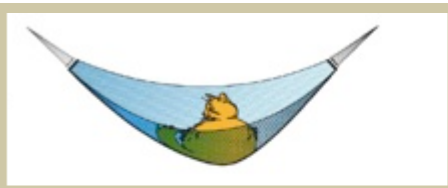
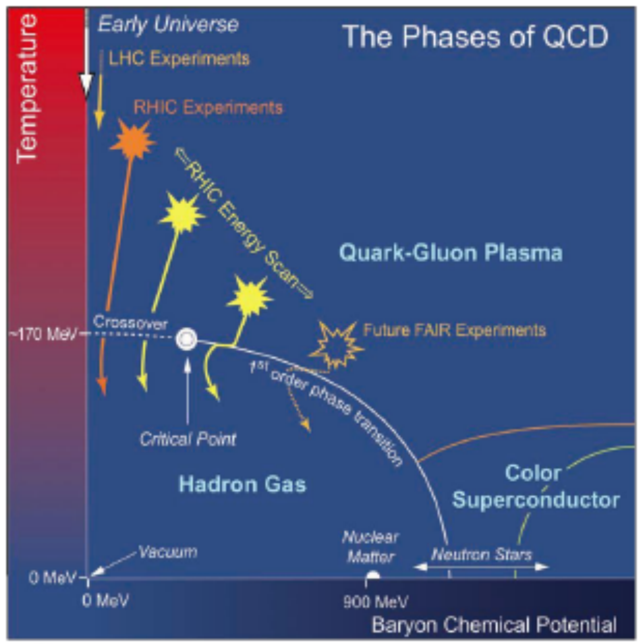
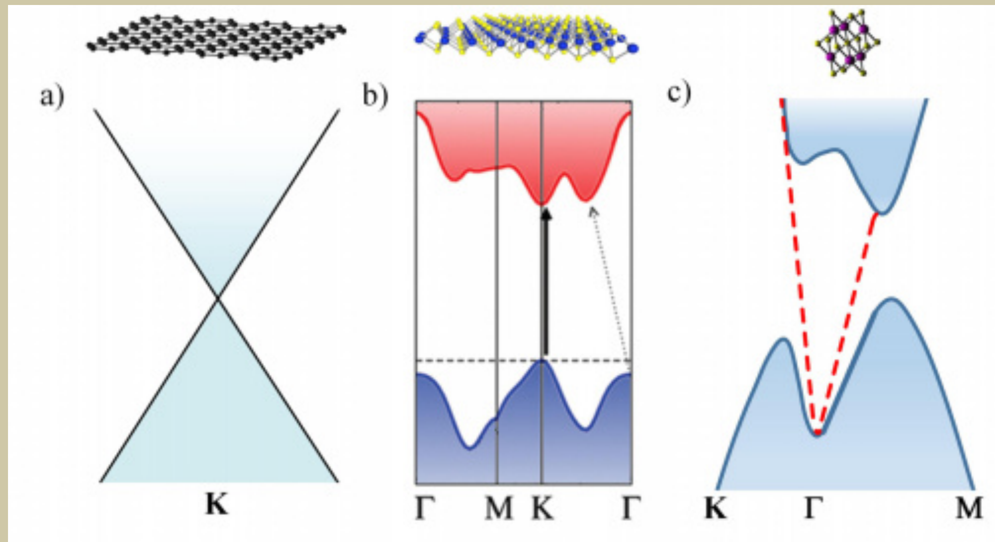






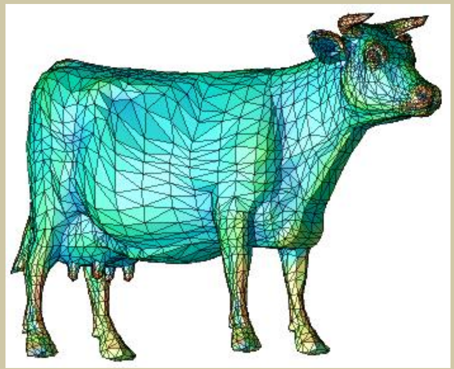
Elastic gauge fields, Hall viscosity and the chiral anomaly in Weyl Semimetals

María A. H. Vozmediano
Instituto de Ciencia de Materiales de Madrid
CSIC

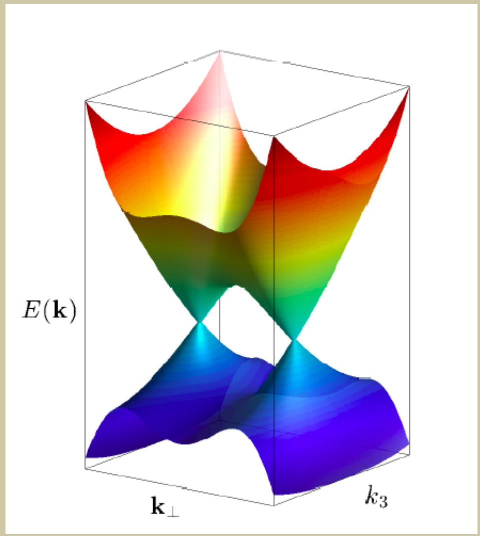
Key issue:

Local lattice deformations couple to the electronic degrees of freedom of **Weyl matter** as fictitious gauge fields. New unexpected anomaly-related response functions.

Tool: effective actions



Geometry



$$L = \bar{\Psi}_k(\gamma^\mu k_\mu - m - b_\mu \gamma^\mu \gamma^5)\Psi_k$$

Players:

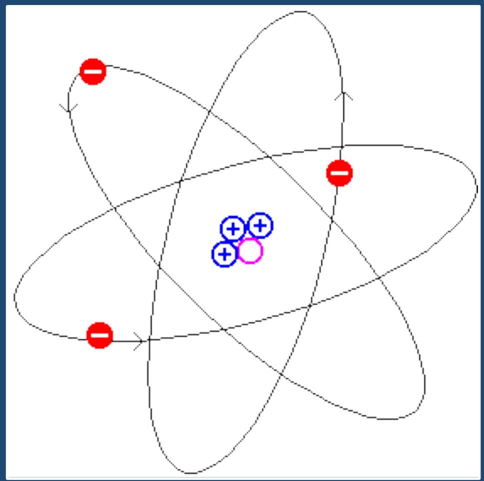
- Weyl matter
- Hall conductivity
- Elastic gauge fields

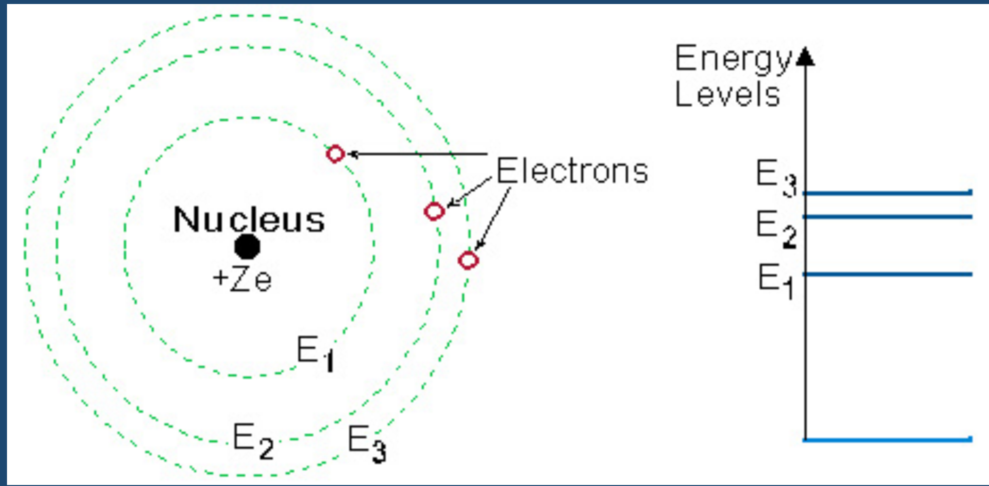
Main results:

- Elastic gauge fields in Weyl semimetals.
- Hall viscosity from Hall conductivity in Dirac matter
- Coefficient related to AAA triangle graph

Hall viscosity from elastic gauge fields in Dirac crystals, A. Cortijo, K. Landsteiner, Y. Ferreiros, and MAHV, arXiv:1506.05136 (2015).

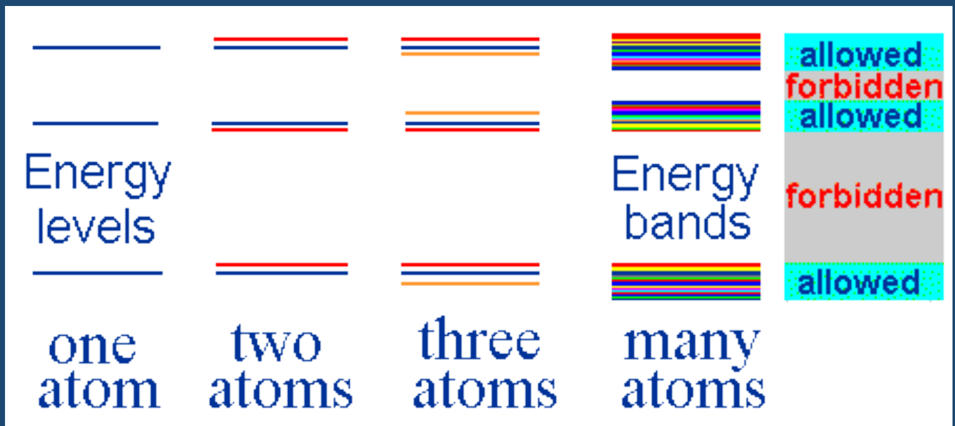
High school solids

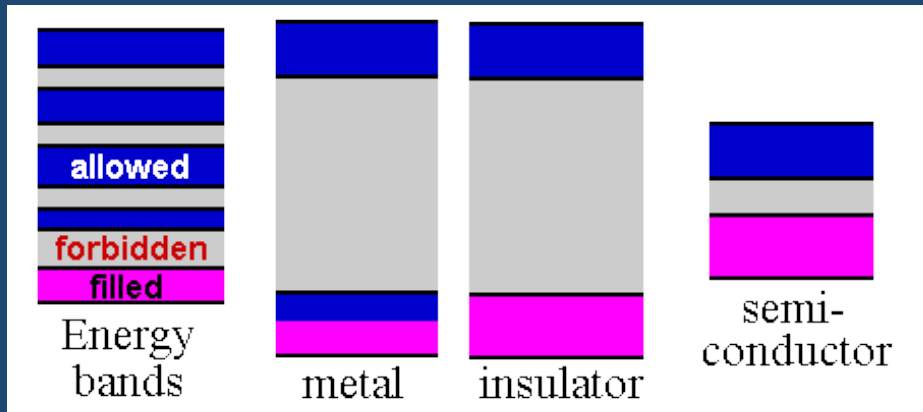




Discrete energy levels separated by forbidden regions

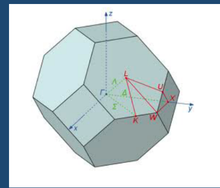
Crystal: many atoms in a periodic lattice.





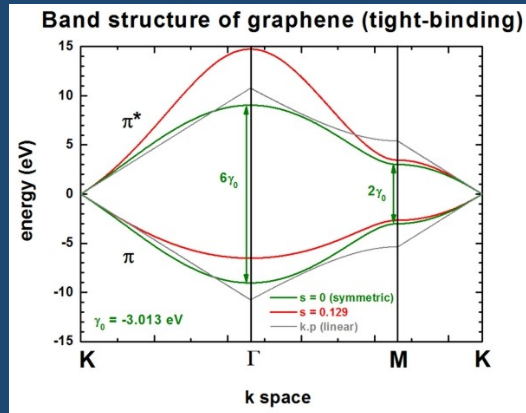
Dictionary

- Band theory: electrons in a periodic potential. The bands retain the symmetry properties of the original orbitals (parity, inversion, etc.)



- Brillouin zone: support of non-equivalent k values.
  In 2D periodic BZ a torus.

- Each discrete level E_n of the atom or molecule forms a band . $\varepsilon_n(k)$



- Fermi surface: fill the bands with electrons.

$$\varepsilon_n(k) = \varepsilon_F$$

The vacuum

- Continuum model: expand dispersion relation around a point at the Fermi surface.

Band topology

Wave function:

$$\Psi_n(\mathrm{r}) = \sum_{k \in B} e^{ikr} u_n(k)$$

Bloch

$$u_n(k+T) = u_n(k)$$

A fiber bundle over BZ

- Berry connection:

$$\vec{A}_n(\mathbf{k}) = \left\langle u_n(k) \middle| \vec{\nabla}_k \middle| u_n(k) \right\rangle$$

Encodes (most) topological properties of the system.

- Chern number (2D):

$$\mathrm{C}_n = \frac{1}{2\pi i} \int_{BZ} d^2\vec{k}.\vec{B}_n(\mathbf{k})$$

- An observable consequence:

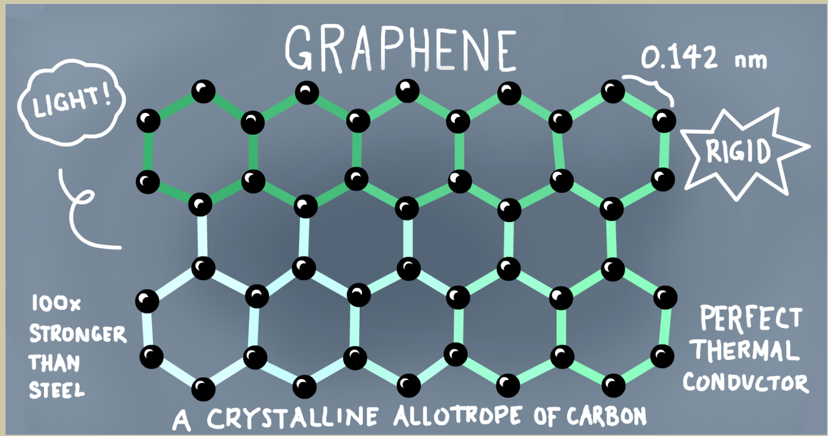
$$\sigma_H = \sum_{nfilled} C_n \frac{e^2}{\hbar}$$

At least two bands needed to have non trivial topology

D. Carpentier
arXiv:1408.1867

Questions

- How did Dirac arise in condensed matter? -> Dirac matter
- How did topology pop up? -> Topological matter

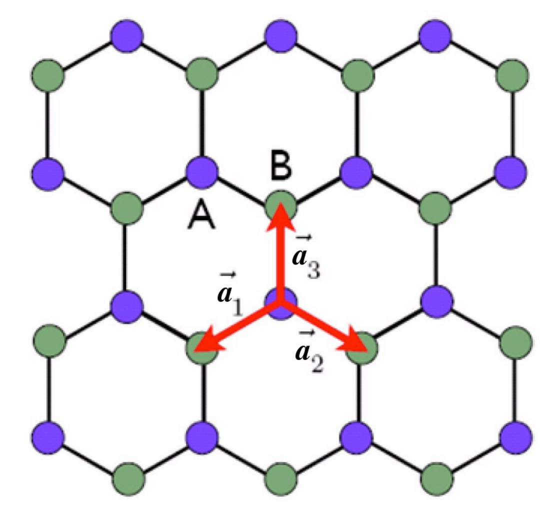


Graphene as a prototypical example

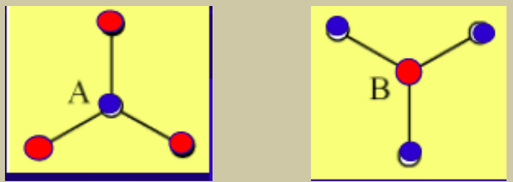
- Are Dirac material physical realizations of QED or are they only analogs? -> Similarities and differences between QFT and CM

Main differences due to the finite bandwidth

Tight binding approach



Honeycomb lattice:



- Two atoms per unit cell
- Two bands

Wave function

$$\Psi = \left(\Psi_A, \Psi_B\right)$$

$$H = -t \sum_{<ij>} a_i^{\dagger} b_j + cc.$$

t~2.7 eV

$$H = \sum_{\vec{k}} \begin{pmatrix} a_{\vec{k}}^{+} & b_{\vec{k}}^{+} \end{pmatrix} \begin{pmatrix} 0 & -t\phi(\vec{k}) \\ -t\phi^{*}(\vec{k}) & 0 \end{pmatrix} \begin{pmatrix} a_{\vec{k}} \\ b_{\vec{k}} \end{pmatrix}$$

$$\phi(\vec{k}) = e^{i\vec{k}\vec{a}_1} + e^{i\vec{k}\vec{a}_2} + e^{i\vec{k}\vec{a}_3}$$

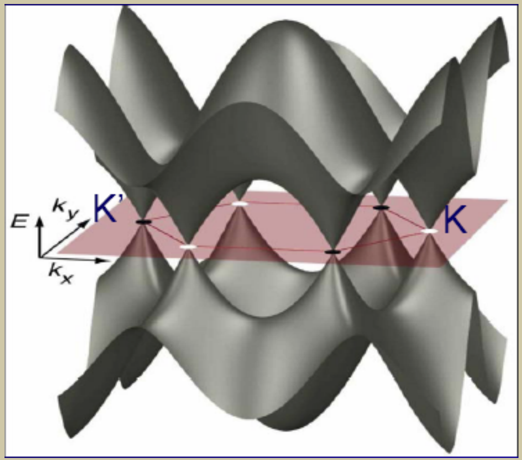
$$H\Psi = E\Psi$$

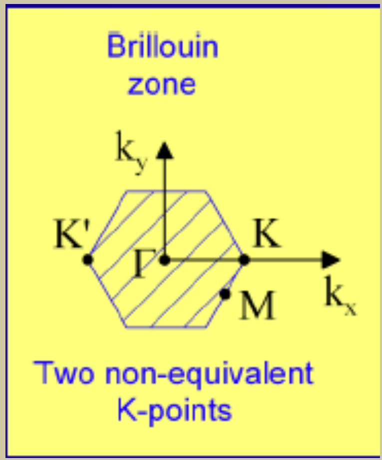
Dispersion relation

$$E(\vec{q}) = \pm t\sqrt{3 + 2\cos(\sqrt{3}q_y a) + 4\cos(\frac{3}{2}q_x a)\cos(\frac{\sqrt{3}}{2}q_y a)}$$

Fermi surface for the neutral material (half filling): E(k)=0

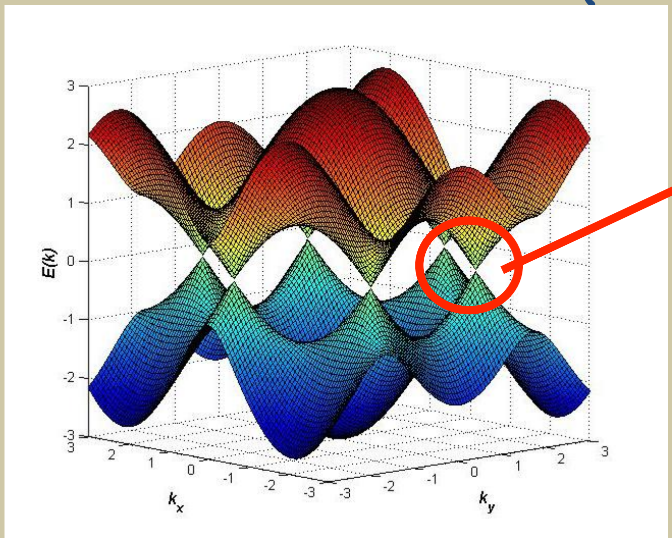
Fermi points and Dirac fermions

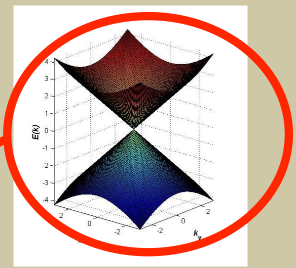




$$E(\mathbf{k}) = 0$$

Get six Fermi points at the six corners of the BZ
(only two are independent)





Continuum limit at K point:
a ⟶ 0 and scale the hamiltonian:

$$H_K^{eff} = v_F \vec{\sigma}.\vec{p}$$

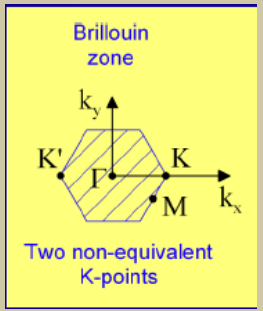
$$v_F = \frac{3ta}{2} \approx 10^6 m/s \approx c/300$$

Massless (2+1) Dirac with v_F

Obstructions to Weyl fermions in crystals

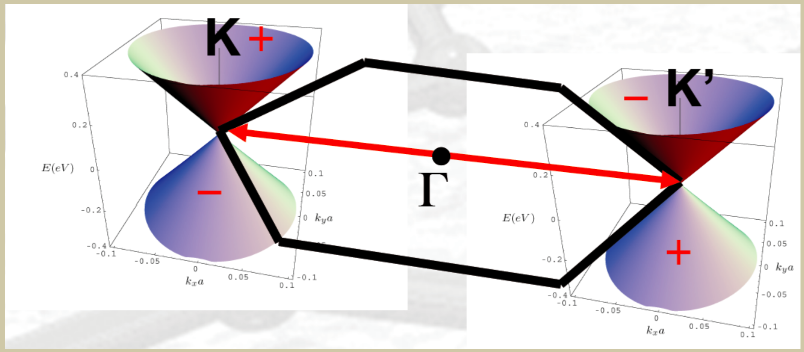
1. Weyl arise in conjugate pairs (Nielsen-Ninomiya). Put them apart in k space.
2. Kramers degeneracy: $\mathcal{T}$ (or $\mathcal{I}$) must be broken (spin).
3. Fine tuning: Fermi level aligned to the nodes.

Expanding around the other Fermi point get



$$H(-K+k) = v_F(-k_x\sigma_x + k_y\sigma_y)$$

The two "flavors" are related by $\mathcal{T}$.
They have opposite helicities and winding numbers.



Essential property: separated in k space

Summary of graphene features

- The electronic properties described by 2D massless spinors.

$$L = \int dt\, d^2x \sum_{i=K,K'} \bar{\Psi}_i(x,t)\gamma^\mu\partial_\mu\Psi_i(x,t)$$

- Spinor structure given by the two sublattices A and B.

- They come in **two flavors** associated to the **two Fermi points** of oposite helicities. (Called valleys in semiconductor language).

$$\Psi_K = \begin{pmatrix} \Psi_A \\ \Psi_B \end{pmatrix} \qquad \Psi_{K'} = \begin{pmatrix} \Psi_A \\ \Psi_B \end{pmatrix}$$

$$H(K) = H^*(-K)$$

Effective Hamiltonian at each valley involves the two in-equivalent representations of the Dirac matrices in 3D .

- Real spin did not play much a role until the recent advent of the topological insulators

- The interacting system behaves as "reduced" QED : $\alpha \sim 2$
- RG analysis: $\alpha = 0$ IR stable fixed point similar to QED(3+1).

Topological aspects

VOLUME 53 24 DECEMBER 1984 NUMBER 26

Condensed-Matter Simulation of a Three-Dimensional Anomaly

Gordon W. Semenoff

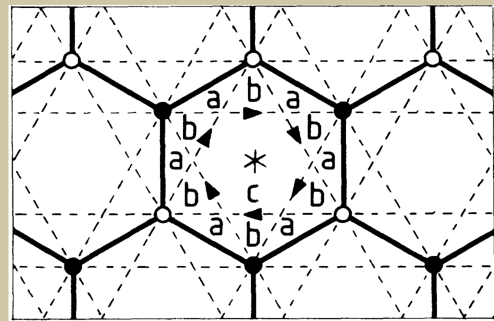
VOLUME 61, NUMBER 18 PHYSICAL REVIEW LETTERS 31 OCTOBER 1988

Model for a Quantum Hall Effect without Landau Levels: Condensed-Matter Realization of the "Parity Anomaly"

F. D. M. Haldane

$$S = \int d^3x \bar{\psi}[(\gamma^\mu(i\partial_\mu + eA_\mu) - m)]\psi$$

$$S_{eff}[A] = \text{sign}(m)\varepsilon^{\mu\nu\lambda}A_\mu\partial_\nu A_\lambda$$



Where does the mass come from?: spin-orbit coupling

$$H_{\text{Dirac}} = \begin{bmatrix} m & k_x - ik_y & & \\ k_x + ik_y & -m & & 0 \\ & 0 & -m & k_x - ik_y \\ & & k_x + ik_y & m \end{bmatrix}$$

Use the spin degree of freedom to get two copies of Dirac. Spin-Hall effect. Same with valleys: VHE. Or with layers in bilayer: LHE. Birth of topological insulators.

Kane & Mele (2005, 2006); Bernevig & Zhang (2006)

Topological insulators in three spatial dimensions

Axion electrodynamics:

$$\mathcal{L} = \frac{1}{4\pi} F_{\mu\nu} F^{\mu\nu} + \frac{e^2}{2\pi^2 \hbar c} \theta \varepsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} + J^{\mu} A_{\mu}$$

$$\vec{\nabla}\mathbf{E} = \rho - \frac{e^2}{4\pi^2 \hbar c} \mathbf{B} \vec{\nabla}\theta, \qquad \vec{\nabla} \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\vec{\nabla}\mathbf{B} = 0, \qquad \vec{\nabla} \times \mathbf{B} = \mathbf{J} + \frac{\partial \mathbf{E}}{\partial t} + \frac{e^2}{4\pi^2 \hbar c} \dot{\theta}\mathbf{B} + \frac{e^2}{4\pi^2 \hbar c} \vec{\nabla}\theta \times \mathbf{E}$$

- **If** $\dot{\theta} = 0 \Rightarrow$ **standard electrodynamics but with**

$$\mathbf{D} = \varepsilon \mathbf{E} + \alpha\theta\mathbf{B}$$

$$\mathbf{H} = \mu^{-1}\mathbf{B} - \alpha\theta\mathbf{E}$$

Magnetoelectric Effect

F Wilczek. Rev. Lett. 58, 1799 (1987)

Topological matter

Kitaev, Moore, Read share Dirac Medal 2015!

Mechanical properties

GRAPHENE'S SUPERLATIVES

- Thinnest imaginable material
- largest surface area (~2,700 m^2 per gram)
- strongest material 'ever measured' (theoretical limit)
- stiffest known material (stiffer than diamond)
- most stretchable crystal (up to 20% elastically)
- record thermal conductivity (outperforming diamond)
- highest current density at room T (106 times of copper)
- completely impermeable (even He atoms cannot squeeze through)
- highest intrinsic mobility (100 times more than in Si)
- conducts electricity in the limit of no electrons
- lightest charge carriers (zero rest mass)
- longest mean free path at room T (micron range)

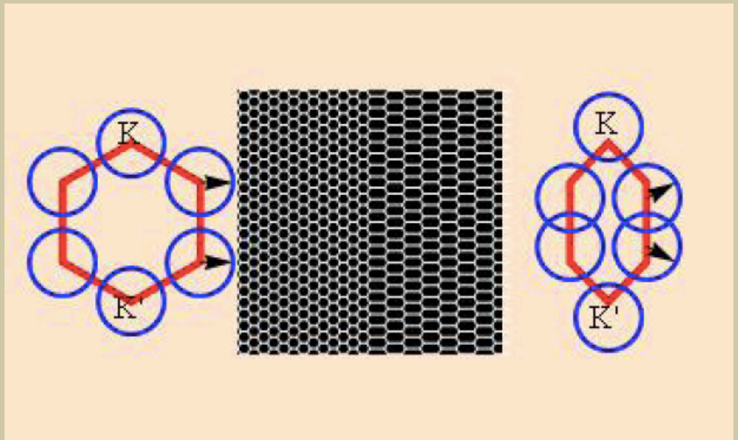
Courtesy of F. Guinea and A. Geim


WHAT...?
IT'S CALLED GRAPHENE. STRONGER THAN MOST STEELS. TRANSPARENT.

WIRED.CO.UK

Modeling lattice deformations in graphene

Tight binding



$$H_{TB} = \sum_{<ij>} t_{ij} a_i^+ b_j$$

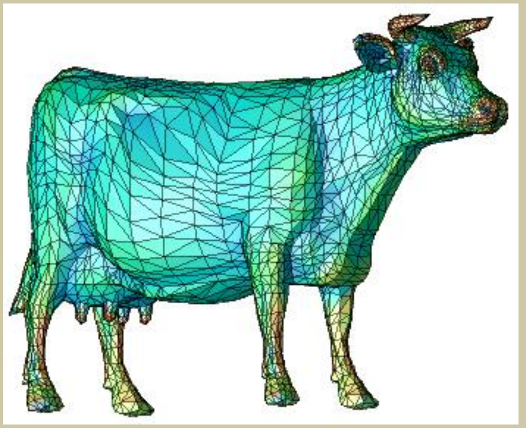
$$\beta = \frac{\partial \log(t)}{\partial \log(a)} \approx 2$$

Elasticity+low energy near a Fermi point

$$H_{TB} = i\int d^2x \, \Psi^+ \sigma^i (\partial_i + iA_i)\Psi$$

$$A_x = \frac{\beta}{a}\left(u_{xx} - u_{yy}\right) \, , A_y = \frac{2\beta}{a} u_{xy}$$

Geometric formalism



Dirac equation in a curved background

$$H = i\int d^2x \sqrt{g}\,\bar{\Psi}\sigma^\mu (\partial_\mu + \Gamma_\mu)\Psi$$

Continuum from the beginning.
Does not see the underlying lattice.

$$\gamma^\mu(r) = \gamma^a e_a^\mu(r)$$

Both predict vector fields coupled to electronic excitations.
Opposite signs at the two Fermi points

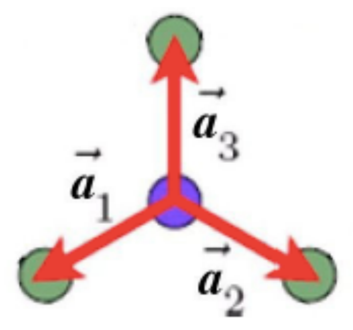
Symmetry approach

Build an effective H at low energy with C_{3v} symmetry

- What can we build with (σ^i, q^i, u^{ij}) ? $H_0 = v_F \vec{\sigma}.\vec{q}$

C_3 invariant tensor:

$$u_{ij} = \tfrac{1}{2}(\partial_i \xi_j + \partial_j \xi_i + \partial_i h \partial_j h), \quad i,j = x,y,$$



$$f^{ijk} = \frac{1}{a^3}\sum_{n=1}^{3} a_n^i a_n^j a_n^k$$

$$A_i \approx f^{ijk} u_{jk}$$

Terms compatible with C_3 symmetry:

Even # indices: contract with the flat metric

- $\sigma_i \partial_j$, the flat Hamiltonian
- $\sigma_i(\partial_j u_{kl})$ the geometric gauge field
- $u_{kl}\sigma_i\partial_j$ the space dependent Fermi velocity

Odd # indices: contract with f or ε_{ijk}

- $\sigma_i u_{jk}$ the trigonal gauge field
- $\sigma_i \partial_k \partial_j$ the trigonal warping term

F. de Juan. M. Sturla, MAHV PRL'12
F. de Juan, J. Mañes, MAHV PRB'13
J. Mañes, F. de Juan, M. Sturla, MAHV PRB13

Physical reality of the elastic gauge fields

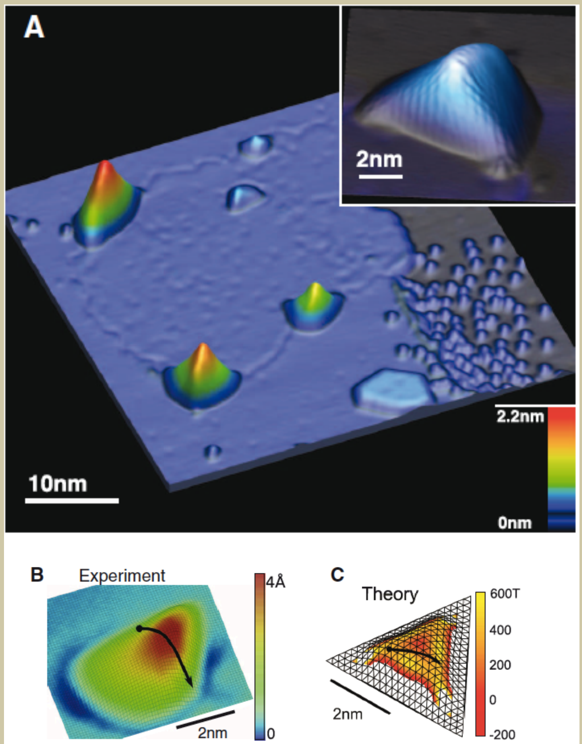


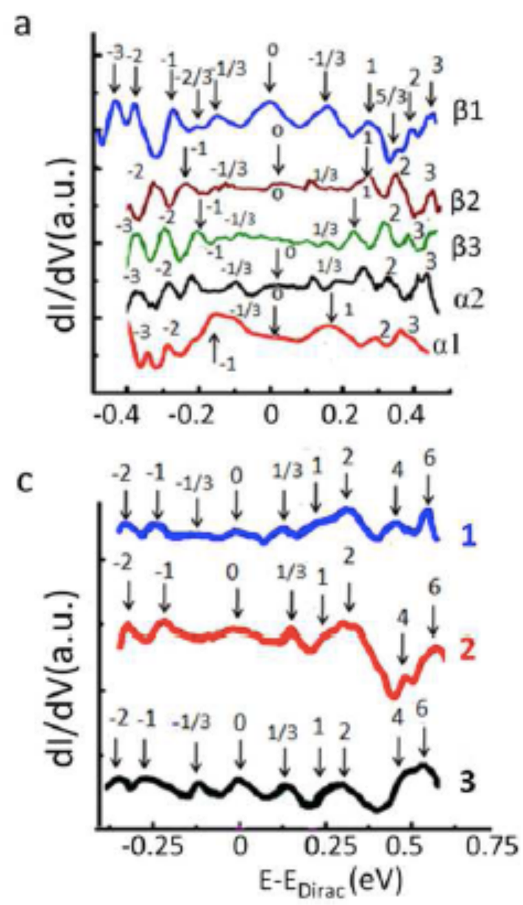
Fig. 3. (**A**) Experimental topographic line scan and experimentally determined B_s profile over the tip trajectory shown by black line in (B). (**B**) STM topography of graphene nanobubble. (**C**) Topography of theoretically simulated graphene nanobubble with calculated B_s color map. (**D**) Simulated topographic line scan and B_s profiles extracted from line shown in (C).

Strain-Induced Pseudo–Magnetic Fields Greater Than 300 Tesla in Graphene Nanobubbles

N. Levy,[1,2]*† S. A. Burke,[1]*‡ K. L. Meaker,[1] M. Panlasigui,[1] A. Zettl,[1,2] F. Guinea,[3] A. H. Castro Neto,[4] M. F. Crommie[1,2]§

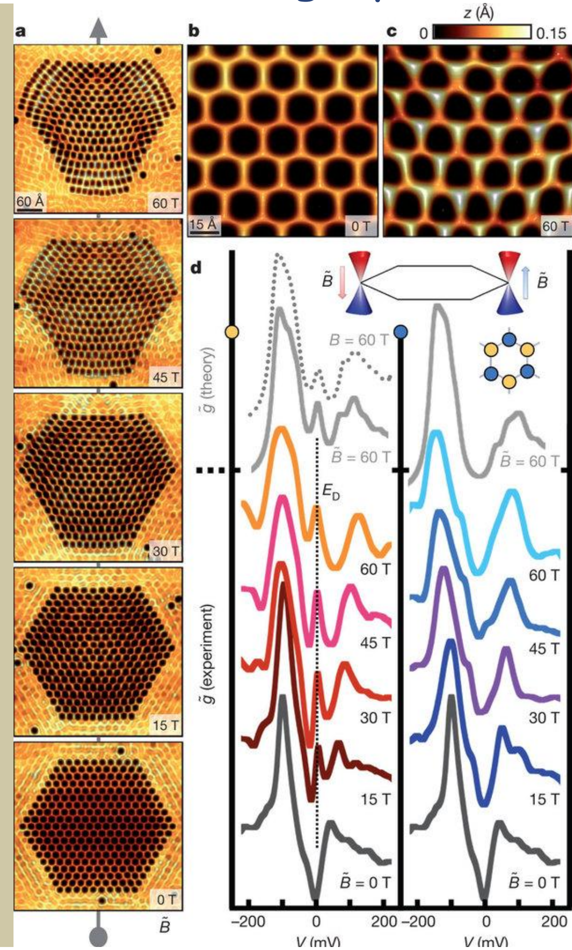
30 JULY 2010 VOL 329 **SCIENCE** www.sciencemag.org

CVD graphene



N. C. Yeh(Caltec), Surface Science 10

Artificial graphene



Manoharan's group, Nature(2011)

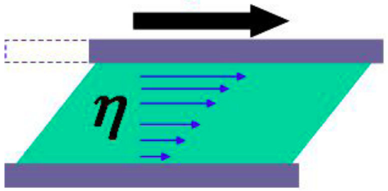
Reorganization of the density of states similar to Landau levels.

Viscosity

Fluids: (characterized by velocity field u)

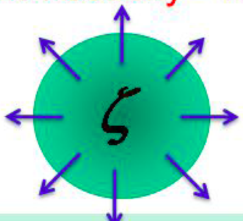
- Shear and bulk. Friction.
- It is zero at zero temperature

Shear viscosity – measures the resistance to flow



act against the buildup of flow anisotropy

Bulk viscosity – measures the resistance to expansion



act against the buildup of radial flow

$$T_{ij} \sim \eta_{ijkl} u_{kl}$$

$$u_{kl} = \frac{1}{2}(\partial_k u_l + \partial_l u_k)$$

Response of the stress tensor to velocity gradient.

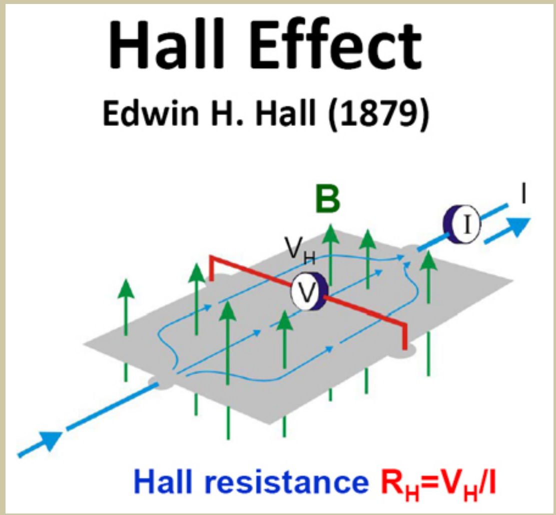
Elasticity: (characterized by strain tensor u_{ij})

- Response of the stress tensor to time varying strain (strain rate):

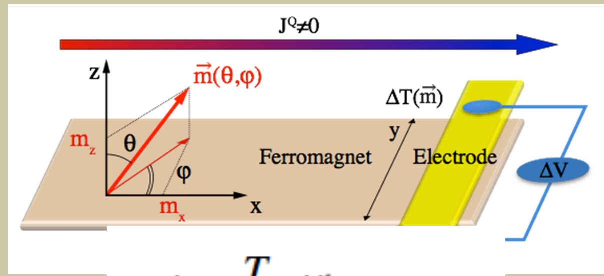
$$T_{ij} \sim \eta_{ijkl} \dot{u}_{kl}$$

(Dimensions of density in units h=1)

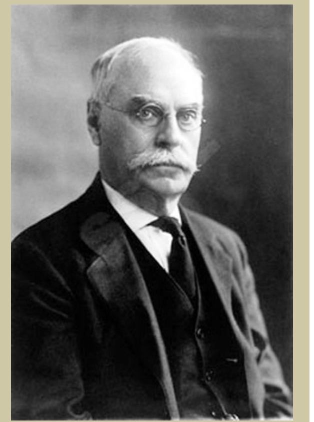
Anomalies and non-dissipative responses



Thermal Hall effect



$$J^i_\varepsilon = \frac{T}{6}\varepsilon^{ijk} b_j(\partial_k T)$$



Hall viscosity

- T broken
- Dissipationless currents
- Chern-Simons effective actions
- Needs coupling e's to elasticity

- **Odd** stress response to a time variation of strain $T_{ij} \sim \eta_{ijkl}\dot{u}_{kl}$
- In FQHS and topological SC proportional to the average angular momentum
- **A new topological quantum number**
- Also interesting in the quark-gluon plasma (Abanov, Gromov 2014)

Ref.: J Avron et al, PRL 75, 697
Review: C. Hoyos, 1403.4739; Torsional: T. Hughes et al, Phys.Rev. D88 (2013) 025040

Hall viscosity from Hall conductivity

Hall conductivity

$$S_{CS} \sim \nu_H \varepsilon^{ijk} A_i \partial_j A_k$$

↓

$$A_1^{el} = \frac{\beta}{a}(u_{11} - u_{22})$$

$$A_2^{el} = -2\frac{\beta}{a} u_{12}$$

Hall viscosity

$$T_{ij} \sim \eta_H^{ijkl} \dot{u}_{kl} \quad , \quad \eta_H^{ijkl} = -\eta_H^{klij}$$

$$S \sim \eta_H^{ijkl} u_{ij} \dot{u}_{kl}$$

(Graphene)

$$S_{CS} = \nu_H \varepsilon^{ijk} A_i^{el} \partial_j A_k^{el} \sim \nu_H \varepsilon^{102} A_1^{el} \partial_0 A_2^{el} + \ldots \sim$$

$$\sim \nu_H \frac{\beta^2}{a^2} 4\, u_{11} \dot{u}_{12} + \ldots$$

$$\eta_H \sim \nu_H \frac{4\beta^2}{a^2}$$

Standard Hall viscosity in B:

$$\frac{\eta_{new}}{\eta_B} \sim \frac{l_B^2}{a^2} \sim \frac{10^4}{B(T)}$$

In graphene Hall conductivity ←→ Hall viscosity
Orders of magnitude bigger than standard

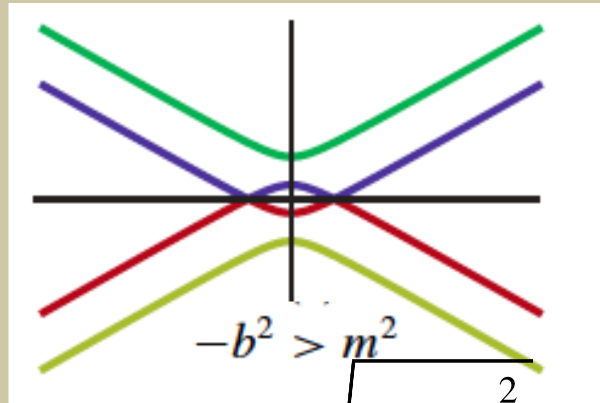
Weyl semimetals (3D graphene)

(3+1) **massless** QED. They exist!!

Minimal model:

$$S=\int d^4k\bar{\Psi}_k(\gamma^\mu k_\mu - m - b_\mu\gamma^\mu\gamma^5 - e\gamma^\mu A_\mu)\Psi_k$$

Axial vector potential



$$\Delta k^\mu \sim b^\mu\sqrt{1-\frac{m^2}{b^2}}$$

(3+1) **massless** QED has **anomalies**

- A realization of Lorentz breaking QED (A. G. Grushin Phys. Rev. B 89, 081407(R) (2014))

Effective action:

$$S_{CS}=\frac{e^2}{16\pi^2}\int d^4x\, \mathrm{K}_i\varepsilon_{ijkl}A_j\partial_k A_l$$

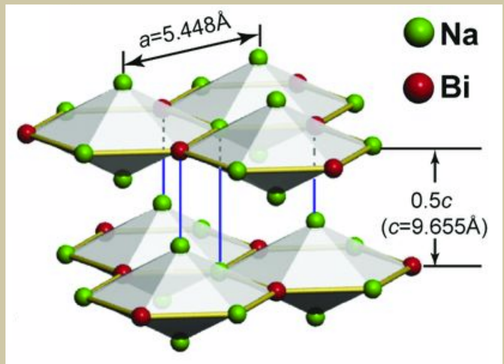
Anomalous Q-Hall conductivity:

$$\sigma_{ij}=\frac{e^2}{2\pi h}\varepsilon_{ijk}K_k$$

K is the vector separating the two Weyl points

Physical reality of WSM

Hassan' s group

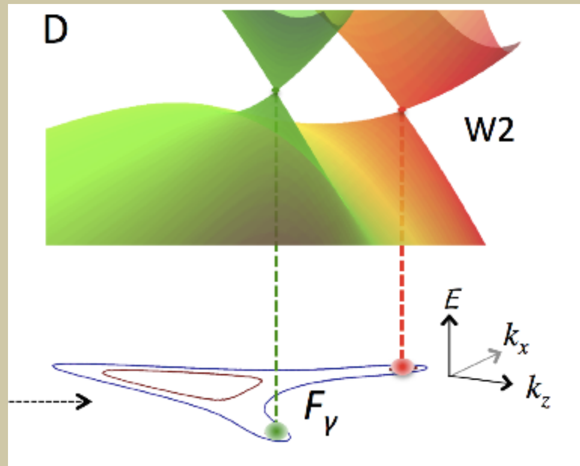


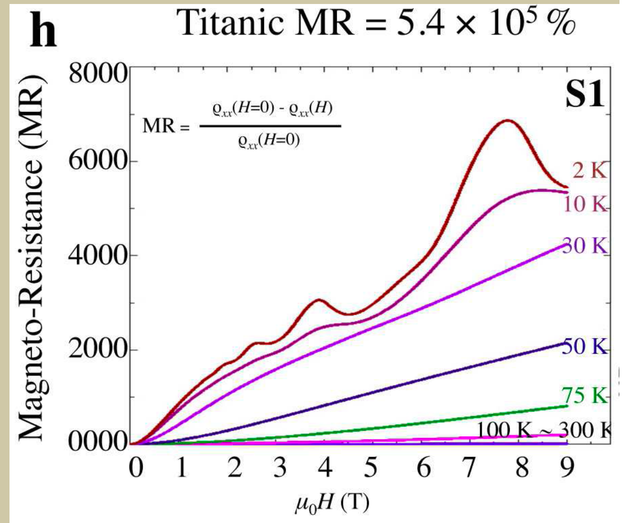
Science 347, 294 (15)

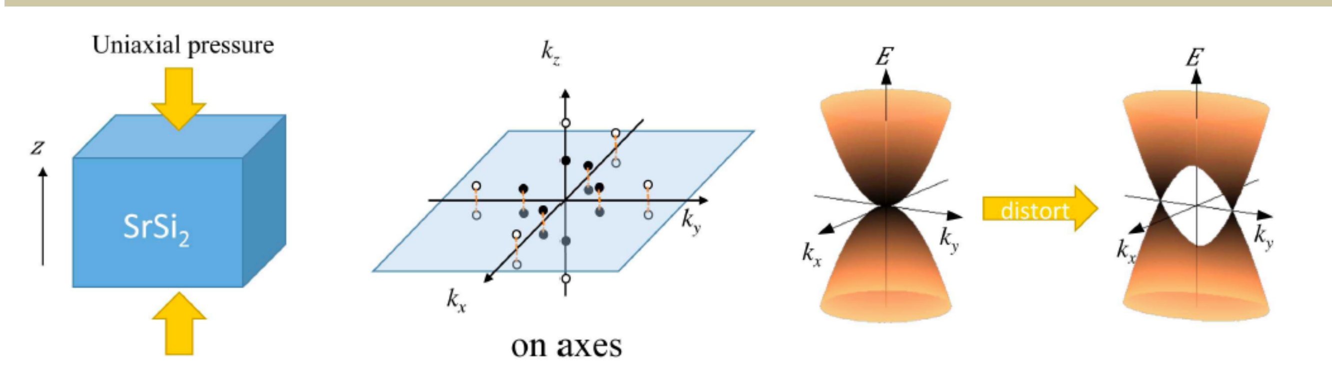
Na3Bi

TaP

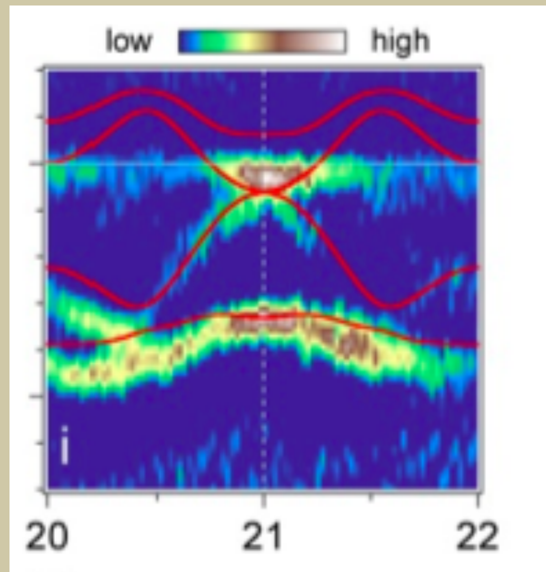
1506.06577Dresden







- A new type of Weyl semimetal with quadratic double Weyl fermions in $SrSi_2$, 1503.05868



TaP

1507.03983 Mesot group

Hasan Group

Laboratory for Topological Quantum Matter & Advanced Spectroscopy

Discovery of Weyl Fermion Semimetal: TaAs, NbAs, $SrSi_2$

Theory: A Weyl Fermion semimetal with surface Fermi arcs in the transition metal monopnictide TaAs class
S.-M. Huang, S.-Y. Xu, I. Belopolski, C.-C. Lee, G. Chang, B. Wang, N. Alidoust, G. Bian, M. Neupane, C. Zhang, S. Jia, A. Bansil, H. Lin, M. Z. Hasan
Paper: *Nature Commun.* **6**:7373 (2015) (submitted Nov. 2014)

ARPES Experiments: Experimental realization of a topological Weyl semimetal phase with Fermi arc surface states in TaAs
S.-Y. Xu, I. Belopolski, N. Alidoust, M. Neupane, C. Zhang, R. Sankar, S.-M. Huang, C.-C. Lee, G. Chang, B. Wang, G. Bian, H. Zheng, D. Sanchez, F.-C. Chou, H. Lin, S. Jia, M. Z. Hasan
Paper: arXiv:1502.03807

Transport Experiments: Tantalum Monoarsenide: an Exotic Compensated Semimetal
C. Zhang, Z. Yuan, S.-Y. Xu, Z. Lin, B. Tong, M. Z. Hasan, J. Wang, C. Zhang, S. Jia
Paper: arXiv:1502.00251

and

Observation of the Adler-Bell-Jackiw chiral anomaly in a Weyl semimetal
C. Zhang, S.-Y. Xu, I. Belopolski, Z. Yuan, Z. Lin, B. Tong, N. Alidoust, C.-C. Lee, S.-M. Huang, H. Lin, M. Neupane, D. S. Sanchez, H. Zheng, G. Bian, J. Wang, C. Zhang, T. Neupert, M. Z. Hasan, S. Jia
Paper: arXiv:1503.02630

Discovery of Weyl semimetal NbAs
S.-Y. Xu, N. Alidoust, I. Belopolski, C. Zhang, G. Bian, T.-R. Chang, H. Zheng, D. S. Sanchez, G. Chang, Z. Yuan, D. Mo, Y. Wu, L. Huang, C.-C. Lee, S.-M. Huang, B. Wang, H.-T. Jeng, T. Neupert, A. Kaminski, H. Lin, S. Jia, and M. Z. Hasan
arXiv:1504.01350

A new type of Weyl semimetal with quadratic double Weyl fermions in SrSi2
S.-M. Huang, S.-Y. Xu, I. Belopolski, C.-C. Lee, G. Chang, B. Wang, N. Alidoust, M. Neupane, H. Zheng, D. Sanchez, A. Bansil, G. Bian, H. Lin, and M. Z. Hasan
arXiv:1503.05868

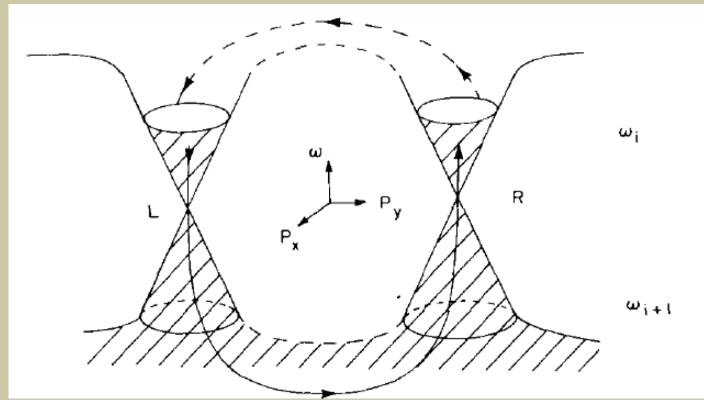
Volume 130B, number 6 PHYSICS LETTERS 3 November 1983

THE ADLER–BELL–JACKIW ANOMALY AND WEYL FERMIONS IN A CRYSTAL

H.B. NIELSEN Masao NINOMIYA

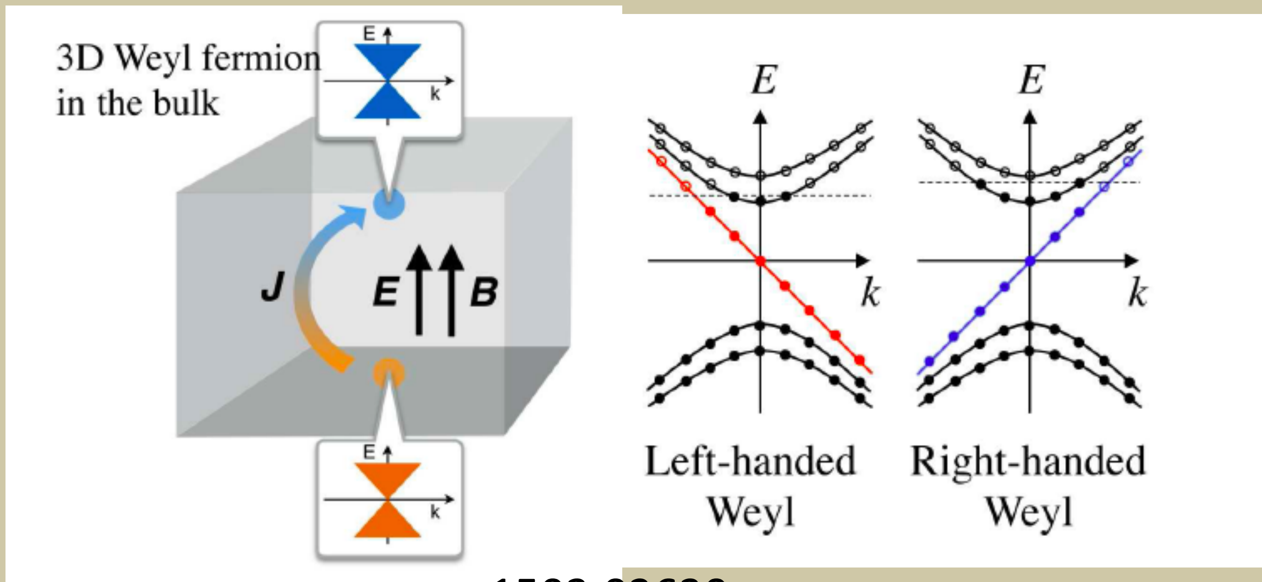
$$[i\partial/\partial t - (P - eA)\boldsymbol{\sigma}]\,\psi_R(x) = 0$$

$$\dot{Q}_5 = \frac{e^2}{4\pi^2}\vec{E}\cdot\vec{B}$$



Chiral anomaly implies charge transfer between the two chiralities with E ıı B

Observation of the ABJ chiral anomaly in a Weyl semimetal

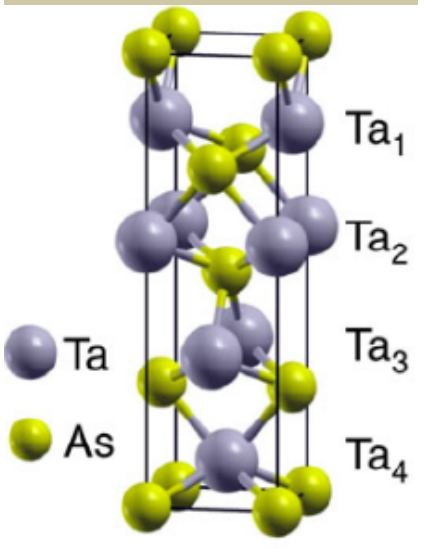


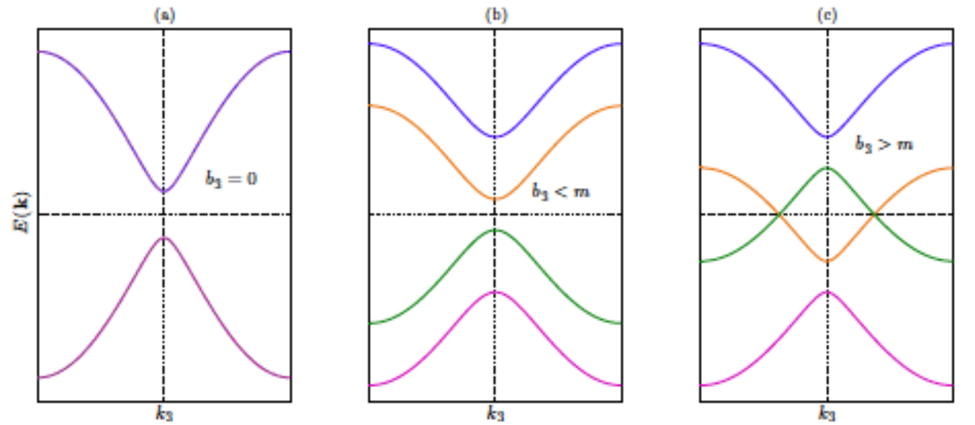
1503.02630

See also:
- 1503.01304
- 1503.08179
- 1504.07398
- PRB88, 104412
- JCCM_MAY_2015_03
- .....

Elastic gauge fields in WSM

$$H = \sum_{i,j} c_i^+ \left(it\alpha_j - r\beta\right) c_{i+j} + (m+3r)\sum_i c_i^+ \beta c_i + \sum_i b_3 c_i^+ \alpha_3 \gamma_5 c_i$$





$$A_1^{el} = \beta\sqrt{b_3^2 - m^2}u_{31}, \quad A_2^{el} = \beta\sqrt{b_3^2 - m^2}u_{32}, \quad A_3^{el} = \beta\frac{r}{2}\frac{b_3^2 - m^2}{t^2}u_{33}.$$

As in graphene, the gauge fields couple with opposite signs to the two chiralities.

$$H_W(\boldsymbol{k}) = \psi^+_{\pm,\boldsymbol{k}} \left(\boldsymbol{\sigma}(v\boldsymbol{k}_\perp \pm \boldsymbol{A}_\perp^{el}) \mp (v_3 k_3 \pm A_3^{el})\sigma_3\right) \psi_{\pm,\boldsymbol{k}},$$

Hall viscosity from Hall conductivity

3D Hall conductivity

$$S_{CS} \sim \nu_H K_i \varepsilon^{ijkl} A_j \partial_k A_l$$

Hall viscosity

$$S \sim \eta_H^{ijkl} u_{ij} \dot{u}_{kl} \text{ , in particular, } S \sim u_{31} \dot{u}_{32}$$

$$T_{ij} = \frac{\delta S}{\delta u_{ij}}$$

Elastic gauge fields

$$A_1^{el} \sim u_{31} \text{ , } A_2^{el} \sim u_{32} \text{ , } A_2^{el} \sim u_{33}$$

$$S_{CS} = \nu_H K_i \varepsilon^{ijkl} A_j^{el} \partial_k A_l^{el} \sim \nu_H K_3 \varepsilon^{3102} A_1^{el} \partial_0 A_2^{el} + \ldots \sim$$

$$\sim \nu_H K_3 u_{31} \dot{u}_{32} + \ldots$$

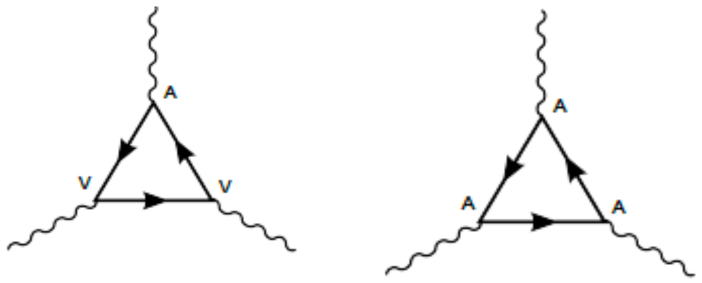
$$\eta_H \sim \nu_H \frac{K\beta^2}{24a^2}$$

Hall conductivity → Hall viscosity

Standard phonon HV for WSM was zero at zero T and μ

The AAA triangle anomaly

$$S = \int d^4x \bar{\psi}\gamma^\mu(i\partial_\mu + eA_\mu + b_\mu\gamma^5)\psi.$$



$$\Gamma_a[A] \qquad \Gamma_a[A^5] = \frac{1}{3}\Gamma[A]$$

High energy implications

No axial fields in the standard model of particle physics.

- The Hall viscosity comes from the AAA diagram. A test of the factor 1/3.
- Axial Magnetic Effect: $\vec{J}_\varepsilon = \sigma_{AME}\vec{B}_5$

Summary

- New mechanism for Hall viscosity in topological matter
- In WSM is was zero.
- In graphene, $$\frac{\eta_{new}}{\eta_B} \sim \frac{l_B^2}{a^2} \sim \frac{10^4}{B(T)}$$
- Other anomaly related responses.
- Straintronics in Dirac crystals.

CSIC

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This work:
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K. Landsteiner
Y. Ferreiros

F. de Juan

Karl Landsteiner

J. L. Mañes

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Observation of the chiral anomaly in solids

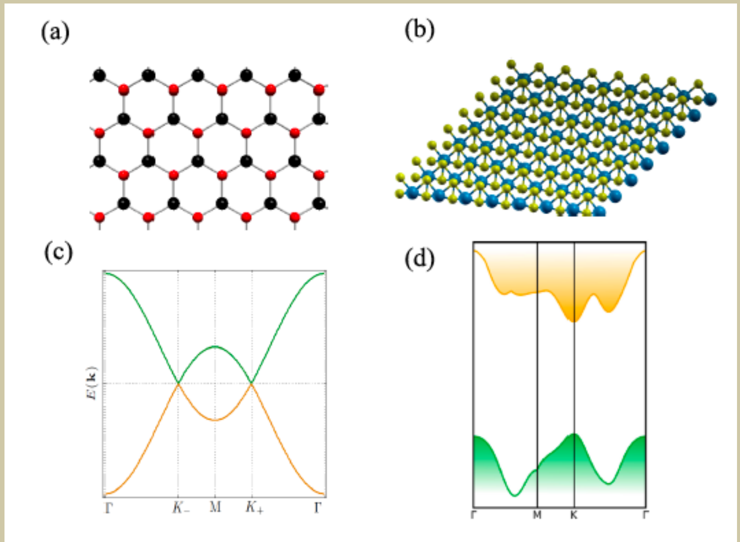
1. Signature of the chiral anomaly in a Dirac semimetal: a current plume steered by a magnetic field.
Authors: Jun Xiong, Satya K. Kushwaha, Tian Liang, Jason W. Krizan, Wudi Wang, R. J. Cava, N. P. Ong.
arXiv:1503.08179

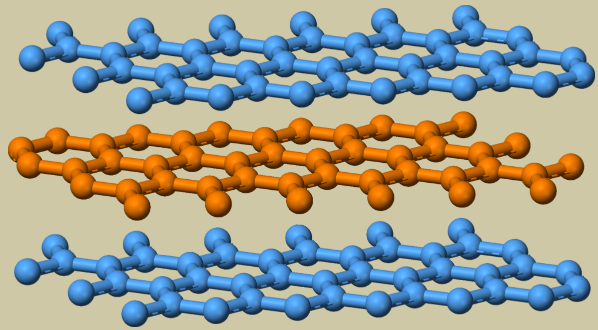
2. Chiral anomaly and classical negative magnetoresistance of Weyl metals
Authors: D. T. Son and B. Z. Spivak.
Phys. Rev. B 88, 104412

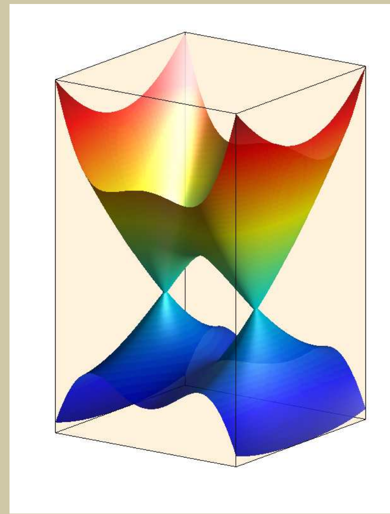
Recommended with a commentary by Patrick Lee, MIT.
|View Commentary|

JCCM_MAY_2015_03

Generality of the effect: Weyl crystals







Layered 3D

G 2D MoS_2

WSM 3D

Recipe

- Need a term in the effective action: $S_{eff}[u] \sim \eta_H u_{11} \dot{u}_{12}$ or $\eta_H u_{31} \dot{u}_{32}$

- Need Hall conductivity:
  - 2D → $S_{eff}[A] \sim \sigma_H \varepsilon_{ijk} A_i \partial_j A_k$
  - 3D → $S_{eff}[A] \sim \sigma_H K_i \varepsilon_{ijkl} A_j \partial_k A_l$

Weyl crystals: Dirac cones located at non equivalent points of the BZ

They will support elastic gauge fields

Dissipationless

$T_{ij} = \eta_{ijkl}\dot{u}_{kl}$

Energy variation under strain:

$\delta E = -T_{ij}\delta u_{ij}$

Thermod: $\delta E = T\delta s - p\delta V$

$T\dot{s} = -\eta_{ijkl}\dot{u}_{ij}\,\dot{u}_{kl}$

Measuring: Phonon Hall viscosity

Barkeshli, Chung, Qi, PRB **85, 245107(2011)**

$$e^{-S_{eff}[\mathbf{u}]} = \int \mathcal{D}c^\dagger \mathcal{D}c e^{-S[\mathbf{u},c,c^\dagger]}.$$

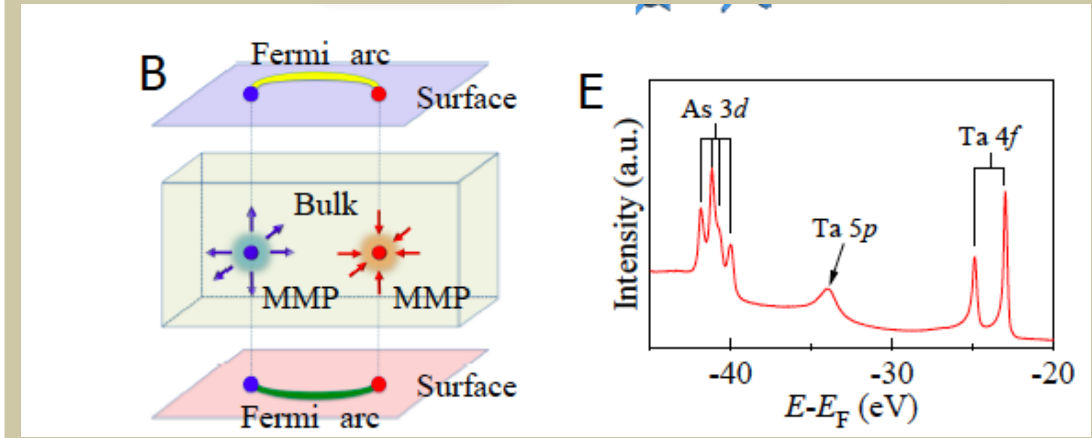
$$S_{eff} = \frac{1}{2}\int d^d x dt (\rho \partial_t u_j \partial_t u_j - \lambda_{ijkl}\partial_i u_j \partial_k u_l),$$

$$\delta S = \frac{1}{2}\int d^{d+1}x d^{d+1}x' \eta_{ab}(x-x') u_a(x) \dot{u}_b(x'),$$

$$\ddot{u}_i = c_t^2 \nabla^2 u_i + (c_l^2 - c_t^2)\partial_i \nabla \cdot \boldsymbol{u} + \eta \nabla^2 \epsilon^{ij} \dot{u}_j/\rho,$$

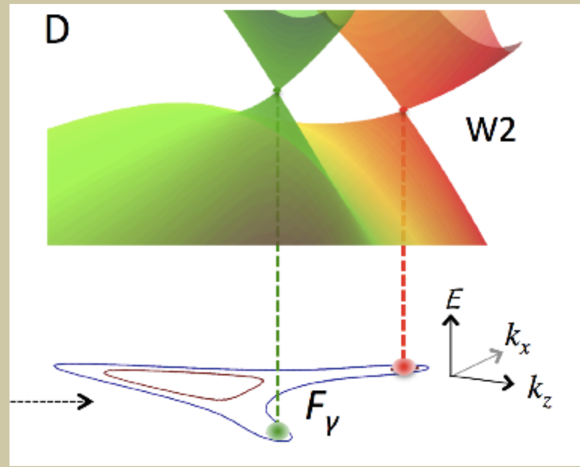
- HV mixes longitudinal and transverse ac. phonon modes.
- Characteristic frequency w_V
- Shifts the frequencies by $(w/w_V)^2$

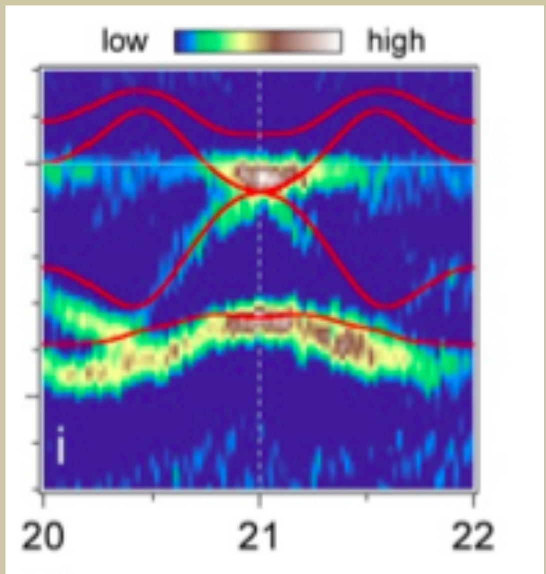
$$\Delta\omega/\omega(\eta = 0) \sim x(\omega(\eta = 0))^2$$



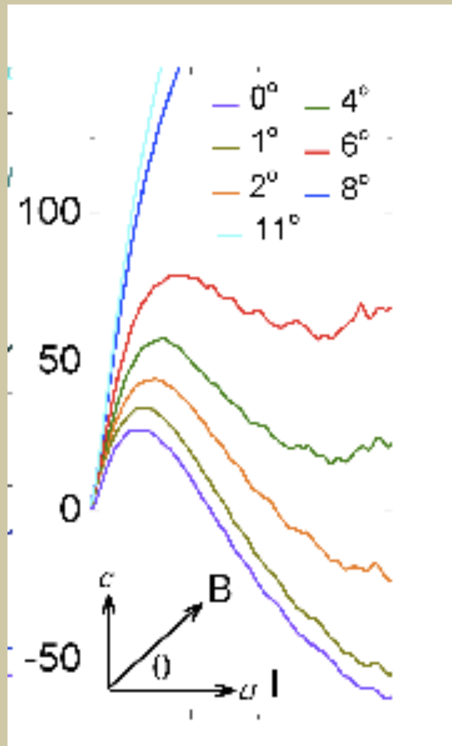
TaSe

TaP
1506.06577Dresden





TaP
1507.03983
Mesot group



Large unsaturated negative magnetoresistance

$$\sigma(H) = \left(\sigma_0 + a\sqrt{H}\right)\left(1 + C_W H^2\right) + \frac{1}{\rho + A\,H^2} + \frac{1}{\rho' + A'\,H^2},$$

Band topology

Wave function:

$$\Psi_n(\mathrm{r}) = \sum_{k \in B} e^{ikr} u_n(k)$$

Bloch

$$u_n(k+T) = u_n(k)$$

(A fiber bundle over BZ)

- Berry connection:

$$\vec{A}_n(\mathbf{k}) = \left\langle u_n(k) \middle| \vec{\nabla}_k \middle| u_n(k) \right\rangle$$

Encodes (most) topological properties of the system.

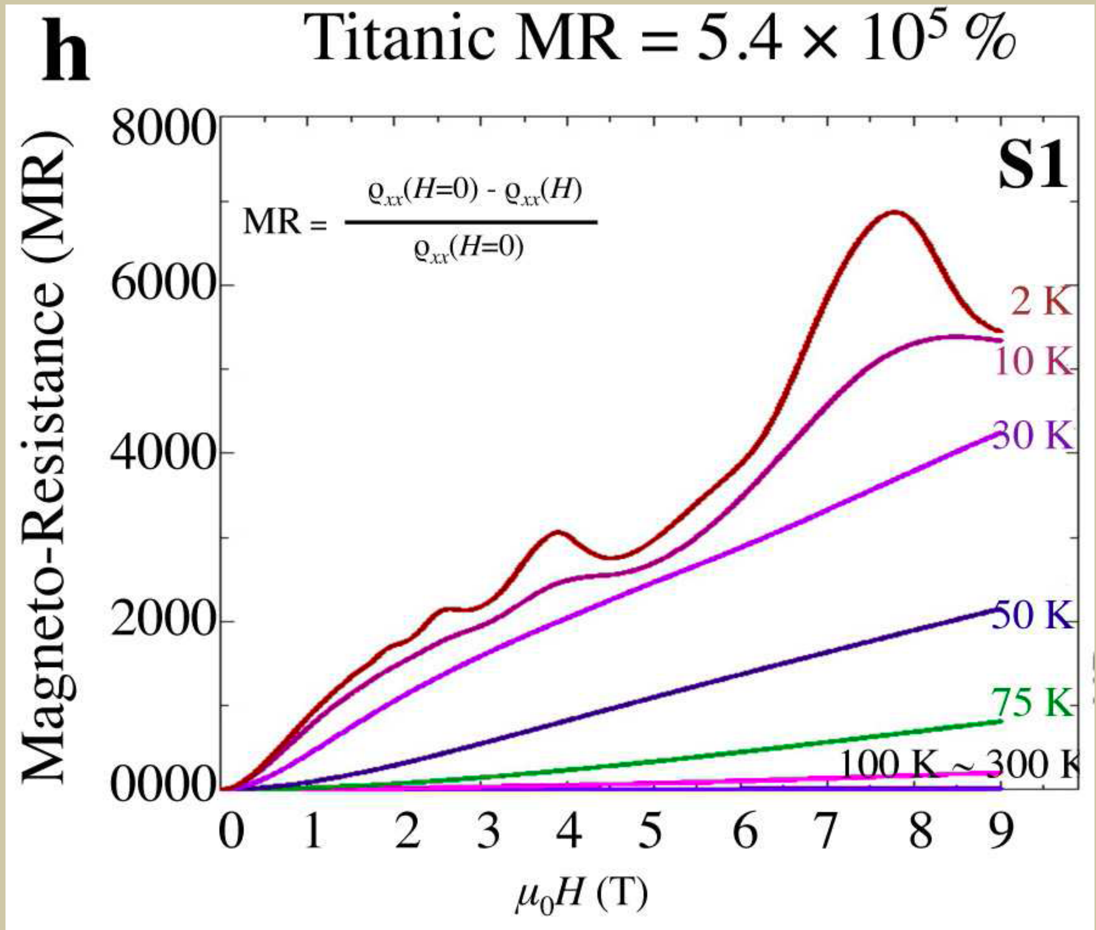
- Chern number (2D):

$$\mathrm{C}_n = \frac{1}{2\pi i} \int_{BZ} d^2\vec{k}.\vec{B}_n(\mathbf{k})$$

- An observable consequence:

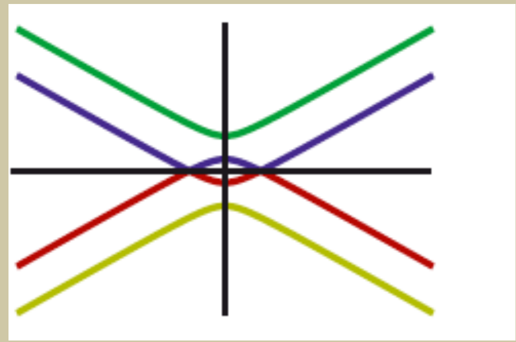
$$\sigma_H = \sum_{nfilled} C_n \frac{e^2}{\hbar}$$

At least two bands needed to have non trivial topology


h
Titanic MR = 5.4 × 10⁵ %
S1
MR = ϱxx(H=0) - ϱxx(H) / ϱxx(H=0)
Magneto-Resistance (MR)
8000
6000
4000
2000
0000
2 K
10 K
30 K
50 K
75 K
100 K ~ 300 K
0
1
2
3
4
5
6
7
8
9
μ0H (T)

Axial magnetic effect

$$S = \int d^4 k \bar{\Psi}_k (\gamma^\mu k_\mu - m - b_\mu \gamma^\mu \gamma^5) \Psi_k$$



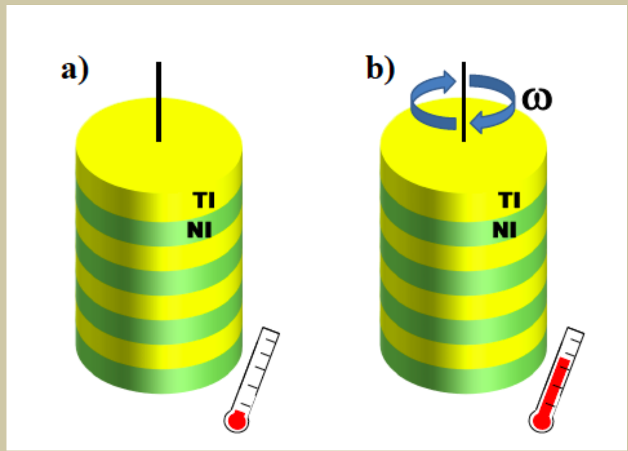
$-b^2 > m^2 \;, b_0 = 0$

b_0: Energy separation → spin-orbit coupling
b_i: k separation → density of magnetic impurities -> b(x) → B_5(x).

$$T^{0i} = J^i_\epsilon = \sigma_{AME} B^i_5.$$

$$\sigma_{AME} = \frac{\mu^2 + \mu_5^2}{4\pi^2} + \frac{T^2}{12},$$

$$L_z = \frac{N_f}{6} T^2 b_z \mathcal{V},$$



Experimental confirmation of the gravitational anomaly?

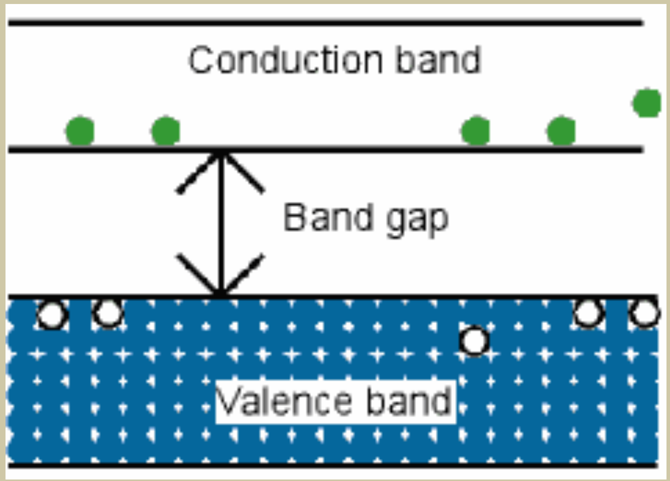
Cortijo, Grushin, Landsteiner, MAHV, PRB 89, 081407R (2014)

Topological insulators

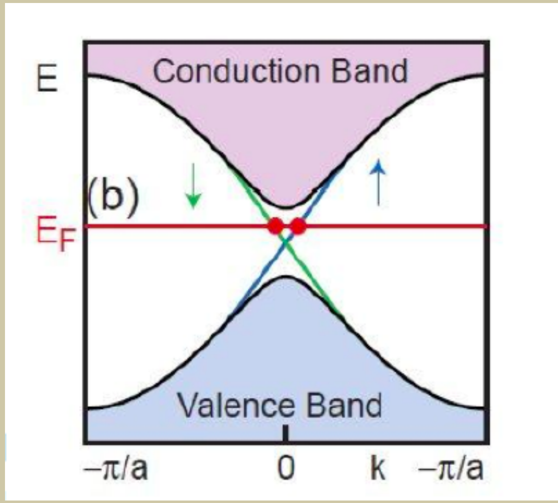
Insulators in the bulk and conducting at the edges

The first non trivial insulator: Quantum Hall system

Band insulators

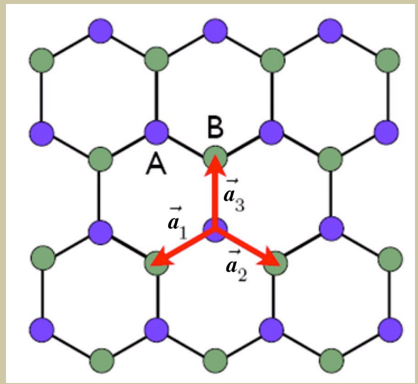


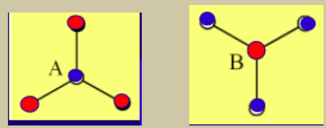
Topological insulators



Massless Dirac matter appears at the interface between a normal (vacuum) and a topological insulator. 2D (1D) edge states at the surface of 3D (2D) TI. There are also crystalline 3D TI.

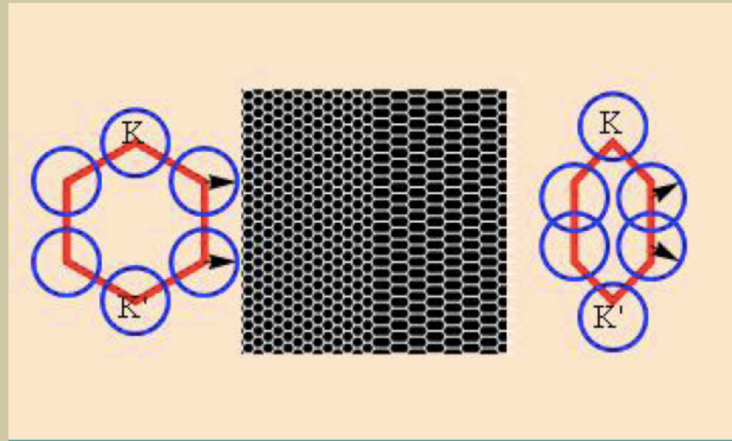
Elastic gauge fields in graphene





$$\Psi = (\Psi_A, \Psi_B)$$

$$H = -t \sum_{<ij>} a_i^{\dagger} b_j$$



$$H \equiv \begin{pmatrix} 0 & t_1 e^{i\vec{k}_1\vec{a}_1} + t_2 e^{i\vec{k}_2\vec{a}_2} + t_3 e^{i\vec{k}_3\vec{a}_3} \\ t_1 e^{-i\vec{k}_1\vec{a}_1} + t_2 e^{-i\vec{k}_2\vec{a}_2} + t_3 e^{-i\vec{k}_3\vec{a}_3} & 0 \end{pmatrix} \approx \begin{pmatrix} 0 & \frac{3\bar{t}a}{2}(k_x + ik_y) + \Delta t \\ \frac{3\bar{t}a}{2}(k_x + ik_y) + \Delta t & 0 \end{pmatrix}$$

Elasticity+low energy near a Fermi point

$$H_{K,u}^{eff} = v_F \vec{\sigma} \cdot (\vec{k} + \vec{A})$$

$$A_x = \frac{\beta}{a}\left(u_{xx} - u_{yy}\right) \,, A_y = \frac{2\beta}{a} u_{xy}$$

u_{ij} strain tensor, β elastic parameter

Generalizable to materials with Fermi points at non equivalent points in BZ

Hall viscosity

ODD (HALL) VISCOSITY

1. Hall viscosity from effective field theory Authors: A.Nicolis & Dam Thanh Son, arXiv:cond-mat/1103.2137 Recommended with a Commentary by David Khmelnitskii, Trinity College, Cambridge | View Commentary (pdf) | JCCM_JUNE2011_02

- Hydrodynamic formulation of electronic fluids (long wavelength dynamics). Derivative expansion. Ideal hydro: dissipationless.

- Dissipation comes as first order corrections. Parametrized by kinetic coefficients (viscosities, thermal coefficients..) **T broken-> dissipationless coefficients**. In (3+1) related to anomalies.

- Avron, Seiler, and Zograf, 1995. First definition. QHE. (**Berry phase in the space H[u_{ij}]**)

- **Lorentz shear modulus**: homogeneous electron gas in a magnetic field: Tokatly Vignale 2007

- **Hall viscosity**: N. Read. 2009. FQHE. $\eta^H = \frac{1}{2}\bar{n}\bar{s}\hbar,$ (**Kubo formula <$T_{ij}T_{kl}$>**)

- Dirac fermions (torsional viscosity): Hughes, Leigh, and Fradkin, 2011
- Relativistic hydro and quark-gluon plasma: Abanov, Gromov 2014

Deformed graphene: TB beyond linear approximation

$$H = -\sum_{n=1}^{3}(t+\delta t_n)\begin{pmatrix} 0 & e^{-i(\vec{K}+\vec{q})\cdot\vec{d}_n} \\ e^{i(\vec{K}+\vec{q})\cdot\vec{d}_n} & 0 \end{pmatrix}$$

Expand in q:
Dirac fermions

Expand in δt_n:
Gauge fields

$$H_{trigonal} = f^{ijk}\sigma^i q^j q^k$$

Expand in both:

$$A^i = \frac{2}{3a}\epsilon^{ij} f^{jkl} u^{kl}$$

$$H_{v_F} = \sigma^i q^j u^{kl} f^{ijkl}$$

Related works

J. L. Mañes 2007 *Phys. Rev. B* ***76 045430***

Winkler R and Zulicke U 2010 *Phys. Rev. B* ***82 245313***

T. L. Linnik arXiv1111.3924

F. de Juan, M. Sturla, MAHV, work in progress.