Interaction effects in quantum point contacts & quantum wires

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Overview

- Part I: Nonlinear magnetotransport in non-centrosymmetric („chiral“) interacting quantum wires (single wall nanotubes, to be specific) (A. De Martino, A. Tsvelik)
- Part II: Transport through (short) quantum point contact at first quantization plateau: interaction effects and the „0.7 anomaly“ (A.M. Lunde, A. De Martino, K. Flensberg)

Nonlinear magnetotransport

- Linear transport: Onsager-Casimir relation

$$G(B) = G(-B)$$

- Out of equilibrium: odd-in-B part allowed

$$I_e(V, B) = -I_e(V, -B)$$

this contribution is generally even in voltage!

- Fundamentally interesting because nonzero effect requires combined presence of
  - Electron-electron interactions
  - Chirality (handedness): broken inversion symmetry
  - Magnetic field: broken time reversal symmetry

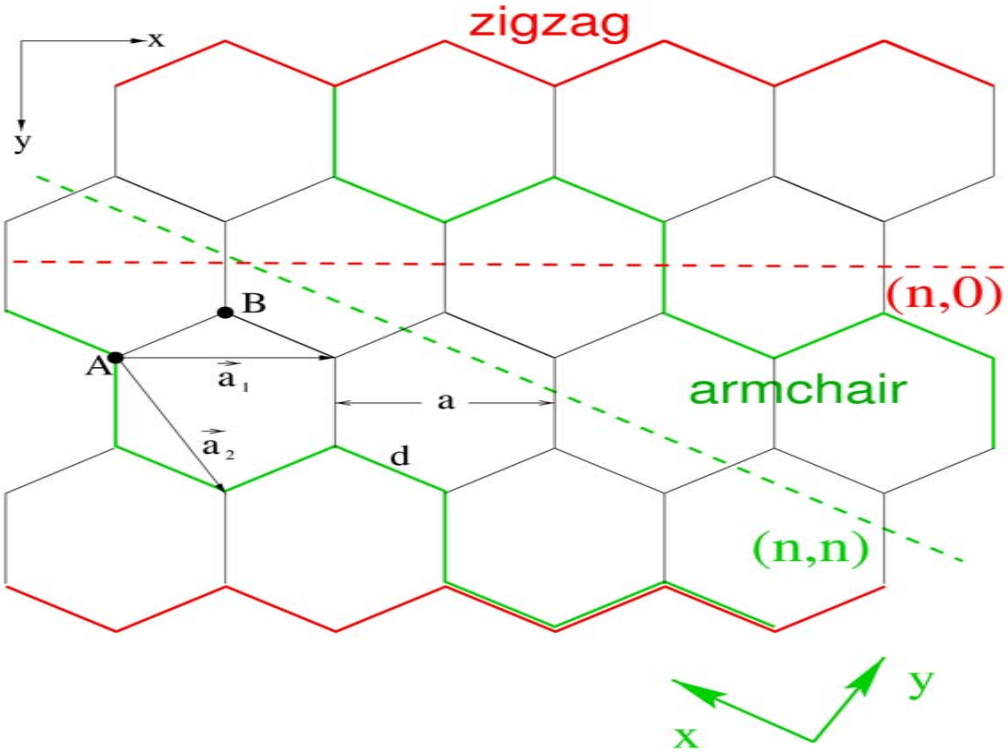
Sanchez & Büttiker, PRL 2004
Spivak & Zyuzin, PRL 2004

Chiral quantum wires: SWNTs

How does chirality enter low-energy theory?

(n,m) indices: wrapping of graphene sheet onto cylinder

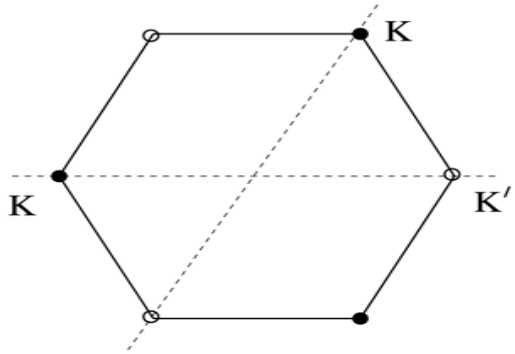
Chiral angle θ: defined with respect to zigzag (n,0) tube



Band structure: Graphene

Two independent corner points K, K´ in first Brillouin zone

- Lowest-order k*p scheme: Dirac light cone dispersion
- Deviations at higher energies: trigonal warping
- Transverse momentum quantization in SWNT: slicing of Dirac cone



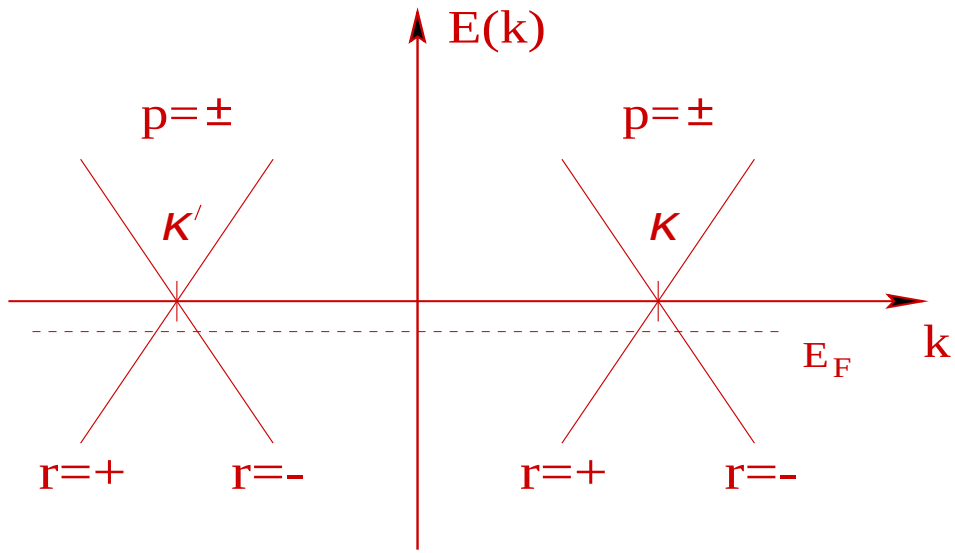
$$E(\vec{q}) = v|\vec{q}|$$

$$\vec{q} = \vec{k} - \vec{K}$$

$$v = 10^6 m/\sec$$

Metallic SWNTs

Within lowest order
k*p scheme:



- Transverse momentum quantization: keep only $k_\perp = 0$
- Ideal 1D quantum wire: 2 spin-degenerate bands
- Low-energy theory: restrict to these 2 bands, but include (long-ranged) Coulomb interactions

Egger & Gogolin, PRL 1997, EPJB 1998
Kane, Balents & Fisher, PRL 1997

Bosonized form

Four bosonic fields, index $a = c+, c-, s+, s-$

Low-energy theory: Luttinger liquid

$$H = \sum_a \frac{v_a}{2} \int dx \left[g_a \Pi_a^2 + g_a^{-1} (\partial_x \varphi_a)^2 \right]$$

$$g_{a \neq c+} \cong 1 \qquad g \equiv g_{c+} \approx 0.2$$

$$v_{c+} = v / g, \; v_{a \neq c+} = v$$

exactly solvable Gaussian model, leads to spin-charge separation and quasi-particles with fractional charge & fractional statistics

Beyond lowest-order k*p scheme?

Dirac cone approximation: chirality drops out

To go beyond, one must include both

- Trigonal warping: anisotropic & nonlinear dispersion relation
- Transverse momentum quantization: in parallel magnetic field *B*, including tube curvature

$$k_{\perp} = eBR^{2}/2h \pm (a/R)\cos 3\theta$$

- Net effect: R/L movers have different velocity

$$\delta = \frac{v_R - v_L}{v_R + v_L} = \frac{B}{B_0}\sin 6\theta$$

$$B_0 \propto k_F R$$

How to include in low energy theory?

- Luttinger liquid theory now comes with R/L moving plasmon velocities, but still exactly solvable Gaussian theory

$$v_{c+,R/L} / v = g^{-1} \pm \delta$$

$$v_{a \neq c+,R/L} = v_{R/L} = v(1 \pm \delta)$$

- Consider long SWNT & good contacts
  - Effect requires at least two impurities
  - Here: 2 impurities separated by distance *d*
  - Nonequilibrium Keldysh approach

Odd-in-B current in a chiral SWNT

De Martino, Egger & Tsvelik, PRL 2006

Analytical result:

$$I_e \propto \sin(2k_F d) T_0^{2g-1} e^{-gT_0} \sin\left(\frac{(1-g^2)B}{gB_0}\sin(6\theta)U\right)$$

$$\times \mathrm{Im}\left[e^{iU}\frac{\Gamma(1+g-iU/T_0)}{\Gamma(g)\Gamma(2-iU/T_0)}F\left(g,1+g-iU/T_0;2-iU/T_0;e^{-2T_0}\right)\right]$$

with dimensionless temperature/voltage

$$T_0 = \frac{2\pi k_B T}{\hbar v/gd},\quad U = \frac{|eV|}{\hbar v/gd}$$

Requires interactions (g<1) and chirality (sin6θ≠0)

odd in magnetic field *B*, even in bias voltage *V*

changes sign with handedness (enantioselective)

Available experimental results

Measured: $\alpha(T) = \left[\frac{I_e(V,T,B)}{V^2 B} \right]_{V,B \to 0}$

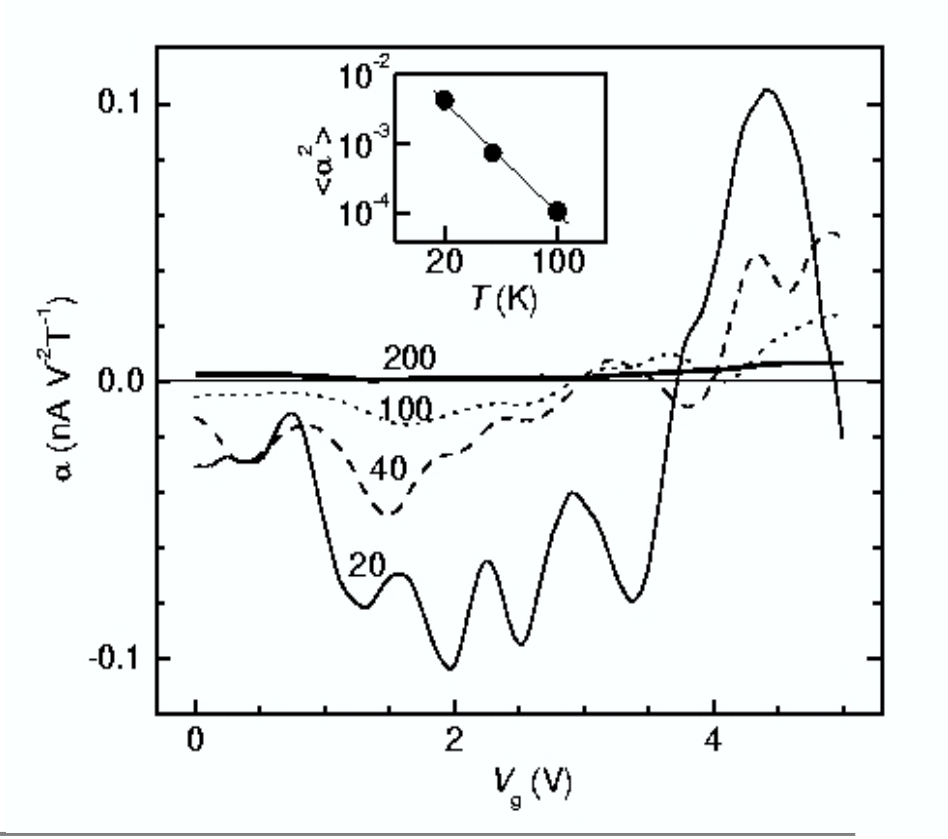
Wei, Cobden et al., PRL 2005

for individual SWNT

- Oscillatory dependence on gate voltage
- Exponentially small at high T, increases when lowering T. Our theory:

$$\alpha(T) \propto T^{(g-1)/2}$$

- Sign does not change with temperature



Oscillations in $I_e(V)$

Zero temperature limit:

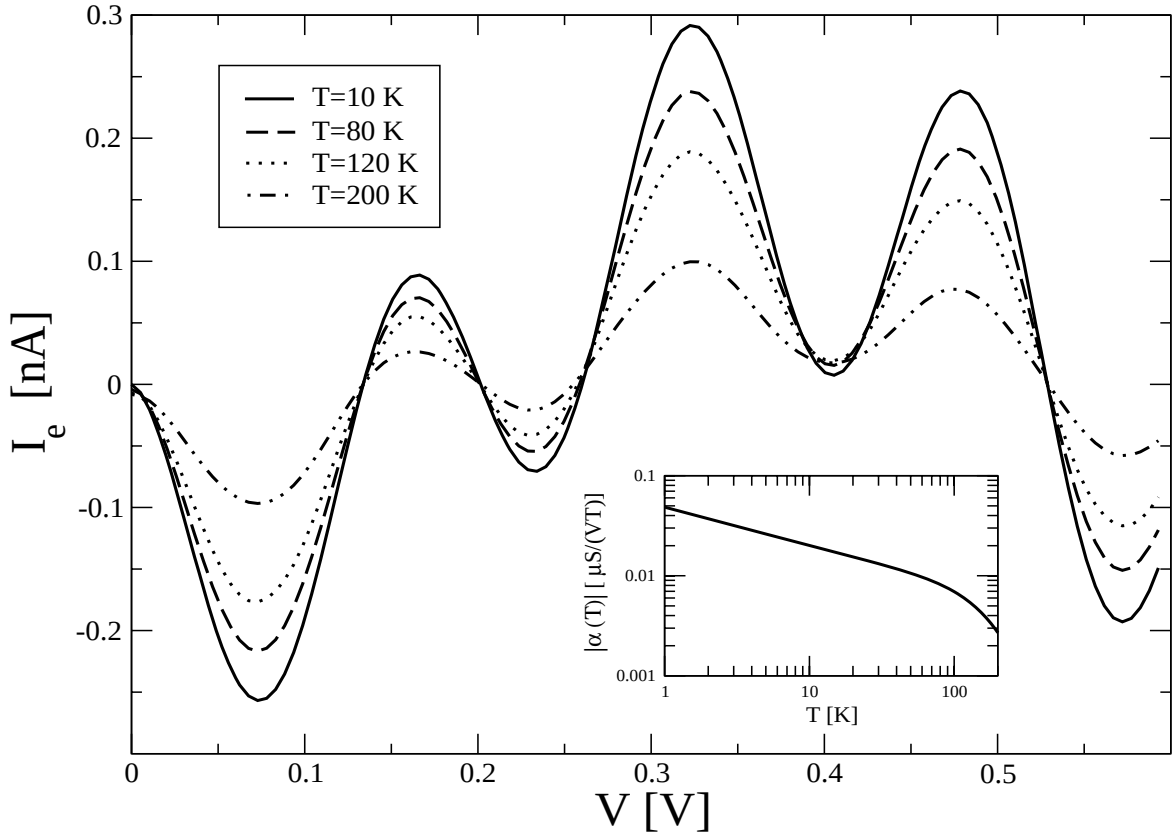
$$I_e \propto \sin\left[\frac{(1-g^2)B}{gB_0}\sin(6\theta)U\right] U^{g-1/2} J_{g-1/2}(U)$$

predicts oscillations as function of V with periods:

$$\Delta V_1 = \frac{hv}{egd}$$ yields Luttinger parameter

$$\Delta V_2 = \frac{B_0 g \Delta V_1}{B(1-g^2)\sin(6\theta)}$$ yields chirality

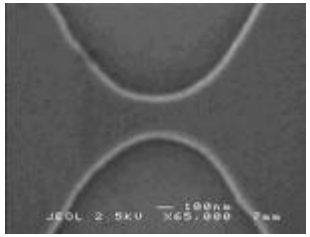
Low-voltage limit: Power-law scaling $I_e(V \to 0) \propto |V|^{2g}$



d=20nm
g=0.23
B=16T
(10,4) SWNT

direct observation of interaction physics possible

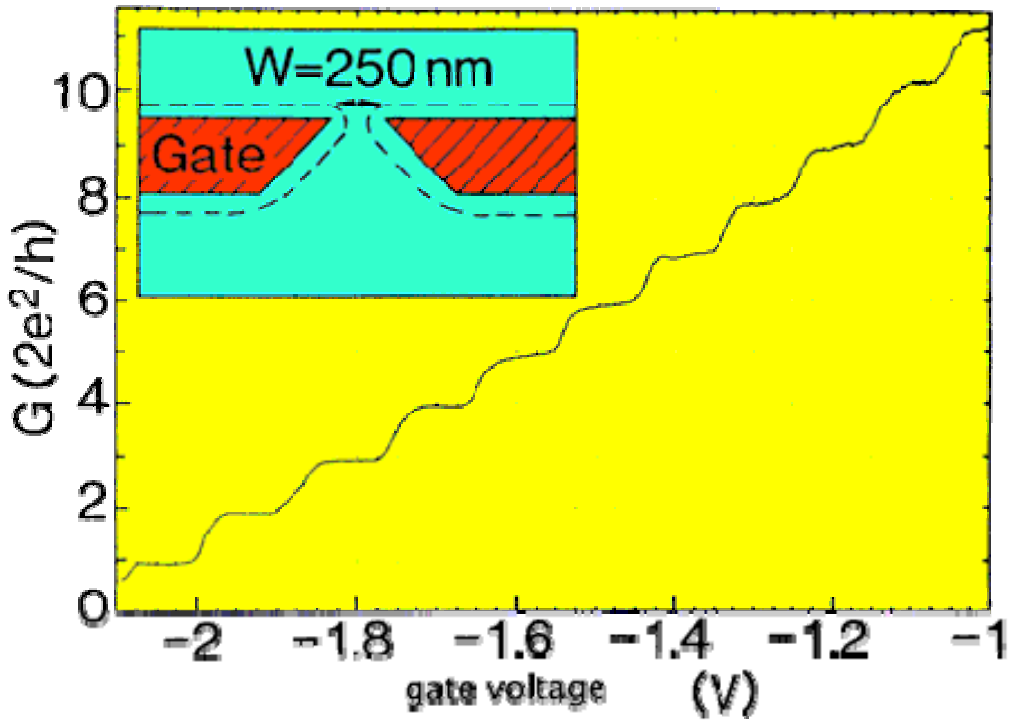
Part II: Quantum point contacts



- 0.7 anomaly in quantum point contact: experimental facts and current understanding
- Non momentum-conserving interaction processes
- Perturbation theory: Low temperature corrections
- Self-consistent approach: High temperature regime

Conductance quantization $G_0 = 2e^2 / h$

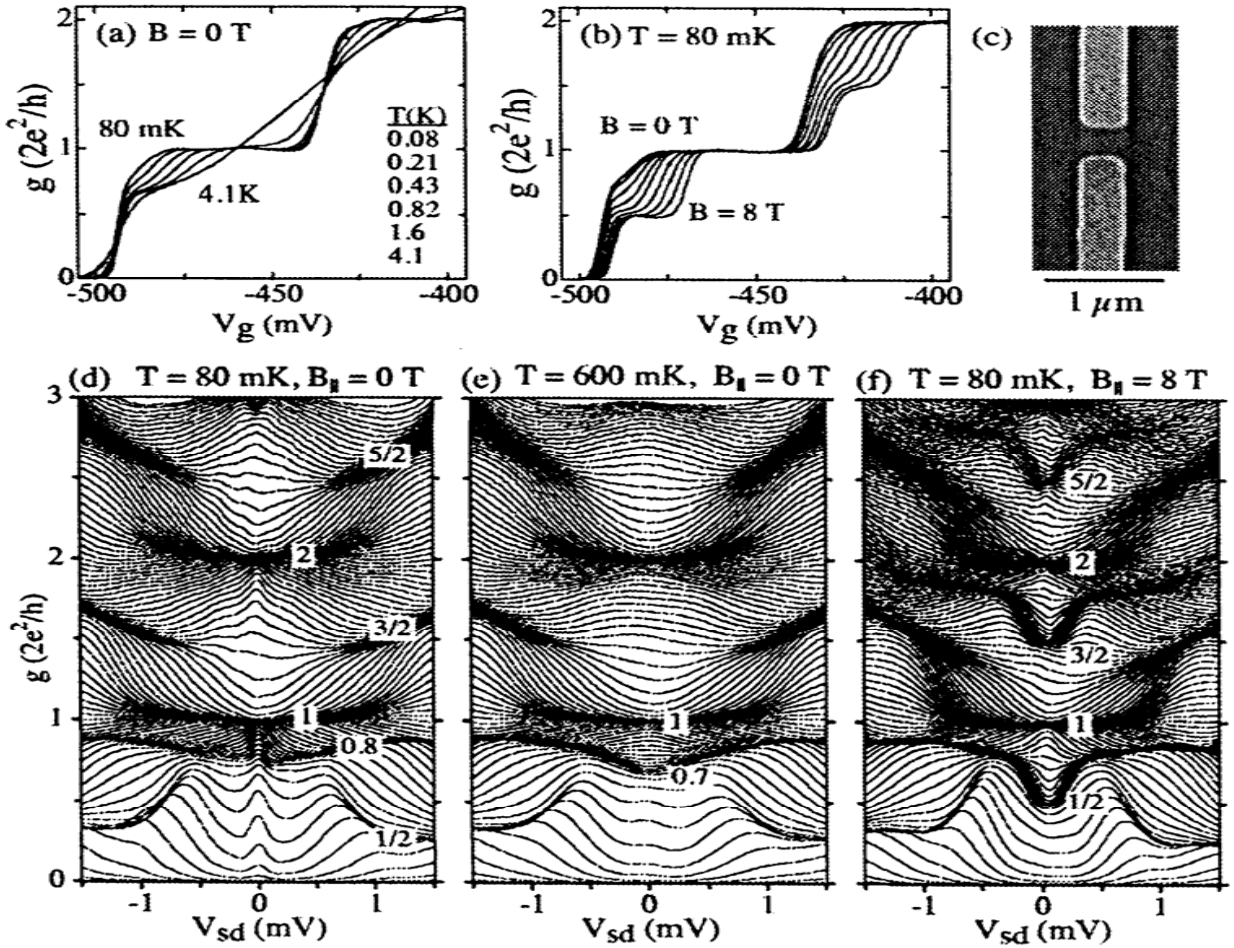
Conductance through quantum point contact in clean 2DEG is quantized: subsequent occupation of transverse energy bands



Wharam et al., J.Phys.C 1988
Van Wees et al., PRL 1988

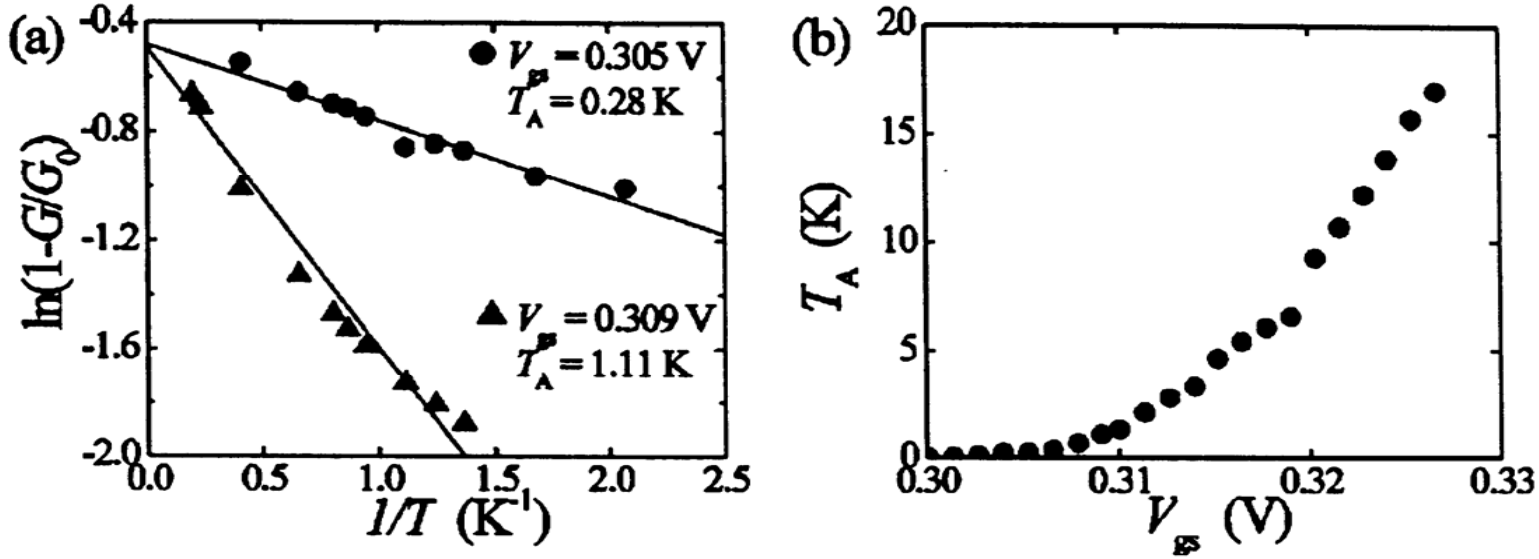
0.7 anomaly

Cronenwett et al., PRL 2002



0.7 anomaly

Kristensen et al., PRB 2000



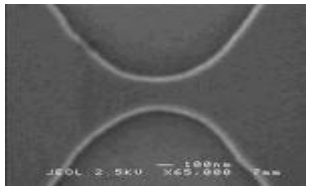
fits reasonably on activated T dependence…

0.7 anomaly: Some experimental facts

- At first plateau: shoulder-like feature around $G \approx 0.7G_0$
- Feature goes away as temperature approaches zero
- *T* dependence (roughly) activated, saturation of conductance at $G \approx 0.5G_0$ for high *T*
- Suppressed shot noise & enhanced thermopower at anomalous plateau
- Related features in longer quantum wires also exist, here we discuss short point contacts
- Reported by many groups since 1996 & in different material systems

Thomas et al., PRL 1996, Kristensen et al., PRB 2000, Cronenwett et al. PRL 2002, etc. etc.

Theories for the 0.7 anomaly



- Phenomenological models: density-dependent static spin gap provides reasonable fits to data

  Bruus et al., Physica E 2001

  - *But: no spin polarization expected from (local) interaction or impurity in presence of unpolarized external leads*
  - *Same problem in spin-symmetry broken mean field theory, gets wrong G(T)* *Lassl et al., PRB 2007*
- Kondo-type theories: Assume (quasi-)bound state in contact region *Rejec & Meir, Nature 2006*
  - *But: Different spin density functional calculations give different answers, artifically broken spin symmetry*

    Ihnatsenka & Zozoulenko, cond-mat/0761657
  - *But: no saturation of the conductance at high T under Kondo scenario*

Theories…

- Phonon backscattering *Seelig & Matveev, PRL 2003*
- Wigner crystal formation (spin incoherent Luttinger liquid) in long quantum wires

  Matveev PRL 2004, Kindermann & Brouwer, PRB 2005

  - *Seems to yield qualitatively correct G(T)*
  - *Not applicable to (short) quantum point contacts*
- consistent theory of 0.7 anomaly due to interaction effects possible?

  (without assuming spin polarization or localized states in the contact)

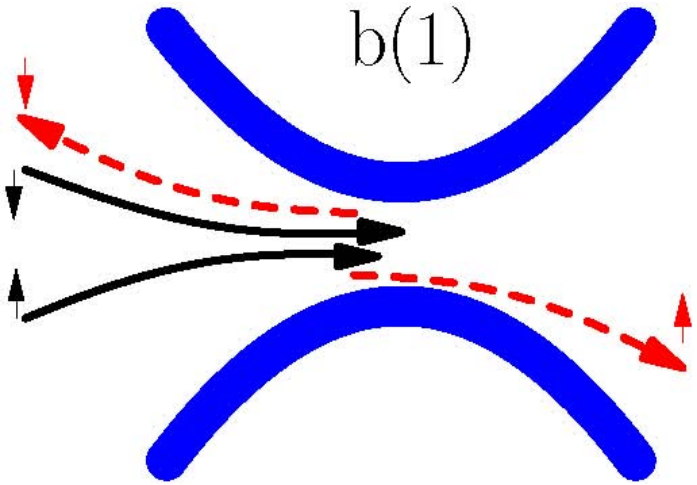
Electron-electron interactions

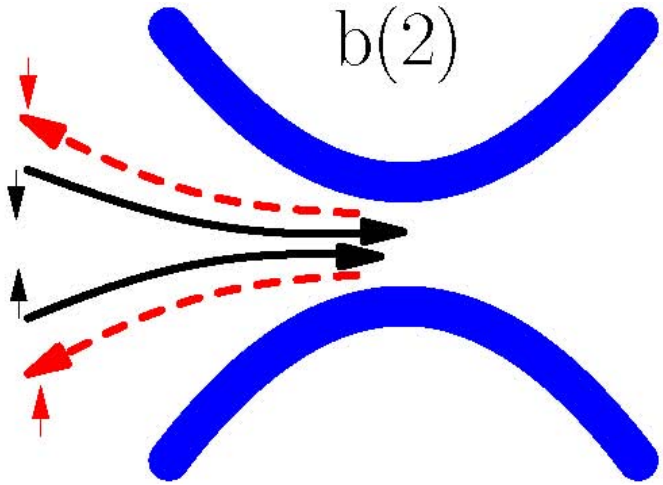
- Broken translational invariance implies existence of non momentum-conserving interaction processes
- First plateau: single channel description, all other channels are „closed“
- Away from contact: interactions are screened off by closed channels

$$H_{\text{int}} = \frac{1}{2}\sum_{\sigma\sigma'}\int dx_1 dx_2 \; W(x_1, x_2)\Psi^{+}_{\sigma}(x_1)\Psi^{+}_{\sigma'}(x_2)\Psi_{\sigma'}(x_2)\Psi_{\sigma}(x_1)$$

Interaction pair potential

Interaction processes





1-electron backscattering,
so far overlooked

2-electron backscattering,
Umklapp-type process

Meidan & Oreg, PRB 2005

Requires opposite spins due to Pauli principle,
spin is crucial!

Interactions

- Momentum conserving, e-e forward & backward scattering processes
  - *cause no interaction corrections to transport at lowest order in temperature, but are important at higher T*
- Non momentum conserving processes:
  - *Lead to interaction corrections at very low T*
  - *Amplitudes are Fourier components of pair potential*

Perturbation theory

Interaction corrections to IV characteristics

$$\frac{I(V,T)}{G_0 V} = 1 - (A_1 + A_2)\left(\frac{\pi T}{T_F}\right)^2 - \left(\frac{A_1}{4} + A_2\right)\left(\frac{eV}{kT_F}\right)^2 + O(W^4)$$

$$A_{1,2} = \frac{W_{1,2}^2 k_F^4}{48\pi^2 \varepsilon_F^2}$$

- *No contribution from other interaction processes than b(1) and b(2) to this order*
- *No correction at T=V=0, Fermi liquid behavior*
- *Reduced conductance at finite T,V*
- *No effect for spin-polarized half-plateau (large B)*

Thermopower

- Non-interacting thermopower is exponentially small on plateau

$$S_{th}^{n.i.} \propto e^{-T_F/T}$$

- Interaction correction dominates completely

$$S_{th} = \left(\frac{\Delta\mu / e}{\Delta T} \right)_{I=0} = \frac{k_B}{e} \frac{2\pi^4}{5} \left(A_1 + A_2 \right) \left(T / T_F \right)^3$$

- Consistent with experimental data on 0.7 anomaly

Appleyard et al., PRB 2000

dc shot noise

Backscattering noise power

$$S_B = 2e\left(2I_{bs,2}\coth\left(\frac{eV}{kT}\right) + I_{bs,1}\coth\left(\frac{eV}{2kT}\right)\right)$$

Schottky formula, reflects backscattered charge

2e for *b(2)* but *e* for *b(1)* process

yields suppression of nonequilibrium shot noise

$$S_I = S - 4G(V,T)kT$$

against non-interacting prediction for $eV/kT < 6.5$

$$S_I^{n.i.} = 2G_0R\left(eV\coth\left(\frac{eV}{2kT}\right) - 2kT\right) \qquad R = I_{bs}/G_0V$$

again consistent with experiment

Roche et al., PRL 2004, DiCarlo et al., PRL 2006

Low vs high temperatures

- Interaction corrections small for $T < T^* \approx \frac{T_F}{\sqrt{A_1 + A_2}}$
- For $T^* < T < T_F$: perturbation theory breaks down. High-*T* behavior difficult!
- Study this question for contact interaction

$$W(x_1, x_2) = \frac{2\pi^{3/2}}{m} \lambda \, \delta(x_1)\delta(x_2)$$

- Estimate for GaAs material parameters and QPC saddle potential with width ~ length :

$$\lambda \approx 1$$

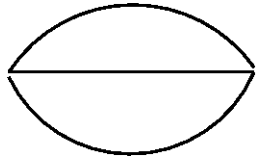
Sloggett, Milstein & Sushkov, cond-mat/0606649

Self-consistent second-order approach

- Dyson equation for Keldysh Green´s function

$$G(x,x';\omega)=G_0(x,x';\omega)+G_0(x,0;\omega)\Sigma(\omega)G(0,x';\omega)$$

- Second-order retarded self energy:



$$\Sigma^r(\omega)\propto\lambda^2\int_0^\infty dt e^{i\omega t}\left(G^<(-t)G^>(t)G^>(t)-[<\Leftrightarrow>]\right)$$

Self consistency: full *G* is used

- Linear response regime:

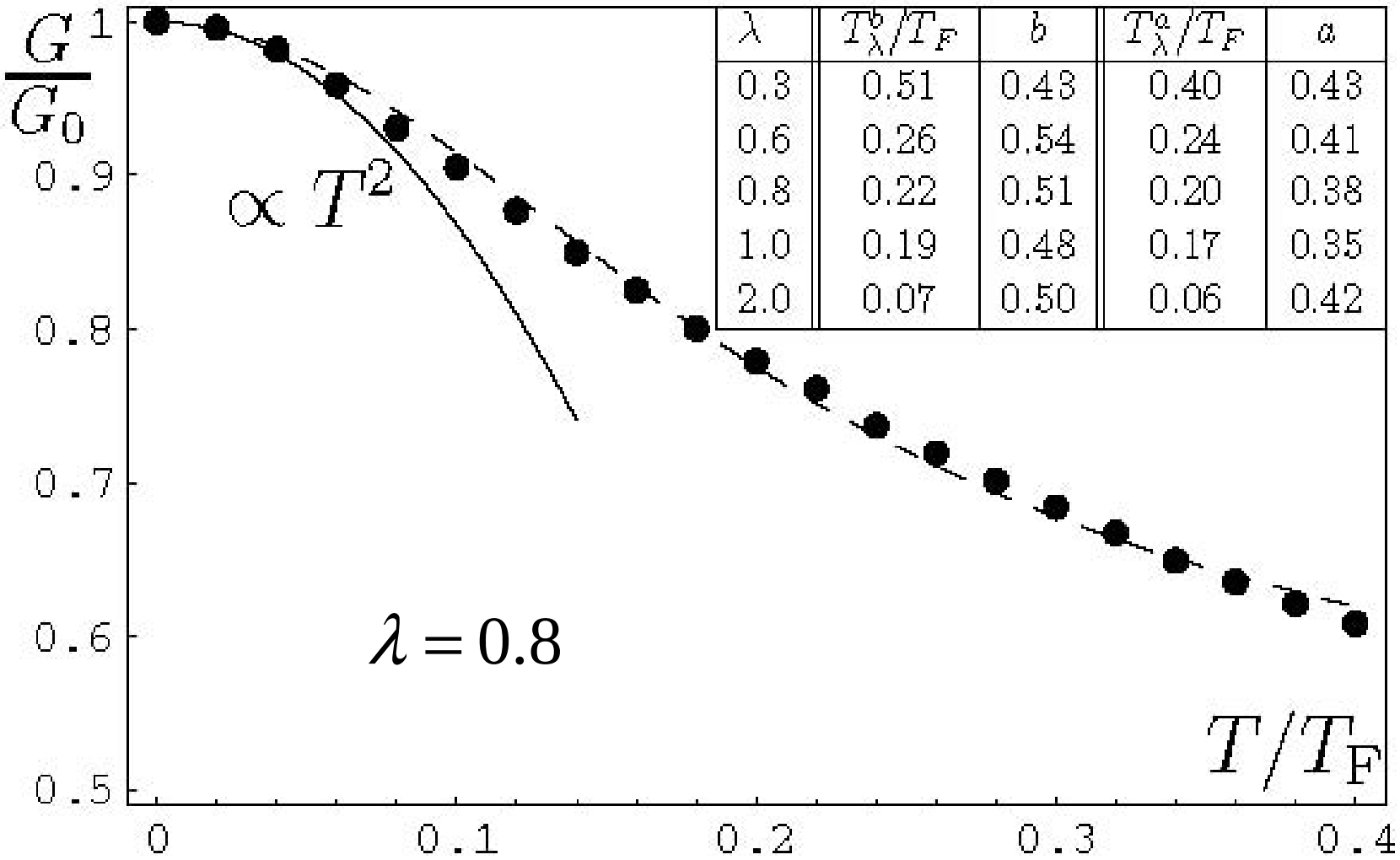
$$G^{</>}(\omega)=\pm iA(\omega)f(\pm\omega)$$

Iterative numerical scheme

- start with non-interacting local spectral function $A(\omega) = A_0(\omega)$
- compute retarded self energy
- obtain new estimate for local spectral function from Dyson eq.
- iterate until convergence has been achieved
- obtain linear conductance

$$G/G_0 = \int d\omega(-\partial_\omega f)\frac{A(\omega)}{A_0(\omega)}$$

Numerical results



λ	T^b_λ/T_F	b	T^a_λ/T_F	a
0.3	0.51	0.43	0.40	0.43
0.6	0.26	0.54	0.24	0.41
0.8	0.22	0.51	0.20	0.38
1.0	0.19	0.48	0.17	0.35
2.0	0.07	0.50	0.06	0.42

Fitting functions

- Data for all interactions scale nicely on

$$\frac{G}{G_0} = b + \frac{1-b}{1+\left(T/T_\lambda^b\right)^2}$$

- Also good fit to activated *T* dependence $\frac{G}{G_0} = 1-(1-a)e^{-T_\lambda^a/T}$

  but no good fit to Kondo-type function
- High-*T* saturation at $G/G_0 \approx 1/2$
- Crossover scales: $T_\lambda^a \approx T_\lambda^b \approx T^* \propto 1/\lambda$

consistent with experimental observations

Conclusions: Part I

Magnetotransport: Linear in B current

- Only present out of equilibrium (current is even in voltage) and with at least two impurities and with interactions and for chiral tubes!
- Prediction: Oscillations with bias voltage, power law scaling in *T* dependence

De Martino, Egger & Tsvelik, PRL 2006

Conclusions: Part II

Consistent theory of 0.7 anomaly appears possible: e-e interactions crucial

- Transport properties (linear & non-linear conductance, thermopower, shot noise) are consistently described by our model
- Incoherent Fermi liquid, with full relaxation between right- and left-movers at high *T*

Lunde, De Martino, Egger & Flensberg, preprint