Effect of the Neutrino Electromagnetic Form Factors on Neutrino Interaction in Dense Matter

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OUTLINE

- The Aim of Research
- Formalism (Model of matter, Neutrino interaction with matter)
- Results and Discussion
- Conclusion

MOTIVATION

- In Standard Model neutrino have zero charge radius (R), zero magnetic moment (μ_ν) and massless.
- However, there are experimental evidences found that $\mu_\nu < 10^{-10}\,\mu_B$ at the 90% confidence level **[Z. Daraktchieva et al (MUNU Collaboration) PLB 564, 190 (2003)]**
- LAMF experiment also found that neutrino have $R^2 = (22.5 \pm 67.5) \times 10^{-12}\ \mathrm{MeV}^{-2}$ **[P.Vilain et al, PLB 345, 115 (1995)]**
- **According to this results**

OUR AIM

1. We observe the effect of neutrino and anti-neutrino electromagnetic form factors on neutrino mean free path (λ).

2. We also observe effect the weak magnetism of nucleons on neutrino mean free path

RELATIVISTIC NUCLEAR MODEL

- **The Effective Lagrangian of E-RMF Model**

$$\mathcal{L}^{nuc} = \mathcal{L}_N + \mathcal{L}_M,$$

where the nucleon part, up to order $\nu = 3$, has the form

$$\mathcal{L}_N = \bar{\psi}[i\gamma^\mu(\partial_\mu + i\bar{v}_\mu + ig_\rho\bar{b}_\mu + ig_\omega V_\mu)$$

$$+ g_A\gamma^\mu\gamma^5\bar{a}_\mu - M + g_\sigma\sigma]\psi - \frac{f_\rho g_\rho}{4M}\bar{\psi}\bar{b}_{\mu\nu}\sigma^{\mu\nu}\psi,$$

with

$$\psi = \begin{pmatrix} p \\ n \end{pmatrix}, \qquad \bar{v}_\mu = -\frac{i}{2}(\xi^\dagger\partial_\mu\xi + \xi\partial_\mu\xi^\dagger) = \bar{v}_\mu^\dagger,$$

RELATIVISTIC NUCLEAR MODEL

$$\bar{a}_\mu = -\frac{i}{2}(\bar{\xi}^\dagger \partial_\mu \bar{\xi} - \bar{\xi} \partial_\mu \bar{\xi}^\dagger) = \bar{a}_\mu^\dagger,$$

$$\bar{\xi} = \exp(i\bar{\pi}(x)/f_\pi), \quad \bar{\pi}(x) = \tfrac{1}{2}\vec{\tau} \cdot \vec{\pi}(x),$$

$$\bar{\pi}(x) = \tfrac{1}{2}\vec{\tau} \cdot \vec{\pi}(x),$$

$$\bar{b}_{\mu\nu} = D_\mu \bar{b}_\nu - D_\nu \bar{b}_\mu + i g_\rho [\bar{b}_\mu, \bar{b}_\nu], \quad D_\mu = \partial_\mu + i \bar{v}_\mu$$

$$V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu,$$

$$\sigma^{\mu\nu} = \tfrac{1}{2}[\gamma^\mu, \gamma^\nu].$$

R. J. Furnstahl, et. Al, Nucl. Phys. A. 598, 539(1996); Nucl. Phys. A. 615, 441 (1997)

RELATIVISTIC NUCLEAR MODEL

$$
\begin{aligned}
\mathcal{L}_M = {} & \frac{1}{4} f_\pi^2 \mathrm{Tr}(\partial_\mu \bar{U} \partial^\mu \bar{U}^\dagger) + \frac{1}{4} f_\pi^2 \mathrm{Tr}(\bar{U}\bar{U}^\dagger - 2) \\
& + \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - \frac{1}{2} \mathrm{Tr}(\bar{b}_{\mu\nu} \bar{b}^{\mu\nu}) - \frac{1}{4} V_{\mu\nu} V^{\mu\nu} \\
& - g_{\rho\pi\pi} \frac{2 f_\pi^2}{m_\rho^2} \mathrm{Tr}(\bar{b}_{\mu\nu} \bar{v}^{\mu\nu}) + \frac{1}{2} \left(1 + \eta_1 \frac{g_\sigma \sigma}{M} \right. \\
& \left. + \frac{\eta_2}{2} \frac{g_\sigma^2 \sigma^2}{M^2} \right) m_\omega^2 V_\mu V^\mu + \frac{1}{4!} \zeta_0 g_\omega^2 (V_\mu V^\mu)^2 \\
& + \left(1 + \eta_\rho \frac{g_\sigma \sigma}{M} \right) m_\rho^2 \mathrm{Tr}(\bar{b}_\mu \bar{b}^\mu) \\
& - m_\sigma^2 \sigma^2 \left(1 + \frac{\kappa_3}{3!} \frac{g_\sigma \sigma}{M} + \frac{\kappa_4}{4!} \frac{g_\sigma^2 \sigma^2}{M^2} \right),
\end{aligned}
$$

where

$$\bar{U} = \bar{\xi}^2, \quad \bar{v}_{\mu\nu} = \partial_\mu \bar{v}_\nu - \partial_\nu \bar{v}_\mu + i[\bar{v}_\mu, \bar{v}_\nu] = -i[\bar{a}_\mu, \bar{a}_\nu].$$

R. J. Furnstahl, et. Al, Nucl. Phys. A. 598, 539(1996); Nucl. Phys. A. 615, 441 (1997)

PARAMETER SET

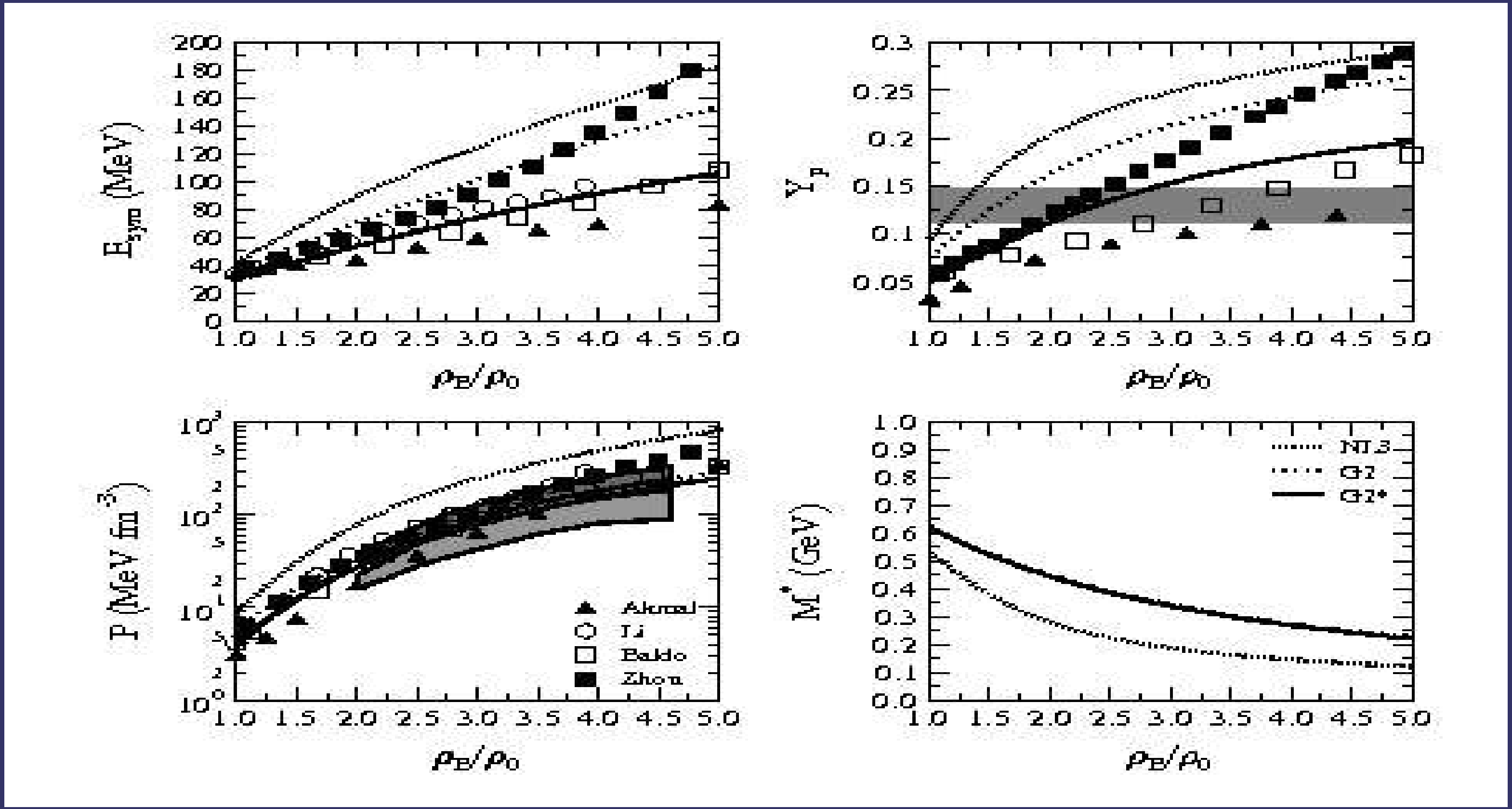
Parameter	G2	NL-3	G2*
m_S/M	0.554	0.541	0.554
$g_S/(4\pi)$	0.835	0.813	0.835
$g_V/(4\pi)$	1.016	1.024	1.016
$g_R/(4\pi)$	0.755	0.712	0.938
κ_3	3.247	1.465	3.247
κ_4	0.632	−5.668	0.632
ζ_0	2.642	0	2.642
η_1	0.650	0	0.650
η_2	0.110	0	0.110
η_ρ	0.390	0	4.490

Why we choose this parameter sets ?

A. Sulaksono, C. K. Williams, P.T.P. Hutauruk and T. Mart
Phys Rev C73, 025803(2006)

PARAMETER SET

Because G_2^* parameter set predicts a soft EOS at high density and has coincides with the direct URCA prosess threshold.



Shaded region data experiment from P. Danielewicz

LAGRANGIAN DENSITY

- **The Lagrangian density of neutrino matter interactions for each constituent**

$$\mathcal{L}^{j}_{\rm int} = \frac{G_F}{\sqrt{2}}\left(\bar{\nu}\,\Gamma^{\mu}_{W}\,\nu\right)\left(\bar{\psi}\,J^{Wj}_{\mu}\,\psi\right) + \frac{4\pi\alpha}{q^2}\left(\bar{\nu}\,\Gamma^{\mu}_{\rm EM}\,\nu\right)\left(\bar{\psi}\,J^{{\rm EM}j}_{\mu}\,\psi\right),$$

$$\Gamma^{\mu}_{W} = \gamma^{\mu}(1-\gamma^{5}),$$

$$\Gamma^{\mu}_{\rm EM} = f_{m\nu}\gamma^{\mu} + g_{1\nu}\gamma^{\mu}\gamma^{5} - (f_{2\nu} + i g_{2\nu}\gamma^{5})\frac{p^{\mu}}{2m_e},$$

$$J^{Wj}_{\mu} = F^{Wj}_{1}\gamma_{\mu} - G^{j}_{A}\gamma_{\mu}\gamma^{5} + i F^{Wj}_{2}\frac{\sigma_{\mu\nu}q^{\nu}}{2M},$$

$$J^{{\rm EM}j}_{\mu} = F^{{\rm EM}j}_{1}\gamma_{\mu} + i F^{{\rm EM}j}_{2}\frac{\sigma_{\mu\nu}q^{\nu}}{2M},$$

FORM FACTORS OF NUCLEON

Target	F_1^V	G_A	F_2^V
n	-0.5	$-g_A/2$	$-1/2(\mu_p - \mu_n) - 2\sin^2\theta_w \mu_n$
p	$0.5 - 2\sin^2\theta_w$	$g_A/2$	$1/2(\mu_p - \mu_n) - 2\sin^2\theta_w \mu_p$
e	$0.5 + 2\sin^2\theta_w$	$1/2$	0
μ	$-0.5 + 2\sin^2\theta_w$	$-1/2$	0

Target	F_1^{EM}	F_2^{EM}
n	0	μ_n
p	1	μ_p
e	1	0
μ	1	0

For neutrinos, while for anti-neutrinos the sign are changed, $g_A \rightarrow -g_A$

$\mu_p = 1.793$ $\quad$ $g_A = 1.260$

$\mu_n = -1.913$ $\quad$ $\sin^2\Theta_W = 0.231$

P. Vogel and J. Engel, PRD 39, 11 (1989)

FORM FACTORS OF NEUTRINO

$$\Gamma^{\mu}_{W} = \gamma^{\mu}(1-\gamma^{5}), \tag{6}$$

while the electromagnetic properties of Dirac neutrinos are described in terms of four form factors $f_{1\nu}$, $g_{1\nu}$, $f_{2\nu}$, and $g_{2\nu}$, which stand for the Dirac, anapole, magnetic, and electric form factors, respectively. The electromagnetic vertex Γ^{μ}_{EM} contains electromagnetic form factors [29,30]. Explicitly, it reads

$$\Gamma^{\mu}_{EM} = f_{m\nu}\gamma^{\mu} + g_{1\nu}\gamma^{\mu}\gamma^{5} - (f_{2\nu} + i g_{2\nu}\gamma^{5})\frac{P^{\mu}}{2m_e}, \tag{7}$$

where $f_{m\nu} = f_{1\nu} + (m_\nu/m_e)f_{2\nu}$, $P^{\mu} = k^{\mu} + k^{\mu\prime}$, and m_ν and m_e are the neutrino and electron masses, respectively. In the static limit, the reduced Dirac form factor $f_{1\nu}$ and the neutrino anapole form factor $g_{1\nu}$ are related to the vector and axial vector charge radii $\langle R_V^2\rangle$ and $\langle R_A^2\rangle$ through [29]

$$f_{1\nu}(q^2) = \tfrac{1}{6}\langle R_V^2\rangle q^2 \quad \text{and} \quad g_{1\nu}(q^2) = \tfrac{1}{6}\langle R_A^2\rangle q^2, \tag{8}$$

where the neutrino charge radius is defined by $R^2 = \langle R_V^2\rangle + \langle R_A^2\rangle$. In the limit of $q^2 \to 0$, $f_{2\nu}$ and $g_{2\nu}$, respectively, define the neutrino magnetic moment and the Charge Parity (CP) violating electric dipole moment [29,31], i.e.,

$$\mu_\nu^m = f_{2\nu}(0)\mu_B \quad \text{and} \quad \mu_\nu^e = g_{2\nu}(0)\mu_B,$$

FORMALISM

- **Neutrino and Anti-Neutrino Differential Cross Section**

$$\left(\frac{1}{V}\frac{d^3\sigma}{d^2\Omega' dE'_\nu}\right) = -\frac{1}{16\pi^2}\frac{E'_\nu}{E_\nu}\left[\left(\frac{G_F}{\sqrt{2}}\right)^2 \left(L_\nu^{\mu\nu}\Pi^{\rm Im}_{\mu\nu}\right)^{(W)} + \left(\frac{4\pi\alpha}{q^2}\right)^2 \left(L_\nu^{\mu\nu}\Pi^{\rm Im}_{\mu\nu}\right)^{(EM)} + \frac{8G_F\pi\alpha}{q^2\sqrt{2}}\left(L_\nu^{\mu\nu}\Pi^{\rm Im}_{\mu\nu}\right)^{(INT)}\right].$$

A. Sulaksono, C. K. Williams, P. T. P. Hutauruk and T. Mart , PRC 73, 025803 (2006)

- **Neutrino and Anti-Neutrino Mean Free Path**

LEPTON TENSOR OF NEUTRINO

$$L_\nu^{\mu\nu(W)} = 8\left[2k^\mu k'^\nu - (k^\mu q^\nu + k'^\nu q^\mu) + g^{\mu\nu}(k\cdot q) - i\epsilon^{\alpha\mu\beta\nu}k_\alpha k'_\beta\right],$$

$$\begin{aligned} L_\nu^{\mu\nu(EM)} &= 4(f_{m\nu}^2 + g_{1\nu}^2)[2k^\mu k'^\nu - (k^\mu q^\nu + k'^\nu q^\mu) + g^{\mu\nu}(k\cdot q)] \\ &- 8i\, f_{m\nu} g_{1\nu} \epsilon^{\alpha\mu\beta\nu}(k_\alpha k'_\beta) \\ &- \frac{f_{2\nu}^2 + g_{2\nu}^2}{m_e^2}(k\cdot q)[4k^\mu k'^\nu - 2(k^\mu q^\nu + q^\mu k'^\nu) + q^\mu q^\nu], \end{aligned}$$

$$f_{m\nu} = f_{1\nu} + (m_\nu/m_e) f_{2\nu},$$

$$L_\nu^{\mu\nu(INT)} = 4(f_{m\nu} + g_{1\nu})[2k^\mu k'^\nu - (k^\mu q^\nu + k'^\nu q^\mu) + g^{\mu\nu}(k\cdot q) - i\epsilon^{\alpha\mu\beta\nu}k_\alpha k'_\beta],$$

$f_{1\nu}$, $g_{1\nu}$, $f_{2\nu}$, $g_{2\nu}$ are Dirac, anapole, magnetic, electric form factors respectively

POLARISATION TENSOR

$$\Pi_{\mu\nu}^{\mathrm{Im}(W)j} = \left(F_1^{Wj\,2} + G_A^{j\,2}\right)\Pi_{\mu\nu}^{Vj} + \left(G_A^{j2} + \frac{q^2}{2mM}F_1^{Wj}F_2^{Wj}\right)\Pi^{Aj}g_{\mu\nu} - 2\left(F_1^{Wj}G_A^j + \frac{m}{M}F_2^{Wj}G_A^j\right)\Pi_{\mu\nu}^{V-Aj} + \frac{F_2^{Wj\,2}}{M^2} \times\left[\left(m^2 + \frac{q^2}{4}\right)(q^2 g_{\mu\nu} - q_\mu q_\nu) - \frac{q^2}{8}\Pi_{\mu\nu}^{Vj}\right],$$

$$\Pi_{\mu\nu}^{\mathrm{Im}(\mathrm{EM})j} = F_1^{\mathrm{EM}j\,2}\Pi_{\mu\nu}^{Vj} + \frac{q^2}{2mM}F_1^{\mathrm{EM}j}F_2^{\mathrm{EM}j}\Pi^{Aj}g_{\mu\nu} + \frac{F_2^{\mathrm{EM}j\,2}}{M^2}\left[\left(m^2 + \frac{q^2}{4}\right) \times (q^2 g_{\mu\nu} - q_\mu q_\nu) - \frac{q^2}{8}\Pi_{\mu\nu}^{Vj}\right],$$

$$\Pi_{\mu\nu}^{\mathrm{Im}(\mathrm{INT})j} = \left(F_1^{Wj}F_1^{\mathrm{EM}j} + \frac{q^2}{4M^2}F_2^{Wj}F_2^{\mathrm{EM}j}\right)\Pi_{\mu\nu}^{Vj} + \left[\frac{F_2^{Wj}F_2^{\mathrm{EM}j}}{4M^2}\left(1 + \frac{q^2}{4m^2}\right) - \frac{\left(F_1^{Wj}F_2^{\mathrm{EM}j} + F_2^{Wj}F_1^{\mathrm{EM}j}\right)}{4mM}\right] \times (q^2 g_{\mu\nu} - q_\mu q_\nu)\Pi^{Aj} + \left(\frac{m}{M}F_2^{\mathrm{EM}j}G_A^j - F_1^{\mathrm{EM}j}G_A^j\right)\Pi_{\mu\nu}^{V-Aj},$$

CONTRACTION RESULTS

$$\left(L_{\nu}^{\mu\nu}\Pi_{\mu\nu}^{\rm Im}\right)^{(W)} = -8q^2 \sum_{j=n,p,e^-,\mu^-} \left[A_W^j\left(\Pi_L^j + \Pi_T^j\right) + B_{1W}^j\Pi_T^j + B_{2W}^j\Pi_A^j + C_W^j\Pi_{VA}^j\right],$$

$$\left(L_{\nu}^{\mu\nu}\Pi_{\mu\nu}^{\rm Im}\right)^{(\rm EM)} = q^2 \sum_{j=n,p,e^-,\mu^-} \left[A_{\rm EM}^j\left(\Pi_L^j + \Pi_T^j\right) + B_{\rm 1EM}^j\Pi_T^j + B_{\rm 2EM}^j\Pi_A^j\right],$$

$$\left(L_{\nu}^{\mu\nu}\Pi_{\mu\nu}^{\rm Im}\right)^{(\rm INT)} = -4q^2 \sum_{j=n,p,e^-,\mu^-} \left[A_{\rm INT}^j\left(\Pi_L^j + \Pi_T^j\right) + B_{\rm 1INT}^j\Pi_T^j + B_{\rm 2INT}^j\Pi_A^j + C_{\rm INT}^j\Pi_{VA}^j\right]$$

CONTRACTION RESULTS

$$\Pi_L = \frac{q^2}{2\pi|\vec{q}|^3}\left[\frac{1}{4}(E_F - E^*) + \frac{q_0}{2}\left(E_F^2 - E^{*2}\right) + \frac{1}{3}\left(E_F^3 - E^{*3}\right)\right],$$

$$\Pi_{VA} = \frac{iq^2}{8\pi|\vec{q}|^3}\left[\left(E_F^2 - E^{*2}\right) + q_0(E_F - E^*)\right].$$

$$\Pi_A = \frac{i}{2\pi|\vec{q}|}M^{*2}(E_F - E^*).$$

CONTRACTION RESULTS

$$A^j_W = \left(\frac{2E(E-q_0)+\frac{1}{2}q^2}{|\vec{q}|^2}\right)\left[F_1^{Wj2} + G_A^{j2} - \frac{F_2^{Wj2}q^2}{4M^2}\right],$$

$$B^j_{1W} = \left[F_1^{Wj2} + G_A^{j2} - \frac{F_2^{Wj2}q^2}{4M^2}\right], \quad (29)$$

$$B^j_{2W} = -\left[G_A^{j2} + \frac{q^2}{2mM}F_1^{Wj}F_2^{Wj} - \frac{F_2^{Wj2}q^2}{4M^2}\left(1+\frac{q^2}{4m^2}\right)\right],$$

$$C^j_W = -2(2E-q_0)\left[F_1^{Wj}G_A^j + \frac{m}{M}F_2^{Wj}G_A^j\right],$$

$$A^j_{EM} = \left[\left(\frac{2E(E-q_0)+\frac{1}{2}q^2}{|\vec{q}|^2}\right)(bq^2-a) + \frac{1}{2}bq^2\right] \times \left[F_1^{EMj2} - \frac{F_2^{EMj2}q^2}{4M^2}\right],$$

$$B^j_{1EM} = -\frac{1}{2}(bq^2+a)\left[F_1^{EMj2} - \frac{F_2^{EMj2}q^2}{4M^2}\right],$$

$$B^j_{2EM} = \frac{1}{2}(bq^2+a)\left[\frac{q^2}{2mM}F_1^{EMj}F_2^{EMj} - \frac{F_2^{EMj2}q^2}{4M^2}\left(1+\frac{q^2}{4m^2}\right)\right].$$

CONTRACTION RESULTS

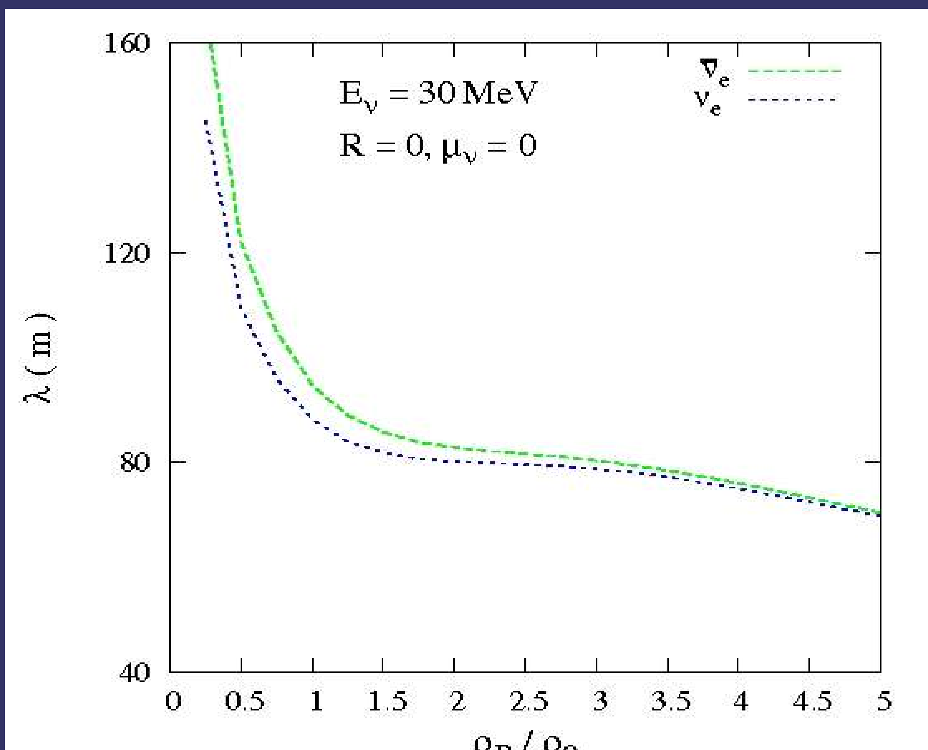
$$A^j_{EM} = \left[\left(\frac{2E(E - q_0) + \frac{1}{2}q^2}{|\vec{q}|^2} \right)(bq^2 - a) + \frac{1}{2}bq^2 \right] \times \left[F_1^{EMj\,2} - \frac{F_2^{EMj\,2} q^2}{4M^2} \right],$$

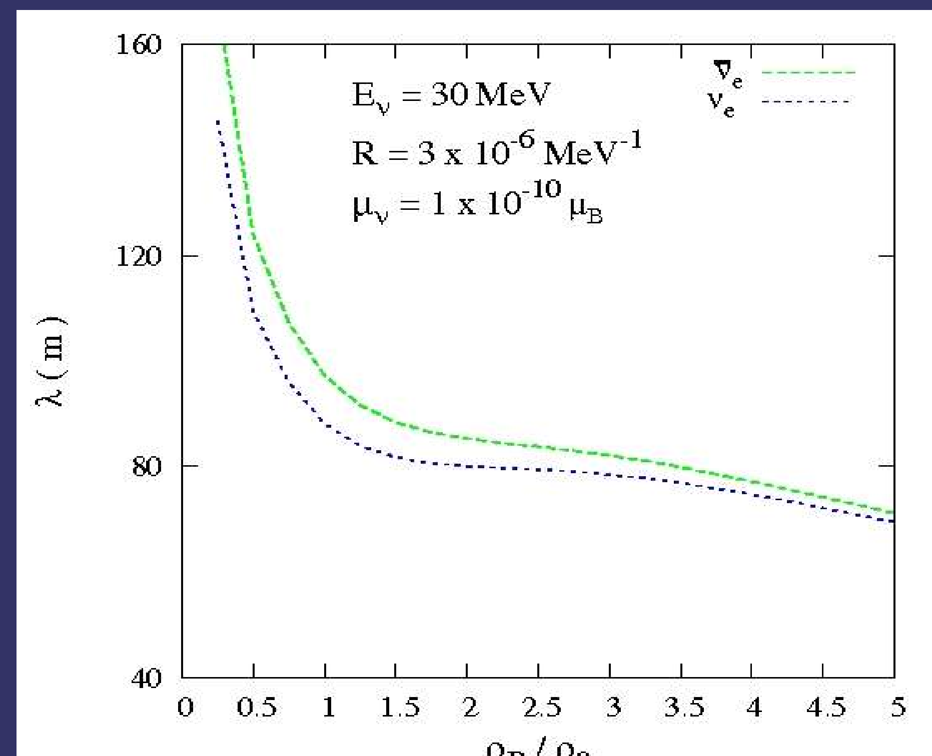
$$B^j_{1EM} = -\frac{1}{2}(bq^2 + a)\left[F_1^{EMj\,2} - \frac{F_2^{EMj\,2} q^2}{4M^2} \right],$$

$$B^j_{2EM} = \frac{1}{2}(bq^2 + a)\left[\frac{q^2}{2mM} F_1^{EMj} F_2^{EMj} - \frac{F_2^{EMj\,2} q^2}{4M^2}\left(1 + \frac{q^2}{4m^2} \right) \right],$$

RESULTS AND DISCUSSION

- ν_e and $\bar{\nu}_e$ Mean Free Path Results

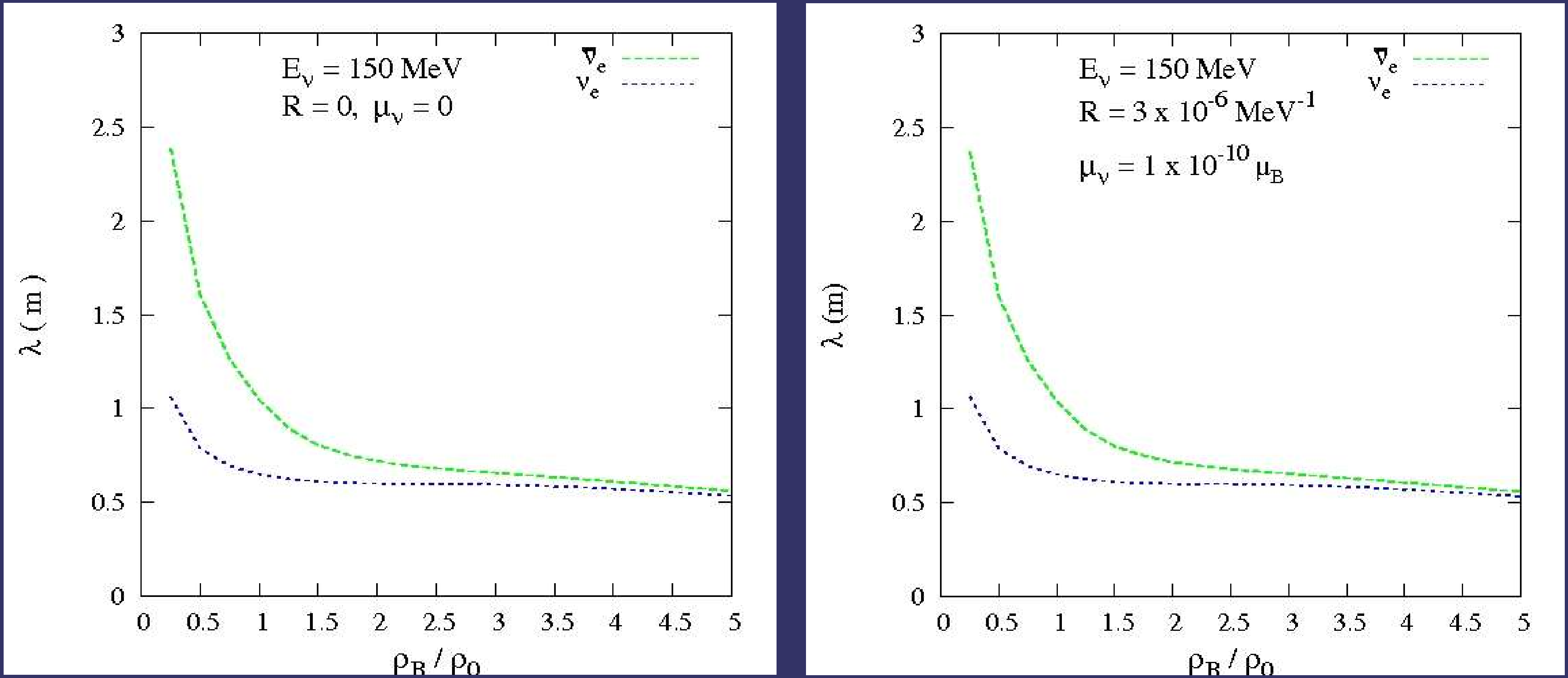




$\bar{\nu}_e$ mean free path is bigger than ν_e ⟷ the effects of weak magnetism of nucleon.

RESULTS AND DISCUSSION

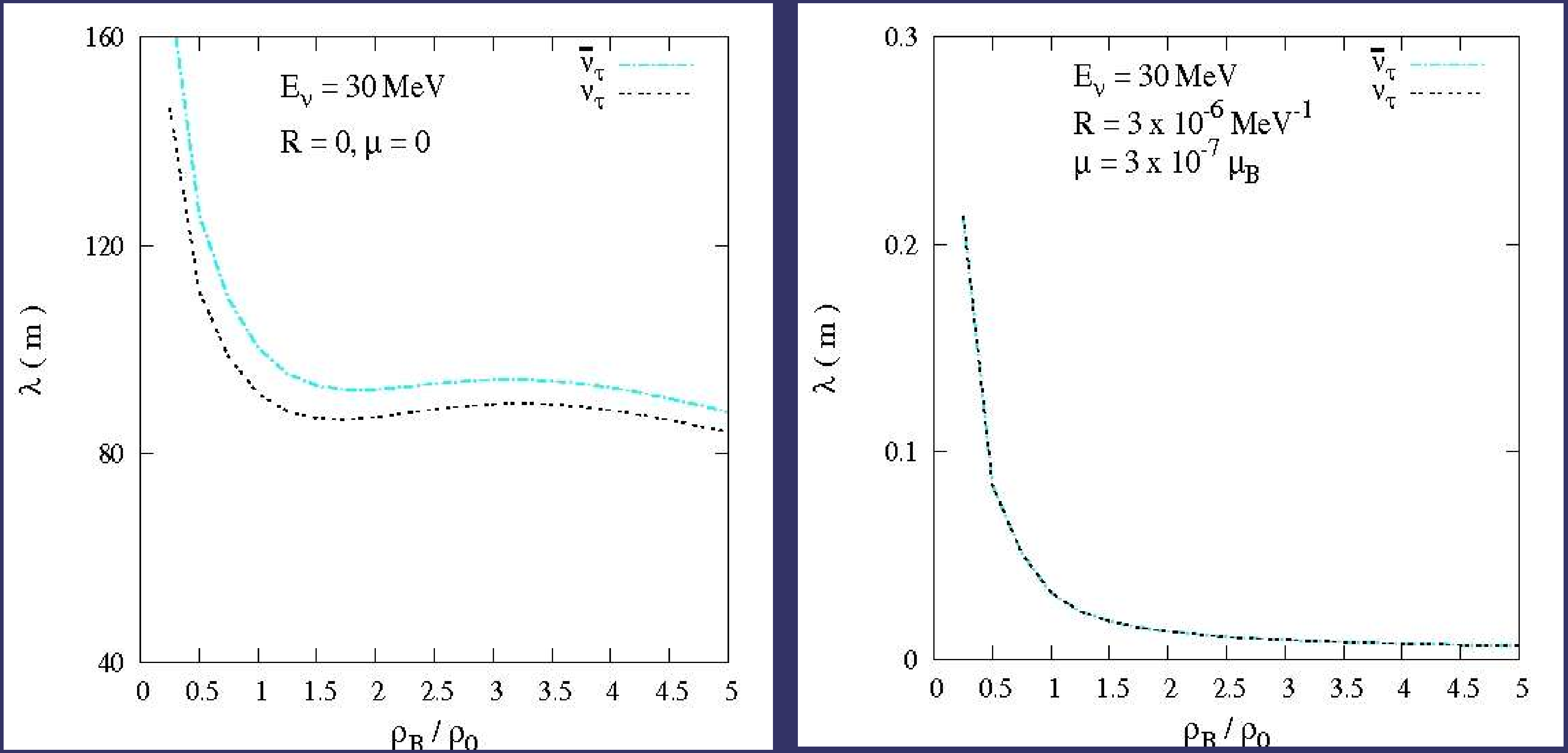
- ν_e and $\bar{\nu}_e$ Mean Free Path Results



If the E_ν increase, neutrino and anti neutrino decrease. The effect moment magnetic and charge radius is too small

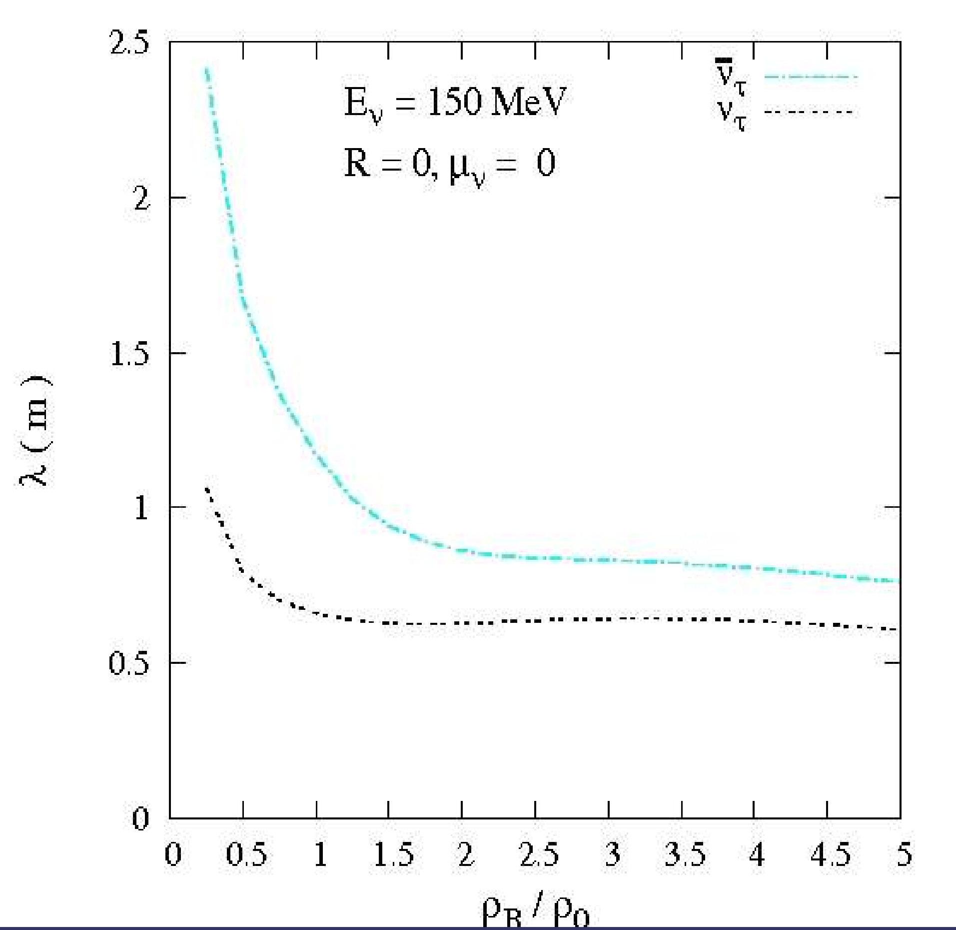
RESULTS AND DISCUSSION

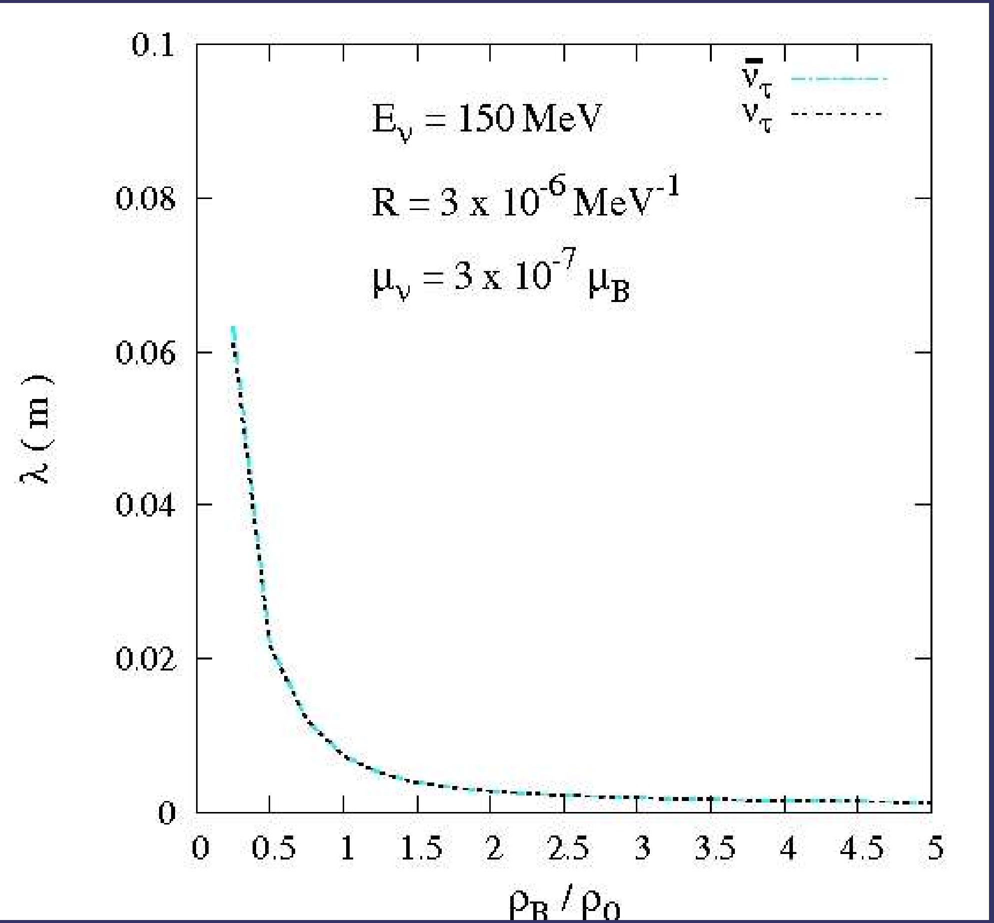
- ν_τ and $\bar{\nu}_\tau$ Mean Free Path Results



RESULTS AND DISCUSSION

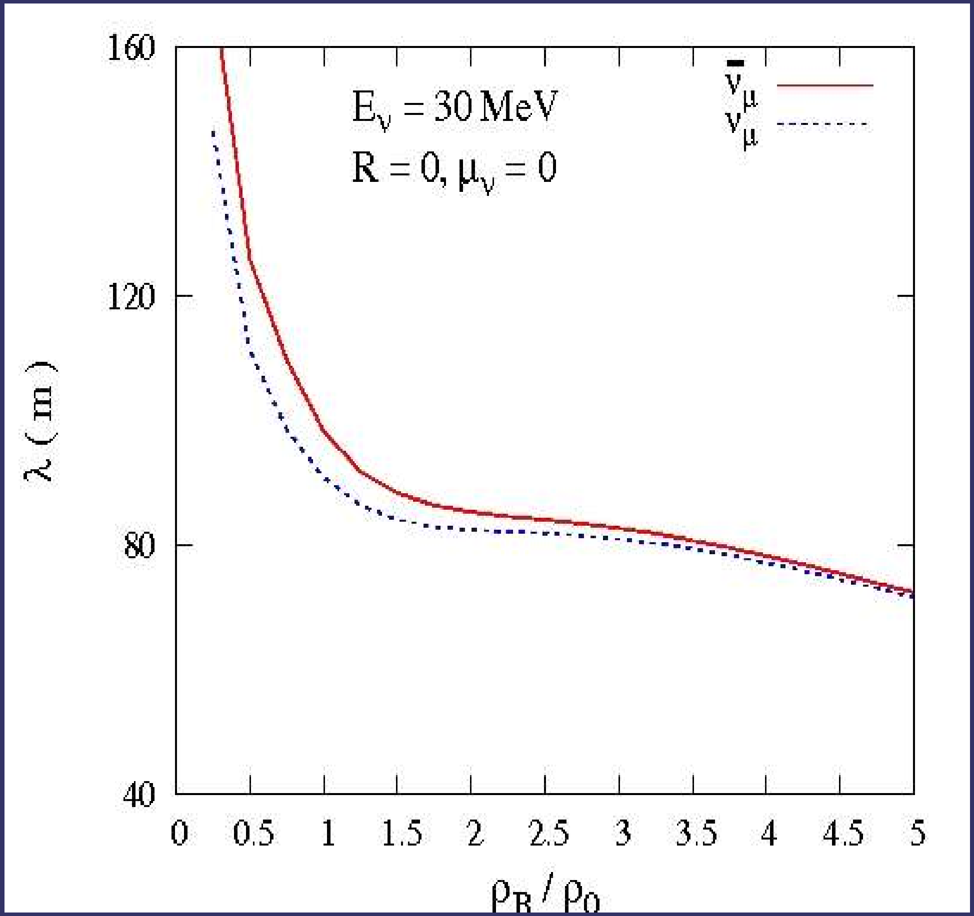
- ν_τ and $\bar{\nu}_\tau$ Mean Free Path Results

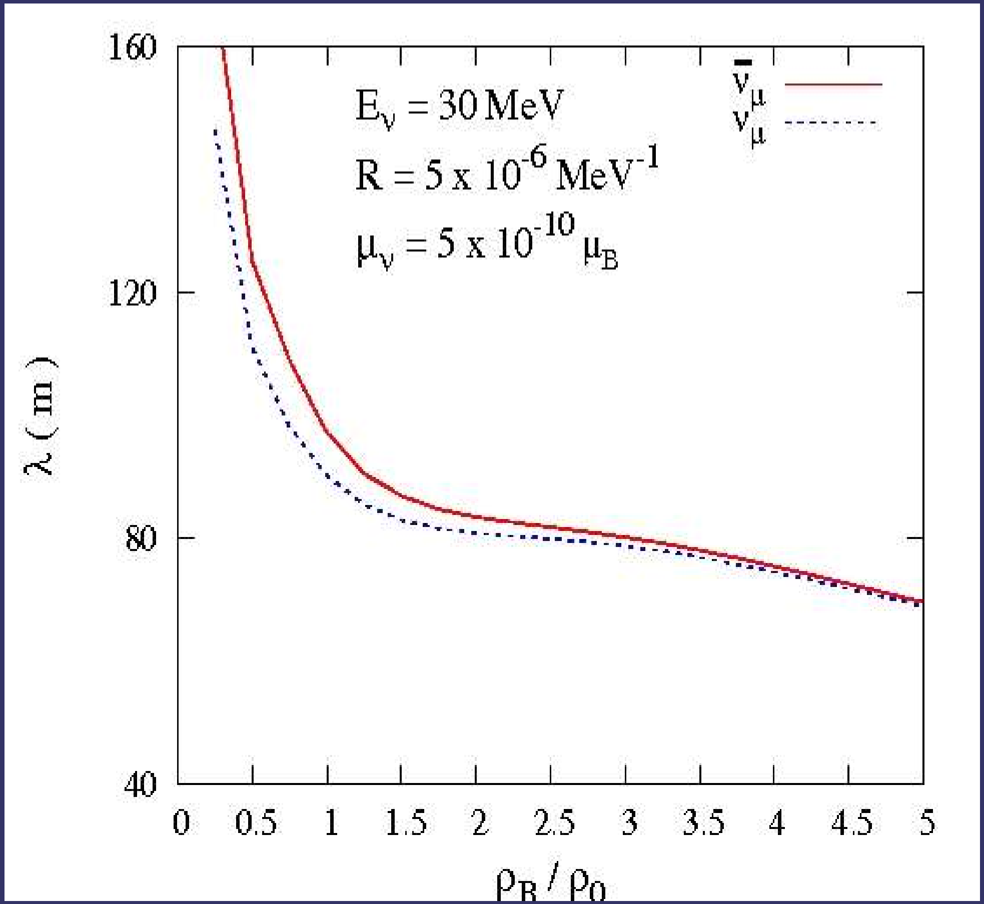




RESULTS AND DISCUSSION

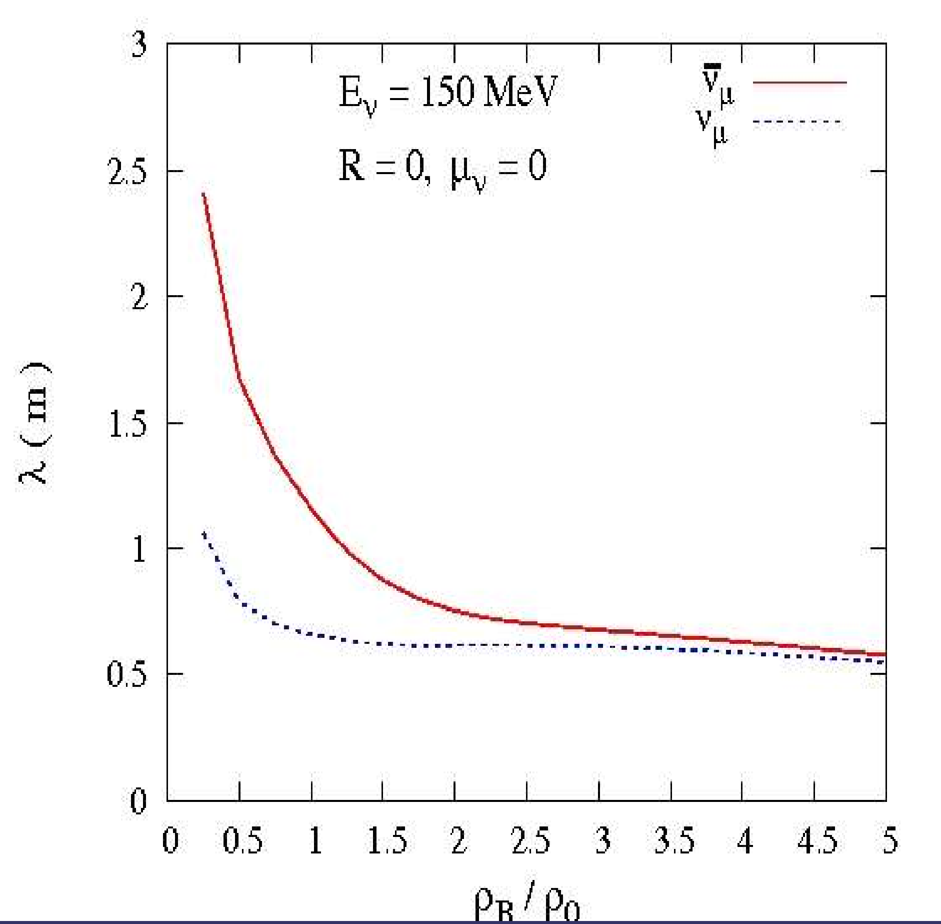
- ν_μ and $\bar{\nu}_\mu$ Mean free Path Results

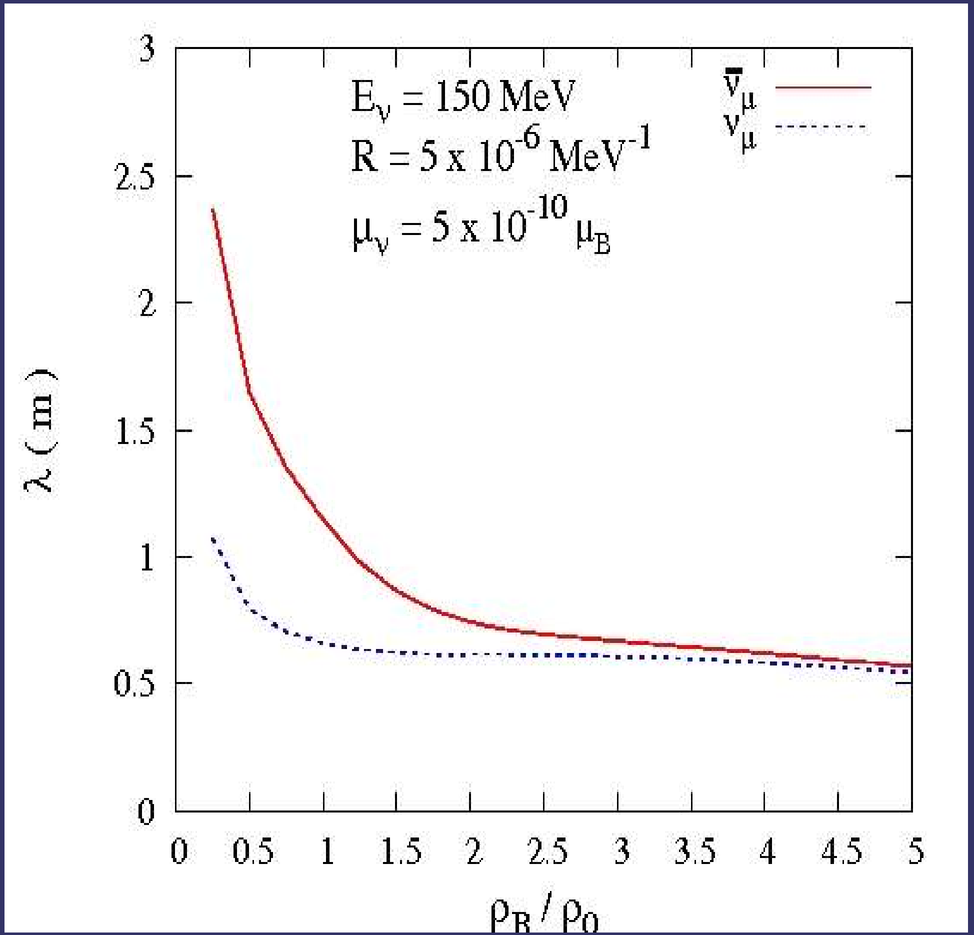




RESULTS AND DISCUSSION

- ν_μ and $\bar{\nu}_\mu$ Mean free Path Results





CONCLUSION

- **The effect of electromagnetic form factor depend on the moment magnetic and charge radius of neutrino**

$$\nu_\tau,\ \bar{\nu}_\tau < \nu_\mu,\ \bar{\nu}_\mu < \nu_e,\ \bar{\nu}_e$$

- **The effect of weak magnetism make a differentiable for neutrino and anti neutrino mean free path. [similar results of C.J. Horowitz. et.al.PRC68,025803 (2003)]**
- **Neutrino and antineutrino mean free path decrease if energy neutrino increase.**
- **In general, the neutrino and anti neutrino have influence on neutrino and antineutrino mean free path but too small.**

THE END

THANK YOU FOR YOUR ATTENTION