $H \equiv \pi_1(1400)$ electroproduction cross sections

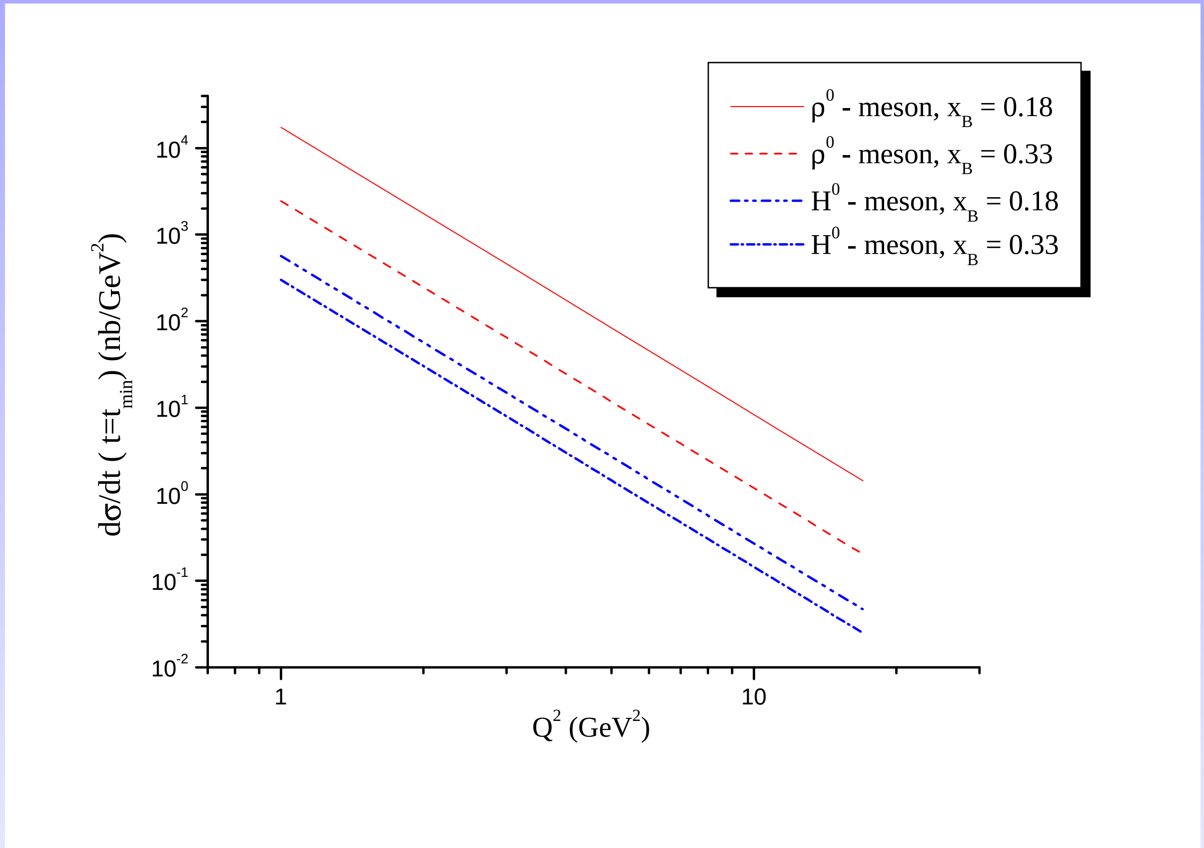



Figure 1:

seems visible $\rightarrow$ COMPASS, e-RHIC

# Extension

- What can pQCD say about other exclusive reactions at large $Q^2$ such as those of

$\bar{p}N \to \gamma^*\gamma$ and $\bar{p}N \to \gamma^*\pi$

*PANDA-PAX programs at GSI-FAIR*

New factorization $P \to \gamma$, $P \to \pi$ TDA

**Transition Distribution Amplitudes**

$$\langle \pi(p')|\, \epsilon^{ijk} u^i_\alpha(z_1\, n) u^j_\beta(z_2\, n) d^k_\gamma(z_3\, n)\, |p(p,s)\rangle\Big|_{z^+=0,\, z_T=0}$$

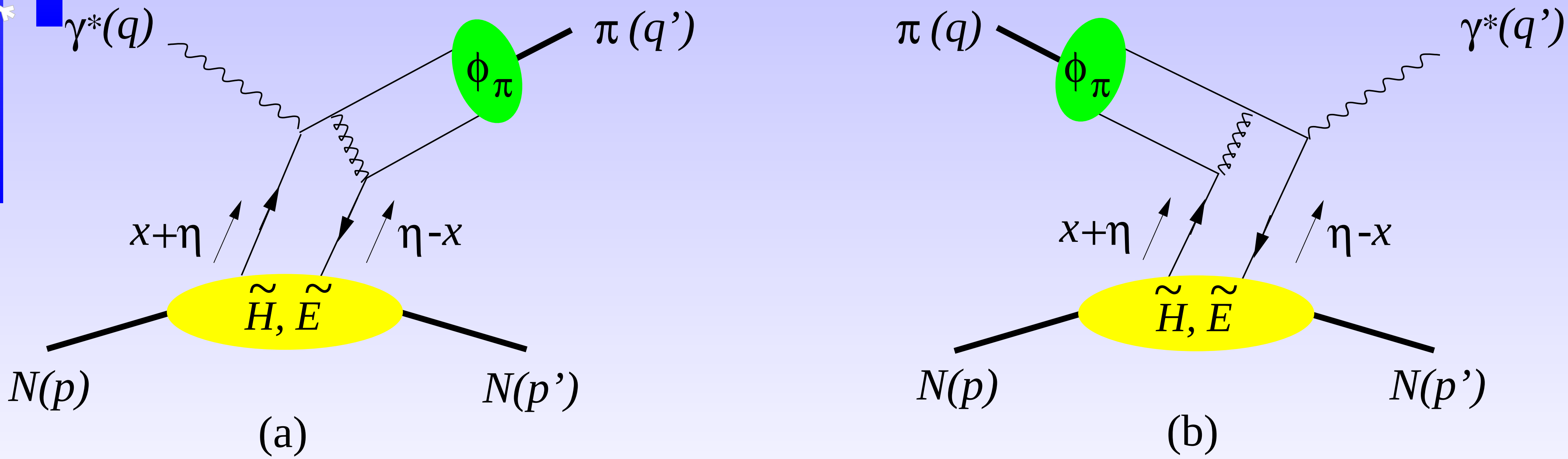
$$\langle \gamma(p',\epsilon')|\, \epsilon^{ijk} u^i_\alpha(z_1\, n) u^j_\beta(z_2\, n) d^k_\gamma(z_3\, n)\, |p(p,s)\rangle\Big|_{z^+=0,\, z_T=0}$$

# Arguments for Factorization

PROOFS EXIST for

• Factorization of deep exclusive $\pi$ electroproduction on meson target. *Collins Frankfurt Strikman*

• Time inversion : Factorization of $\pi M \to \gamma^* M'$ on meson target. *Berger Diehl BP*



# Arguments for Factorization (continued)

- Choose N = $\pi$ and N' = $\rho$

$\longrightarrow$ Factorization of $\pi\pi \longrightarrow \gamma^* \rho$

- Change $\rho \longrightarrow \gamma$

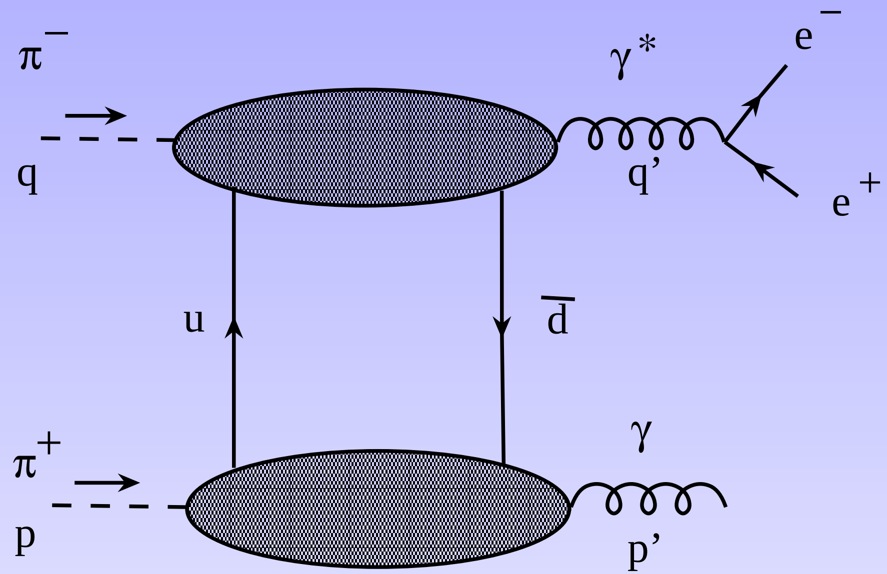
*Remember : Photon structure function factorizes in the same way as meson structure function !*

$\longrightarrow$ Factorization of TDA in

$$\pi\pi \longrightarrow \gamma^* \gamma$$

in the forward direction (*where cross section is bigger.*)

# The factorization of $\pi\ \pi \to \gamma^*\ \gamma$



# Arguments for Factorization - continued

- Change Meson $\rightarrow$ Baryon

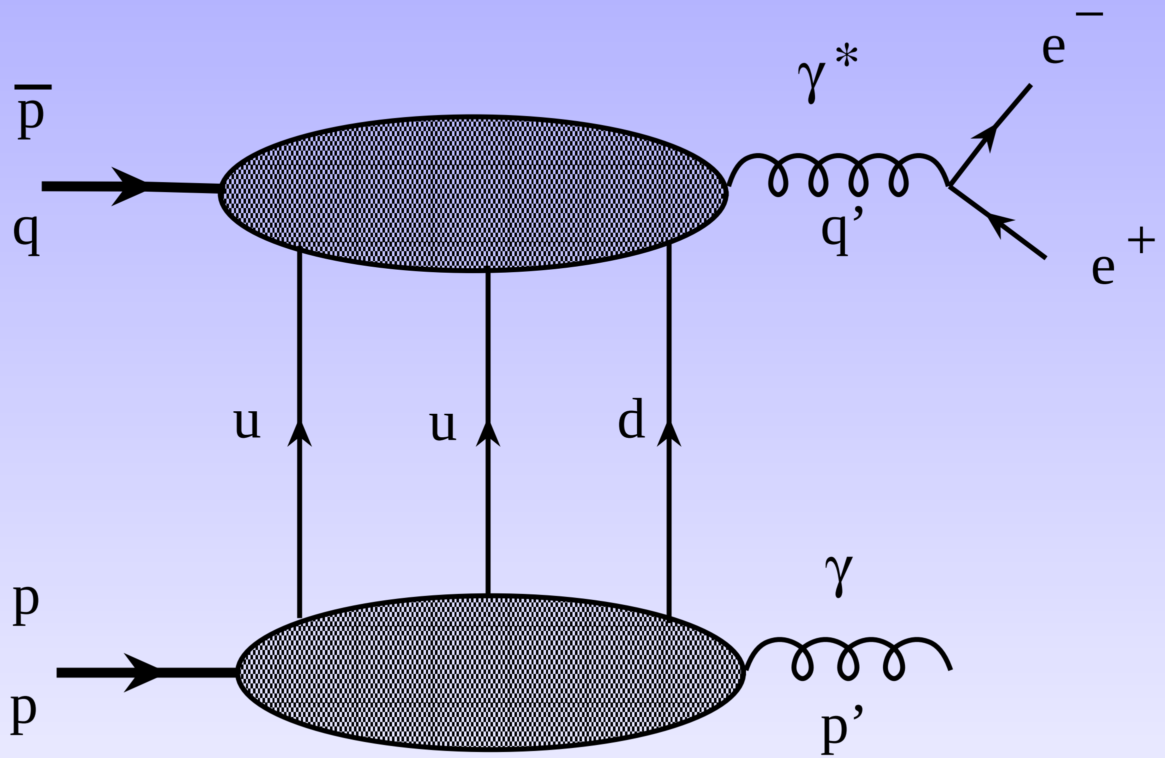
*More problematic since 3 quark exchange !*

BUT Remember : Baryon Form Facor factorizes in the same way as Meson Form Factor !

$\longrightarrow$ Factorization of the $p \longrightarrow \gamma$ TDA in $\bar{p}p \longrightarrow \gamma^*\gamma$

*This is NOT a proof ... Hope for a technical derivation*

# The factorization of $\bar{N}\ N \to \gamma^* \gamma$



# From DAs to TDAs

- Recall definition of Distribution Amplitudes

$$4\langle 0|\epsilon^{ijk}u^i_\alpha(z_1\,n)u^j_\beta(z_2\,n)d^k_\gamma(z_3\,n)|B(p,s)\rangle = f_N$$

$$V(\hat{p}\,C)_{\alpha\beta}(\gamma^5\,B)_\gamma + A(\hat{p}\,\gamma^5\,C)_{\alpha\beta}B_\gamma + T(p^\nu i\sigma_{\mu\nu}\,C)_{\alpha\beta}(\gamma^\mu\,\gamma^5\,B)$$

$i,j,k =$ color indices $\qquad n =$ light cone + direction

- Define Transition Distribution Amplitudes

$$4\langle\pi^0(p')|\,\epsilon^{ijk}u^i_\alpha(z_1\,n)u^j_\beta(z_2\,n)d^k_\gamma(z_3\,n)\,|p(p,s)\rangle\Big|_{z^+=0,\,z_T=0} =$$

$$\frac{-f_N}{2f_\pi}\Big[V_1^0(\hat{P}C)_{\alpha\beta}(B)_\gamma + A_1^0(\hat{P}\gamma^5C)_{\alpha\beta}(\gamma^5B)_\gamma -$$

$$3T_1^0(P^\nu i\sigma_{\mu\nu}C)_{\alpha\beta}(\gamma^\mu B)_\gamma] + V_2^0(\hat{P}C)_{\alpha\beta}(\hat{\Delta}_TB)_\gamma +$$

$$A_2^0(\hat{P}\gamma^5C)_{\alpha\beta}(\hat{\Delta}_T\gamma^5B)_\gamma + T_2^0(\Delta_T^\mu P^\nu\sigma_{\mu\nu}C)_{\alpha\beta}(B)_\gamma$$

$$+T_3^0(P^\nu\sigma_{\mu\nu}C)_{\alpha\beta}(\sigma^{\mu\rho}\Delta_T^\rho B)_\gamma + \frac{T_4^0}{M}(\Delta_T^\mu P^\nu\sigma_{\mu\nu}C)_{\alpha\beta}(\hat{\Delta}_TB)_\gamma$$

$B =$ nucleon spinor.

• **Fourier transform** each TDA, $\rightarrow$ momentum fractions representation

$$F(z_i P{\cdot}n) = \int\limits_{-1+\xi}^{1+\xi} d^3x\, \delta\Big(\sum x_i - 2\xi\Big) e^{-iPn\Sigma x_i z_i} F(x_i, \xi, t, Q^2)$$

• **Factorize** process amplitude :

$$\mathcal{M}(Q^2, \xi, t) = \int dx\, dy\, \phi(y_i) T_H(x_i, y_i, Q^2) F(x_i, \xi, t)$$

# Evolution equations

•QCD radiative corrections $\longrightarrow$ logarithmic scaling violations.

• The scale dependence of $N \to \pi$ or $N \to \gamma$ TDAs is governed by evolution equations = an extension of DGLAP/ERBL equations for DAs and GPDs

• Start with quark fields having definite chirality or helicity $q^{\uparrow(\downarrow)} = \frac{1}{2}\left(1 \pm \gamma^5\right) q$

• Separate "minus" components $\longrightarrow$ dominant twist-2 with $\hat{n} = n^\mu \gamma_\mu$

# Evolution equations (2)

- Two relevant operators in our problem :

$$B^{1/2}_{\alpha\beta\gamma}(z_i) = \epsilon^{ijk}(\hat{n}q_i^{\uparrow})_\alpha(z_1 n)(\hat{n}q_j^{\downarrow})_\beta(z_2 n)(\hat{n}q_k^{\uparrow})_\gamma(z_3 n)$$

$$B^{3/2}_{\alpha\beta\gamma}(z_i) = \epsilon^{ijk}(\hat{n}q_i^{\uparrow})_\alpha(z_1 n)(\hat{n}q_j^{\uparrow})_\beta(z_2 n)(\hat{n}q_k^{\uparrow})_\gamma(z_3 n)$$

- They obey renormalisation group equation

$\mu\frac{d}{d\,\mu}\,B = H\cdot B$ with

$$H = -\frac{\alpha_s}{2\pi}\left[(1+1/N_c)\,H_h + 3C_F/2\right]$$

- $H_{3/2} = \mathcal{H}^v_{1\,2} + \mathcal{H}^v_{2\,3} + \mathcal{H}^v_{1\,3}$ with $\mathcal{H}^v_{1\,2}B(z_i) =$

$$-\int_0^1 \frac{d\alpha}{\alpha}\left\{\bar{\alpha}\left[B(z^\alpha_{1\,2}, z_2, z_3) - B(z_1, z_2, z_3)\right]\right.$$

$$\left. +\bar{\alpha}\left[B(z_1, z^\alpha_{2\,1}, z_3) - B(z_1, z_2, z_3)\right]\right\}$$

# Evolution equations (3)

• $H_{1/2} = H_{3/2} - \mathcal{H}^e_{1\,2} - \mathcal{H}^e_{2\,3}$ where $\mathcal{H}^e_{1\,2} B(z_i) =$

$$\int_0^1 d\alpha_1\, d\alpha_2\, d\alpha_3\, \delta(1-\alpha_1-\alpha_2-\alpha_3) B(z_{1\,2}^{\alpha_1}, z_{2\,1}^{\alpha_2}, z_3)$$

• Derive the corresponding equation for the matrix element of operators $B$ from the RGE

$$Q\frac{d}{dQ}\,F^{\uparrow\downarrow\uparrow}(x_i) = -\frac{\alpha_s}{2\pi}\Big[\frac{3}{2}\,C_F\,F^{\uparrow\downarrow\uparrow}(x_i) - (1+\frac{1}{N_c})\mathcal{A}\Big]$$

$$\begin{aligned}
\mathcal{A} = \Bigg[\Bigg(&\int_{-1+\xi}^{1+\xi} dx'_1 \left[\frac{x_1\rho(x'_1,x_1)}{x'_1(x'_1-x_1)}\right]_+ + \int_{-1+\xi}^{1+\xi} dx'_2 \left[\frac{x_2\rho(x'_2,x_2)}{x'_2(x'_2-x_2)}\right]_+\Bigg) F^{\uparrow\downarrow\uparrow}(x'_1,x'_2,x_3)\\
&+\Bigg(\int_{-1+\xi}^{1+\xi} dx'_1 \left[\frac{x_1\rho(x'_1,x_1)}{x'_1(x'_1-x_1)}\right]_+ + \int_{-1+\xi}^{1+\xi} dx'_3 \left[\frac{x_3\rho(x'_3,x_3)}{x'_3(x'_3-x_3)}\right]_+\Bigg) F^{\uparrow\downarrow\uparrow}(x'_1,x_2,x'_3)\\
&+\Bigg(\int_{-1+\xi}^{1+\xi} dx'_2 \left[\frac{x_2\rho(x'_2,x_2)}{x'_2(x'_2-x_2)}\right]_+ + \int_{-1+\xi}^{1+\xi} dx'_3 \left[\frac{x_3\rho(x'_3,x_3)}{x'_3(x'_3-x_3)}\right]_+\Bigg) F^{\uparrow\downarrow\uparrow}(x_1,x'_2,x'_3)\\
&+\frac{1}{2\xi-x_3}\Bigg(\int_{-1+\xi}^{1+\xi} dx'_1\,\frac{x_1}{x'_1}\rho(x'_1,x_1) + \int_{-1+\xi}^{1+\xi} dx'_2\,\frac{x_2}{x'_2}\rho(x'_2,x_2)\Bigg) F^{\uparrow\downarrow\uparrow}(x'_1,x'_2,x_3)\\
&+\frac{1}{2\xi-x_1}\Bigg(\int_{-1+\xi}^{1+\xi} dx'_2\,\frac{x_2}{x'_2}\rho(x'_2,x_2) + \int_{-1+\xi}^{1+\xi} dx'_3\,\frac{x_3}{x'_3}\rho(x'_3,x_3)\Bigg) F^{\uparrow\downarrow\uparrow}(x_1,x'_2,x'_3)\Bigg]\Bigg\}
\end{aligned}$$

• with integration region restricted by:
$\rho(x,y) = \theta(x \geq y \geq 0) - \theta(x \leq y \leq 0)$,
and $x_i' \in [-1+\xi, 1+\xi]$

• Different evolution in the various $x_i$ sectors.
When $x_i > 0 \rightarrow$ usual ERBL ($x_i \rightarrow x_i/2\xi$ rescaling).

• Other regions need further study !

# CHIRAL LIMIT of $p \to \pi$ TDA

- Soft pion theorems$\to$

$$< \pi^a(k)|O|P(p,s) >= \frac{-i}{f_\pi} < 0|[Q_5^a, O]|P(p,s) >$$

$$+ \frac{ig_A}{4f_\pi p\cdot k} \sum_{s'} \bar{u}(p,s)\hat{k}\gamma_5\tau^a u(p,s') < 0|O|P(p,s') >$$

1st term $\to$ TDA at threshold ; 2nd term $\to$ nucleon pole.

- Since $[Q_5^b, \psi] = i\frac{\tau^b}{2}\gamma^5\psi$

# CHIRAL LIMIT ($\xi \to 1$)

$$V_1^0(x_1,x_2,x_3) \quad \to V(x_1,x_2,x_3)$$
$$= \quad \left(\phi_N(x_i) + \phi_N(x_2,x_1,x_3)\right)/2$$

$$A_1^0(x_1,x_2,x_3) \quad \to A(x_1,x_2,x_3)$$
$$= \quad \frac{1}{2}\left(\phi_N(x_i) - \phi_N(x_2,x_1,x_3)\right)$$
$$T_1^0(x_i) \to T(x_i) = \quad \frac{1}{2}\left(\phi_N(x_i) + \phi_N(x_2,x_3,x_1)\right)$$

*where* $\phi_N(x_1,x_2,x_3) =$ *standard leading twist DA*

# Interpretation

- The proton **DA** selects the valence contribution and analyses it from large angle scattering (and Form Factors)

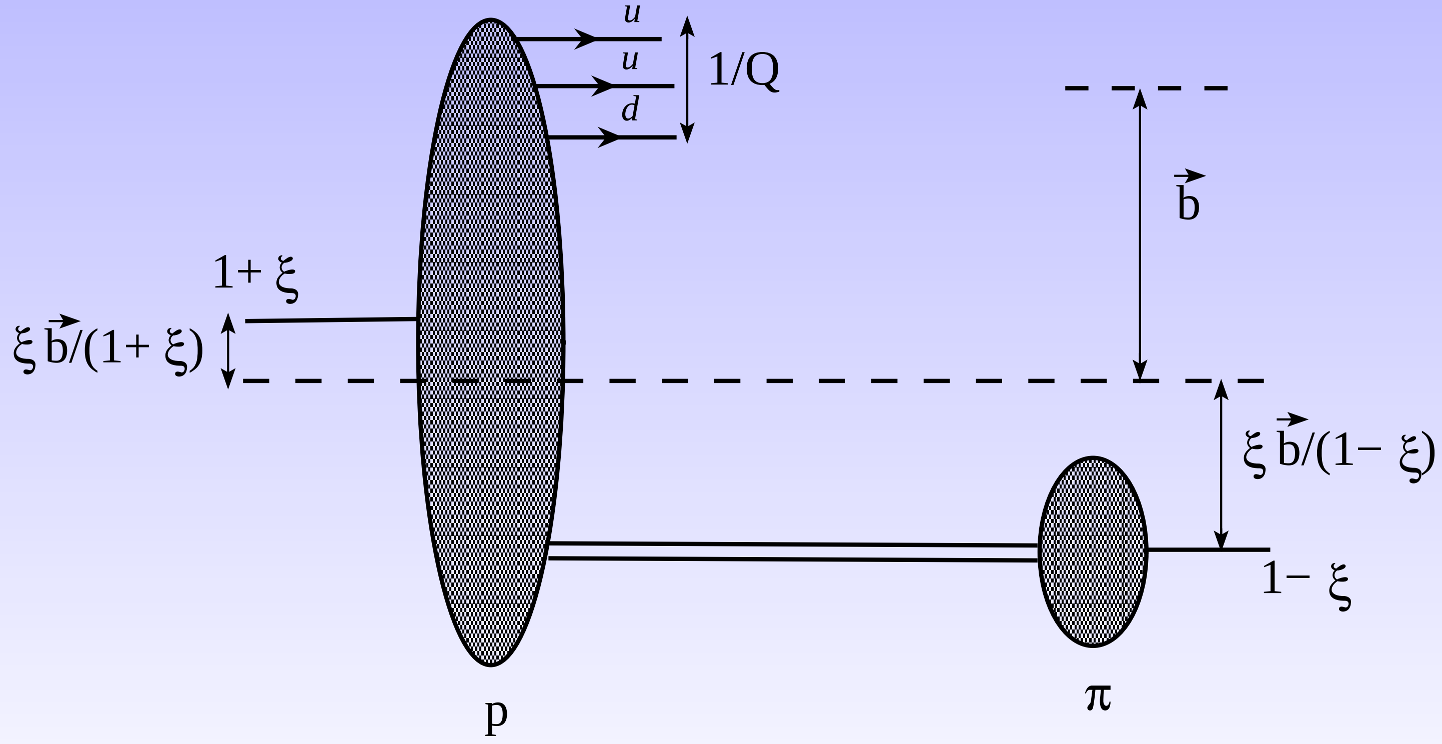
- The proton $\longrightarrow \pi$ **TDA** allows a pion (cloud) around the valence contribution.

- The proton $\longrightarrow \gamma$ **TDA** allows a photon (cloud) around the valence contribution.

- The proton $\longrightarrow \rho$ **TDA**...

# Impact parameter interpretation

- As for GPDs and GDAs, Fourier transform $t \longrightarrow b_T$
- Transverse picture of *pion cloud* in the proton



# To test these ideas: Model-independent predictions

• scaling law for the amplitude : $\mathcal{M}(Q^2, \xi) \sim \frac{\alpha_s(Q^2)^2}{Q^4}$ , ( up to logarithmic corrections ).

• Ratio : $\frac{d\sigma(\bar{p}p \to l^+ l^- \pi^0)/dQ^2}{d\sigma(\bar{p}p \to l^+ l^-)/dQ^2}$ almost $Q^2$ independent.

• $\gamma^*_T$ dominates $\rightarrow \frac{d\sigma(p\bar{p} \to l^+ l^- \pi)}{\sigma d\theta} \sim 1 + cos^2\theta$ ($\theta$ = lepton angle in $\gamma^*$ CMS)

• Choose $V$, $A$ and $T$ $\rightarrow$ Estimate threshold cross section in terms of e-m form factor

# This description also applies to crossed reactions

• Backward VCS $\gamma^* P \to P'\gamma$
Data exist (JLab) for $Q^2$ up to 1 $GeV^2$.

Data from HERMES ?

• and backward meson electroproduction
$\gamma^* P \to P'\pi$ ; $\gamma^* P \to P'\rho$ ...

• Data exist (JLab) Analysis to be done

# $\gamma^*\gamma$ collisions

• One may describe along the same lines the crossed reactions

$$\gamma^*\gamma \rightarrow \pi^+\pi^- \quad (1)$$

$$\gamma^*\gamma \rightarrow \pi^\pm\rho^\mp \quad (2)$$

and

$$\gamma^*\gamma \rightarrow \rho^+\rho^- \quad (3)$$

in the near forward region and for large virtual photon invariant mass $Q$, which may be studied in detail at intense electron colliders such as BABAR and BELLE.

• wait for talk by Jean Philippe Lansberg ...

# CONCLUSIONS on TDAs

• FAIR will help to understand the deep structure of the proton

• Transition Distribution Amplitudes will reveal the dynamics of the *next to lowest* Fock state

• $\bar{p}p \longrightarrow \gamma^* \pi$ explores the pion cloud.

• $\bar{p}p \longrightarrow \gamma^* \rho$ explores the $\rho$ cloud.

• $\bar{p}p \longrightarrow \gamma^* \gamma$ explores the photon cloud.

• Detectors should be ready to measure these reactions !

• If *Polarized* beam and target $\longrightarrow$ spin structure too!

• NOT SO SMALL CROSS-SECTIONS AND BIG REWARDS.

# CONCLUSIONS

- Exclusive Hard Reactions are revealing much about Hadron structure
- Theoretical progress ongoing ...
- Extremely Good Expriments are being done and prepared
- Nature seems to help us with early scaling !