# Spin Ice in d=2 and 3:
# Magnetic monopoles and dipolar nanoarrays

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## Outline

- Spin ice in a nutshell

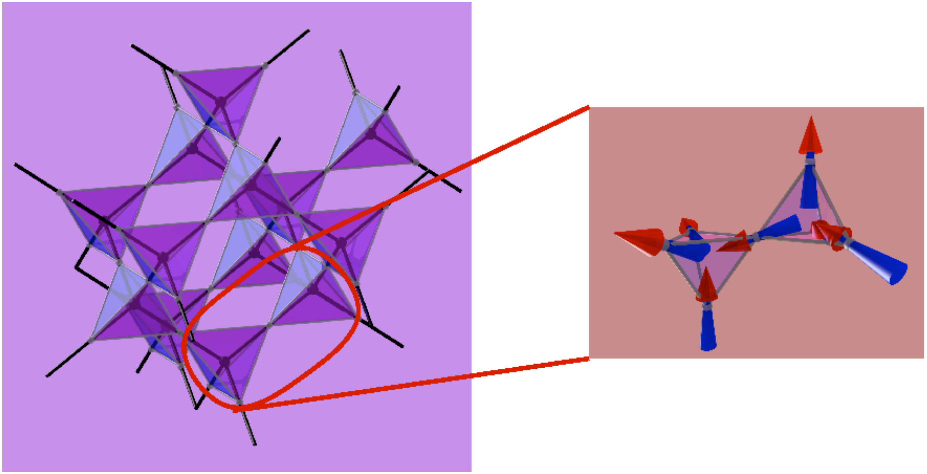
- A useful perspective: decomposing dipoles into "dumbells"

- Dipole fractionalisation and the emergence of magnetic monopoles

- Monopole physics: From induced liquid-gas phase transitions to the Stanford monopole experiment

- Artificial spin ice: dipolar nanoarrays

## The Structure of Spin Ice



- pyrochlore lattice of rare earth atoms ($Dy_2Ti_2O_7$ with spin $S = 15/2$, and $Ho_2Ti_2O_7$ with spin $S = 8$)

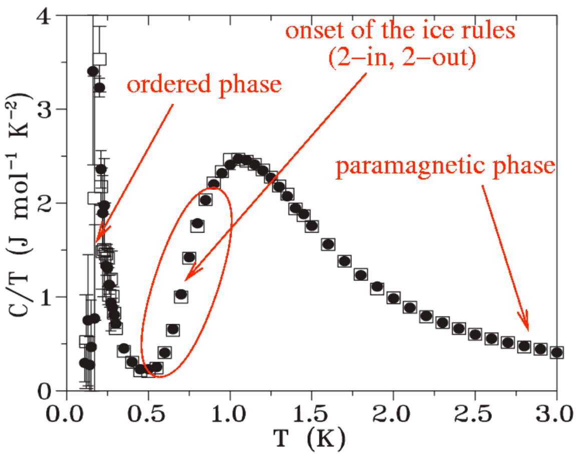
- crystal fied anysotropy along the local [111] axis $\Delta_{\mathrm{CEF}} \sim 200$ K $\longrightarrow$ classical Ising spins below $T \sim 10$ K

## The Phase Diagram [Melko, Gingras, J. Phys.: C. M. 16, R1277 (2004)]

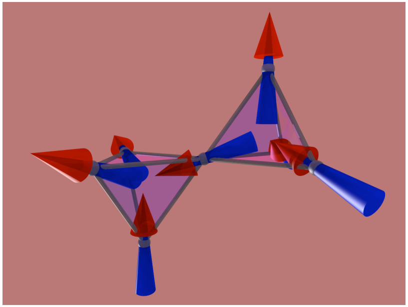
Exchange and dipolar interactions

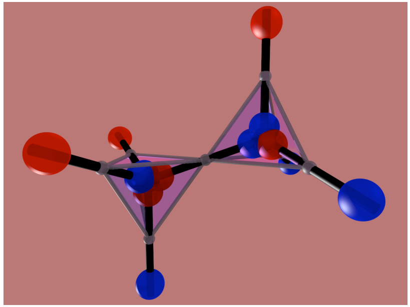
$$H = \frac{J}{3}\sum_{\langle ij\rangle} S_i S_j + Da^3 \sum_{(ij)} \left[ \frac{\hat{e}_i \cdot \hat{e}_j}{|\mathbf{r}_{ij}|^3} - \frac{3\left(\hat{e}_i \cdot \mathbf{r}_{ij}\right)\left(\hat{e}_j \cdot \mathbf{r}_{ij}\right)}{|\mathbf{r}_{ij}|^5} \right] S_i S_j$$

where $S_i = \pm 1$ are the normalized Ising spins, $\hat{e}_i$ is the unit vector in the local [111] direction, and $J \sim 1 - 2$ K and $D = 1.41$ K.



# The Dumbell Picture





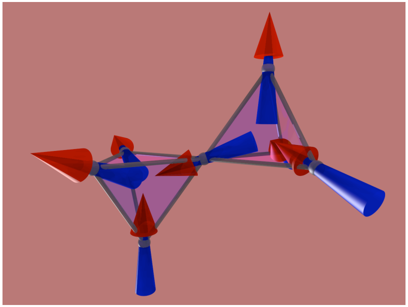
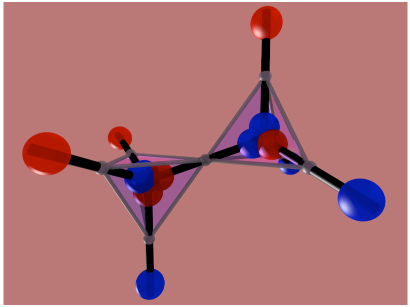
- replace each dipole $\vec{d}$ by two equal and opposite magnetic charges $\pm q$ separated by the bond length $a$ ($q = d/a$)

- renormalize the onsite Colomb interaction so as to give the correct nearest-neighbour interaction between dipoles:

$$v(r_{ij}) = \begin{cases} \frac{\mu_0}{4\pi}\frac{q_i q_j}{r_{ij}} & i \neq j \\ v_o(\frac{\mu}{a})^2 = \frac{J}{3} + 4\frac{D}{3}(1+\sqrt{\frac{2}{3}}) & i = j, \end{cases}$$

- Thanks to projective equivalence, this dumbell model reproduces the energies of spin-ice configurations quantitatively up to quadrupolar corrections

# The Dumbell Picture

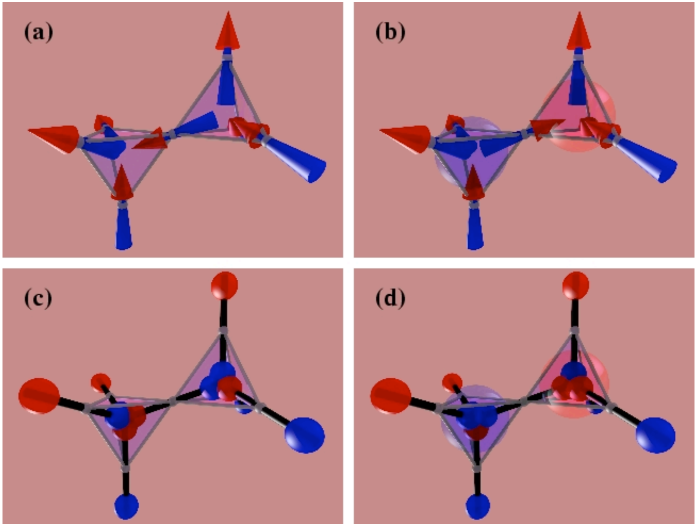
 

- with a simple resummation $q_i \to Q_\alpha = \sum_{i \in \text{tretrahedron } \alpha} q_i$, the energy of a generic dumbell configuration can be rewritten as

$$v(r_{ij}) \longrightarrow V(r_{\alpha\beta}) = \begin{cases} \frac{\mu_0}{4\pi} \frac{Q_\alpha Q_\beta}{r_{\alpha\beta}} & \alpha \neq \beta \\ \frac{1}{2} v_o Q_\alpha^2 & \alpha = \beta \end{cases}$$

$\Longrightarrow$ the 2-in, 2-out ice rules appear naturally: the lowest energy configurations are the ones with $Q_\alpha = 0$ everywhere!

# Low Temperature Defects (i.e., Violations to the Ice Rule)

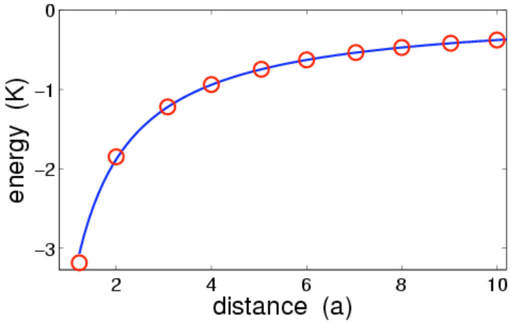


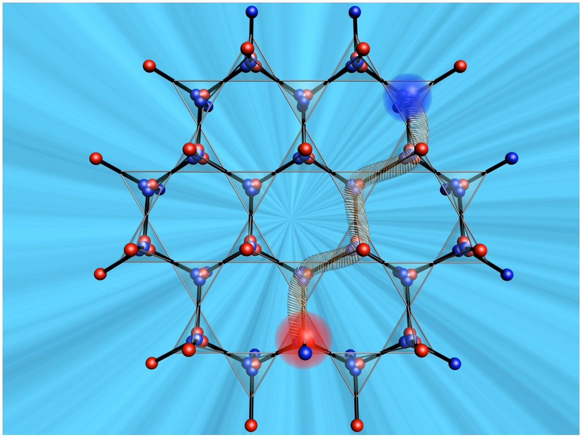

- single spin flip in a spin-ice configuration: naively a dipolar excitation

- two neighbouring opposite charges $Q_\alpha = \pm 2d/a$ in the dumbell picture...

- the two charges can be separated at the expense of a purely Coulomb interaction (not confining in $3D$) $\Rightarrow$ *the dipolar excitation fractionalise into two magnetic monopoles!* (notice the presence of $\mu_0$ instead of $\varepsilon_0$ in the Coulomb interaction)

This can be explicitly checked by numerical evaluation in specific spin-ice configurations



projective equivalence $\Rightarrow$ vanishing string tension!

Intuitive picture: separating the two poles of a magnet, while the string in between is energetically immaterial

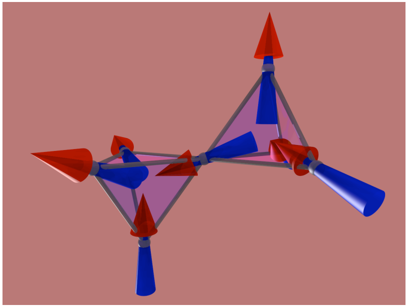
## The Physics of Magnetic Monopoles in Spin Ice

- they are real monopoles (sinks and sources of the magnetic field $\vec{H}$), i.e., they would be felt by independent test charges

- they are classical in nature, in that the Dirac string is energetically immaterial but observable (thus no quantisation condition)

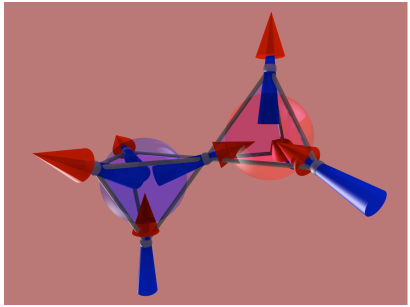
- How can we observe them?
  - *indirectly*, by looking for signatures in spin-ice physics (e.g., ionic liquid behaviour)
  - *directly*, e.g., via scattering or Stanford search type experiments

## A Liquid-Gas Phase Transition in Spin Ice



$$\vec{B}$$

$$\Uparrow$$

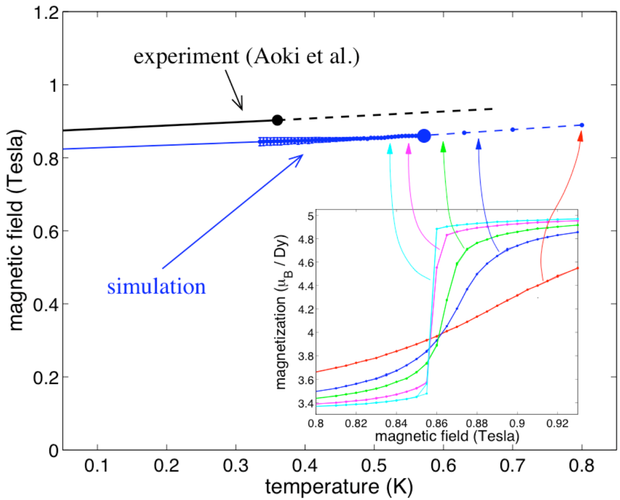


- As the field $B$ is increased, the system attains first a partial ordering known as kagome ice, within the ice rules (Left Panel).

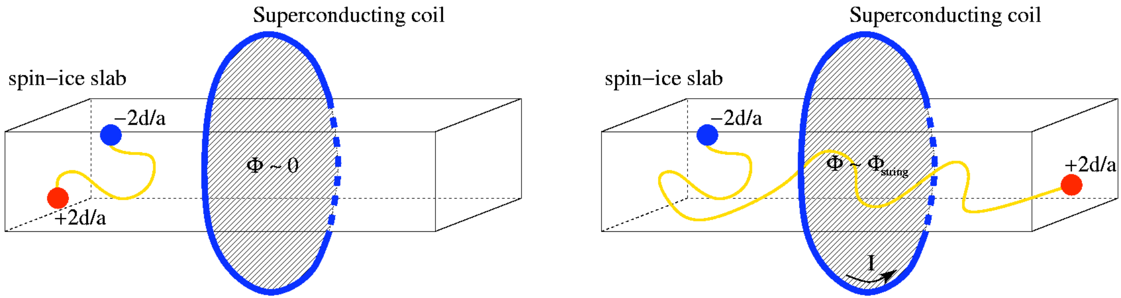
- When the field is strong enough, every spin acquires a positive projection in the direction of the field and we obtain densely packed magnetic monopoles (Right Panel).

- ionic liquid of monopoles as a function of temperature and chemical potential ($B$) $\longrightarrow$ liquid-gas behavior (M. E. Fisher et al.)

- confirmed numerically – observed in actual experiments! (Hiroi, Maeno groups)



# Detection Using a Superconducting Coil




- the passage of a monopole through a supercondicting coil induces a long-lived current, whose strength is proportional to the magnetic charge $q_m = 2d/a$ [Cabrera, PRL 48, 1378 (1982)]

- this current sets up a magnetic flux equal and opposite to the one carried by the dipole string connecting the two monopoles, which is strictly confined within the spin-ice slab

$$\Downarrow$$

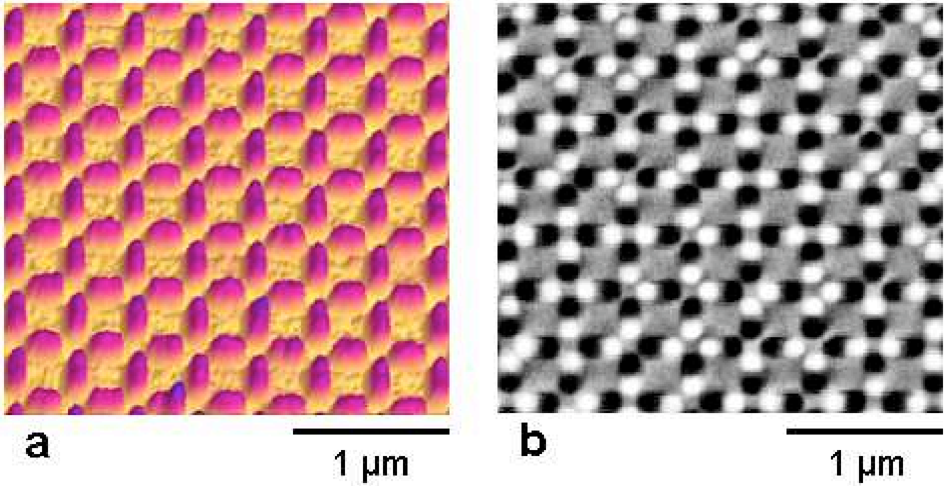
*Spin-ice monopoles can be detected in much the same way as 'ordinary' monopoles would!*

# Conclusions I

- the low-energy excitations in spin ice are classical magnetic monopoles
- these monopoles interact via a magnetic Coulomb interaction $V(r) = \mu_0 Q^2/(4\pi r)$
- they are sources and sinks of the magnetic field $\vec{H}$
- the monopoles live at the end points of observable, yet tensionless Dirac strings, hence they are not quantised
- they can be experimentally observed, either indirectly (e.g., in the recently discovered liquid-gas phase transition) or directly (e.g., monopole-search type experiments)
- other experiments: scattering, transport and noise measurements, flux detection – further ideas welcome
- Monopoles in nanoarrays?

# *Artificial spin ice*


Wang et al. Nature **439**, 303 (2006)

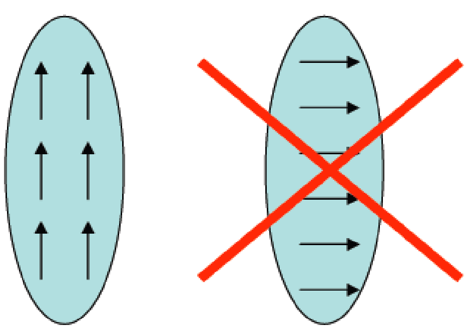


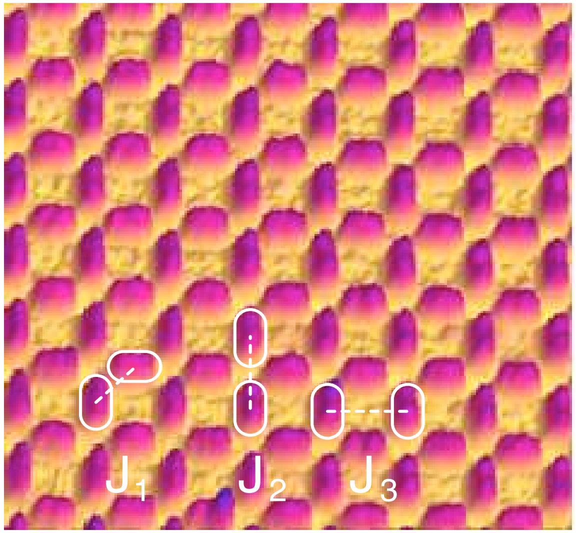

- Frustrated dipolar nanoarray
- any ice regime at low $T$?

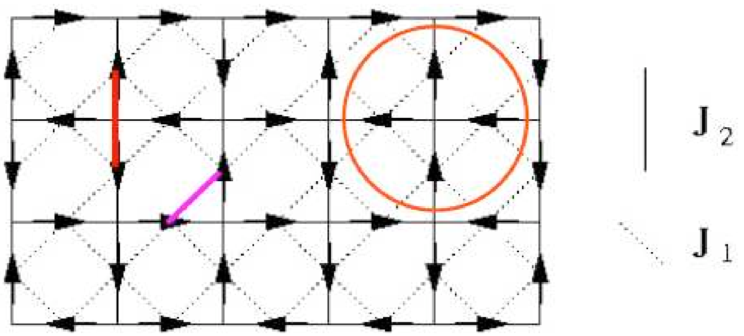
$\Rightarrow$ models for thermodynamics and dynamics

# *The dipolar array*

- permalloy islands with shape anisotropy
- dipolar interactions between islands ⇒ ice model?

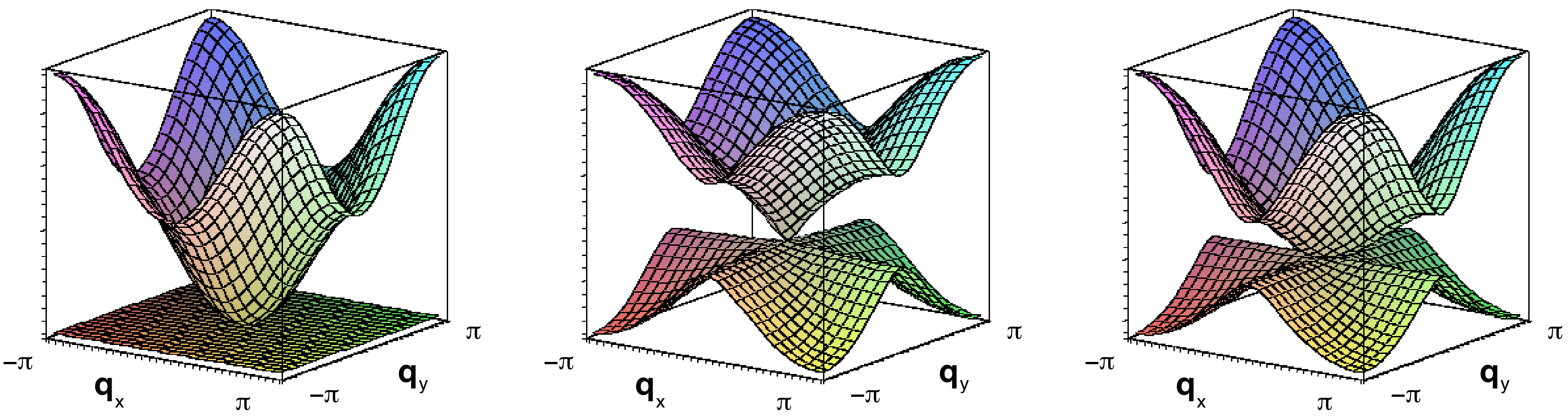




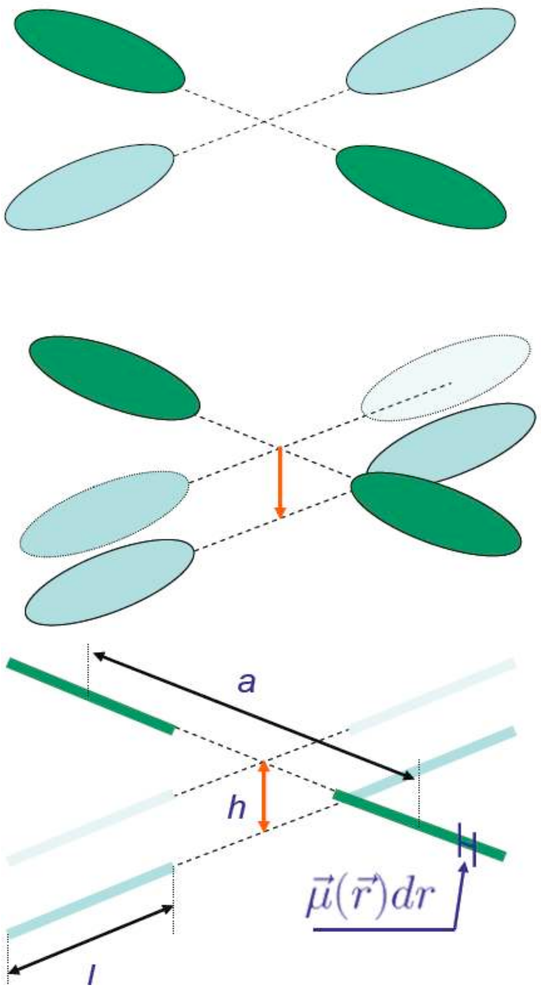
# *Interaction spectrum in mean-field theory*

- ice model has flat band (degeneracy!)
- dipolar interactions closer to F-model
- dipolar model orders at $T \approx 1.7 J_1$

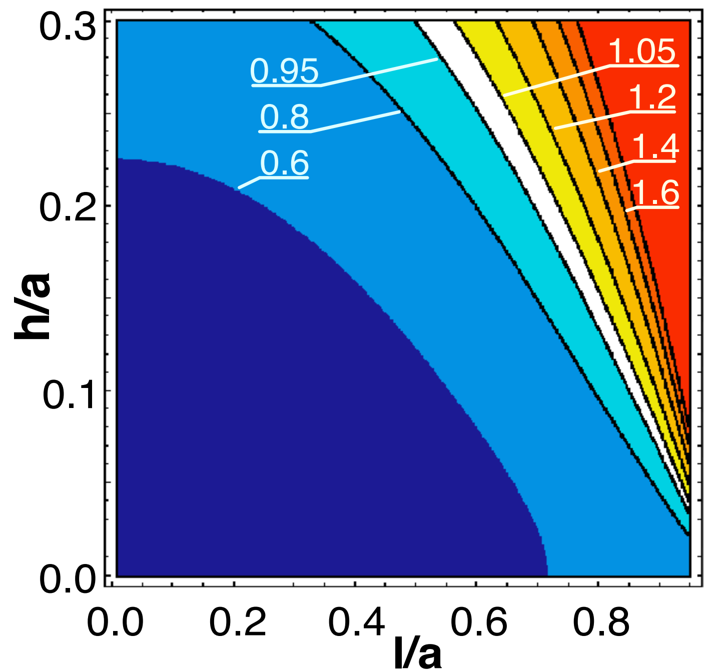


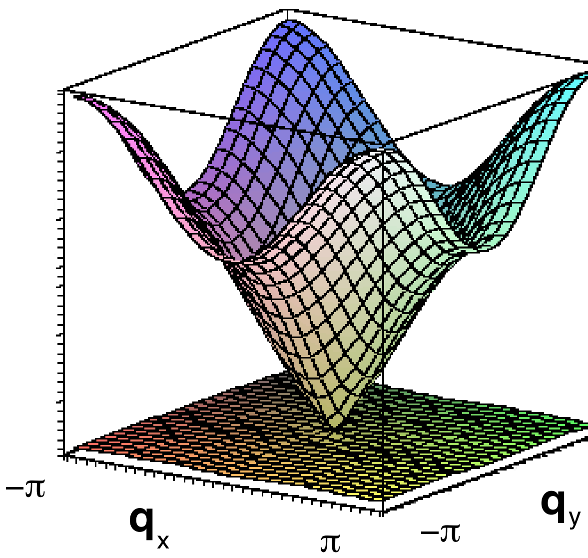
# *Curing $J_1 \neq J_2$: lower dipoles in one direction*

height offset $h$

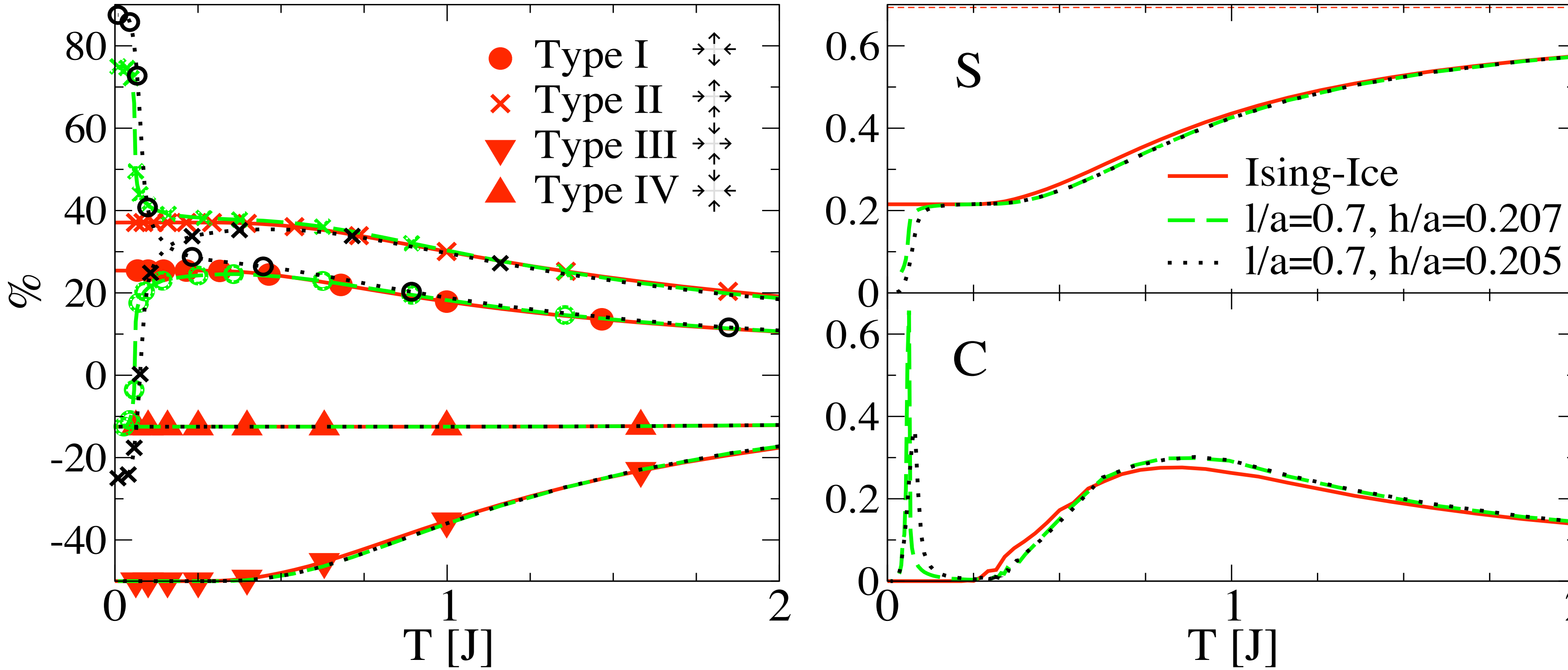



$J_1/J_2 = 1$ delicate ($l/a, h/a$ variable)



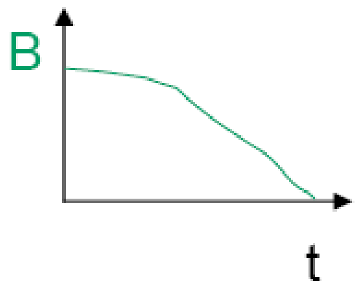
# *Spin ice regime for optimal model*

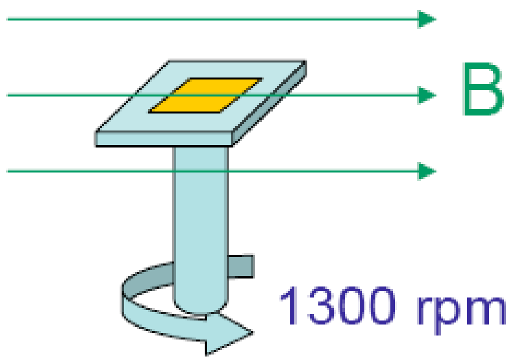



- low-$T$ regime with ice rules satisfied and near-degenerate state exists

# *Analysis of experimental results*

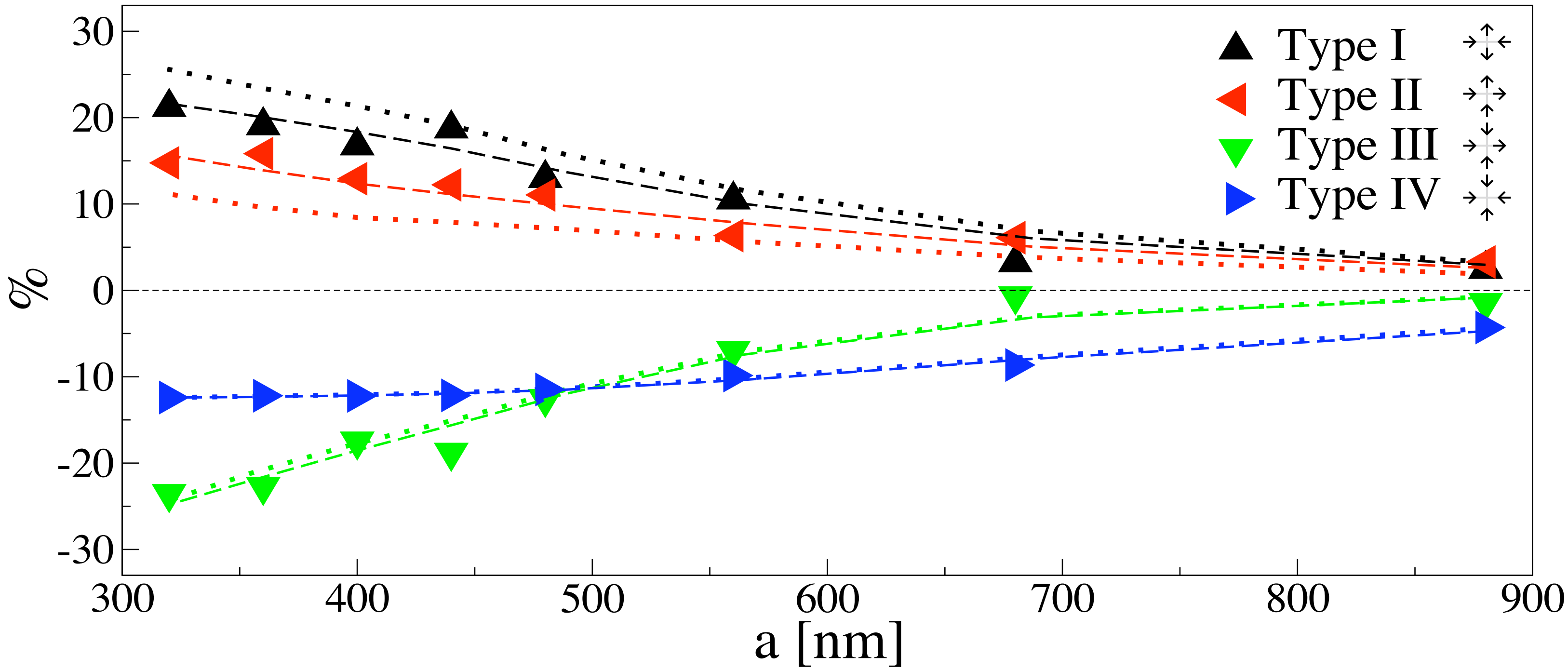
- nominal interaction energy scale: $\sim 10^5$ K
- no ordering observed
- experimental protocol: rampdown
- need to study dynamics!



# *Simple model for dynamics*

- spins flip randomly in field
- two parameters: sweep rate, minimal energy gain
- dotted line: best fit
- dashed line: includes finite-element results




- interaction energies vary by factor 30

# Conclusions

- Magnetic monopoles exist in $d = 3$ spin ice
- but not (yet) in $d = 2$ artificial spin ice