Thermo-electric transport in holographic systems with momentum dissipation

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Based on:

"Thermo-electric transport in gauge/gravity models with momentum dissipation", arXiv:1406.4134, JHEP 1409 (2014) 160.

"Analytic DC thermo-electric conductivities in holography with massive gravitons", arXiv:1407.0306, Phys. Rev. D 91 (2015), 025002.

"Bounds on intrinsic conductivities in momentum dissipating holography", arXiv:1411.6631.
with A. Amoretti, A Braggio, N. Maggiore and D. Musso .

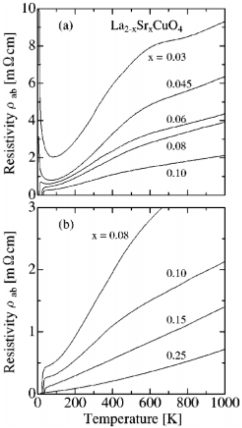
Outline

Fermi liquids and strange metals

- Almost all the transport properties deviate from the Fermi liquid behaviour

	Fermi Liquid	Strange Metals
ρ	T^2	T e.g. Hussey review, '08
$s \equiv \frac{\alpha_{xy}}{\alpha_{xx}}$	T	$s \sim A - BT$ Orbetelli '92
$\tan\theta_H \equiv \frac{\sigma_{xy}}{\sigma_{xx}}$	$\frac{1}{T^2}$	$\frac{1}{T^2}$ e.g. Hussey review, '08
Kohler's rule	$\frac{\Delta\rho}{\rho} \sim \frac{B^2}{\rho^2}$	$\frac{\Delta\rho}{\rho} \sim \tan^2\theta_H$ Harris '92

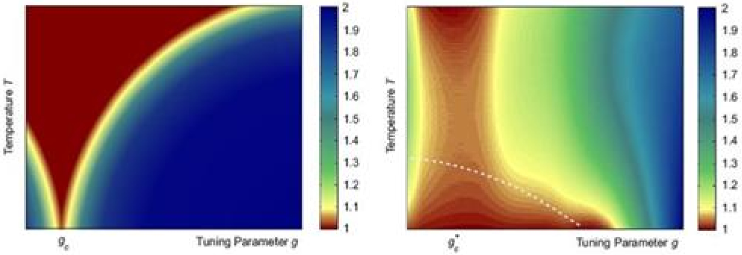
Linear resistivity



- Cuprate resistivity increases without saturation at least to 1000K. Takenaka et al. '03.

Quantum critical point

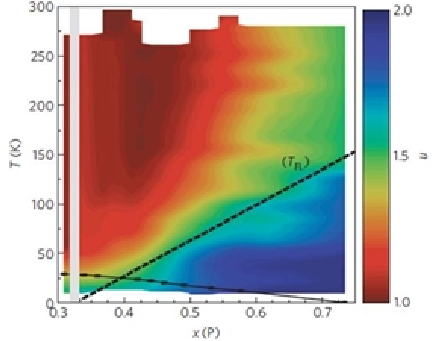
- Linear resistivity related to a critical point. Temperature is the only scale.
- Resistivity near a critical point (left), Cuprates (right).



- Analytis et al. Nature '15

Quantum critical point

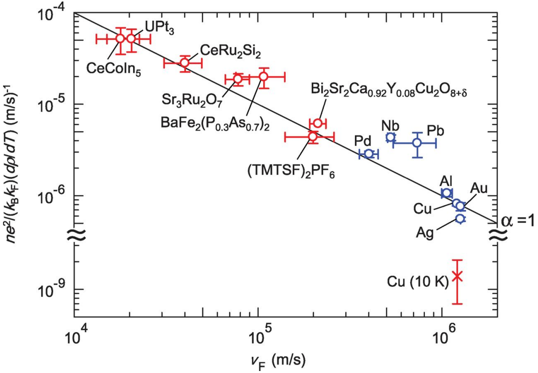
- Resistivity (Pnictides)



- Analytis et al. Nature '15

Scattering rates

- Scattering rates of metals with T-linear resistivity. $(\tau T)^{-1} = k_B/\hbar$



- Bruin et al. Science '13

Mott-Ioffe-Regel (Mir) bound

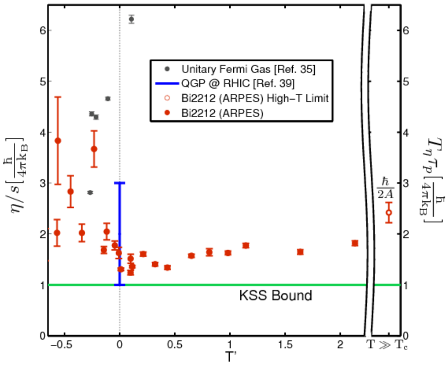
- Drude formula: $\sigma = \frac{ne^2\tau}{m} = k_F^2 \frac{e^2 l}{2\pi\hbar k_F} \gtrsim \frac{e^2}{h}$
- l = mean free path, τ = relaxation time.
- When $l \sim \frac{1}{k_F}$, minimal conductivity or maximal resistivity.
- The bound is violated (not saturated) by strongly correlated systems.

Bound on viscosity

- Bound on shear viscosity $\eta/s \geq \hbar/(4\pi k_B)$ Kovtun, Son, Starinets '05
- Other quantities saturate a bound?

Bound on viscosity

- QGP, Unitary Fermi gas, Arpes on optimally doped cuprates almost saturate the bound.



- Rameau et al. PRB '14

DC thermo-electric response

- Definition of the transport quantities,

$$\begin{pmatrix} J \\ Q \end{pmatrix} = \begin{pmatrix} \sigma & \alpha \\ T\alpha & \bar{\kappa} \end{pmatrix} \begin{pmatrix} E \\ -\nabla T \end{pmatrix}$$

- $\kappa = \bar{\kappa} - \frac{\alpha^2 T}{\sigma}$ thermal conductivity, α Seebeck coefficient.

Einstein relation

- Charge conservation $\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{j} = 0$
- $\mathbf{j} = -\sigma \nabla \mu$
- Relation between ρ and μ: $\nabla \rho = \chi \nabla \mu$
- Diffusion equation: $\frac{\partial \rho}{\partial t} = D \nabla^2 \rho, \quad D = \frac{\sigma}{\chi}$.
- Energy conservation. Diffusion equation for energy: $\frac{\partial \epsilon}{\partial t} = D \nabla^2 \epsilon, \quad D = \frac{\kappa}{c_\rho}$.

Einstein relation

- Diffusion equation: $\frac{\partial n_A}{\partial t} = D_{AB}\nabla^2 n_B$. $n = (\rho, \epsilon)$.
- The eigenvalues D_+ and D_- satisfy:

$$D_+ D_- = \frac{\sigma}{\chi}\frac{\kappa}{c_\rho}$$

$$D_+ + D_- = \frac{\sigma}{\chi} + \frac{\kappa}{c_\rho} + \frac{T(\zeta\sigma - \chi\alpha)^2}{c_\rho \chi^2 \sigma}.$$

Hartnoll argument

- Hartnoll argument Hartnoll., '14
- $\frac{\eta}{s} \sim \frac{\epsilon \tau}{k_B n} \gtrsim \frac{\hbar}{k_B}$.
- In a relativistic system with $\mu = 0$, $\frac{\eta}{s} = D\frac{T}{c^2}$.
- $\frac{D}{c^2} \gtrsim \frac{\hbar}{k_B T}$.
- In the incoherent regime (momentum not conserved) (charge and energy) diffusion constants D_+ and D_- saturate the bound Hartnoll '14:

$$D_+, D_- \geq C\frac{\hbar v^2}{k_B T}$$

Holography

What can be said in the holographic framework?

see also Blake & Tong '13 Donos & Gauntlett '14 Blake & Donos '14 Hartnoll & Karch '15 Blake, Donos & Lohitsiri '15

Momentum dissipation in holography

1. Inhomogeneous lattices: Horowitz, Santos & Tong '12...
2. Breaking translations to a helical Bianchi VII subgroup Donos & Gauntlett '12...
3. Random-field disorder Hartnoll & Herzog '08...
4. Breaking diffeomorphism in the bulk: Q-Lattices, axions and massive gravity Donos & Gauntlett '13, Vegh '13, Andrade & Withers '13...

We use massive gravity

- simple to solve
- we can obtain general physical statements

Massive gravity and momentum dissipation

- Breaking diffeomorphisms in the bulk by adding a mass term for the graviton

$$S = \int d^4x\sqrt{-g}\left[R - \Lambda - \frac{1}{4}F^2 + \beta\left([\mathcal{K}]^2 - [\mathcal{K}^2]\right)\right]$$

where $\mathcal{K}^{2\,\nu}_{\mu} \equiv f_{\mu\rho}g^{\rho\nu}$, $\mathcal{K} \equiv \sqrt{\mathcal{K}^2}$

- the fixed metric $f_{\mu\nu}$ controls how diffeomorphisms are broken
- Holographic dictionary $\Rightarrow \quad \partial_\mu T^{\mu\nu} \neq 0$
- we want to dissipate momentum but to conserve energy (elastic processes)

$$f_{xx} = f_{yy} = 1 \ , \ \text{and zero otherwise}$$

DC thermo-electric response

- in a system with a $U(1)$ gauge field A and a killing vector ∂_t you can define two radially conserved quantities (independent on the radial AdS coordinate z) Donos & Gauntlett, '14
- concerning the DC response, these two quantities can be identified with the electric current J^i and the heat current $Q^i \equiv T^{ti} - \mu J^i$ at the conformal boundary $z = 0$
- due to their radial independence we can express these quantities in terms of horizon data (thermodynamics)

DC thermo-electric response

- The DC electric conductivity σ_{DC} splits into two parts Blake & Tong, '13

$$\sigma_{DC} = \sigma_{\text{ccs}} + \frac{\rho^2 \tau}{\mathcal{E} + P}$$

- The thermal $\bar{\kappa}_{DC}$ and thermoelectric α_{DC} DC conductivity are affected only by the Drude part A.A. et al., '14

$$\alpha_{DC} = \frac{\mathcal{S} \rho \tau}{\mathcal{E} + P} \qquad \bar{\kappa}_{DC} = \frac{\mathcal{S}^2 T \tau}{\mathcal{E} + P}$$

DC thermo-electric response

- In the hydrodynamic regime ($|\beta| \ll T^2$) a dissipation rate τ^{-1} can be defined Davison, '13

$$\partial_t T^{tt} = 0, \quad \partial_t T^{ti} = \tau^{-1} T^{ti}$$

$$\tau^{-1} \equiv -\frac{\mathcal{S}\beta}{2\pi(\mathcal{E} + P)}$$

Bound in holography

- $$D_c = \frac{\sigma}{\chi} = -\frac{\sqrt{4\pi^2 T^2 - 3\beta} - 2\pi T}{\beta},$$
  $$D_h = \frac{\kappa}{c_\rho} = -\frac{\sqrt{4\pi^2 T^2 - 3\beta}}{\beta}.$$
- no bound in the simple massive gravity model.

The dilaton model

- Dilaton model Gubser, Rocha '10, Davison, Schalm, Zaanen '14
- The action of the dilaton model
- $$S_d = \int \sqrt{-g}[R + 6cosh(\phi) - \frac{e^{\phi}}{4} F_{\mu\nu} F^{\mu\nu} - \frac{3}{2}\partial_\mu \phi \partial^\mu \phi + \mathcal{M}_\beta(g)] + S_{c.t.},$$

The dilaton model

- The solution:

$$ds^2 = \frac{g(z)}{z^2}\left(-h(z)dt^2 + \frac{dz^2}{g(z)^2 h(z)} + dx^2 + dy^2\right) \ ,$$

$$A_t = \sqrt{\frac{3Q(Qz_h+1)}{z_h}\left(1+\frac{\beta z_h^2}{(Qz_h+1)^2}\right)}\frac{z_h - z}{z_h(Qz+1)} \ ,$$

$$\phi(z) = \frac{1}{3}\log g(z) \ , \qquad g(z) = (1+Qz)^{\frac{3}{2}} \ ,$$

$$h(z) = 1 + \frac{\beta z^2}{(Qz+1)^2} - \frac{z^3(Qz_h+1)^3}{z_h^3(Qz+1)^3}\left(1+\frac{\beta z_h^2}{(Qz_h+1)}\right) \ .$$

Thermodynamic quantities

- $$T = \frac{3(1+Qz_h)^2 + \beta z_h^2}{4\pi(1+Qz_h)^{\frac{3}{2}} z_h},$$
$$\mathcal{S} = \frac{4\pi}{z_h^2}(Qz_h+1)^{\frac{3}{2}} \ ,$$
$$\mu = \sqrt{\frac{3Q(Qz_h+1)}{z_h}\left(1+\frac{\beta z_h^2}{(Qz_h+1)^2}\right)} \ ,$$
$$\rho = \frac{\mu}{z_h}(Qz_h+1) \ .$$

Transport quantities

- $$\sigma = \frac{2\beta\left(Qz_h+1\right) - 3Qz_h\left(\beta + \left(\frac{1}{z_h}+Q\right)^2\right)}{2\beta\sqrt{Qz_h+1}}\,,$$
  $$\alpha = -\frac{2\sqrt{3}\pi\sqrt{Q\left(Qz_h+1\right)\left(Q\left(Qz_h+2\right)+\beta z_h+\frac{1}{z_h}\right)}}{\beta z_h}\,,$$
  $$\kappa = \frac{4\pi\left(Qz_h+1\right)\left(3\left(Qz_h+1\right)^2+\beta z_h^2\right)}{z_h^2\left(\beta z_h\left(Qz_h-2\right)+3Q\left(Qz_h+1\right)^2\right)}\,.$$

The critical limit

- Strange metal phase at high $\frac{T}{\mu}$. Let us consider $\mu = 0$.
- A trivial solution $Q = 0$ and a non trivial solution $Qz_h + 1 = |\beta|^{1/2} z_h$.
-
$$T = \frac{|\beta|^{1/4}}{2\pi z_h^{1/2}}, \quad \rho = 0, \quad \mathcal{S} = 8\pi^2 |\beta|^{1/2} T$$
- The susceptibilities:
$$\zeta = 0, \quad \chi = |\beta|^{1/2}, \quad c_\rho = 2\pi |\beta|^{-1/2} T$$
- The transport coefficients:
$$\rho = 2\pi |\beta|^{-1/2} T, \quad s = 0, \quad \kappa = 16\pi^2 |\beta|^{-1/2} T^2$$

The diffusion constants

- $$D_c^{cr} = \frac{\sigma}{\chi} = \frac{1}{2\pi T} \qquad D_h^{cr} = \frac{k}{c_\rho} = \frac{2\pi T}{|\beta|}.$$
- The incoherent regime: $\frac{T}{\beta} \to 0$
- Bound on the sum: $D_h^{cr} + D_c^{cr} \geq \frac{1}{2\pi T}$
- See also Kovtun, '15 where a bound on the sum was proposed.

Conclusions

- At finite density thermodynamics and transport are intimately related.
- Sum of Diffusion constants seems to be bounded in a specific model.
- Study the system in presence of a magnetic field : new measurable quantities (Nerst,....).
- To get phenomenological insight we need data clean from spurious effects: working directly with experimentalists!.

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