Finite temperature monopole correlators in holographic liquids

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Outline

- Definition of magnetic monopole operator
- Motivations from cond-matt and AdS/CFT point of view
- Introduction to bulk magnetic monopole: a specific top-down approach
- Monopole correlators in BH/ME phases
- Monopole correlators in a compressible finite-density phase (Fermi liquid phase)
- Comparison of the results:

  QFT vs $T = 0$ AdS/CFT approach vs finite T AdS/CFT approach
- Conclusions and outlook

Monopole operators

- Magnetic monopole operator $\mathcal{M}(x_m)$ [Borokhov-Kapustin-Wu'02]
  - In a $\mathrm{U}(1)$ $(2+1)$d gauge theory: It is a localized defect that inserts a magnetic flux at a certain point x_m
  - in a path integral formalism: In a $(2+1)$d field theory with a global $\mathrm{U}(1)$: Apply an external gauge field a_μ coupled to the current j_μ

$$S_{QFT} \to S_{QFT} - \int d^3x\, a_\mu\, j^\mu$$

and promote a_μ to be dynamical with boundary condition

$$df = q_m \delta^{(3)}(x - x_m)$$

where $f = da$, then

$$\langle \mathcal{M}(x_m) \rangle = \frac{1}{\mathcal{Z}} \int_{df = q_m \delta^{(3)}(x - x_m)} [Da][D\varphi] e^{S_{QFT} - \int d^3 x\, a_\mu\, j^\mu}$$

Monopole operators

- In a $(2+1)$d field theory global topological $\mathrm{U}(1)$ conserved current

$$\tilde{j}^{\mu} \sim \epsilon^{\mu\nu\lambda} f_{\mu\nu}$$

  BUT now:

$$\langle \partial_{\mu} \tilde{j}^{\mu}(x) \mathcal{M}(x_m) \rangle \sim q_m \delta^{(3)}(x - x_m) \langle \mathcal{M}(x_m) \rangle + \dots .$$

  "particle" with magnetic charge q_m with respect to $\tilde{j}^{\mu}$

- Vortex-creating operator [Borokhov-Kapustin-Wu'02]: insertion at x_m of a vortex charge q_m (magnetic flux through S^2 centred at x_m)

- Topological disorder operator

Monopole operators: why?

- non-local and non-perturbative (Dirac quantization): intrinsically interesting objects
- from a condensed matter point of view: order parameter in quantum phase transition
  - continuous quantum phase transition from an antiferromagnet to a valence bond solid phase: described by a $\mathbb{C}P^N$ model with a condensation of monopoles at the critical point [Read'89][Read'90][Murthy-Sachdev'90]
  - phase transition between compressible phases (superfluid and solid phase) in a doped $(2+1)$d conformal field theory [Sachdev '12]

Goal and strategy

- Idea:
  monopole operators relevant in a holographic approach to study quantum matter phases
  [Faulkner-Iqbal'12][Sachdev'12] [Iqbal'14][Filev'14]

- Goal:
  - How does the system react to an injection of magnetic flux?
  - Different matter phases are characterized by different answers to this question
  - Relevant observables: monopole two-point functions

- Strategy: Computation of monopole correlators in AdS/CFT

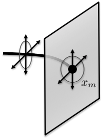
Bulk monopoles at large N: general idea

- The idea is [Sachdev'12] [Witten'03]:

  On the bulk a particle electrically charged under A is dual to a (heavy) operator which carries a charge under j (the dual boundary current)

  On the bulk a particle magnetically charged under A is dual to a (heavy) operator which carries a magnetic charge under j (picture by [Iqbal'14])

- This is what we define a bulk magnetic monopole



- We will work with a top-down construction employed by Iqbal [Iqbal'14]

Bulk monopoles: a top-down model I

- Background: D3/ probe-D5
  - $N \gg 1$ D3-branes: $\mathrm{AdS}_5 \times \mathrm{S}^5$ background
  - D5-brane: probe and intersecting the D3's along $(2+1)$-*d* (embedding: $\mathrm{AdS}_4 \times \mathrm{S}^2$)

	t	x	y	$x_\perp$	u	ψ	θ	ϕ	$\tilde{\theta}$	$\tilde{\phi}$
N D3 (background)	×	×	×	×						
probe D5 (background)	×	×	×		×		×	×		

- Field theory content: $\mathcal{N}=4$ SYM in $(3+1)$-*d* with fundamental matter charged under $\mathrm{SU}(N)$ living in the $(2+1)$-d defect with a $\mathrm{U}(1)_B$ baryon number current
  [DeWolfe-Freedman-Ooguri'01][Erdmenger-Guralnik-Kirsch'02][Karch-Katz'02]
- We focus on $(2+1)$-d defect field theory and $\mathrm{U}(1)_B$ baryon number current

Bulk monopoles: a top-down model II

- add an extra probe D3: wrapping an $S^2 \subset S^5$ and its boundary (s, θ, ϕ) ends on D5-brane
- D3 describes a world-line $\mathcal{C}$ in AdS_4

	t	x	y	$x_\perp$	z	ψ	θ	ϕ	$\tilde{\theta}$	$\tilde{\phi}$
N D3s	×	×	×	×						
probe D5	×	×	×		×		×	×		
probe D3	×					×	×	×		

- $D(p-2)$- branes ending on Dp-branes appear as magnetic charges in the Dp world-volume [Strominger'99]

Bulk monopole is a wrapped D3 ending on D5:

magnetically charged point-particle in AdS_4 [Iqbal'14]

From bulk to boundary monopoles: a top-down model III

- When one end of the D3 world-line reaches the boundary at some point x_m, this corresponds to an insertion of a magnetic charge at a point x_m: boundary monopole operator

- In the large N limit, boundary monopole correlators computed by the D3 action [Iqbal'14]:

$$\langle \mathcal{M}(\Delta x) \mathcal{M}^\dagger(0) \rangle \sim e^{-S_{D3}[\Delta x]}$$

  where Δx is separation among the two D3 ends.

- This is the object we are going to compute at finite T

How do we proceed?

- Our background is thermal $\mathrm{AdS}_5 \times \mathrm{S}^5$
- Insert the probe D5: We solve for the various D5 embedding
- Insert the probe D3 (bulk monopole): Compute the action with the condition that its boundary ends on the D5.
- Repeat all the steps for various embeddings and with/without charge on the D5 world-volume.

Monopole correlators

- Action for the D3 monopole: [Iqbal'14]

$$S_{D3} = T_3 \int_{D3} C_4 - T_3 \int_{D3} \sqrt{-\det\left(\gamma_3 + 2\pi\alpha' F_3\right)} + q_m \int_C \tilde{A} \,.$$

Third term: magnetic coupling between the D3 brane and the gauge field living on the D5

- The gauge invariance of the D3 and D5 action with respect to the gauge transformation of C_4 requires a 3-form Lagrange multiplier at the boundary of the D3

$$q_m \int_{\partial D3} K_3 \,, \qquad K_3 \sim \tilde{A} \wedge V_2$$

with V_2 the unit volume on S^2 and $q_m = \frac{T_3}{2\pi\,\alpha'\,T_5} = 2\pi$.

- $\tilde{A}$ is the 4d magnetic dual of A, the D5 world-volume gauge field

$$(d\tilde{A})_{LM} \sim \sqrt{-\det\left(\gamma_5 + 2\pi\alpha'\,F_5\right)} \left(\left(\gamma_5 + 2\pi\alpha'\,F_5\right)^{-1}\right)^{NP} \epsilon_{LMNP} \,,$$

At leading order: $d\tilde{A} \sim \star_4 F$ and is dual to the topological boundary current

Warming up: $T = 0$ conformal phase I [Iqbal'14]

- Background metric at $T = 0$

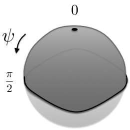
$$ds^2 = \frac{u^2}{R^2}(-dt^2 + dx^2 + dy^2 + dx_\perp^2) + \frac{R^2}{u^2}\left(du^2 + u^2\, d\Omega_5^2\right) \,,$$
$$d\Omega_5^2 = d\psi^2 + \sin^2\psi\, d\Omega_2^2 + \cos^2\psi\, d\tilde{\Omega}_2^2$$

where R is the AdS radius.

- D5 embedding: $\mathrm{AdS}_4 \times \mathrm{S}^2$ specified by

$$\psi = \frac{\pi}{2}\,, \qquad x_\perp = 0$$

- monopole D3: boundary wraps S^2 times a world-line in AdS_4 and then it extends from $\psi = 0$ to $\psi = \frac{\pi}{2}$ picture by [Iqbal'14]



Warming up: $T = 0$ conformal phase I [Iqbal'14]

- Here the D3 action

$$S_{D3} = -T_3 \int_{D3} \sqrt{-\det \gamma_3}$$

- The D3 extends in (ψ, θ, ϕ) directions, and also where it ends on the D5 -brane, i.e. at $\psi = \frac{\pi}{2}$

$$S_{D3} = T_3 \Omega_2 R^3 \int_0^{\frac{\pi}{2}} \sin^2 \psi \, d\psi \int_{\mathcal{C}} ds$$

- it is a 4d magnetic monopole with an effective mass

$$m_{bm} R = \frac{2}{\pi} \int_0^{\frac{\pi}{2}} \sin^2 \psi \, d\psi$$

- in a conformal phase we expect [Borokhov-Kapustin-Wu'02]:

$$\langle \mathcal{M} (\Delta x) \, \mathcal{M}^\dagger (0) \rangle \sim \frac{1}{|\Delta x|^{2\Delta}}$$

This is what we get with the dimension of the unit-charge monopole operator at strong coupling given by

$$\Delta = T_3 \Omega_2 R^3 m_{bm} R = \frac{N}{2R}$$

Charge (un)gapped phase at finite T: The background

- $\mathrm{AdS}_5 \times \mathrm{S}^5$ metric at finite T:

$$ds^2 = \frac{u^2}{R^2}(-h(u)\,dt^2 + dx_3^2) + \frac{R^2}{u^2}\left(\frac{du^2}{h(u)} + u^2\,\left(d\psi^2 + \sin^2\psi\,d\Omega_2^2 + \cos^2\psi\,d\tilde{\Omega}_2^2\right)\right)\,,$$

  where $h(u) = 1 - (u_0/u)^4$ with horizon at $u = u_0$

- The profile for D5 is described by $\psi(u)$
- The general action for the D5 brane is

$$S_{D5} = T_5 \int_{D5} 2\pi\alpha' F \wedge C_4 - T_5 \int_{D_5} \sqrt{-\det\left(\gamma_5 + 2\pi\alpha' F\right)}\,.$$

- For now: Switch off F

BH/ME phase at finite T: The background

- Helpful coordinates [Mateos et al '07]

$$(u_0 \varrho)^2 = u^2 + \sqrt{u_0^4 - u^4}\,, \qquad \chi = \cos\psi$$

with horizon at $\varrho = 1$.

- The $\mathrm{AdS}_5 \times \mathrm{S}^5$ metric is

$$ds^2 = \frac{1}{2}\left(\frac{u_0 \varrho}{R}\right)^2 \left[-\frac{f^2}{\tilde{f}}\, dt^2 + \tilde{f}\, dx_3^2\right] + \frac{R^2}{\varrho^2}\left(d\varrho^2 + \varrho^2\, d\Omega_5^2\right)\,,$$

where $f = 1 - 1/\varrho^4$ and $\tilde{f} = 1 + 1/\varrho^4$.

- Temperature

$$T = \frac{u_0}{\pi R^2}$$

BH/ME phase: D5 probe

- The Euclidean D5-action for the D5 embedding specified by $\chi(\upsilon)$

$$\frac{S_{D5}}{\mathcal{N}_5} = \int d\upsilon \left(1 - \frac{1}{\upsilon^4}\right) \sqrt{(1+\upsilon^4)\,(1-\chi^2)\,(1-\chi^2+\upsilon^2\dot{\chi}^2)}$$

- At the boundary: Asymptotic behaviour for χ

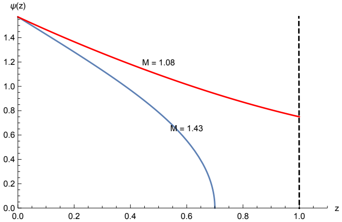
$$\chi(\upsilon) \sim \frac{m}{\upsilon} + \frac{c}{\upsilon^2} + \ldots\,, \qquad \upsilon \to \infty$$

- (m, c) are dimensionless and related to the boundary mass M_q and the "condensate" $\langle \mathcal{O}_m \rangle$ of $U(1)_B$ flavour charged d.o.f. [Kobayashi et al. 06] [Mateos et al. 06] [Mateos et al'07]

$$M_q = \frac{\upsilon_0}{2\sqrt{2}\pi\ell_s^2} m \quad \Leftrightarrow \quad m = \frac{\bar{M}}{T} = \frac{2\sqrt{2}}{\sqrt{\lambda}} \frac{M_q}{T}$$

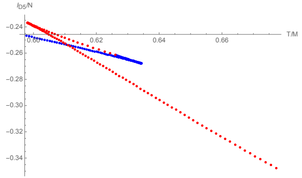
$$\langle \mathcal{O}_m \rangle = -\frac{4\pi\ell_s^2}{\sqrt{2}} \Omega_2 T_5 u_0^2 c$$

BH/ME phase: D5 embedding at finite T



- At finite temperature:
  - Minkowski embedding (ME): D5 ends at a finite value of the radial direction υ_m, before to meet the horizon
  - Black-hole embedding (BHE): D5 reaches and enters the horizon $\upsilon = 1$ at some angle $0 < \psi \leq \frac{\pi}{2}$ $(0 \leq \chi < 1)$
- At zero temperature: ME: D5 ends at a finite value of the radial direction υ_m, before to meet the Poincare horizon
- At $T = 0$ and $T \neq 0$: there is always the constant solution: $\psi = \frac{\pi}{2}$

- at finite T there is a 1st order phase transition [Mateos et al. 07]



- at low T: D5 is in a ME: mass gap in “quark-antiquark” flavored charges (ME phase)
- at high T: D5 is in BHE: gapless spectrum (BH phase)

Warning: this is not a confined/deconfined phase transition (there is a horizon now)

Monopole correlators in BH/ME phase

- Here the D3 action

$$S_{D3} = -T_3 \int_{D3} \sqrt{-\det \gamma_3}$$

- The D3 extends in (s, ψ, θ, ϕ) directions, and it ends on the D5 -brane, i.e. at $\psi = \psi(\upsilon)$

$$S_{D3} = \mathcal{N}_3 \int_{\mathcal{C}} ds \int_0^{\psi(\upsilon)} \sin^2 \psi \, d\psi \, \sqrt{G_{MN} \dot{X}^M \dot{X}^N}$$

- it is a 4D magnetic monopole particle with an effective mass

$$m_{bm}(\upsilon) R = \frac{2}{\pi} \int_0^{\psi(\upsilon)} \sin^2 u \, du$$

- Recall: ψ controls the size of the S^2 wrapped by the monopole D3

Monopole correlators in BH/ME phase

- For the actual calculations: D3 as a point particle in the effective conformally rescaled metric

$$\bar{G}_{MN} \equiv m_{bm}^2(\upsilon) G_{MN}$$

- Geodesic extends along $(x(s), \upsilon(s))$
- Compare against the disconnected configuration
- D3 disconnected configuration

$$\frac{S_{D3}}{\mathcal{N}_3} = 2\int_{\upsilon_m(1)}^{\infty} d\upsilon \sqrt{\bar{G}_{\upsilon\upsilon}} = 2\int_{\upsilon_m(1)}^{\infty} d\upsilon \, \frac{m_{mb}(\upsilon) R}{\upsilon}$$

Monopole correlators in BH/ME phase

- Then the monopole action becomes

$$\frac{S_{D3}}{\mathcal{N}_3} = 2\int_{\upsilon_*}^{\infty} d\upsilon \sqrt{\frac{\bar{G}_{\upsilon\upsilon}}{1 - P^2 \bar{G}^{xx}}} = 2\int_{\upsilon_*}^{\infty} d\upsilon \frac{m_{bm} R}{\upsilon\sqrt{1 - \frac{2\hat{P}^2}{\upsilon^2 \tilde{f}}}} ,$$

  where P is a conserved momentum

$$P \equiv \dot{x}\, \bar{G}_{xx} , \qquad \hat{P} = \frac{P}{\pi\, m_{bm}\, R\, T}$$

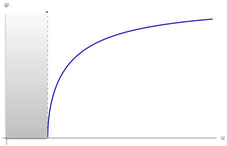
- υ_* is the turning point:

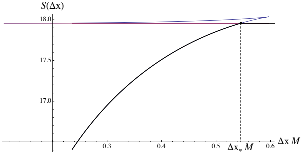
$$P^2 = \bar{G}_{xx}(\upsilon_*) \qquad \Rightarrow \qquad \upsilon_*^2 = \hat{P}^2 + \sqrt{\hat{P}^4 - 1} \quad \text{and} \quad \hat{P} \geq 1$$

  The temperature sets a lower bound on the momentum
- This should be plotted against

$$\Delta x = 2\int_{\upsilon_*}^{\infty} \sqrt{\frac{\bar{G}_{\upsilon\upsilon}}{1 - P^2 \bar{G}^{xx}}}\, \bar{G}^{xx}\, P\, d\upsilon \quad \Rightarrow \quad \Delta x\, \bar{M} = \frac{4}{\pi}\int_{\upsilon_*}^{\infty} \frac{1}{\upsilon^3 \tilde{f}} \frac{m\, \hat{P}}{\sqrt{1 - \frac{2\hat{P}^2}{\upsilon^2 \tilde{f}}}}\, d\upsilon$$

Monopole correlators in a charge gapped (ME) phase: T=0 [Iqbal'14]





- at T=0 in a charge gapped phase:

  At $u = u_m$, $\psi(u_m) = 0$ and the effective mass for the monopole D3 goes to zero (wrapped S^2 shrinks to zero moving into the bulk)

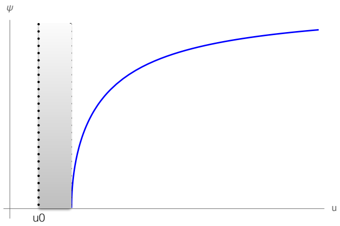
  closing off of the D5 ⇔ 4d "confinement" ⇔ condensate of monopoles

- first order phase transition from a connected to a disconnected configuration at $T = 0$ picture by [Iqbal'14]: The correlator develops a vev

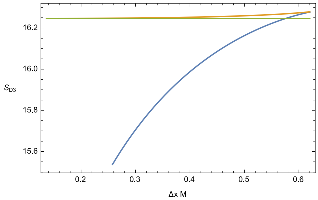
- From a QFT approach: in a gapped phase and at large separation [Iqbal'14]:

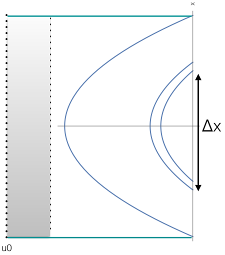
$$\langle \mathcal{M}(\Delta x) \mathcal{M}^\dagger(0) \rangle \sim \langle \mathcal{M} \rangle^2 \neq 0$$

Monopole correlators in a ME phase at finite T

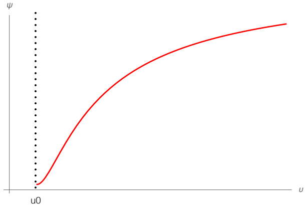


- at non-zero T in a ME phase: at u_m the monopole mass is zero: "thermal monopole condensate"
- 1st order phase transition to a disconnected configuration

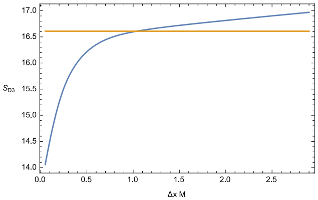


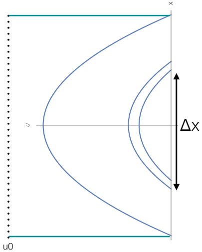


Monopole correlators in a BH phase at finite T



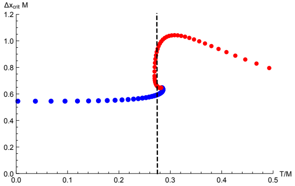
- at non-zero T in a BHE phase: at u_0 (horizon) the monopole mass is non zero: there is no monopole condensate
- 1st order phase transition to a disconnected configuration

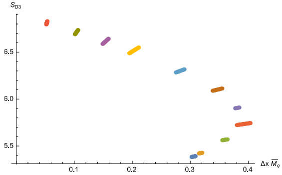




Monopole correlators in BH/ME phase at finite T

- Meaning of the monopole transition: The correlator develops a vev
- Here: The first order phase transition persists at arbitrary high T: it will always saturates at any T
- How the D5 phase transition affects $\Delta x_{\rm crit}$? $\Delta x_{\rm crit}$ depends on T and at high T tends to 0 (plot on the left)
- The transition point moves when increasing T (plot on the right)





Finite density liquid phase at finite T: The background

- Now: Switch on a gauge field on the D5 world-volume

$$2\pi\alpha' F = R^2 a_t'(\upsilon) d\upsilon \wedge dt$$

- The general action for the D5 brane is

$$S_{D5} = T_5 \int_{D5} 2\pi\alpha' F \wedge C_4 - T_5 \int_{D_5} \sqrt{-\det\left(\gamma_5 + 2\pi\alpha' F\right)} \,.$$

Finite-density liquid phase: D5 probe

- The D5-action for the D5 embedding specified by $\chi(\upsilon)$

$$S_{D5} = -\mathcal{N}_5 V_3 \int d\upsilon (1-\chi^2)\chi^2 \tilde{f} \sqrt{\frac{1}{2}\pi^2 T^2 \frac{f^2}{\tilde{f}} \left(1 + \frac{\upsilon^2 \chi'^2}{1-\chi^2}\right) - a_t'^2}$$

- The charge density is defined as

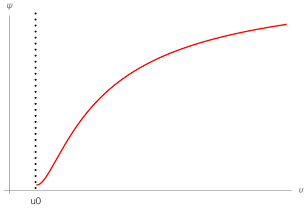
$$\rho = \frac{1}{V_3} \frac{2\pi}{\sqrt{\lambda}} \frac{\partial S_{D5}}{\partial\left(\partial_\upsilon a_t\right)} = 2\pi^2 N T^2 \frac{\upsilon^2 \tilde{f} a_t' (1-\chi^2)}{\sqrt{\frac{1}{2}\pi^2 T^2 \frac{f^2}{\tilde{f}} \left(1 + \frac{\upsilon^2 \chi'^2}{1-\chi^2}\right) - a_t'^2}}$$

- Maxwell e.o.m.: The charge density is conserved

$$\partial_\upsilon \rho = 0$$

Finite-density liquid phase: D5 probe

- Solve e.o.m. for χ:
  - at finite *T* and charge density ρ: we can only have a BHE up to $\psi = \frac{\pi}{2}$ $(\chi = 0)$ (no explicit bulk sources)
  - at $T = 0$ and charge density ρ: only the massless embedding $\psi = \frac{\pi}{2}$ $(\chi = 0)$



Monopole correlators in a finite density phase

- Here the D3 action: geometrical + magnetic coupling term

$$S_{D3} = -T_3 \int_{D3} \sqrt{-\det \gamma_3} + q_m \int_{\mathcal{C}} \tilde{A}$$

- The magnetic coupling

$$d\tilde{A} = \rho \, dx \wedge dy$$

- The D3 extends in (s, ψ, θ, ϕ) directions, and it ends on the D5 -brane, i.e. at $\psi = \psi(\upsilon(s))$

$$S_{D3} = \mathcal{N}_3 \int_{\mathcal{C}} ds \int_0^{\psi(\upsilon(s))} \sin^2 \psi \, d\psi \sqrt{G_{MN} \dot{X}^M \dot{X}^N} + i q_m \int_{\mathcal{C}} \tilde{A}$$

- it is a 4D magnetic monopole particle with an effective mass

$$m_{bm}(\upsilon) = \frac{1}{R} \int_0^{\psi(\upsilon(s))} \sin^2 u \, du$$

Monopole correlators in a finite density phase

- Now the geodesic extends along $(x(s), y(s), \upsilon(s))$, and explicitly the Euclidean action is

$$\frac{S_{D3}}{\mathcal{N}_3} = \int_{\mathcal{C}} ds \; - i\,\tilde{q}_m\,\rho \int_{\mathcal{C}} ds\, y(s)\,\dot{x}(s)\,.$$

- the conserved momenta:

$$P_x = \bar{G}_{xx}\,\dot{x}(s) - i\,\omega\,y(s)\,, \qquad P_y = \bar{G}_{xx}\,\dot{y}(s) + i\,\omega\,x(s)$$

  where the frequency is nothing but

$$\omega \sim q_m\rho$$

- It is helpful to change the parametrization of the curve $\mathcal{C}$ such as

$$\bar{G}_{xx}\frac{d}{ds} = \frac{d}{d\eta}\,, \tag{1}$$

  and rewrite the momenta P_x , P_y as

$$P_x = x'(\eta) - i\omega\,y(\eta)\,, \qquad P_y = y'(\eta) + i\omega\,x(\eta)\,.$$

Monopole correlators at finite density phase I

- Solving the geodesic equation with boundary conditions

$$\text{at}\quad \eta = 0 \qquad x(0) = 0\,, \qquad \upsilon(0) = \upsilon_*$$
$$\text{at}\quad \eta = \eta_i \qquad x(\eta_i) = \pm\frac{\Delta x}{2}\,, \qquad y(\eta_i) = 0\,, \qquad \upsilon(\eta_i) = 0$$

we get

$$x(\eta) = \beta \sinh(\omega\eta)\,, \qquad y(\eta) = y_0 + i\beta\cosh(\omega\eta)\,,$$
$$\Delta x = 2\beta\sinh\left(\omega\eta_i\right)\,, \qquad y_0 = -i\beta\cosh(\omega\eta_i)\,.$$

- The final action for the bulk monopole:

$$\frac{S_{D3}}{\mathcal{N}_3} = 2\int_{\upsilon_*}^{\infty} d\upsilon\sqrt{\frac{\bar{G}_{\upsilon\upsilon}}{1-(\omega\,P_x)^2\,\bar{G}^{xx}}} - i\,\tilde{q}_m\,\rho\int_{\mathcal{C}} y\,\dot{x}$$

and

$$\omega\,\eta_i = \int_{\upsilon_*}^{\infty} d\upsilon\,\omega\,\bar{G}^{xx}\sqrt{\frac{\bar{G}_{\upsilon\upsilon}}{1-(\omega\,P_x)^2\,\bar{G}^{xx}}}$$

Monopole correlators at finite density phase: massless case

- Computing explicitly the action

$$\frac{S_{D3}}{\mathcal{N}_3} = 2\int_{\upsilon_*}^{\infty} d\upsilon \frac{m_{bm}R}{\upsilon\sqrt{1-\frac{2\mathcal{P}^2}{\tilde{f}\upsilon^2(m_{bm}R)^2}}} + \frac{\pi}{2}\frac{\mathcal{P}^2}{Q}\left(\sinh(2\bar{\omega}\bar{\eta}_i) - 2\bar{\omega}\bar{\eta}_i\right)$$

- with turning point

$$\upsilon_*^2 = \frac{\mathcal{P}^2}{(m_{bm}R)^2} + \sqrt{\frac{\mathcal{P}^4}{(m_{bm}R)^4} - 1}$$

- to be plotted against

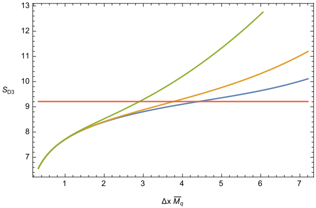
$$\Delta x\,\bar{M} = 2\frac{\mathcal{P}}{Q} m \sinh(\bar{\omega}\bar{\eta}_i)\,, \qquad Q \sim \frac{\rho}{T^2}$$

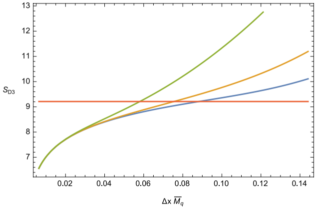
- where

$$\bar{\omega}\bar{\eta}_i = \omega\eta_i = \frac{2}{\pi}\int_{\upsilon_*}^{\infty} d\upsilon \frac{Q}{\upsilon^3\tilde{f}\, m_{bm}R\sqrt{1-\frac{2\mathcal{P}^2}{\tilde{f}\upsilon^2(m_{bm}R)^2}}}$$

$$\mathcal{P} = \bar{P}_x\bar{\omega}\,, \qquad \bar{P}_x = \bar{M}\,P_x \qquad \bar{\omega} = \frac{\omega m}{\pi\bar{M}^2}$$

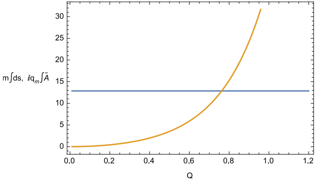
Finite density phase for massless case: Results I

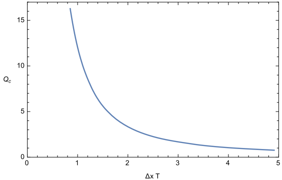




- On the left: monopole correlators at fixed large temperature for three different charges, blue (small) to green (large)
- On the right: monopole correlators at fixed small temperature for three different charges, blue (small) to green (large)
- Straight line is the corresponding disconnected configuration

Finite density phase for massless case: Results II





- On the left: At any T and any given separation, there is a critical charge above which the magnetic coupling is dominant
- On the right: the critical charge is rapidly increases as separation decreases
- By fitting the curve at large separation: the bulk monopole action is a Gaussian function of Δx
- at T=0 from holographic approach: [Iqbal'14]

$$\langle \mathcal{M}\,(\Delta x)\,\mathcal{M}^{\dagger}\,(0)\rangle \sim e^{-\frac{|q_m \rho|}{4}(\Delta x)^2}$$

- Compare with a QFT approach: the monopole correlator falls off exponentially [Kaul et al. 08][Kim et al '94][Herbut et al. '03][Hermele et al. '04]

Summary and...

- I reviewed a top-down approach proposed by Iqbal to engineer a bulk monopole operator: wrapped D3 ending on the boundary of a probe D5
- We computed the bulk monopole action at finite T: in the large N limit we computed the monopole D3 action at finite T
  - gapped and ungapped phase: at any T the monopole develops a vev
  - finite density phase: for the massless case the monopole correlator decays exponentially

... outlook

- For the finite density case: BHE case: is there a competing effect among the two terms?
- what about a superfluid phase?
- a monopole D5 in D3/probe D7
- non-abelian monopoles? higher point functions?

The general action for the D5 brane is

$$S_{D5} = T_5 \int_{D5} 2\pi\alpha' F \wedge C_4 - T_5 \int_{D_5} \sqrt{-\det\left(\gamma_5 + 2\pi\alpha' F\right)} \,. \tag{2}$$

where C_4 is the Ramond-Ramond 4-form, $F = dA$ the field strength corresponding to the gauge field living on the D5 world-volume, and γ_5 the induced D5 world-volume metric. Notice that for the brane configurations investigated in this work, only the second DBI term contributes, since C_4 is proportional to the volume of $\mathbb{R}^4 \subset \mathrm{AdS}_5$ and the product of the two S^2 in S^5. The general action for the bulk monopole was computed in **[?]**,

$$S_{D3} = T_3 \int_{D3} C_4 - T_3 \int_{D3} \sqrt{-\det\left(\gamma_3 + 2\pi\alpha' F\right)} + q_m \int_C \tilde{A} \,. \tag{3}$$

$$S_K = \int_{D5} K_3 \wedge dF + q_m \int_{\partial D3} K3 \,, \tag{4}$$

with

$$\delta_\Lambda C_4 = \Lambda_3 \,, \qquad \delta_\Lambda K_3 = -2\pi\alpha' T_5 \Lambda_3 \,. \tag{5}$$

Notice that this also fixes the value of the coupling q_m to be

$$q_m = \frac{T_3}{2\pi\,\alpha'\,T_5} = 2\pi \,. \tag{6}$$

Adopting the same ansatz as in **[?]**, that is

$$K_3 = \frac{1}{4\pi} \tilde{A} \wedge V_2 \tag{7}$$

Bonus track: Constant

$$\mathcal{N}_5 = 2\pi R^2 u_0^2 T_5 = \frac{N}{4\pi}\frac{\lambda}{R^2} T^2$$

$$\mathcal{N}_3 = 2\pi^2 T_3 R^4 = N$$