Testing AdS/CFT with flavours on a computer

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Dublin Institute for Advanced Studies

Workshop on Holography and Condensed Matter
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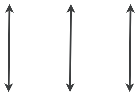
Outline

AdS/CFT correspondence

Type IIB String Theory on

Nc D3

$AdS_5 \times S^5$



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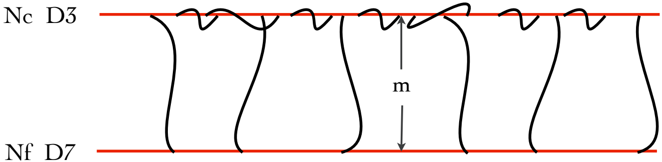
- Gubser-Klebanov-Polyakov-Witten formula:

$$\langle e^{\int d^d x \phi_0(x) \langle \mathcal{O}(x) \rangle} \rangle_{\rm CFT} = \mathcal{Z}_{\rm string}[\phi_0(x)]$$

Adding flavours

Generalizing the correspondence



	0	1	2	3	4	5	6	7	8	9
D3	-	-	-	-	·	·	·	·	·	·
D7	-	-	-	-	-	-	-	-	·	·

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$$m_q \int d^2\theta \, \tilde{Q} \, Q \rightarrow \text{SYM} \quad \text{with} \quad m_q = m/2\pi\alpha'$$

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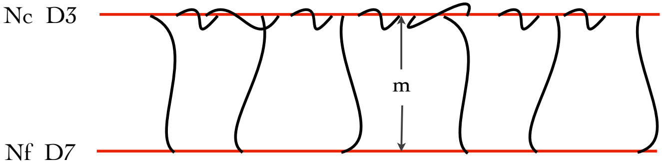

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- Numerous applications: thermal and quantum phase transitions, chiral symmetry breaking, magnetic catalysis etc.
- Can we test if AdS/CFT really works in this case?

Computer simulations of holographic gauge theories

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- Natural candidate is the D0/D4 system, T-dual to the D3/D7 and D3/D5 systems. (Same “class of universality”)
- The field theory is the Berkooz-Douglas matrix model - a flavoured version of the BFSS-matrix model.

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- Dimensionally reduce $\mathcal{N} = 1$ 10D SYM to 1D:

$$S_E = \frac{1}{g^2}\int d\tau\,\mathrm{Tr}\left\{\frac{1}{2}(D_\tau X^i)^2 - \frac{1}{4}[X^i, X^j]^2 + \frac{1}{2}\psi^T C_9\, D_\tau\psi - \frac{1}{2}\psi^T C_9\,\gamma^i[X^i,\psi]\right\}\ ,$$

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- The model enjoys a global $SO(9)$ symmetry and has flat directions associated to the Cartan modes:

$$[X^i, X^j] = 0$$

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- Small curvature and string coupling require $1 \ll g_{eff} \ll N^{\frac{4}{7}}$.

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- We focus on the studies performed in reference 1506.01366.

Discretisation

- Following Catterall and Wiseman we consider a basis in which $C_9 = \sigma_1 \otimes 1_8$ and discretise:

$$\psi^T C_9 \, \mathcal{D}_t \, \psi \;\rightarrow\; (\psi^T_{1\,m} \,, \quad \psi^T_{2\,m}) \cdot \begin{pmatrix} 0 & 1_8 \, (\mathcal{D}_-)_{mn} \\ 1_8 \, (\mathcal{D}_+)_{mn} & 0 \end{pmatrix} \cdot \begin{pmatrix} \psi_{1\,n} \\ \psi_{2\,n} \end{pmatrix}$$

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- The resulting lattice theory is free of fermion doubling.

RHMC

- We employ the RHMC method [hep-lat/0409133] (Clark et al. 2004).

$$|\mathrm{Pf}(\mathcal{M})| = \det(\mathcal{M}^\dagger \mathcal{M})^{1/4} \propto \int D\bar{\xi} D\xi\, e^{-\xi^\dagger (\mathcal{M}^\dagger \mathcal{M})^{-1/4} \xi}$$

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- The pseudo fermionic force is then:

$$\frac{\partial S_{\mathrm{ps.f}}}{\partial u} = -\sum_{i=1}^{\#} \alpha_i \, h_i^\dagger \frac{\partial (\mathcal{M}^\dagger \mathcal{M})}{\partial u} h_i \, ,$$

- where h_i satisfy $(\mathcal{M}^\dagger \mathcal{M} + \beta_i) h_i = \xi_i$ and can be obtained by a multi-shift solver.

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- At low T only the internal energy can be obtained from AdS/CFT

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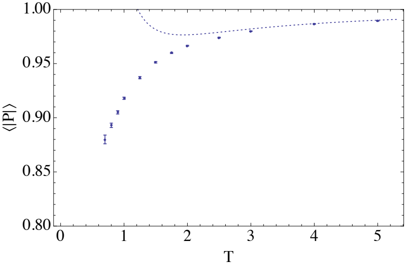

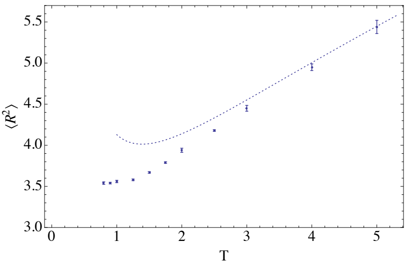

- Plots of the expectation value of the Polyakov loop $\langle |P| \rangle$ and the extent of space $\langle R^2 \rangle$ as functions of temperature.
- The dashed curves represent the predictions of the high temperature expansion.
- Excellent agreement with the results of 0707.4454 and 1503.08499.

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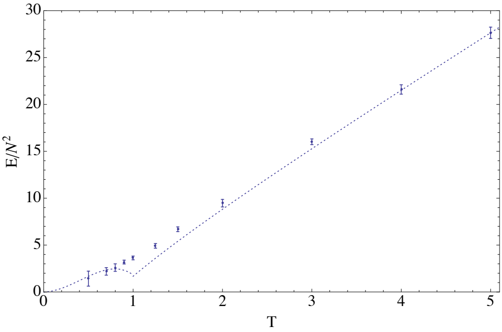


- At high T the plot agrees with the predictions of 0710.2188. At low T the curve represents the AdS/CFT result including α' corrections:

$$\frac{1}{N^2}\frac{E}{\lambda^{1/3}} = \left(\frac{2^{21}3^{12}5^2}{7^19}\pi^{14}\right)^{1/5}\left(\frac{T}{\lambda^{1/3}}\right)^{\frac{14}{5}} - 5.58\left(\frac{T}{\lambda^{1/3}}\right)^{\frac{23}{5}}$$

obtained in 0811.3102 (Hanada et al. 2008)

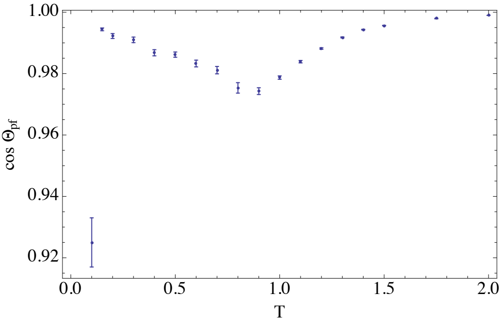
Sign Problem

- There is a special unitary transformation S transforming $C_9 \to 1_{16}$
- In this basis $\mathcal{M}(X) = \mathcal{M}_{kin} + \mathcal{M}_{pot}(X)$ with $\mathcal{M}_{kin}^{\dagger} = -\mathcal{M}_{kin}$ and $\mathcal{M}_{pot}(X)^{\dagger} = \mathcal{M}_{pot}(X)$
- Since $\mathcal{M}_{pot}(-X) = -\mathcal{M}_{pot}(X)$ it follows that $\mathcal{M}(-X) = -\mathcal{M}(X)^{\dagger}$ and therefore $\mathrm{Pf}(\mathcal{M}(-\mathcal{X})) = \mathrm{Pf}(\mathcal{M}(\mathcal{X}))^{*}$
- The symmetry $S_{\mathrm{bos}}[-X] = S_{\mathrm{bos}}[X]$ allows us to write:

$$\mathcal{Z} \propto \int \mathcal{D}X\, \mathrm{Pf}(\mathcal{M})\, e^{-S_{\mathrm{bos}}[X]} = \int \mathcal{D}X\, \cos\Theta_{Pf}\, |\mathrm{Pf}(\mathcal{M})|\, e^{-S_{\mathrm{bos}}[X]}$$

- Now as long as $-\frac{\pi}{2} < \Theta_{Pf} < \frac{\pi}{2}$ the cosine is positive and the effective action defines a true probability distribution

Phase



- A plot of $\cos\Theta_{Pf}$ for $N = 3$ and $\Lambda = 4$. The phase remains small for all T, but drops at very low temperatures possibly due to strong lattice effects.

Berkooz-Douglas matrix model

- Original motivation to introduce M_5 brane density to the BFSS matrix model hep-th/9610236 (Berkooz & Douglas 1996).
- Obtained by reducing the D5/D9 system (Van Raamsdonk, 2002):

$$\begin{aligned}\mathcal{L} &= \frac{1}{g^2}\mathrm{Tr}\left(\frac{1}{2}D_0X^aD_0X^a + \frac{i}{2}\lambda^{\dagger\,\rho}D_0\lambda_\rho + \frac{1}{2}D_0\bar{X}^{\rho\dot{\rho}}D_0X_{\rho\dot{\rho}} + \frac{i}{2}\theta^{\dagger\dot{\rho}}D_0\theta_{\dot{\rho}}\right)\\ &+\frac{1}{g^2}\mathrm{tr}\left(D_0\bar{\Phi}^\rho D_0\Phi_\rho + i\chi^\dagger D_0\chi\right) + \mathcal{L}_{\mathrm{int}}\end{aligned}$$

where:

$$\begin{aligned}\mathcal{L}_{\mathrm{int}} = &\frac{1}{g^2}\mathrm{Tr}\left(\frac{1}{4}[X^a,X^b][X^a,X^b] + \frac{1}{2}[X^a,\bar{X}^{\rho\dot{\rho}}][X^a,X_{\rho\dot{\rho}}] - \frac{1}{4}[\bar{X}^{\alpha\dot{\alpha}},X_{\beta\dot{\alpha}}][\bar{X}^{\beta\dot{\beta}},X_{\alpha\dot{\beta}}]\right)\\ &-\frac{1}{g^2}\mathrm{tr}\left(\bar{\Phi}^\rho(X^a-m^a)(X^a-m^a)\Phi_\rho\right)\\ &+\frac{1}{g^2}\mathrm{tr}\left(\bar{\Phi}^\alpha[\bar{X}^{\beta\dot{\alpha}},X_{\alpha\dot{\alpha}}]\Phi_\beta + \frac{1}{2}\bar{\Phi}^\alpha\Phi_\beta\bar{\Phi}^\beta\Phi_\alpha - \bar{\Phi}^\alpha\Phi_\alpha\bar{\Phi}^\beta\Phi_\beta\right)\\ &+\frac{1}{g^2}\mathrm{Tr}\left(\frac{1}{2}\bar{\lambda}^\rho\gamma^a[X^a,\lambda_\rho] + \frac{1}{2}\bar{\theta}^{\dot{\alpha}}\gamma^a[X^a,\theta_{\dot{\alpha}}] - \sqrt{2}\,i\,\varepsilon_{\alpha\beta}\,\bar{\theta}^{\dot{\alpha}}[X_{\beta\dot{\alpha}},\lambda_\alpha]\right)\\ &+\frac{1}{g^2}\mathrm{tr}\left(\bar{\chi}\gamma^a(X^a-m^a)\chi + \sqrt{2}\,i\,\varepsilon_{\alpha\beta}\,\bar{\chi}\lambda_\alpha\Phi_\beta - \sqrt{2}\,i\,\varepsilon_{\alpha\beta}\,\bar{\Phi}^\alpha\bar{\lambda}_\beta\chi\right)\end{aligned}$$

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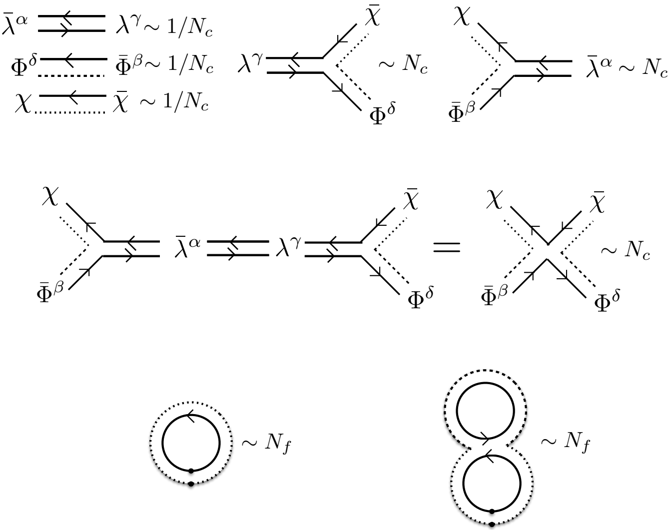

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  - Ignore the backreaction of the flavours on the adjoint fields and reweight the fundamental determinant. Advantage: there is no extra sign problem. Disadvantage: it is computationally very expensive.
  - Consider a full dynamical simulation. Advantage: easier to implement and execute. Disadvantage: There might be an extra sign problem.

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- Therefore quantum effects are essential even in holographic probe brane studies. Example: non-mean field critical exponents.
- We have two options:
  - Ignore the backreaction of the flavours on the adjoint fields and reweight the fundamental determinant. Advantage: there is no extra sign problem. Disadvantage: it is computationally very expensive.
  - Consider a full dynamical simulation. Advantage: easier to implement and execute. Disadvantage: There might be an extra sign problem.
- We were able to show that in a dynamical simulation the path integral again depends only on $\cos\Theta_{Pf}$, which is an encouraging sign.

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  - We also have to worry about α' corrections. Which come of order α'^2 and generally require fitting.

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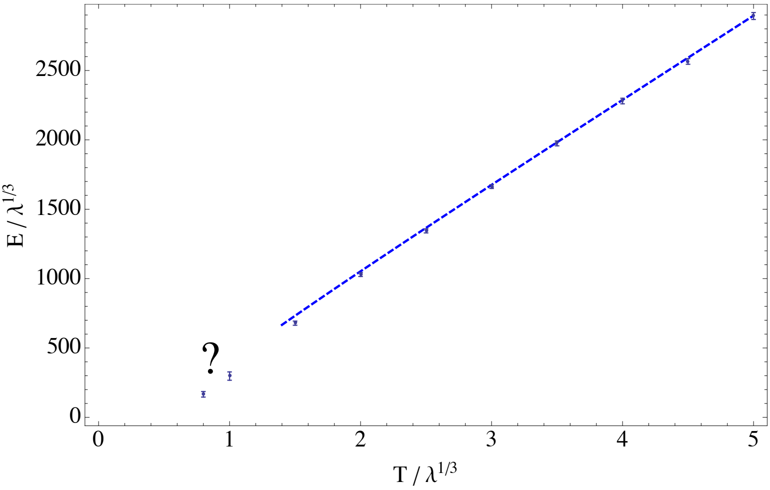
- For the total energy one obtains:

$$E = \Big(6(N_c^2 - 1) + 3N_f\, N_c\Big)\; T + \Big(\#_1\, N_c^2 + \#_2\, N_f\, N_c\Big)\; T^{-1/2} \; ,$$

where $\#_1$ has been determined in 0710.2188 and $\#_2$ has to be determined from simulations of the flavoured bosonic IKKT model.

Some results: Total energy



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- We obtained numerical data for the total energy E_{tot}.

Thank you!