Custom-made holographic matter

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Outline

- Motivation
- Brane setup
- Thermodynamics
- Fluctuations
  - Zero sound
  - Diffusion
- Alternative quantization: anyons
- Creeping towards more realistic matter:
  - mass for the fundamentals [NJ-Ramallo-G. Itsios]
  - internal flux aka Higgs branch [NJ-Ramallo-G. Itsios]
  - Lifshitz $z \neq 1$ and hyperscaling violation $\theta \neq 0$ [NJ-Ramallo-J. Järvelä]
- Einstein relation

Motivation

- We wish to go beyond the paradigm of Landau-Fermi liquid theory and explore new phases of matter at non-zero density
- Physical examples (non-Fermi liquids):
  - Quark-gluon plasma
  - Strange metals
  - Heavy electron systems
- Holography allows to study strongly correlated systems with no quasiparticle descriptions
- We consider different holographic models and ask
  - How do these models depend on particulars or is there some universal behavior?
  - How do these models behave at low temperature?
- Tell us about some exotic fluid, we provide a microscopic model

Brane setup

	x^1	$\cdots$	x^n	x^{n+1}	$\cdots$	x^p	y^1	$\cdots$	y^{q-n}	y^{q-n+1}	$\cdots$	y^{9-p}
Dp :	$\times$	$\cdots$	$\times$	$\times$	$\cdots$	$\times$	$-$	$\cdots$	$-$	$-$	$\cdots$	$-$
Dq :	$\times$	$\cdots$	$\times$	$-$	$\cdots$	$-$	$\times$	$\cdots$	$\times$	$-$	$\cdots$	$-$

- Dp-Dq brane intersection of the type $(n|p \perp q)$:
  - Dp $\rightarrow$ N_c color branes: $(p+1)$-dim. gauge theory in the bulk
  - Dq $\rightarrow$ N_f flavor branes: fundamental hypermultiplets
- Probe approximation $N_f \ll N_c$:
  - Dp $\rightarrow$ represented by a gravity solution
  - Dq $\rightarrow$ a probe in the Dp-brane background
- Coordinates transverse to both branes:
  - $\vec{z} = (z^1, \ldots, z^{9+n-p-q})$ embedding functions
  - $|\vec{z}| = 0 \rightarrow$ massless quarks for now, $m \neq 0$ in a couple of slides

Probe action

- Dq-brane probe action

$$S = -T_{Dq} \int d^{q+1}\xi e^{-\phi} \sqrt{g + F}$$

- Induced metric for Dp-brane background (massless quarks)

$$ds^2_{q+1} = \rho^{\frac{7-p}{2}} \left(-f_p(\rho) dt^2 + dx_1^2 + \ldots + dx_n^2 \right) + \rho^{\frac{p-7}{2}} \left(\frac{d\rho^2}{f_p(\rho)} + \rho^2 d\Omega^2_{q-n-1} \right)$$

$$f_p(\rho) = 1 - \left(\frac{r_h}{\rho} \right)^{7-p} \quad , \quad e^{-2\phi} = \left(\frac{R}{\rho} \right)^{\frac{(7-p)(p-3)}{2}}$$

- r_h is related to the temperature:

$$T = \frac{7-p}{4\pi} r_h^{\frac{5-p}{2}}$$

Probe action

- Ansatz for gauge fields

$$F = -A_t' d\rho \wedge dt + B dx^1 \wedge dx^2$$

- Action

$$S_{Dq} = -\mathcal{N} V_{\mathbb{R}^{(n,1)}} \int d\rho \sqrt{\rho^\lambda + B^2 \rho^{\lambda+p-7}} \sqrt{1 - A_t'^2}$$

- A_t is cyclic variable

$$A_t' = \frac{d}{\sqrt{d^2 + \rho^\lambda + B^2 \rho^{\lambda+p-7}}} \quad , \quad \langle J^t \rangle = \frac{\delta S}{\delta A_t'} = \mathcal{N} d$$

- The dynamics depends solely on p and

$$\boxed{\lambda = 2n + \frac{1}{2}(p-3)(p+q-2n-8)}$$

What is lambda?

SUSY intersections $(n|p \perp q)$ with $n = \frac{p+q-4}{2}$:

$$\boxed{\lambda = q - p + 2}$$

- Dp-D$(p+4) \to (p|p \perp (p+4)) \to \lambda = 6$
  - Examples: D3-D7, D2-D6
- Dp-D$(p+4) \to (p-1|p \perp (p+2)) \to \lambda = 4$
  - Examples: D3-D5, D4-D6
- Dp-D$(p+4) \to (p-2|p \perp p) \to \lambda = 2$
  - Examples: D3-D3, D4-D4

Non-SUSY intersections (#ND=6):

- D4-D8/$\overline{\text{D8}}$ Sakai-Sugimoto model $p = 4, \lambda = 5, q = 8, n = 3$
- D3-D7' $p = 3, \lambda = 4, q = 7, n = 2$
- D2-D8' $p = 2, \lambda = 5, q = 8, n = 2$

Notice that for $p = 3 \to \lambda = 2n$.

Thermodynamics at T=0

- Chemical potential

$$\mu = A_t(\infty) = \gamma d^{2/\lambda} \qquad , \qquad \gamma = \frac{1}{\sqrt{\pi}}\Gamma(1/2-1/\lambda)\Gamma(1+1/\lambda)$$

- Grand potential

$$\Omega = -S^{reg}_{on-shell} = -\frac{2}{2+\lambda}\mathcal{N}\gamma^{-\lambda/2}\mu^{1+\lambda/2}$$

- Energy density

$$\epsilon = \Omega + \mathcal{N}\mu d = \frac{\lambda}{\lambda+2}\mathcal{N}\gamma d^{1+2/\lambda}$$

- Pressure

$$P = -\Omega = \frac{2}{\lambda}\epsilon$$

- Speed of sound

$$\boxed{u_s^2 = \frac{\partial P}{\partial \epsilon} = \frac{2}{\lambda}}$$

- As $P \propto d^{1+\frac{2}{\lambda}}$, λ is interpreted as the polytropic index.

Thermodynamics

- Specific heat for small T:

$$p \neq 3 \quad \rightarrow \quad c_V = T \left(\frac{\partial s}{\partial T} \right) \Big|_d \sim d T^{\frac{p-3}{5-p}}$$

Linear behavior ($\leftrightarrow$ Landau-Fermi liquid) only for $p = 4$.

- The $p = 3$ is special. The entropy is non-vanishing at $T = 0$:

$$s(p=3)/\mathcal{N} = \pi d + \frac{\pi}{2d} (\pi T)^{\lambda} \quad , \quad c_V(p=3) \sim \frac{T^{\lambda}}{d} \sim \frac{T^{2n}}{d}$$

Scaling properties

- The energy scale of the bulk theory is given by [Peet-Polchinski]

$$\mathcal{E} \sim r^{\frac{5-p}{2}}$$

- Consider rescalings $\mathcal{E} \to \Lambda \mathcal{E}$:

$$\rho \to \Lambda_\rho^\Delta \rho \ , \ \Delta_\rho = \frac{2}{5-p}$$

where Δ_ρ is the scaling dimension.
- Scaling dimensions for d and B:

$$\Delta_d = \frac{\lambda}{5-p} \quad , \quad \Delta_B = \frac{7-p}{5-p}$$

- λ determines the scaling dimension of the charge density (recall polytropic index)
- Only for $p=3$ one gets canonical scaling dims. for (n+1) conformal QFT: $\Delta_d = \frac{\lambda}{2} = n$, $\Delta_B = 2$.

Violation of speed of sound bound

- For strongly coupled $(n+1)$d QFT with gravity dual, bound:
[Hohler-Stephanov,Cherman-(Cohen)-Nellore]

$$u_s^2 \leq \frac{1}{n}$$

- We find $u_s^2 = \frac{2}{\lambda}$, so violated iff

$$\frac{\lambda}{2} < n \leftrightarrow (p-3)(p+q-2n-8) < 0$$

- For SUSY $(p+q-2n-8) = -4$, then

$$p > 3 \quad \leftrightarrow \quad \text{violated}$$

- In general, for $p > 3$ the bound is always violated except for two cases $(1|4 \perp 6)$ and $(1|5 \perp 5)$ for which cases $u_s = 1$.

Fluctuation spectrum

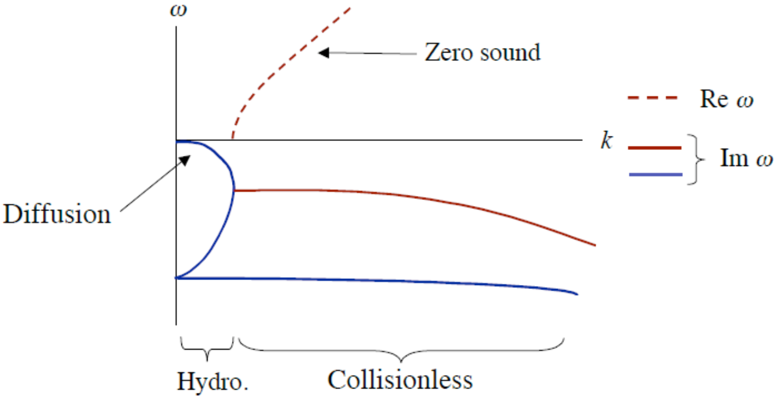
Collective excitations $\leftrightarrow$ poles of the retarded Green's functions $\leftrightarrow$ quasinormal modes $\leftrightarrow$ density waves in the dual field theory

- Perturb as $A_\nu = A_\nu^{(0)} + a_\nu(\rho, x^\mu)$.
- Define symm. $\mathcal{G}$ and antis. $\mathcal{J}$ as $(g^{(0)} + F^{(0)})^{-1} = \mathcal{G} + \mathcal{J}$.
- Yields Lagrangian

$$\mathcal{L} \propto \frac{\rho^\lambda + B^2 \rho^{\lambda+p-7}}{\sqrt{\rho^\lambda + B^2 \rho^{\lambda+p-7} + d^2}} \left(\mathcal{G}^{ac}\mathcal{G}^{bd} - \mathcal{J}^{ac}\mathcal{J}^{bd} + \frac{1}{2}\mathcal{J}^{cd}\mathcal{J}^{ab} \right) f_{cd} f_{ab}$$

- Fourier $a_\nu = a_\nu(\rho, t, x) = \int \frac{d\omega dk}{(2\pi)^2} a_\nu(\rho, \omega, k) e^{-i\omega t + ikx}$
- Solve the equations of motion with the conditions:
  - Infalling boundary conditions at the horizon
  - No sources at the UV boundary
  - Low ω, k

Snapshot of typical QNM: B=0



Zero sound (T=0,B=0)

- Take small $\omega \sim \mathcal{O}(\epsilon), k \sim \mathcal{O}(\epsilon)$:

$$\omega = \omega_R(k) - i\Gamma(k),$$

where $\Gamma(k)$ is the attenuation (decay rate).

$$\boxed{\omega_R = \pm\sqrt{\frac{2}{\lambda}}k} \quad , \quad \boxed{\Gamma = \frac{\pi}{2\mu} \frac{(5-p)^{\frac{3-p}{5-p}}}{\left[\Gamma\left(\frac{1}{5-p}\right)\right]^2} \left(\frac{2}{\lambda}\right)^{\frac{7-p}{2(5-p)}} k^{\frac{7-p}{5-p}}}$$

Same speed as the first sound!

- Reproduces the known cases, for example:
[Karch-Son-Starinets,Brattan&al.,Kulaxizi-Parnachev,Goykhman&al.,...]

$$\omega = \begin{cases} \pm\frac{k}{\sqrt{3}} - \frac{i}{6}\frac{k^2}{\mu} & , \text{ D3-D7 } (p=3, \lambda=6) \\ \pm\frac{k}{\sqrt{2}} - \frac{i}{4}\frac{k^2}{\mu} & , \text{ D3-D5 } (p=3, \lambda=4) \end{cases}$$

Diffusion mode (B=0)

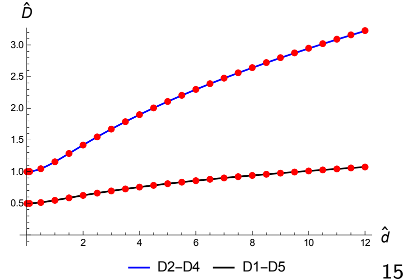
- Purely im. mode with $\omega \sim \mathcal{O}(\epsilon^2)$, $k \sim \mathcal{O}(\epsilon)$:

$$\omega = -iDk^2 \qquad ,$$

where the diffusion constant ($\hat{d} = \frac{d}{r_h^{\lambda/2}} = \left(\frac{7-p}{4\pi T}\right)^{\frac{\lambda}{5-p}} d$):

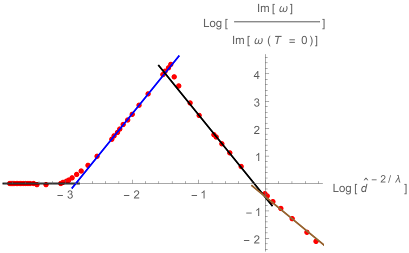
$$\boxed{D = \frac{7-p}{2\pi(\lambda-2)} \frac{\sqrt{1+\hat{d}^2}}{T} F\left(\frac{3}{2}, \frac{1}{2} - \frac{1}{\lambda}, \frac{3}{2} - \frac{1}{\lambda}, -\hat{d}^2\right)}$$

$$D \sim \begin{cases} T^{-1} & , \; T \gg 1 \\ T^{-\frac{7-p}{5-p}} & , \; T \ll 1 \end{cases}$$



Crossover transition

- Following the lowest excitation mode as heating up the system, e.g. in D1-D5:



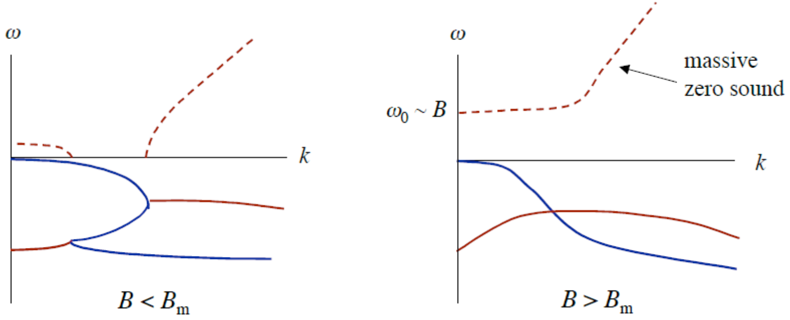
- Crossover transition between collisionless and hydrodynamic regimes, scales as

$$\boxed{\omega_{cross} \sim \frac{T^{\frac{7-p}{5-p}}}{\mu}} \quad , \quad \boxed{k_{cross} \sim \frac{T^{\frac{7-p}{5-p}}}{\mu}}$$

Snapshot of typical QNM, increasing B

- Increasing magnetic field above a critical value gives the zero sound a mass, $\omega = \omega_R - i\Gamma$:

$$\omega_R = \pm\sqrt{\frac{2}{\lambda}k^2 + \frac{B^2}{\mu^2}}, \quad \Gamma = \frac{\pi}{2\mu}\frac{(5-p)^{\frac{3-p}{5-p}}}{\left[\Gamma\left(\frac{1}{5-p}\right)\right]^2}\left(\frac{2}{\lambda}k^2 + \frac{B^2}{\mu^2}\right)^{\frac{p-3}{2(5-p)}}\left(\frac{k^2}{\lambda} + \frac{B^2}{\mu^2}\right)$$



Comments on Hartnoll bound

- In analogy to $\frac{\eta}{s} \geq \frac{1}{4\pi}$, Hartnoll proposed a lower bound on D at high T:

$$\hat{D} \geq \frac{\hbar v_F^2}{k_B} = v_F^2 \ .$$

- For Fermi liquids with quasiparticle description v_F is Fermi velocity. Otherwise, no understanding what v_F should be.
- For us minimum $\hat{D}$ occurs at $\hat{d} = 0$ i.e. high T:

$$\lim_{T\to\infty} \hat{D} = \frac{2}{\lambda - 2}$$

- In general $\hat{D}$ decreases for increasing $\hat{B}$: holographic metals evade all bounds (at least for $B \neq 0$).

Anyons

- When $n = 2$ (i.e. bulk gauge field is 4-dimensional), implement alternative quantization to make A_μ dynamical [Witten,Yee]
- Mix Dirichlet $\mathfrak{n} = 0$ and Neumann $\mathfrak{n} = \infty$ boundary conditions: [NJ-Lifschitz-Lippert]

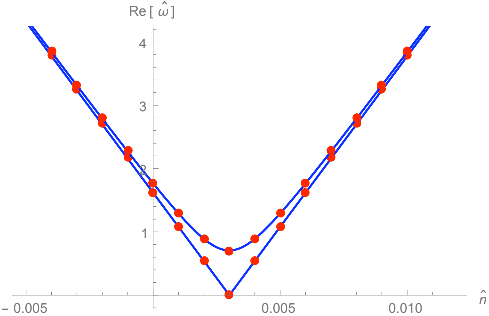
$$\lim_{\rho\to\infty} \left(\mathfrak{n} \rho^{\frac{\lambda}{2}} f_{\rho\mu} - \frac{1}{2} \epsilon_{\mu\alpha\beta} f^{\alpha\beta} \right) = 0$$

- Corresponds to a $SL(2, \mathbb{Z})$ EM transformation: mixes charged current J_μ and magnetic field B
- Changes statistics of the particles: anyons

Anyons: zero sound

- Cheap way of turning on a "magnetic field" w/o touching the background
- Essentially a shift: $B \to B - \mathfrak{n} d$
- Closes the gap for the zero sound

$$\omega_R = \pm\sqrt{\frac{2}{\lambda}k^2 + \frac{(B - \mathfrak{n} d)^2}{\mu^2}}$$

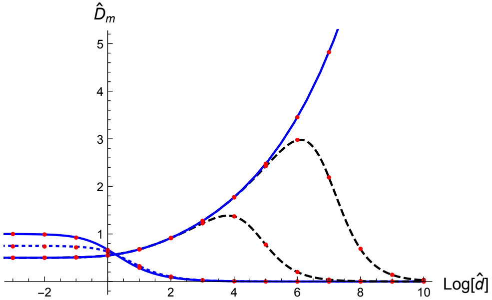


Anyons: diffusion

- Still can find a diffusive mode $\hat{\omega} = -i\hat{D}_{\mathfrak{m}}\hat{k}^2$:

$$\hat{D}_{\mathfrak{m}} = \frac{2\sqrt{1+\hat{d}^2}}{1+\hat{d}^2+\hat{\mathfrak{m}}^2}\left\{\frac{1}{2(6-p)-\lambda}\frac{1}{\sqrt{1+\hat{d}^2}}F\left(\frac{1}{2},\frac{6-p}{\lambda}-\frac{1}{2},\frac{6-p}{\lambda}+\frac{1}{2},-\hat{d}^2\right)\right.$$
$$\left.+\frac{\hat{\mathfrak{m}}^2}{\lambda-2}F\left(\frac{3}{2},\frac{1}{2}-\frac{1}{\lambda},\frac{3}{2}-\frac{1}{\lambda},-\hat{d}^2\right)\right\}$$

where $\mathfrak{m} \sim \frac{1}{\mathfrak{n}}$ and corresponds to $ST^{\mathfrak{m}}$ transformation.



Massive quarks

- Massive quarks in SUSY intersections, speed and attenuation of 0-sound depend on reduced mass:
[Kulaxizi-Parnachev,Davison-Starinets]
[Itsios-NJ-Ramallo to appear]

$$\mathbf{m} = \frac{m}{\mu}$$

$$\omega_R = \pm\sqrt{\frac{2}{\lambda}}\sqrt{\frac{1-\mathbf{m}^2}{1-\frac{2\mathbf{m}^2}{\lambda}}}\,k$$

$$\Gamma = \frac{\pi}{2\mu}\frac{(5-p)^{\frac{3-p}{5-p}}}{\left[\Gamma\left(\frac{1}{5-p}\right)\right]^2}\left(\frac{2}{\lambda}\right)^{\frac{7-p}{2(5-p)}}\frac{\left(1-\mathbf{m}^2\right)^{\frac{6-p}{5-p}-\frac{1}{2}}}{\left(1-\frac{2\mathbf{m}^2}{\lambda}\right)^{\frac{7-p}{2(5-p)}+1}}k^{\frac{7-p}{5-p}}$$

- Speed of sound vanishes when $m = \mu$
- There is a quantum phase transition when $m \to \mu$, exponents:
[Ammon& al.]

$$\boxed{z = 2 \;\;,\;\; \theta = p - 2}$$

Higgs branch

- Consider $\lambda = 4$ ie. Dp-D$(p+2)$ SUSY intersections
- Turn on internal flux q: non-trivial embedding scalar due WZ term but still preserve SUSY
  [Arean-Ramallo-RodriquezGomez,Myers-Wapler,Ammon&al]
- Diffusion constant:

$$\hat{D} = \frac{\sqrt{\hat{d}^2 + (1+\hat{B}^2)(1+\hat{q}^2)}}{1+\hat{B}^2} \int_1^\infty dx \frac{(x^{7-p}+\hat{B}^2)(x^{3-p}+\hat{q}^2)}{(\hat{d}^2 + (x^{7-p}+\hat{B}^2)(x^{3-p}+\hat{q}^2))\sqrt{\hat{d}^2 + x^4 + \hat{B}^2(x^{p-3}+\hat{q}^2)}}$$

- Zero sound mass gap is indep. of q:

$$\hat{\omega}_R = \pm\sqrt{\frac{\sqrt{\pi}\hat{J}}{4\Gamma(5/4)^2}\hat{k}^2 + \frac{\hat{B}^2}{\hat{\mu}^2}} \ , \ \hat{J} = \int_0^\infty dx \frac{x^4 + x^{7-9}\hat{q}^2\hat{d}^{\frac{3-p}{2}}}{\sqrt{1+x^4}(1+x^4+x^{7-p}\hat{q}^2\hat{d}^{\frac{3-p}{2}})}$$

- Generalizes to non-Abelian instantons in Dp-D$(p+4)$?

Lifshitz and hyperscaling violating bg

- Consider background

[∼Dong-Harrison-Kachru-Torroba-Wang]

$$ds^2_{p+2} = r^{-\frac{2\theta}{p}} \left(-f_p r^{2z} dt^2 + r^2 d\vec{x}^2 + \frac{dr^2}{f_p r^2} \right)$$

$$f_p = 1 - \left(\frac{r_h}{r}\right)^{p+z-\theta} \quad , \quad r_h = \left(\frac{4\pi T}{p+z-\theta}\right)^{\frac{1}{z}}$$

- Embed probe q-brane at $d \neq 0$ and $B \neq 0$

[O'Bannon-Hoyos-Wu,Dey-Roy,(Lee-)Pang,Edalati-Pedraza]

- Turns out only three parameters:

$$q \quad , \quad z \quad , \quad \xi \equiv 1 - \frac{\theta}{p}$$

- Can compute thermodynamics, diffusion constant, zero sound dispersions, alternative quantization. . .

[NJ-Järvelä-Ramallo to appear]

Einstein relation

$$\boxed{D\chi = \sigma}$$

- Relates an equilibrium quantity, charge susceptibility $\chi = \frac{\partial d}{\partial \mu}$, to two transport quantities.
- D and σ are computed from different channels: highly non-trivial formula.
- Always holds! For example for $m = 0$ anyonic fluids for $z \neq 1, \theta \neq 0$ analytic match at $B = 0$.

  - Reverse engineering: use Einstein relation $\rightarrow$ $\frac{\sigma_L}{\mathcal{N}} = \frac{\sqrt{1+\hat{B}^2+\hat{d}^2}}{1+\hat{B}^2} r_h^{\xi(q-2)}$.