Magnetic impurities and universal relations in AdS/CMT

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(Werner-Heisenberg-Institut)

Outline

1. Kondo models from holography
   - Model — J.E., Hoyos, O'Bannon, Wu 1310.3271
   - Screening, resistivity
   - Quantum quenches — J.E., Flory, Newrzella, Wu in progress
   - Entanglement entropy — J.E., Flory, Newrzella 1410.7811
     J.E., Hoyos, Newrzella, O'Bannon, Wu in progress
   - Two-point functions — J.E., Hoyos, O'Bannon, Papadimitriou, Probst, Wu in progress

2. S-Wave Superconductivity in Anisotropic Holographic Insulators
   J.E., Herwerth, Klug, Meyer, Schalm 1501.07615
   - Scalar condenses in helical Bianchi VII background
   - Homes' Law

Kondo effect:

Screening of a magnetic impurity by conduction electrons at low temperatures

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Motivation for study within gauge/gravity duality:

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1. Kondo model: Simple model for a RG flow with dynamical scale generation

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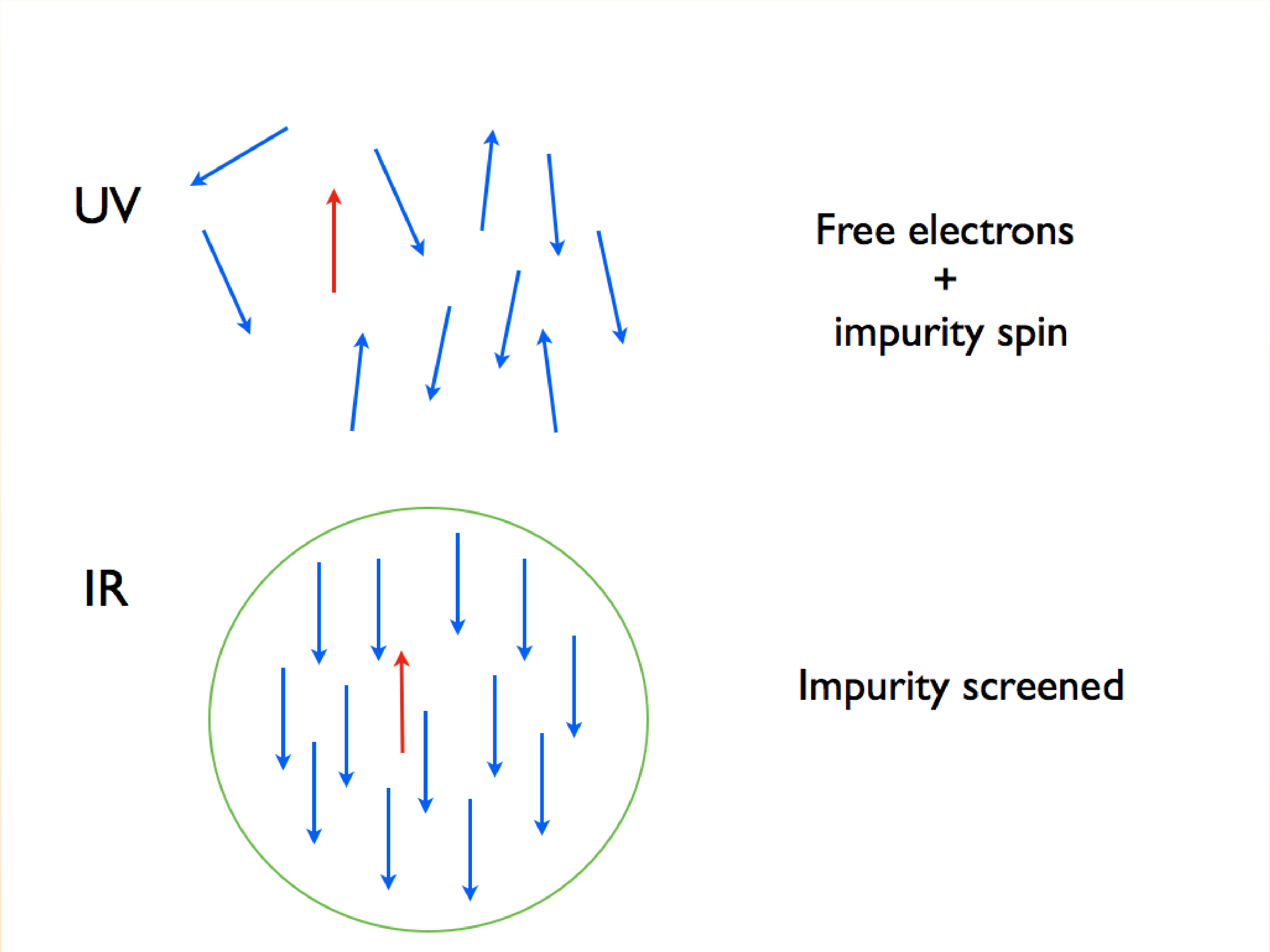
1. Kondo model: Simple model for a RG flow with dynamical scale generation

2. New applications of gauge/gravity duality to condensed matter physics:

- Magnetic impurity coupled to strongly correlated electron system
- Entanglement entropy
- Quantum quench
- Kondo lattice

Kondo effect



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Impurity screened at low temperatures:
Logarithmic rise of resistivity at low temperatures

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Due to symmetries: Model effectively $(1+1)$-dimensional

Hamiltonian:

$$H = \frac{v_F}{2\pi}\psi^\dagger i\partial_x\psi + \lambda_K v_F \delta(x)\vec{S}\cdot\vec{J}, \quad \vec{J} = \psi^\dagger \frac{1}{2}\vec{T}\psi$$

Decisive in development of renormalization group
IR fixed point, CFT approach Affleck, Ludwig '90's

Gauge/gravity requires large N: Spin group $SU(N)$

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In this case, interaction term simplifies introducing slave fermions:

$S^a = \chi^\dagger T^a \chi$

Totally antisymmetric representation: Young tableau with Q boxes

Constraint: $\chi^\dagger \chi = Q$

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Screened phase has condensate $\langle \mathcal{O} \rangle$

Parcollet, Georges, Kotliar, Sengupta cond-mat/9711192
Senthil, Sachdev, Vojta cond-mat/0209144

J.E., Hoyos, O'Bannon, Wu 1310.3271, JHEP 1312 (2013) 086

Coupling of a magnetic impurity to a strongly interacting non-Fermi liquid

J.E., Hoyos, O'Bannon, Wu 1310.3271, JHEP 1312 (2013) 086

Coupling of a magnetic impurity to a strongly interacting non-Fermi liquid

Results:

- RG flow from perturbation by 'double-trace' operator
- Dynamical scale generation
- Holographic superconductor: Condensate forms in AdS_2
- Power-law scaling of conductivity in IR with real exponent
- Screening, phase shift

Kondo models from gauge/gravity duality

J.E., Hoyos, O'Bannon, Wu 1310.3271, JHEP 1312 (2013) 086

J.E., Hoyos, O'Bannon, Wu 1310.3271, JHEP 1312 (2013) 086

Top-down brane realization

	0	1	2	3	4	5	6	7	8	9
N D3	X	X	X	X						
N_7 D7	X	X			X	X	X	X	X	X
N_5 D5	X				X	X	X	X	X	

- 3-7 strings: Chiral fermions ψ in 1+1 dimensions
- 3-5 strings: Slave fermions χ in 0+1 dimensions
- 5-7 strings: Scalar (tachyon)

D3: $AdS_5 \times S^5$

D7: $AdS_3 \times S^5 \to$ Chern-Simons A_μ dual to $J^\mu = \psi^\dagger \sigma^\mu \psi$

D5: $AdS_2 \times S^4 \to \begin{cases} \text{YM } a_t \text{ dual to } \chi^\dagger \chi = q \\ \text{Scalar dual to } \psi^\dagger \chi \end{cases}$

Operator		Gravity field
Electron current J	$\Leftrightarrow$	Chern-Simons gauge field A in AdS_3
Charge $Q = \chi^\dagger \chi$	$\Leftrightarrow$	2d gauge field a in AdS_2
Operator $\mathcal{O} = \psi^\dagger \chi$	$\Leftrightarrow$	2d complex scalar Φ

Bottom-up gravity dual for Kondo model

Action:

$$S = S_{CS} + S_{AdS_2},$$

$$S_{CS} = -\frac{N}{4\pi} \int_{AdS_3} \mathrm{Tr} \left(A \wedge dA + \frac{2}{3} A \wedge A \wedge A \right),$$

$$S_{AdS_2} = -N \int d^3x \, \delta(x) \sqrt{-g} \left[\frac{1}{4} \mathrm{Tr} f^{mn} f_{mn} + g^{mn} \left(D_m \Phi \right)^\dagger D_n \Phi + V(\Phi^\dagger \Phi) \right]$$

$$V(\Phi) = M^2 \Phi^\dagger \Phi$$

Metric:

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = \frac{1}{z^2} \left(\frac{dz^2}{h(z)} - h(z) \, dt^2 + dx^2 \right),$$

$$h(z) = 1 - z^2/z_H^2 \,, \qquad T = 1/(2\pi z_H)$$

‘Double-trace’ deformation by $\mathcal{O}\mathcal{O}^\dagger$

Boundary expansion

$$\Phi = z^{1/2}(\alpha \ln z + \beta)$$

$$\alpha = \kappa\beta$$

κ dual to double-trace deformation

Witten hep-th/0112258

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Φ invariant under renormalization $\Rightarrow$ Running coupling

$$\kappa_T = \frac{\kappa_0}{1 + \kappa_0 \ln\left(\frac{\Lambda}{2\pi T}\right)}$$

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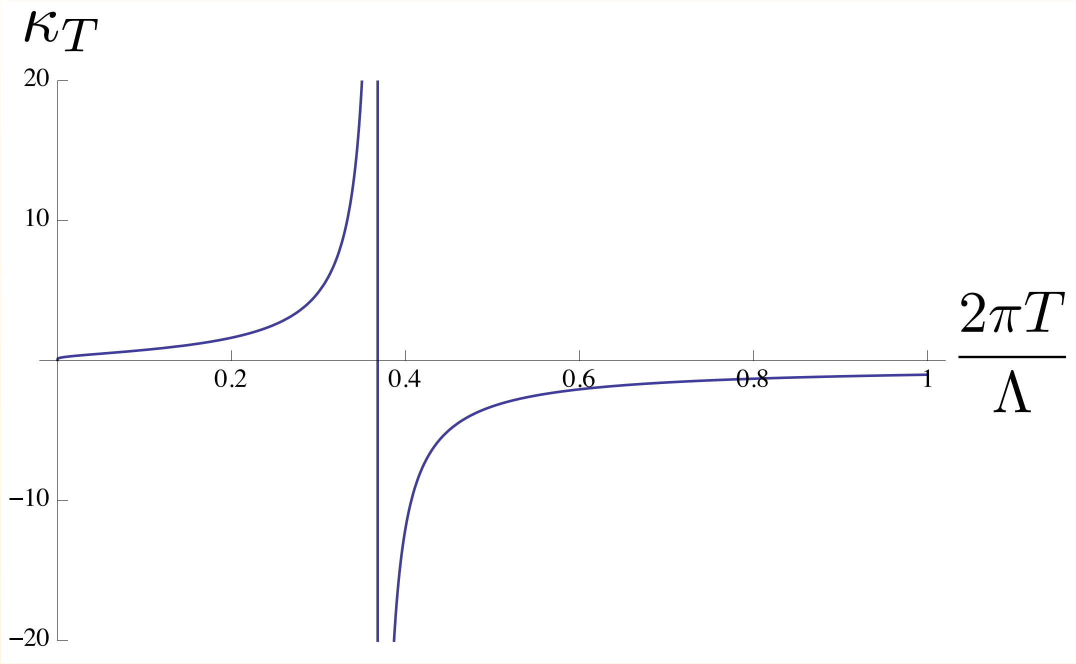
Dynamical scale generation

Scale generation

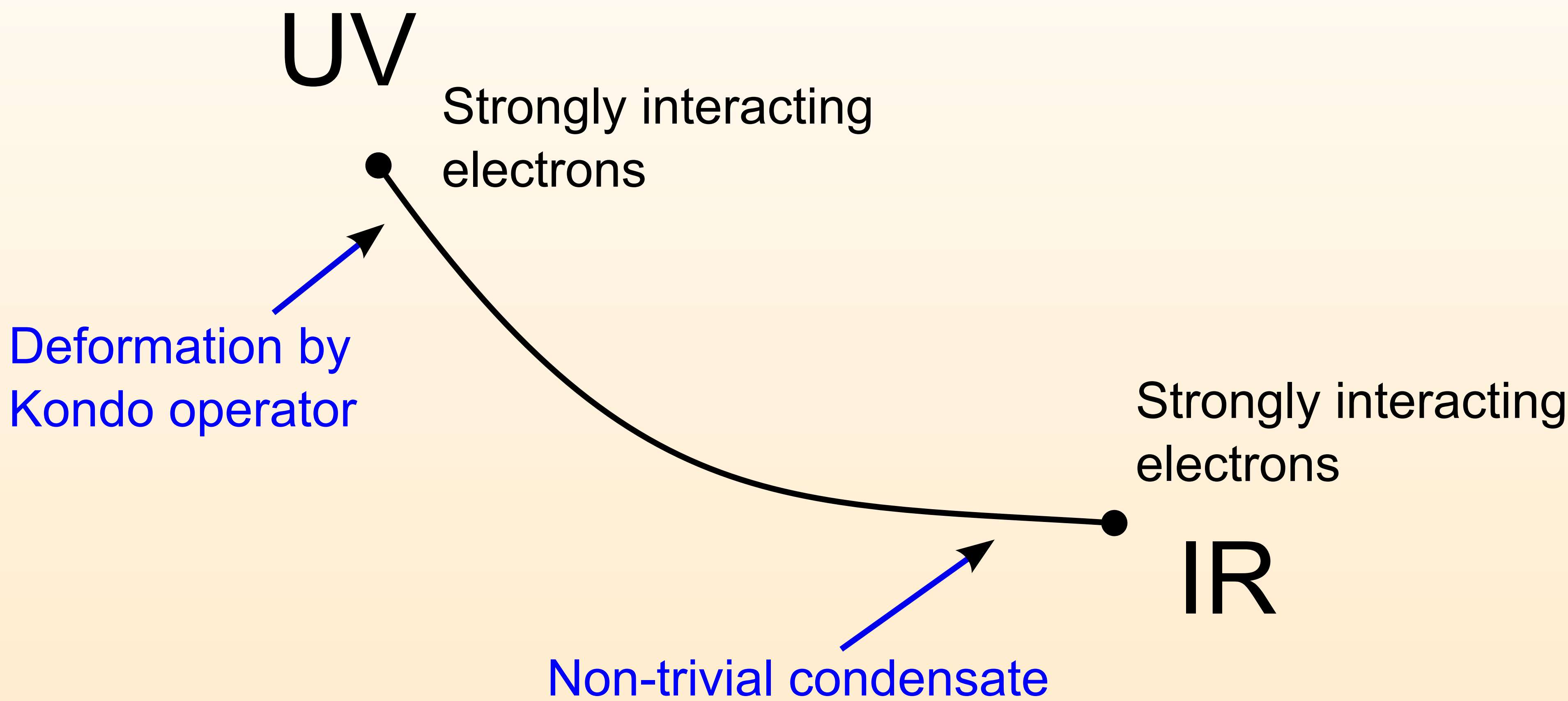


Divergence of Kondo coupling determines Kondo temperature T_K

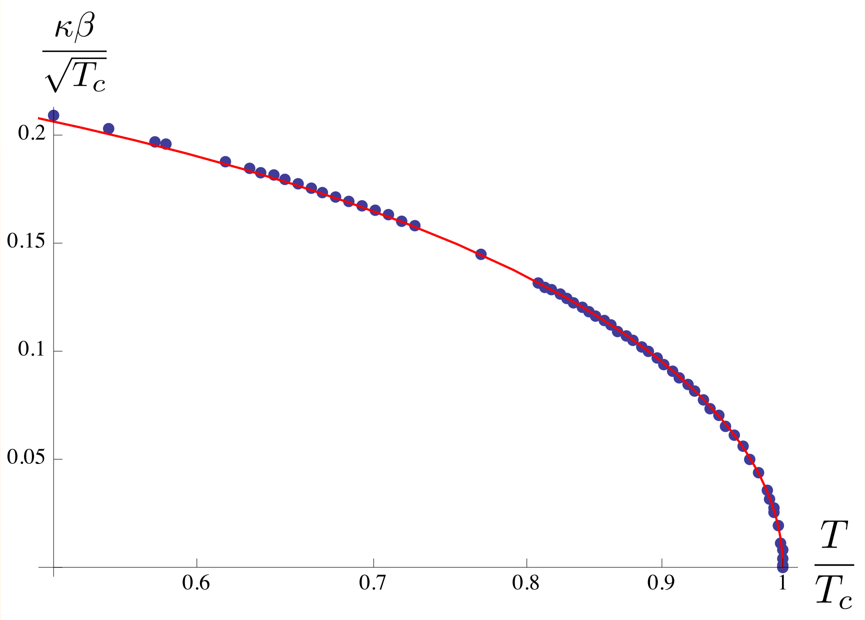
Transition temperature to phase with condensed scalar: T_c

$T_c < T_K$

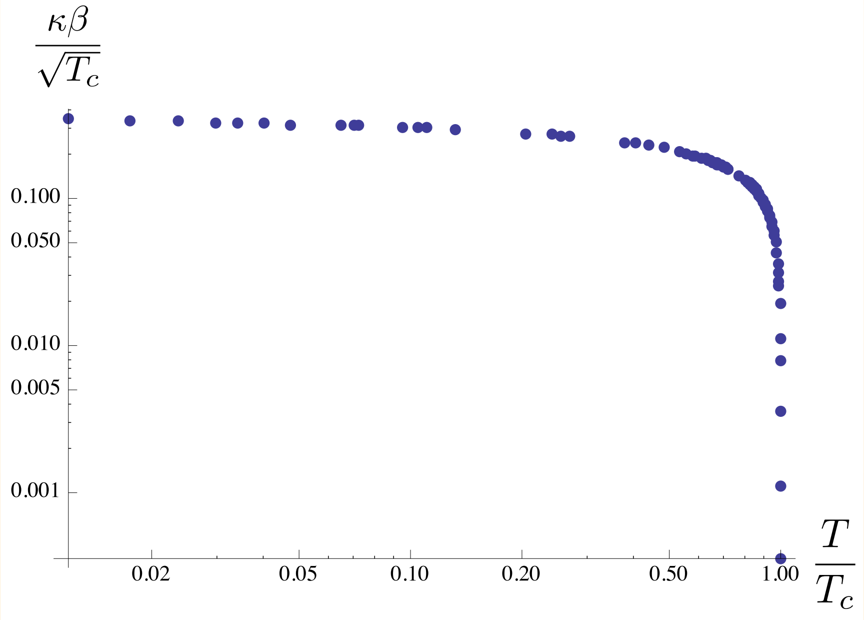
RG flow


UV
Strongly interacting electrons
Deformation by Kondo operator
Strongly interacting electrons
IR
Non-trivial condensate

Normalized condensate $\langle \mathcal{O} \rangle \equiv \kappa\beta$ as function of the temperature



(a)

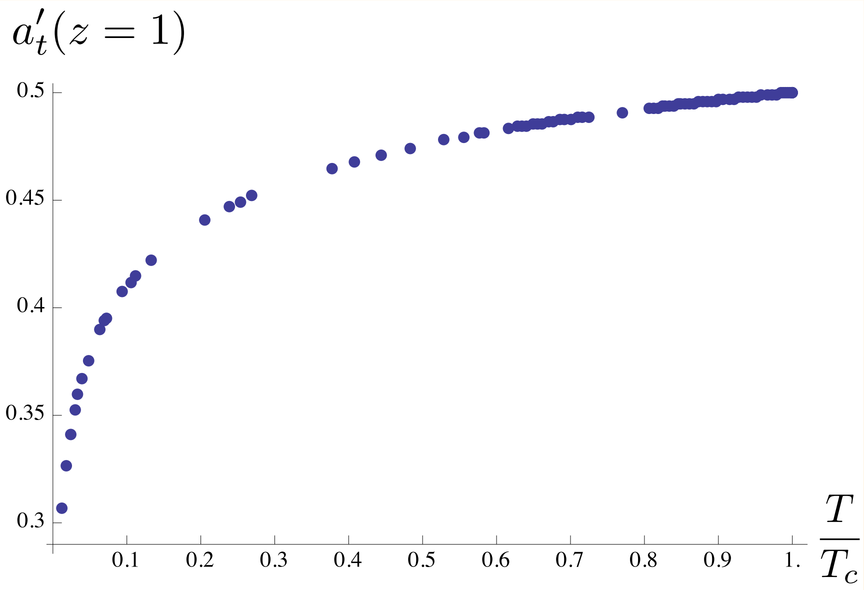


(b)

Mean field transition

$\langle \mathcal{O} \rangle$ approaches constant for $T \to 0$

Electric flux at horizon



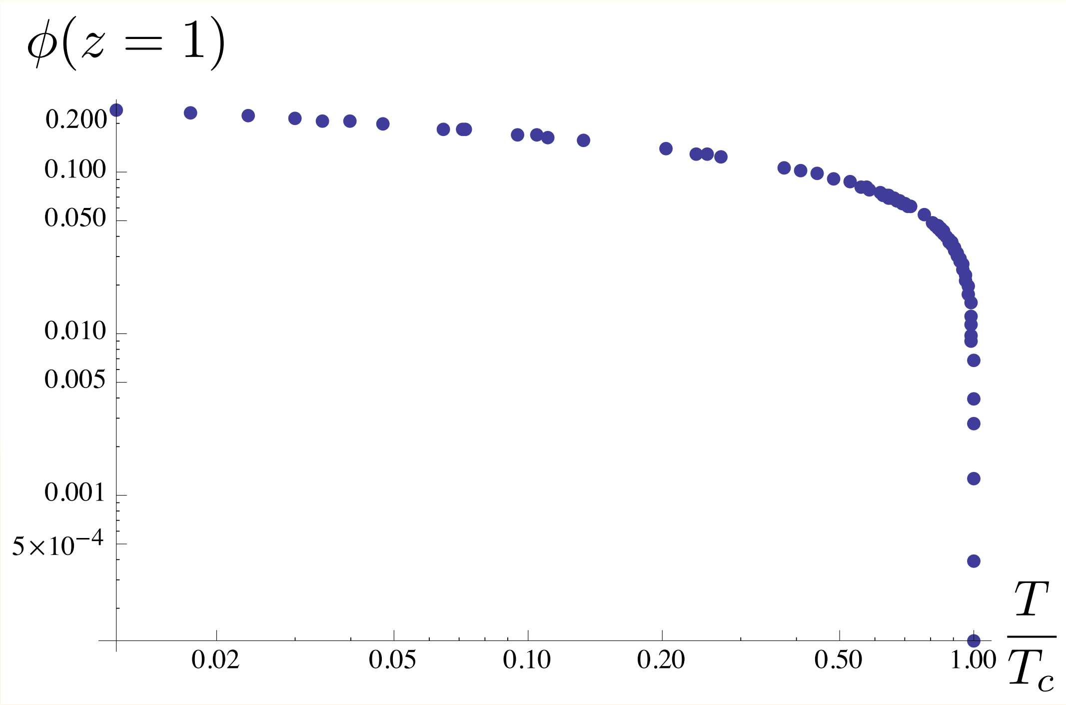
(a)

$$\sqrt{-g}f^{tr}\Big|_{\partial AdS_2} = q,$$ charge density $q = Q/N$

Impurity is screened

Resistivity obtained from leading irrelevant operator

(No logarithmic behaviour due to the large N limit)



Dimension: $\Delta = 1/2 + \sqrt{1/4 + 2\phi_\infty^2} = 1.07$

Entropy density: $s = s_0 + c_s \lambda_{\mathcal{O}} T^{\Delta-1}$

Resistivity: $\rho = \rho_0 + c_\rho \lambda_{\mathcal{O}} T^{\Delta}$

Time dependence

Allow for time dependence of the Kondo coupling and study response of the condensate

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Examples for time dependence of the Kondo coupling:

- Gaussian pulse in IR
- Quench from condensed to normal phase (IR to UV)
- Quench from normal to condensed phase (UV to IR)

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Examples for time dependence of the Kondo coupling:

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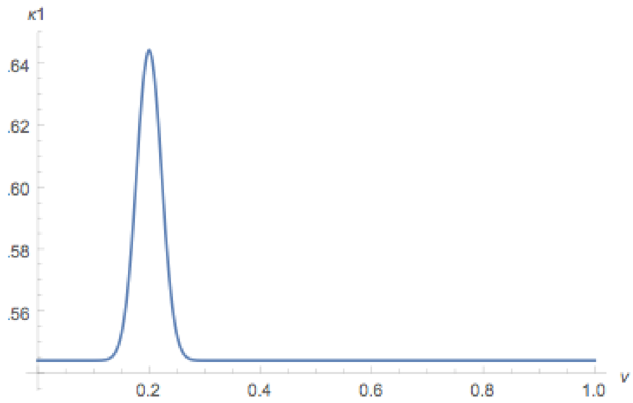
Observations:

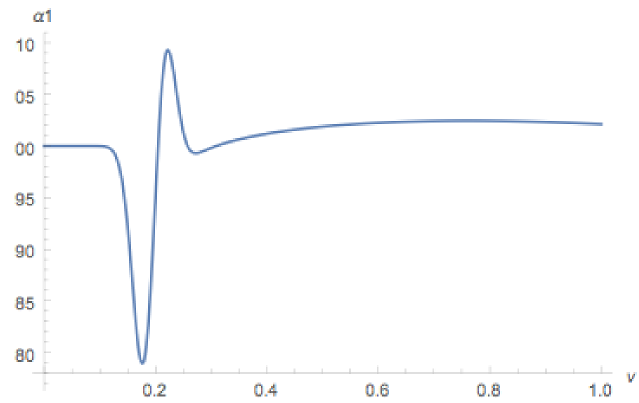
Different timescales depending on whether the condensate is asymptotically small or large

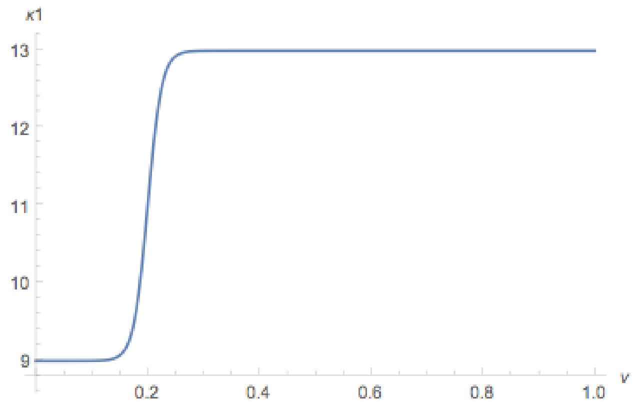
Anderson orthogonality catastrophe? $\tau \sim 1/\langle \mathrm{initial}|\mathrm{final}\rangle$

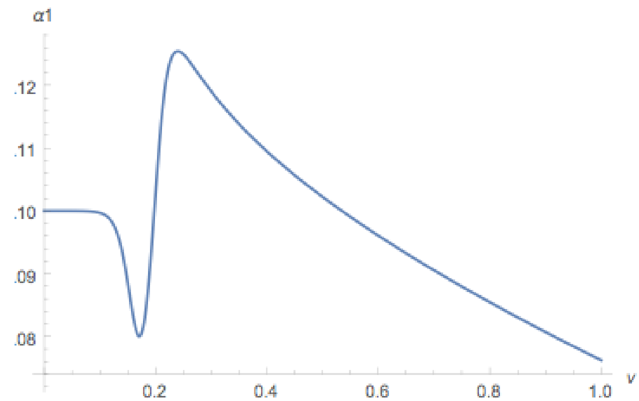
Time dependence

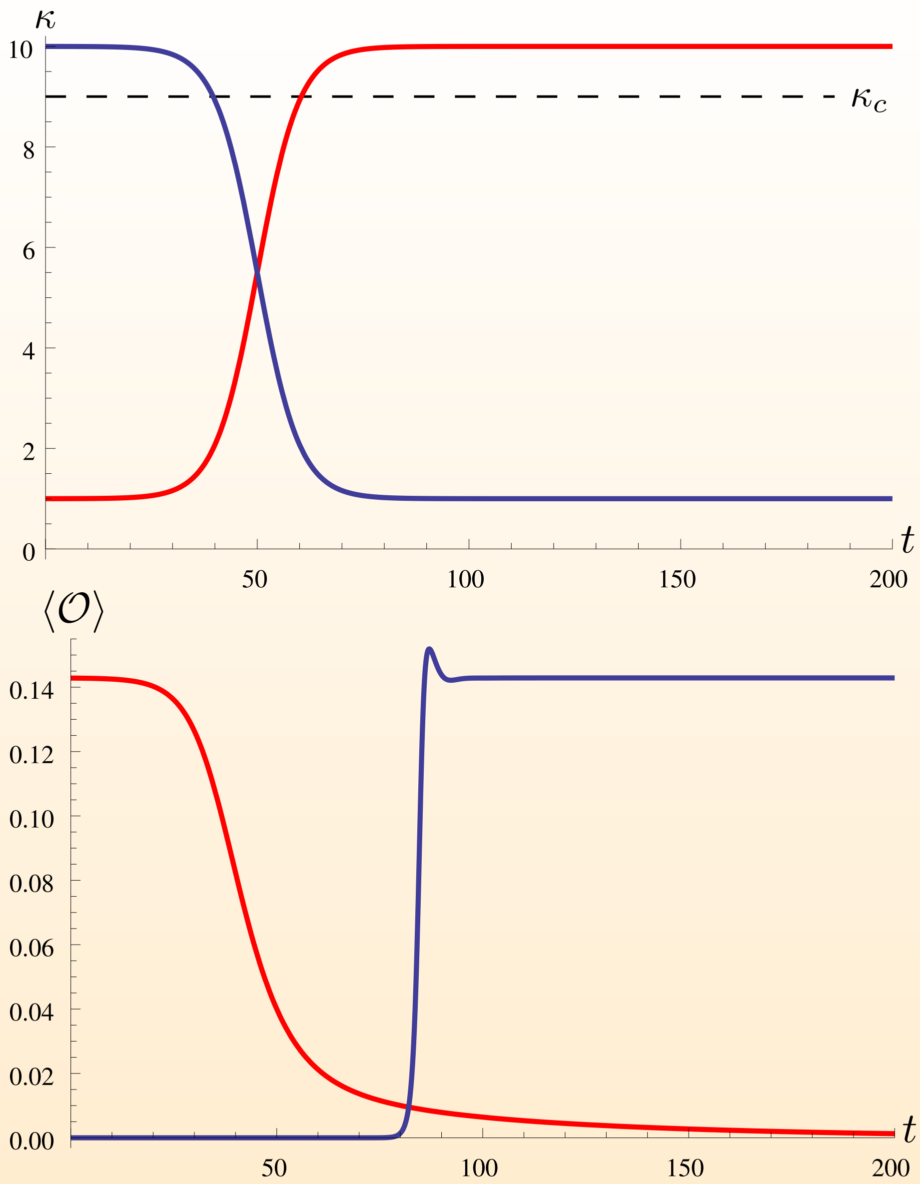
Kondo coupling → Condensate











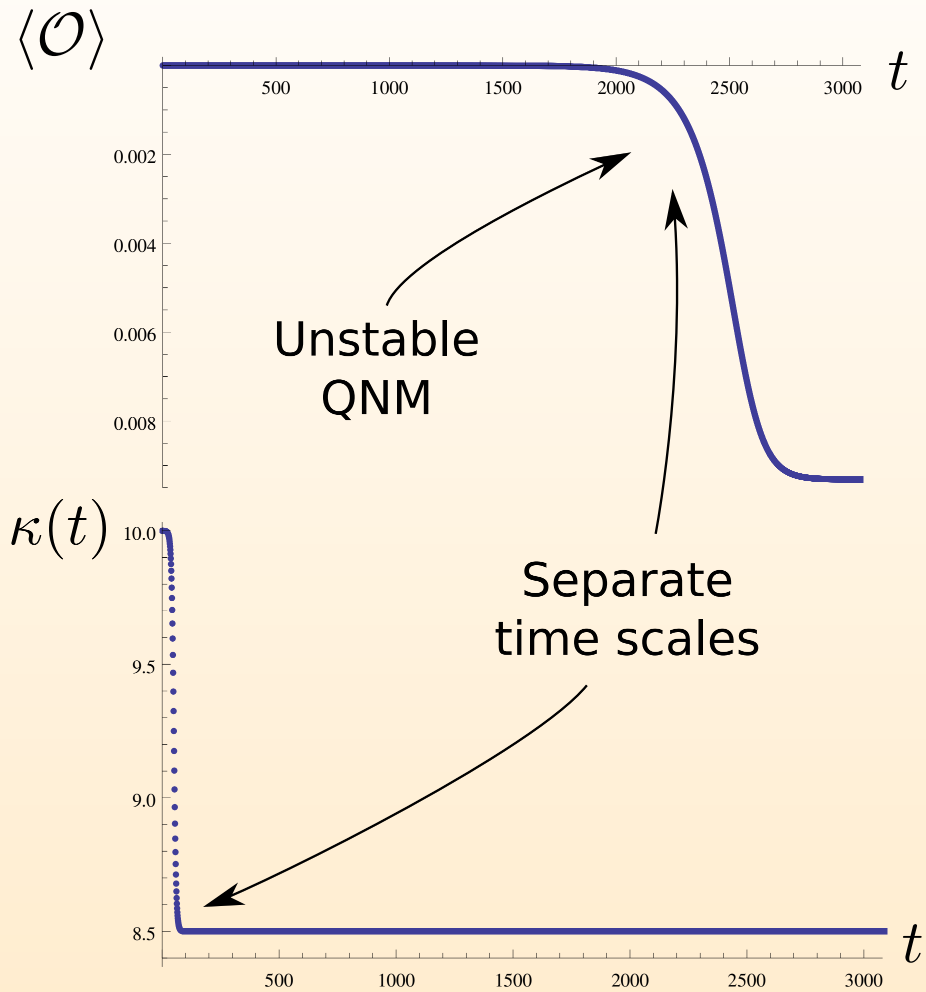
Quantum quenches in holographic Kondo model

To and from condensed phase

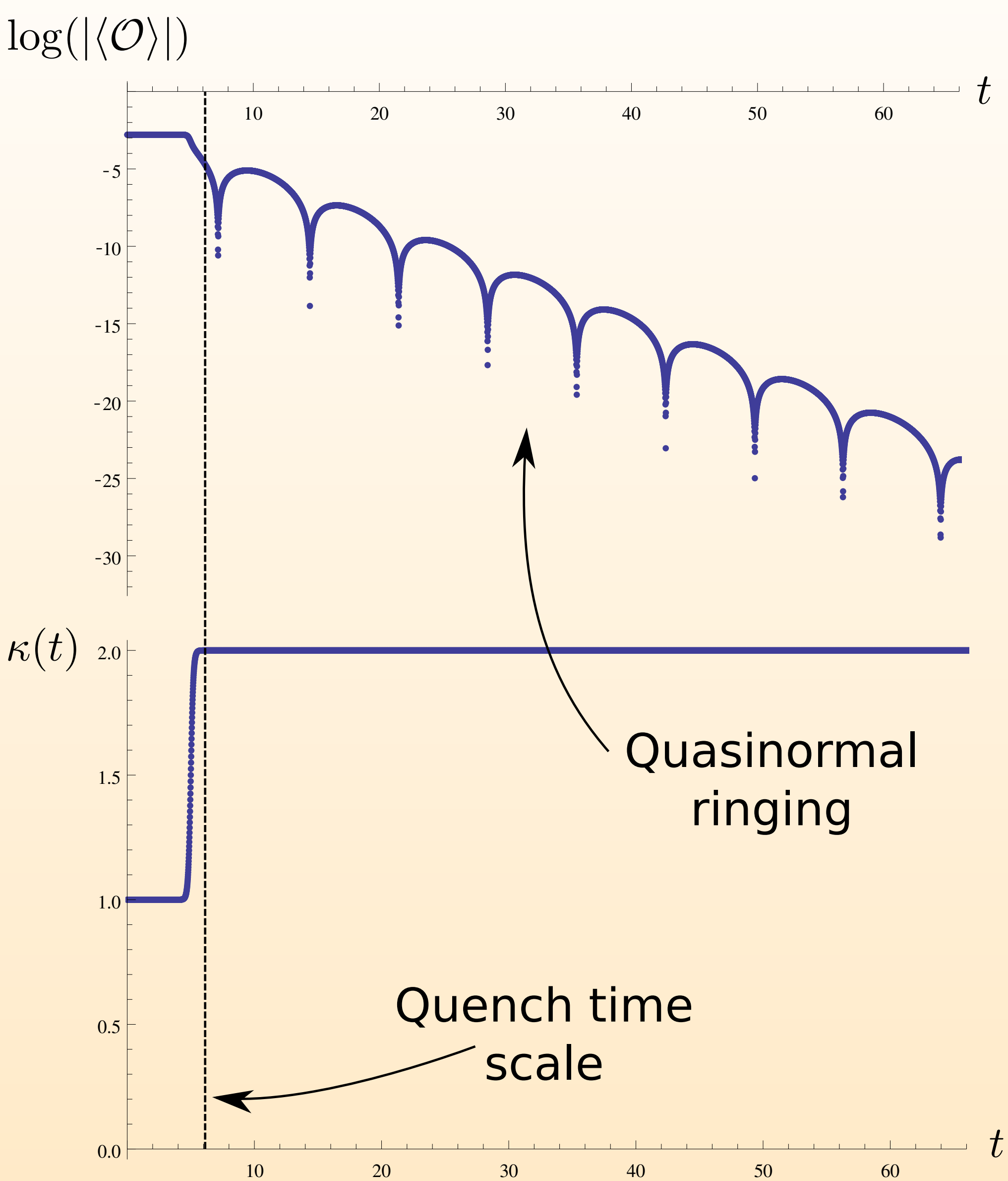
Timescales determined by quasinormal modes

J.E., Flory, Newrzella, Strydom, Wu

Timescales in quantum quench

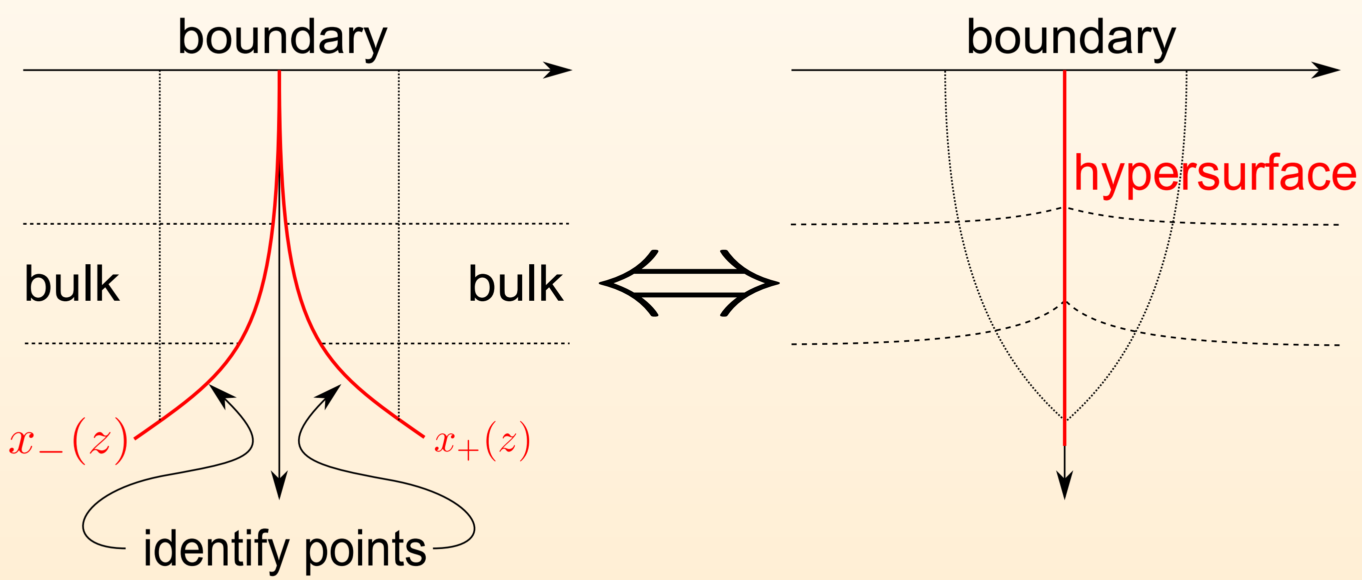


Timescales in quantum quench



Including the backreaction using a thin brane and Israel junction conditions

Israel junction conditions $K_{\mu\nu} - \gamma_{\mu\nu} K = -\frac{\kappa}{2} T_{\mu\nu} \Leftrightarrow$ Energy conditions

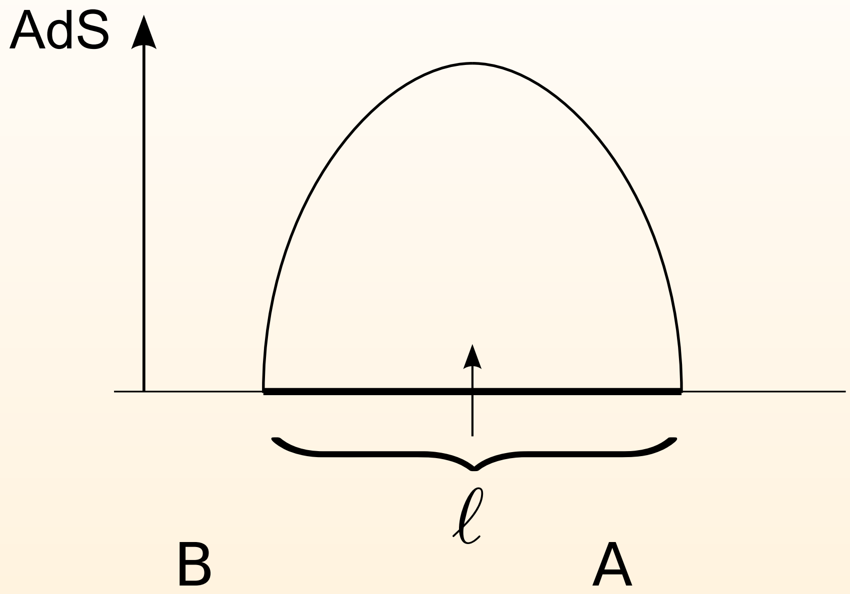


J.E., Flory, Newrzella 1410.7811

In extension of previous work on holographic BCFT

Takayanagi; Fujita, Takayanagi, Tonni 2011; Nozaki, Takayanagi, Ugajin 2012

Entanglement entropy for magnetic impurity



Impurity entropy:

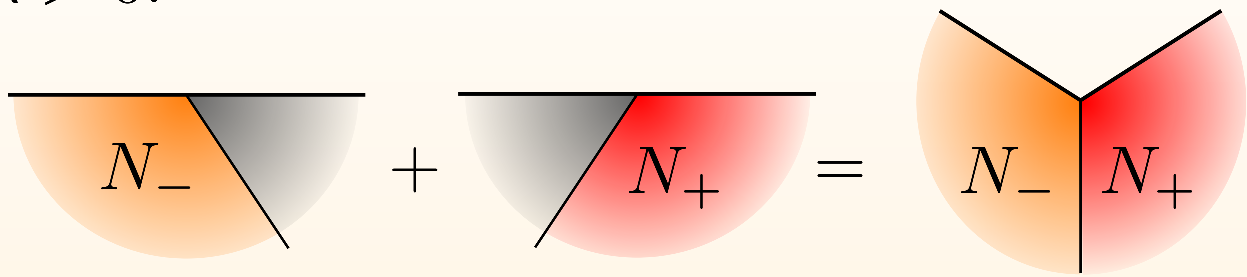
$$S_{\mathrm{imp}} = S_{\mathrm{condensed\,phase}} - S_{\mathrm{normal\,phase}}$$

Subtraction also guarantees UV regularity

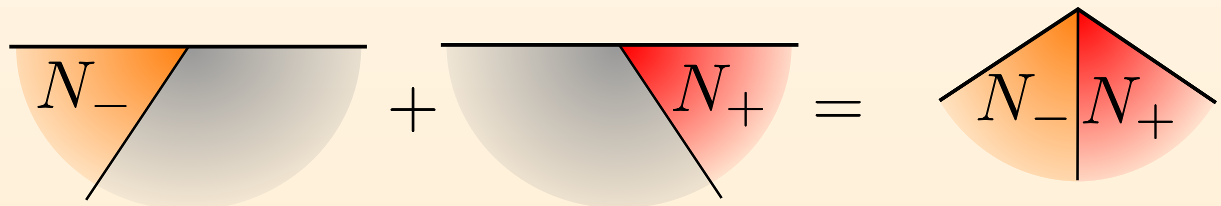
Entanglement entropy for magnetic impurity

J.E., Flory, Newrzella 1410.7811

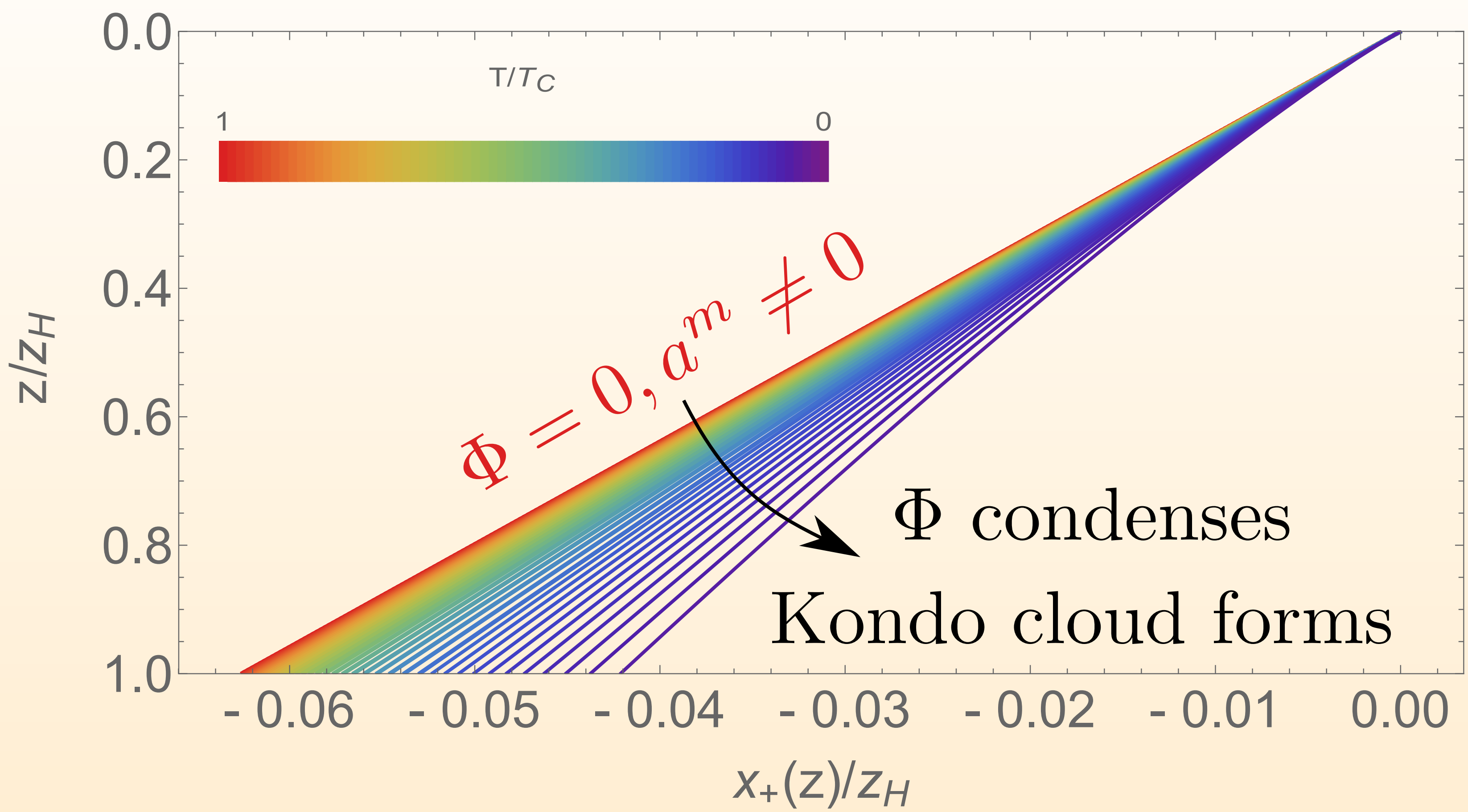
$\lambda > 0$:



$\lambda < 0$:

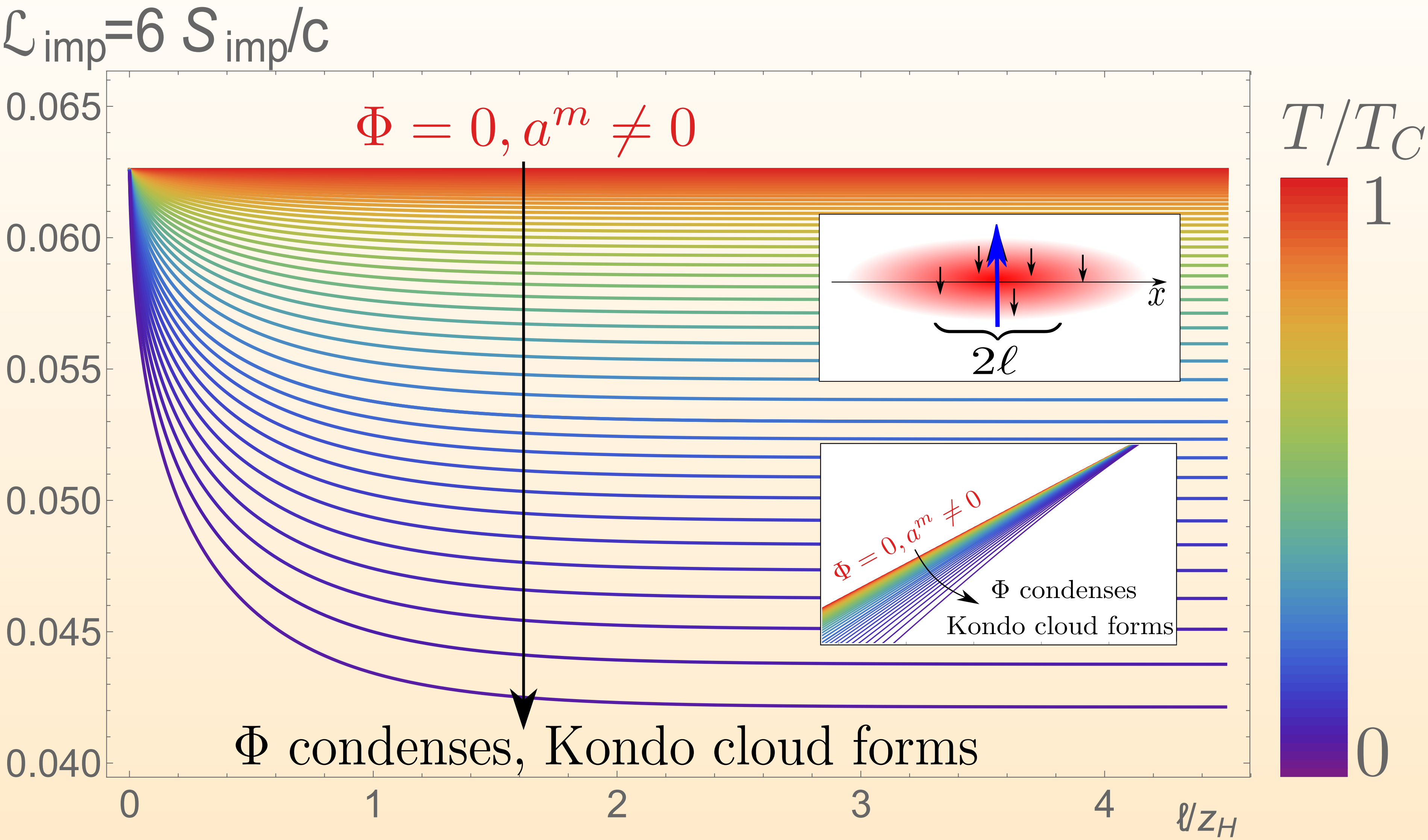


Depending on the brane tension λ, the total space is enhanced or reduced



The larger the condensate, the shorter the geodesic

Impurity entropy from gauge/gravity duality



Entanglement entropy for magnetic impurity: Comparison to field theory

Field theory result

Sorensen, Chang, Laflorencie, Affleck 2007
(Eriksson, Johannesson 2011)

$$\Delta S_{\mathrm{imp}}(\ell) = \frac{\pi^2 \xi_K T}{6} \coth(2\pi \ell T) + C_0$$

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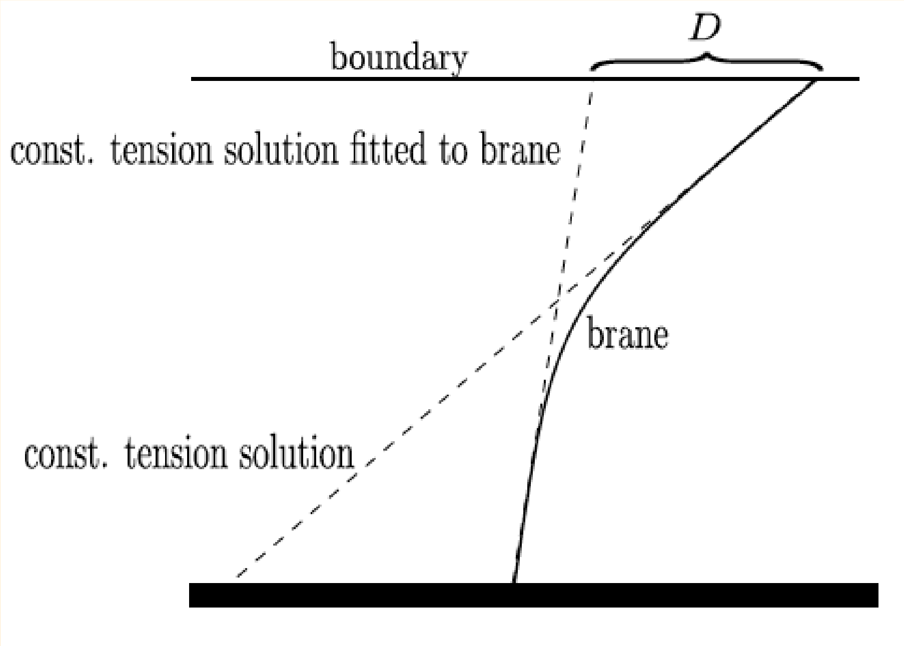
In our gravity approach:



Entanglement entropy for magnetic impurity: Comparison to field theory

On gravity side:

Impurity entropy from difference of entanglement entropies for constant tension branes

$$\Delta S_{\mathrm{imp}}(\ell) = c_0 + S_{BH}(\ell + D) - S_{BH}(\ell)$$

$$S_{BH}(\ell) = \frac{c}{3} \ln \left(\frac{1}{\pi \epsilon T} \sinh(2\pi \ell T) \right)$$

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For $D \ll \ell$:

$$\Delta S_{\text{imp}}(\ell) \sim c_0 + D \cdot \partial_\ell S_{BH}(\ell) = c_0 + \frac{2\pi D T}{3} \coth(2\pi \ell T)$$

Agrees with field theory result subject to identification $D \sim \xi_K$

Universal properties of superconductors

Universality: IR fixed point determines physical properties
Macroscopic properties do not depend on microscopic degrees of freedom

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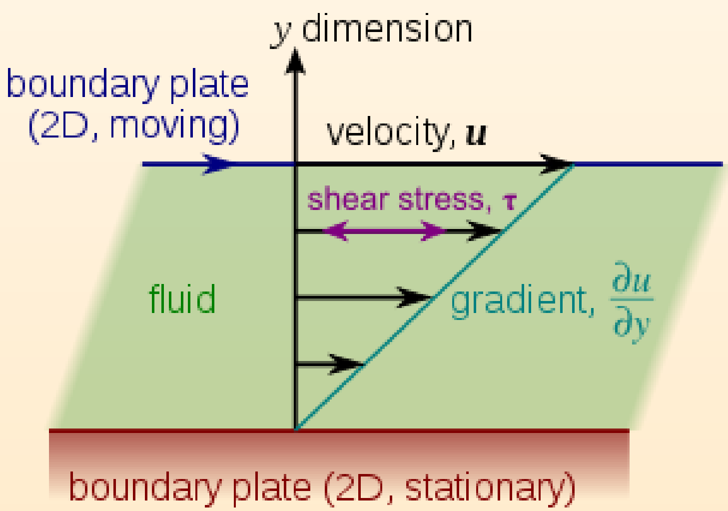
Macroscopic properties do not depend on microscopic degrees of freedom

Example: Universal result from gauge/gravity duality:

Shear viscosity over entropy density:

$$\frac{\eta}{s} = \frac{1}{4\pi}\frac{\hbar}{k_B}$$



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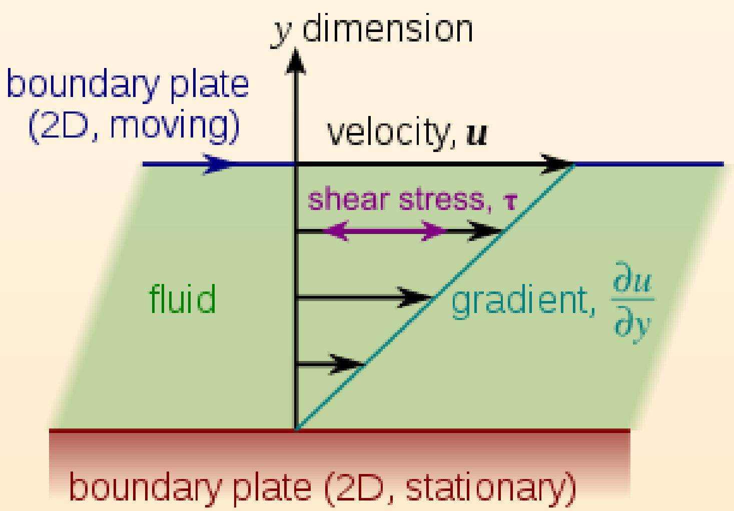


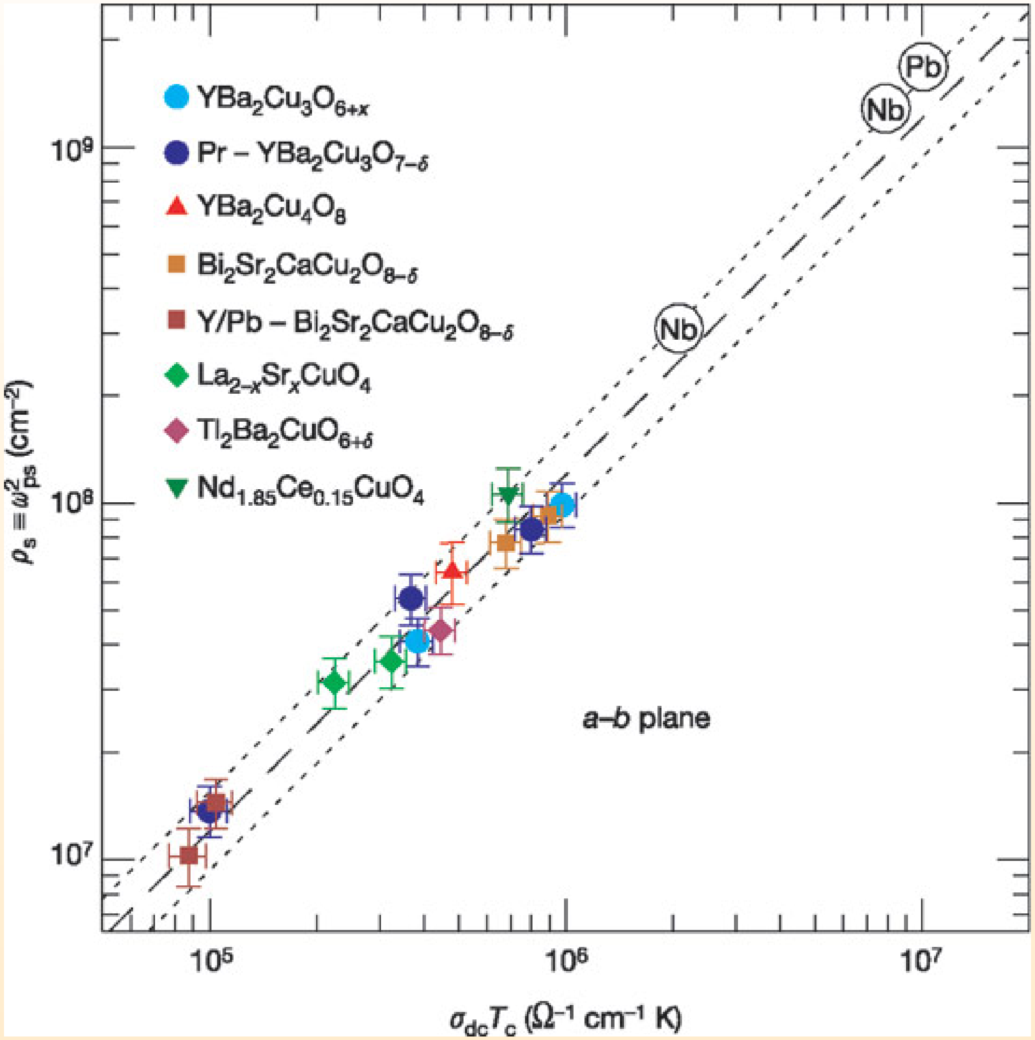
Planckian dissipator: relaxation time $\tau = \frac{\hbar}{k_B T}$

Damle, Sachdev 1997

Universal properties of superconductors

Is there a similiar universal result for applications of the duality within condensed matter physics?

Candidate: Homes' relation
$\rho_s(T=0) = C\,\sigma_{\rm DC}(T_c)\,T_c$



C. Homes et al, Nature 2004

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Homes' relation $\rho_s(T = 0) \, = \, C \, \sigma_{\rm DC} \, T_c$

general form may be deduced from Planckian dissipation Zaanen 2004

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J.E., Herwerth, Klug, Meyer, Schalm arXiv:1501.07615:

Investigation of C in a family of gauge/gravity duality models

In particular region of parameter space:

$$C \approx 6.2$$

BCS superconductor in 'dirty limit': $C = 8.1$,
High-T_c superconductors: $C = 4.4$

Holography: J.E., Kerner Müller 2012

Conditions for identifying ρ_s:

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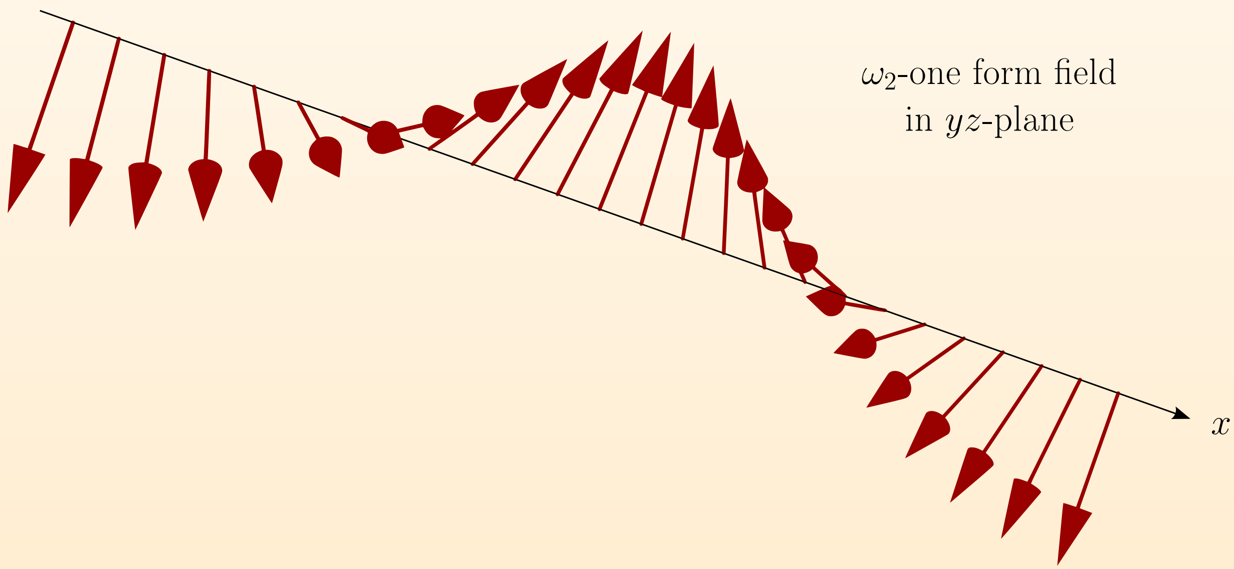
Horowitz, Santos 2013

Use background with helical symmetry

Background: Helical Bianchi VII symmetry

Donos, Gauntlett 2011; Donos, Hartnoll 2012

Model with broken translation symmetry:



Gauge/gravity duality with helical symmetry

Background: (Hartnoll, Donos)

$$S_{\text{helix}} = \int \mathrm{d}^{4+1}x\,\sqrt{-g}\left[R + 12 - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{4}W^{\mu\nu}W_{\mu\nu} - m^2 B_\mu B^\mu\right] - \frac{\kappa}{2}\int B\wedge F\wedge W.$$

$$B = w(r)\omega_2, \qquad w(\infty) = \lambda$$

$$\omega_1 = \mathrm{d}x\,,$$
$$\omega_2 = \cos(px)\,\mathrm{d}y - \sin(px)\,\mathrm{d}z$$
$$\omega_3 = \sin(px)\,\mathrm{d}y + \cos(px)\,\mathrm{d}z$$

S-wave superconductivity in helical symmetry background

S-wave superconductivity in helical symmetry background

Add charged scalar:

$$S_{\text{total}} = S_{\text{helix}} + \int \mathrm{d}^{4+1}x\, \sqrt{-g}\left[-\left|\partial\rho - \mathrm{i}qA\rho\right|^2 - m_\rho^2|\rho|^2 \right]$$

S-wave superconductivity in helical symmetry background

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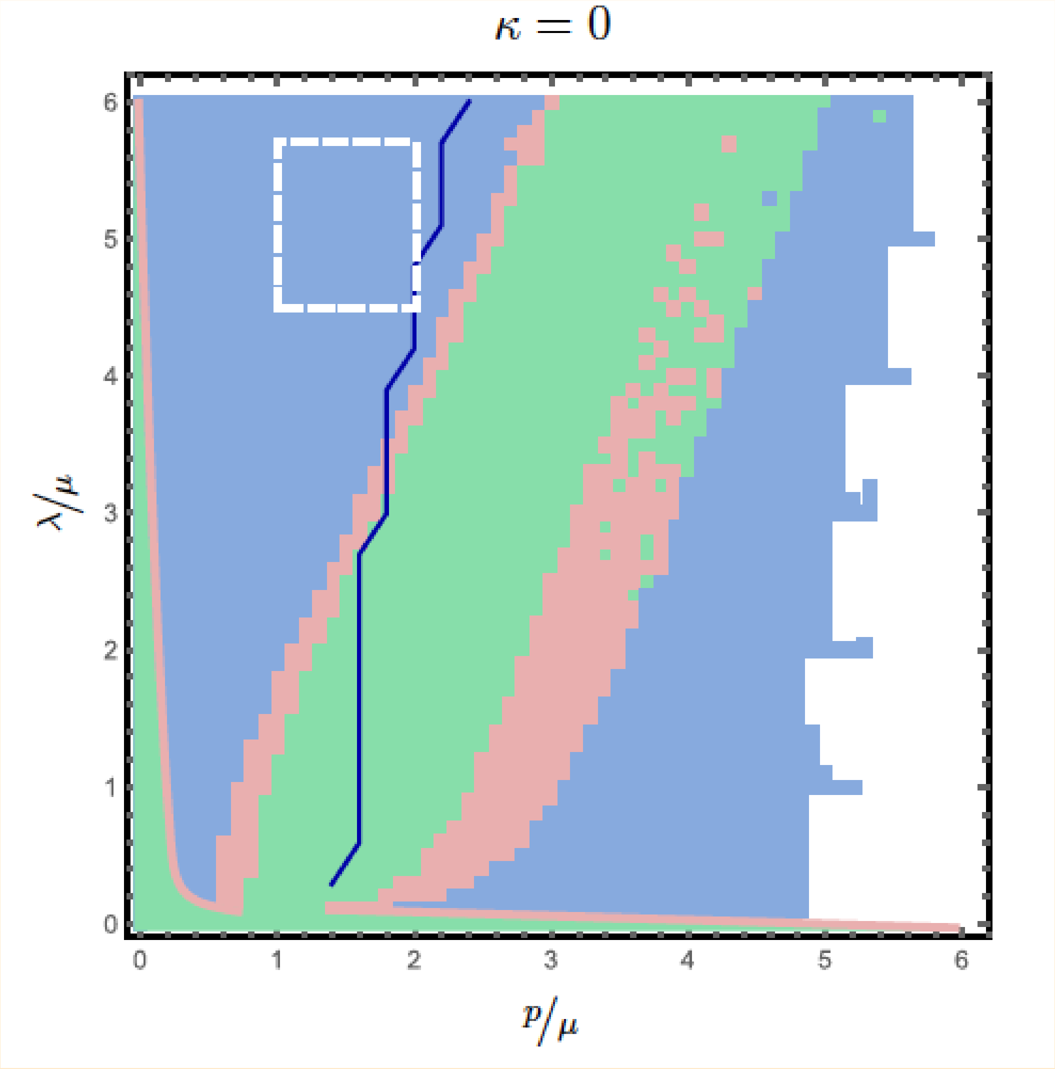
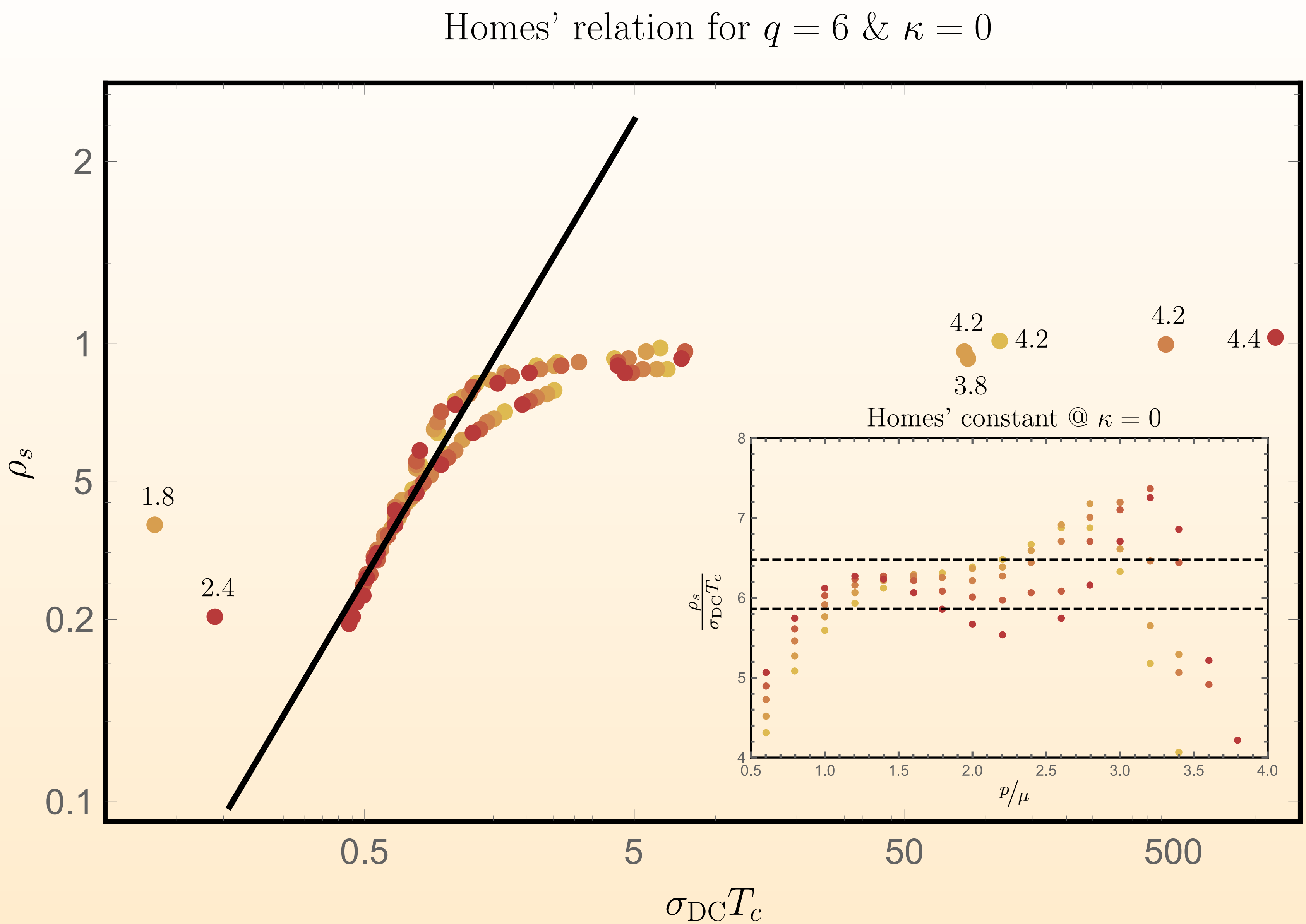
$$S_{\text{total}} = S_{\text{helix}} + \int \mathrm{d}^{4+1}x \sqrt{-g} \Big[- |\partial\rho - \mathrm{i}qA\rho|^2 - m_\rho^2 |\rho|^2 \Big]$$

All charged degrees of freedom condense at $T = 0$

Phase diagram





J.E., Herwerth, Klug, Meyer, Schalm 1501.07615

Conclusions and outlook

- Kondo model:
- Magnetic impurity coupled to strongly coupled system
- Quantum quench
- Entanglement entropy
- Outlook: Two-point and spectral functions
- S-wave superconductor in Bianchi VII background:
- Homes' Relation

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