PHYSICAL IMPLEMENTATIONS OF TOPOLOGICALLY PROTECTED QUBITS

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Kitaev's proposal (quant-ph/9707021)

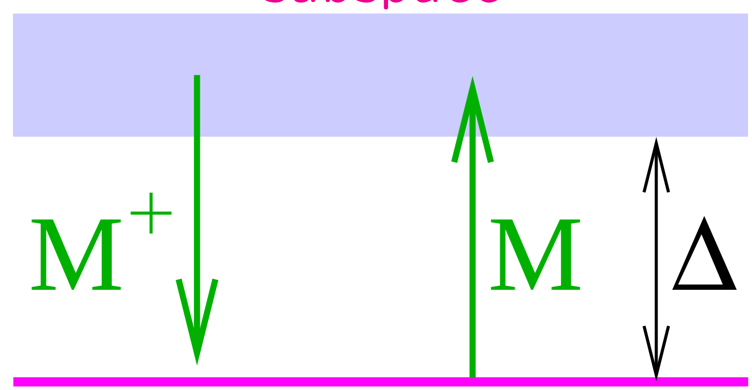
1)Physical operators coupling the computer to its environment are **local**

2)Different internal states used for computation should be:

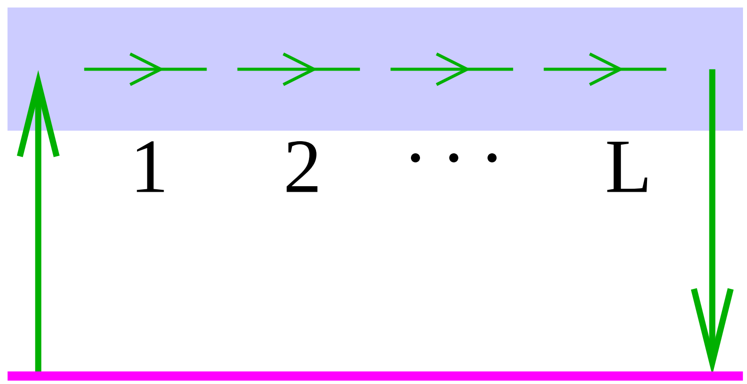
- macroscopically different
- perfectly degenerate, even in the presence of environmental noise

3)computer should be a **large** system, of which only a small number of eigenstates are used for computation.

"Topologically protected" subspace



Constraints on system-environment coupling



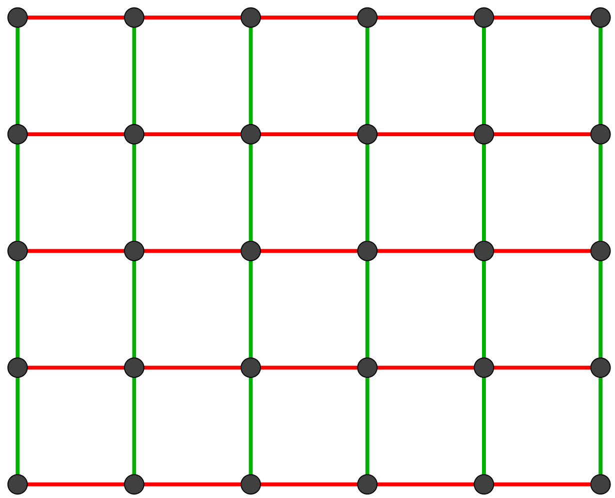
Environment-induced splittings in protected subspace

$$\Delta E \simeq M \left(\frac{M}{\Delta}\right)^{L}$$

Can be very small if L is **large**, provided $\frac{M}{\Delta} < 1$

So we can't use environment to distinguish states in the protected subspace

Assume $[H, P_i] = [H, Q_j] = 0$



$$P_i^2 = 1, \quad [P_i, P_j] = 0$$
$$Q_i^2 = 1, \quad [Q_i, Q_j] = 0$$

$$\{P_{\text{row}}, Q_{\text{column}}\} = 0$$

Can diagonalize *simultaneously*:

$$P_1, P_2, ..., P_M, Q_1Q_2, ..., Q_1Q_N$$

Gives only two-dimensional irreducible representations!

1)Start with $|\uparrow\rangle$, such that:

$$P_i|\uparrow\rangle = \alpha_i|\uparrow\rangle$$
$$Q_1Q_j|\uparrow\rangle = \beta_j|\uparrow\rangle$$

2)Define $|\downarrow\rangle$ as:

$$|\downarrow\rangle = Q_1|\uparrow\rangle$$

3)$|\downarrow\rangle$ satisfies:

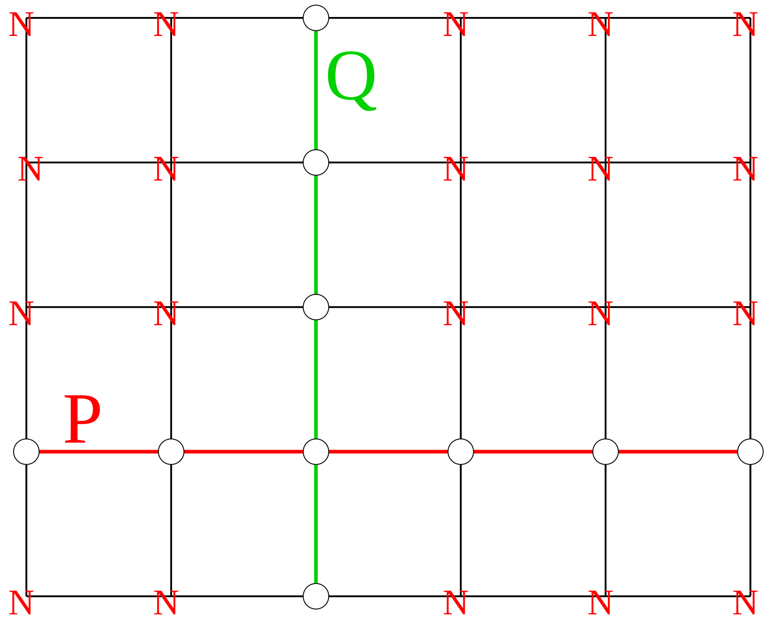
$$P_i|\downarrow\rangle = -\alpha_i|\downarrow\rangle$$
$$Q_1Q_j|\downarrow\rangle = \beta_j|\downarrow\rangle$$

4)Furthermore:

$$Q_j|\uparrow\rangle = \beta_j|\downarrow\rangle$$

Doublets exist as long as one can find at least ONE pair P_i, Q_j, commuting with H.

Example of a static disorder configuration which does NOT lift any degeneracy!



Local noise breaks degeneracies only at high orders (M, or N) in perturbation theory!

Important example

Local noise operators $\mathcal{N}_{ij}$ such that they commute with all P_{row}'s and Q_{column}'s, with the exception of:

$$\{\mathcal{N}_{ij}, Q_j\} = 0$$

$$\mathcal{N}_{ij}|\{\alpha\},\{\beta\},\tau\rangle = |\{\alpha\},\beta_1,...,-\beta_j,...,\beta_N,\tau\rangle$$
$$\mathcal{N}_{i1}|\{\alpha\},\{\beta\},\tau\rangle = |\{\alpha\},-\beta_1,...,-\beta_j,...,-\beta_N,\tau\rangle$$

1)This noise induces no transitions inside the doublets (τ is conserved):

$\longrightarrow$ No relaxation!

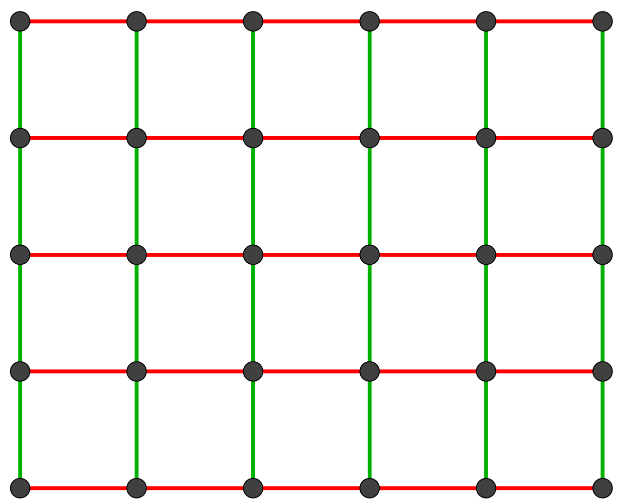
2)Degeneracies are lifted only at order N or larger:

$\longrightarrow$ dephasing rate is *exponentially small*!

Example: X-Z Ising model

Douçot, Feigel'man, Ioffe, Ioselevich, P. R. B. **71**, (2005)

$$H = -J_x \sum_{\langle ij \rangle}^{(h)} \sigma_i^x \sigma_j^x - J_z \sum_{\langle ij \rangle}^{(v)} \sigma_i^z \sigma_j^z$$



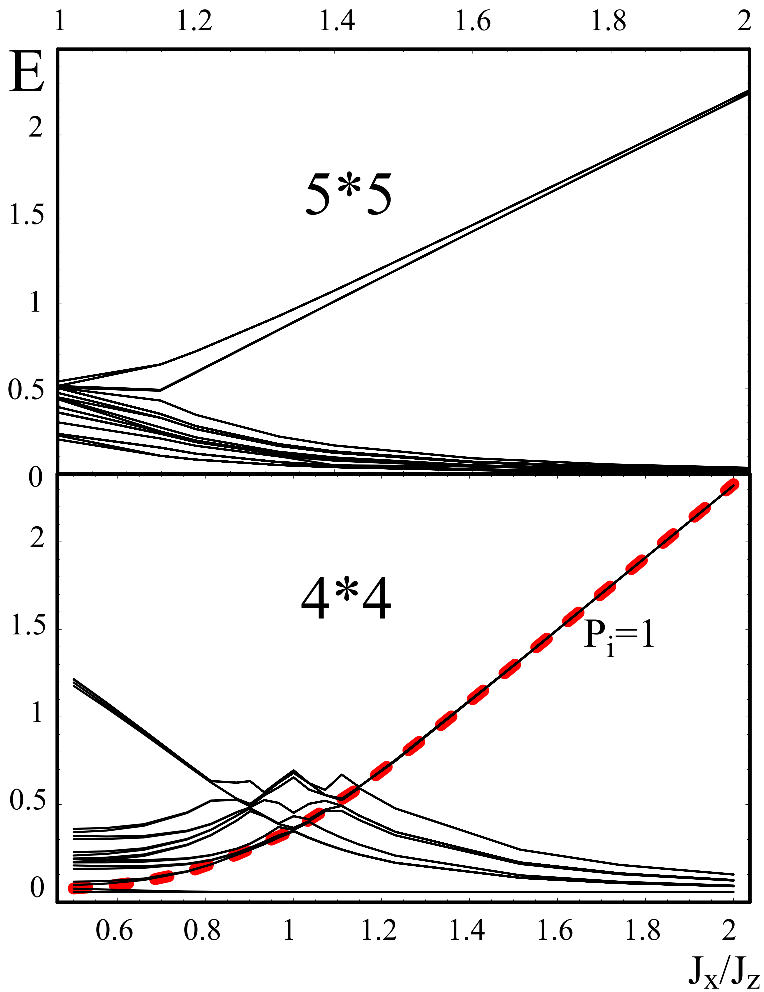
Conservation laws

$$P_{\text{row}} = \prod_{r \in \text{row}} \sigma_r^z$$
$$Q_{\text{column}} = \prod_{r \in \text{column}} \sigma_r^x$$

$$P_i^2 = 1, \quad [P_i, P_j] = 0$$
$$Q_i^2 = 1, \quad [Q_i, Q_j] = 0$$

$$\{P_{\text{row}}, Q_{\text{column}}\} = 0$$

Numerical diagonalizations



Main question: how does energy gap depend on J_x, J_z, M, N ?

Seems to close exponentially with system size

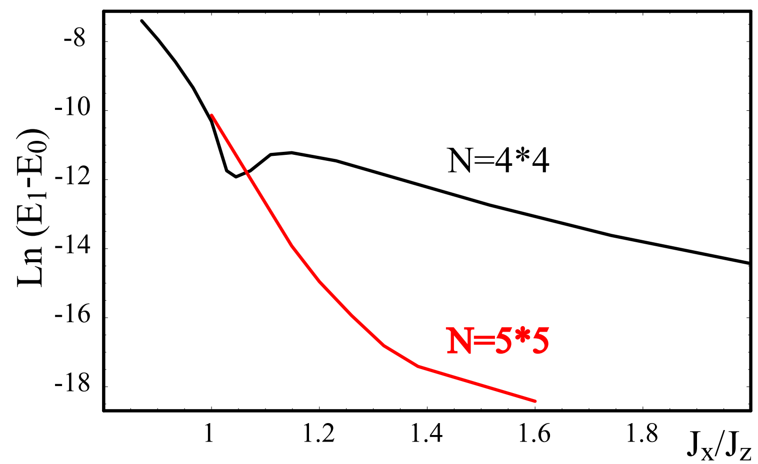
Dorier, Becca, and Mila, P. R. **B 72**, 024448, (2005)

$\longrightarrow$ One should work with a relatively small array, that is $M, N \simeq 5$

Effect of quenched disorder

$$H_{\text{dis}} = \sum_r h_r^z \sigma_r^z + h_r^x \sigma_r^x$$

Random field $h_r^z \in [-0.05, 0.05]$,
$h_r^x = 0$



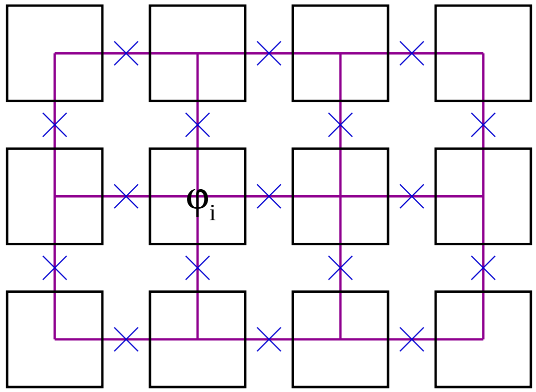
All states are doubly-degenerate, in a way largely unsensitive to noise from environment

Basics of Josephson junction arrays

ϕ_j; local phase of Cooper pair condensate

$\hat{n}_j = \frac{\partial}{i\partial\phi_j}$: number of Cooper pairs on island j

Example of a square array:



$$\Delta\phi_j \Delta n_j \simeq 2\pi$$

$$A_{ij} = \frac{2\pi}{\Phi_0} \int_i^j \vec{A}_{ij}.d\vec{r}$$

$$H = -E_{\mathsf{J}} \sum_{\langle ij \rangle} \cos(\phi_i - \phi_j - A_{ij}) + \frac{E_{\mathsf{C}}}{2} \sum_{ij} (C_{ij}^{-1}) \hat{n}_i \hat{n}_j$$

E_{J}: Josephson coupling energy

E_{C}: Charging energy

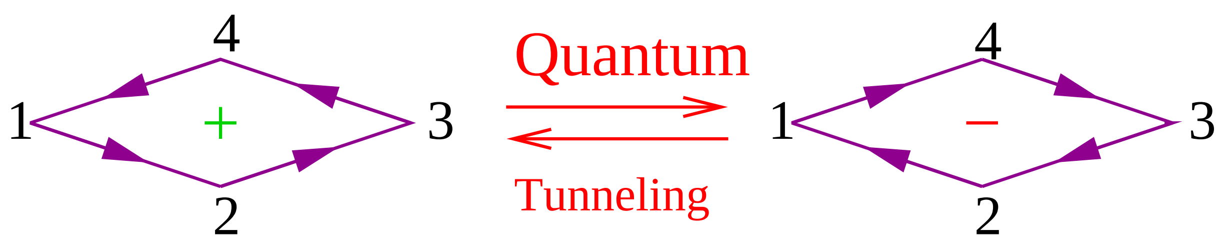
A rhombus with half a flux quantum

Define $\theta_{ij} = \phi_i - \phi_j - A_{ij}$, then:

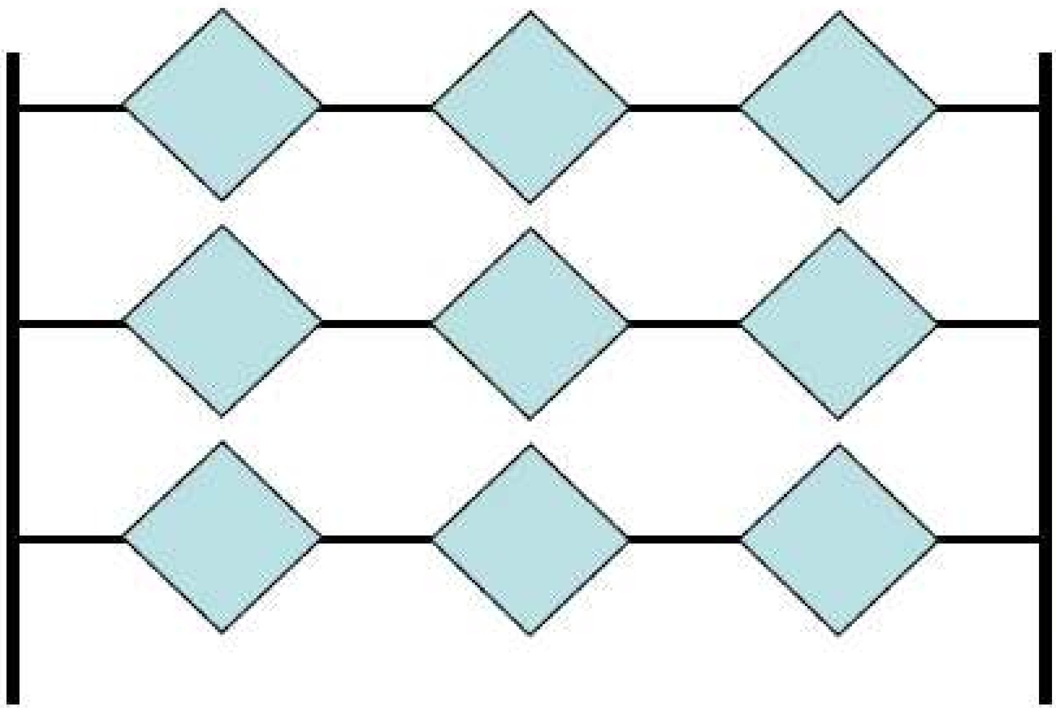
$$\theta_{12} + \theta_{23} + \theta_{34} + \theta_{41} \equiv \pi, \text{ mod } 2\pi$$

$\longrightarrow$ Get two-fold degenerate classical ground-state, with $\theta_{ij} = \pm\frac{\pi}{4}$

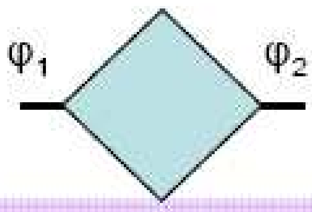
$\longrightarrow$ Quantum fluctuations ($E_C \neq 0$) of phases lift this degeneracy



Simplest protected physical device



$\phi=0$ $\quad$ $\phi=0$ or $\phi=\pi$



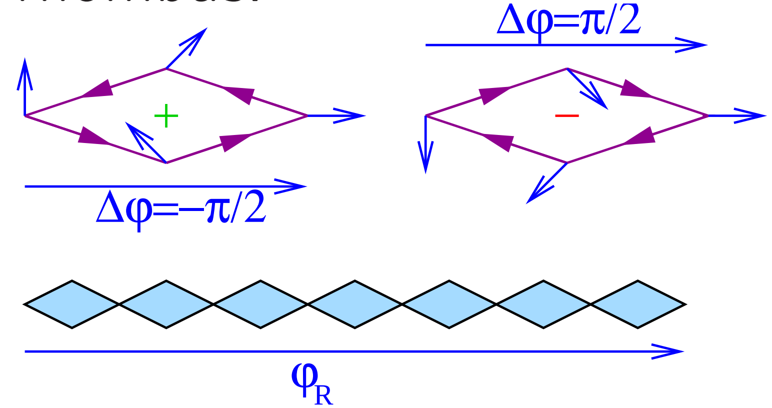
Josephson element that has two (approximately) degenerate classical states with phase differences $\phi_2-\phi_1=0$ or $\phi_2-\phi_1=\pi$ and fluctuates between them → effective 'spin' degree of freedom.

In order to protect against flip errors one needs large potential barrier in the potential V(φ) that separates states with φ=0 and φ=π.

⇨ Need M~N chains to add these potentials and decrease charging energy of φ

Single chain of rhombi: Josephson energy

Classical ground-states for a rhombus:



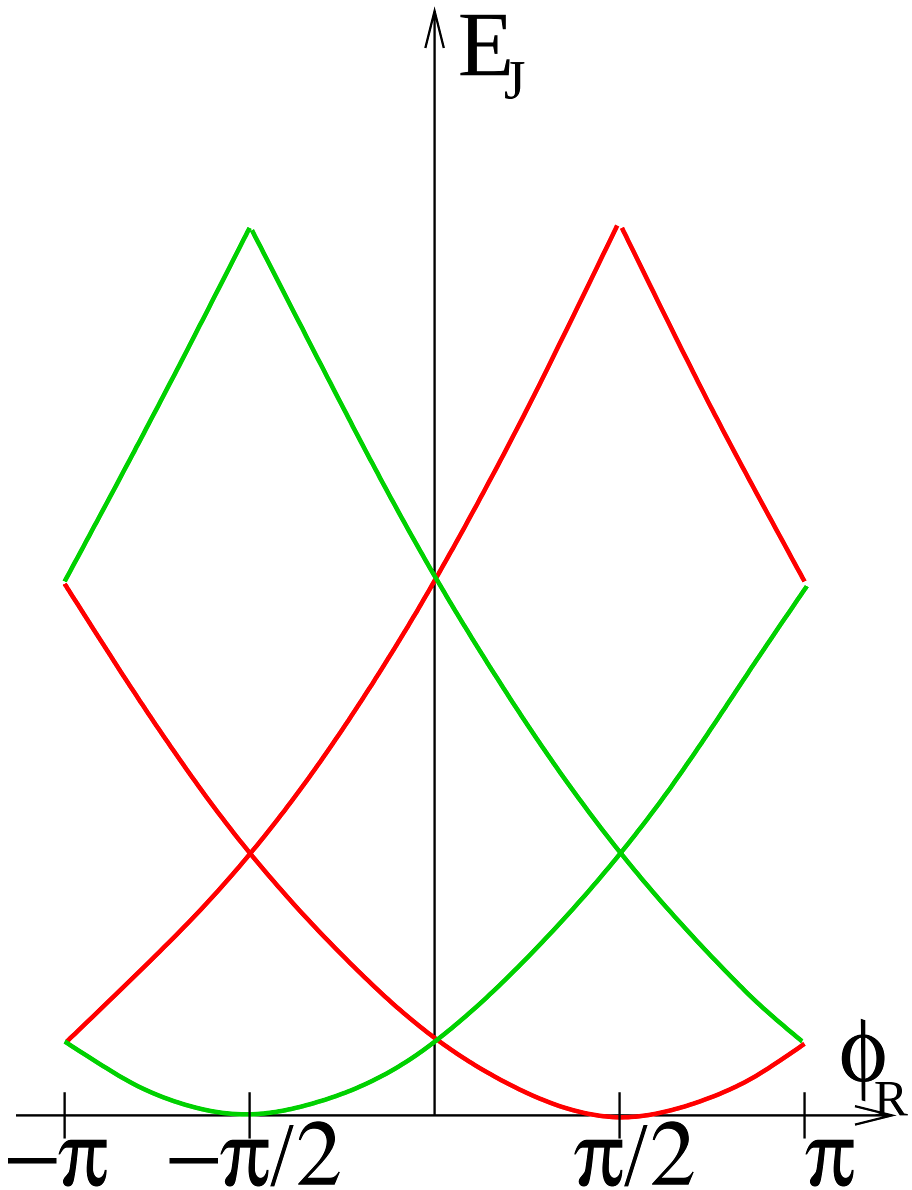
$$\Delta\phi_n = -\frac{\pi}{2}\sigma_n^z$$

$$-\frac{\pi}{2}\sum_{n=1}^{N}\sigma_n^z \equiv \frac{\pi}{2}\prod_{n=1}^{N}\sigma_n^z - \frac{\pi}{2}(N+1) \quad [2\pi]$$

Boundary condition:

$$\phi_{\mathrm{R}} = \frac{\pi}{2}\prod_{n=1}^{N}\sigma_n^z - \frac{\pi}{2}(N+1) + \phi_{\mathrm{el}}$$

$$H_{\mathrm{eff,J}} = \frac{c(N)}{2}\left(\phi_{\mathrm{R}} - \frac{\pi}{2}\prod_{n=1}^{N}\sigma_n^z + \frac{\pi}{2}(N+1)\right)^2$$



Josephson energy in the classical limit:

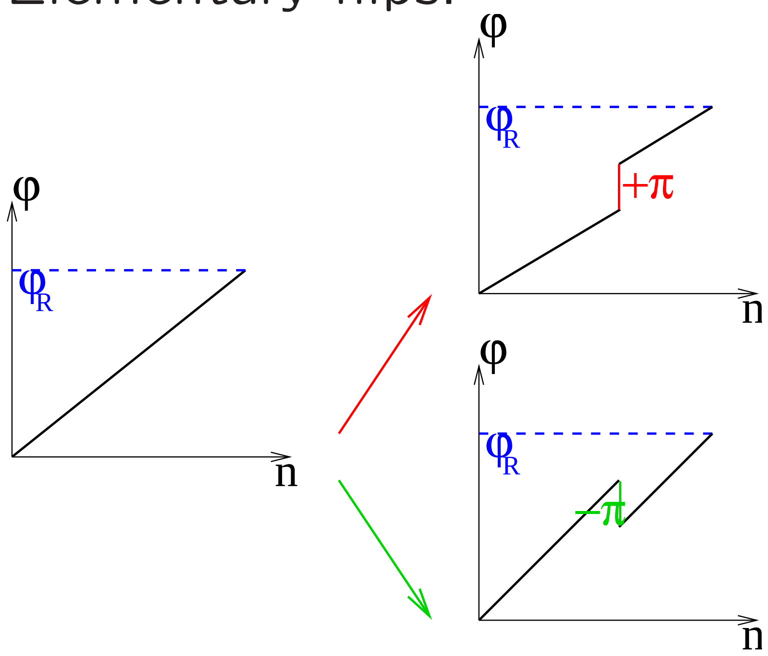
The two colors correspond to the parity of $\prod_{n=1}^{N} \sigma_n^z$ (even,odd)

Effect of quantum fluctuations on low-energy spectrum

If $\frac{E_C}{E_J}$ not very small, single rhombi can flip, with tunneling amplitude:

$$b \simeq (E_J^3 E_C)^{1/4} \exp(-1.61\sqrt{\frac{E_J}{E_C}})$$

Elementary flips:



Model Hamiltonian for low-energy spectrum

CONSTRAINT: $\phi_{\mathsf{R}} = \frac{\pi}{2}\prod_n \sigma_n^z - \frac{\pi}{2}(N+1) + \phi_{\mathsf{el}}$

Introduce translation operator T such that $T|\phi_{\mathsf{el}}\rangle = |\phi_{\mathsf{el}} + \pi\rangle$:

$$H_{\mathsf{eff}} = -a(N)\left(\sum_n \sigma_n^x\right)^2 - b(N)\left(\sum_n \sigma_n^x\right)(T + T^\dagger) + \frac{c(N)}{2}\phi_{\mathsf{el}}^2$$

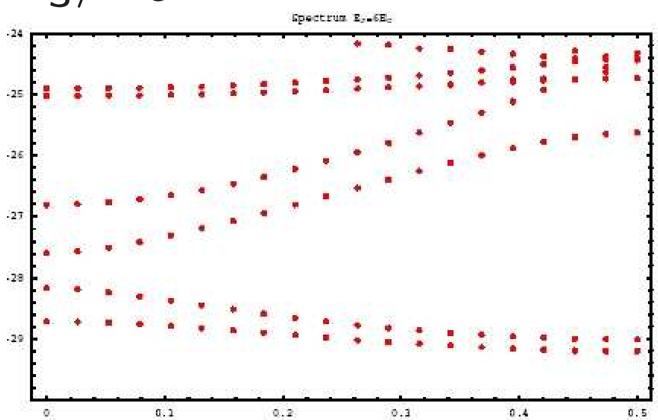
Keeping only two lowest branches in Josephson energy

$$H_{\mathsf{eff}} = -a(N)\left(\sum_n \sigma_n^x\right)^2 - b(N)\sum_n \sigma_n^x + \frac{c(N)}{2}\left(\phi_{\mathsf{R}} - \frac{\pi}{2}\prod_{n=1}^{N}\sigma_n^z + \frac{\pi}{2}(N+1)\right)^2$$

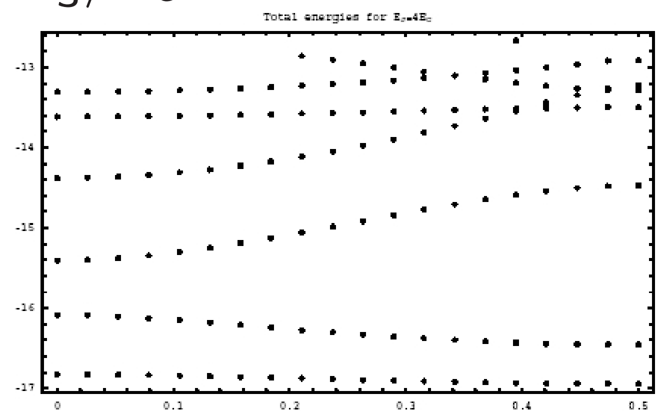
Spectrum depends only on $\Sigma = |\sum_n \sigma_n^x|$

Comparison between effective model and numerical diagonalizations

$E_J/E_C = 6$



$E_J/E_C = 4$



$$E_{\pm}(\Sigma) = a(N)\Sigma^2 + \frac{c(N)}{2}(\phi_R + \frac{\pi^2}{4}) \pm \left(b(N)^2\Sigma^2 + (\frac{\pi}{2}c(N)\phi_R)^2 \right)^{1/2}$$

Non-local symmetries in the low-energy sector

Introduce τ^z, such that $\phi_{\mathsf{R}} = \frac{\pi}{2}\tau^z - \frac{\pi}{2}(N+1)$

(Global qubit state)

$$H_{\text{eff}} = -a(N)\left(\sum_n \sigma_n^x\right)^2 - b(N)\sum_n \sigma_n^x + \frac{\pi^2}{4}\frac{c(N)}{2}\left(\prod_{n=1}^{N}\sigma_n^z\right)\tau^z$$

$$\begin{aligned} P &= \tau^z \\ Q_n &= \sigma_n^x \tau^x \end{aligned}$$

Effet of local noises: $H_{\mathcal{N}} = \sum_n h_n^z \sigma_n^z + h_n^x \sigma_n^x$

$h_n^x \sigma_n^x$ preserves P and Q_m $\longrightarrow$ No degeneracy lifting

$h_n^z \sigma_n^z$ preserves P and $Q_m,\ m \neq n$, but anticommutes with Q_n

$\longrightarrow$ Degeneracy lifting, but at order N in perturbation theory!

Effect of static perturbations (1)

Offset charges and local variations of E_J do not lift doublet degeneracy provided N is odd

Proof: for N odd, total phase variation across the chain is $\pm\frac{\pi}{2}$. So one goes from $\tau^z = 1$ to $\tau^z = -1$ by changing all local phases ϕ_r into $-\phi_r$.

This operation commutes with Josephson energy $-\sum_{\langle r,r'\rangle} E_{\mathrm{J},rr'} \cos(\phi_r - \phi_{r'} - A_{rr'})$ because for half-flux quantum/rhombus, we may choose $A_{rr'} \in \{0, \pi\}$.

To preserve the charging energy (in the presence of offset charges), we need to preserve $\hat{n}_r = \frac{\partial}{i\partial\phi_r}$, so the desired operation reads:

$$\Psi_{\mathrm{new}}(\phi_1, ..., \phi_{3N+1}) = \bar{\Psi}(-\phi_1, ..., -\phi_{3N+1})$$

Remark: for N even, these perturbations lift doublet degeneracy only at order N.

Effect of static perturbations (2)

Effect of area variations $\longrightarrow \Phi \neq \frac{\Phi}{\Phi_0}$ in a given rhombus.

$\longrightarrow$ lifts degeneracy between opposite chiralities

$$\Delta H_{\mathrm{eff}} = \sum_n h_n \sigma_n^z$$

$$h_n \simeq \frac{\Delta\Phi_n}{\Phi_0} E_{\mathrm{J}} \sim 0.01 E_{\mathrm{J}}$$

By choosing E_{C} not too small, we can get $|h_n| < \Delta$

Numerical results for $N = 3$:

$$\begin{aligned} \frac{\Delta}{E_{\mathrm{J}}} &\simeq 0.03 \quad (E_{\mathrm{J}} = 6E_{\mathrm{C}}) \\ \frac{\Delta}{E_{\mathrm{J}}} &\simeq 0.1 \quad (E_{\mathrm{J}} = 4E_{\mathrm{C}}) \end{aligned}$$

Remark: here again, this perturbation lifts doublet degeneracy only at order N.

Read-out

Requires moving the system out of topologically protected subspace

Can be either destructive (critical current measurement) or non destructive (ac impedance measurement)

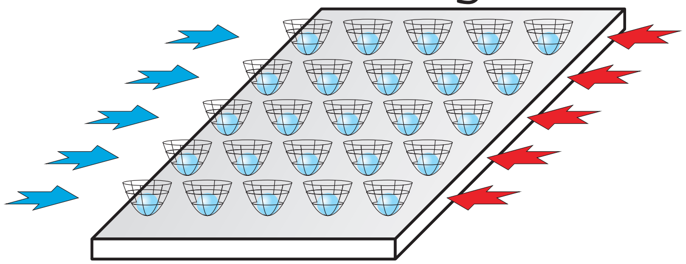
Measuring τ^z

Apply a small uniform magnetic field, which lifts degeneracy between $\tau^z = \pm 1$. This has to be applied fast compared to the inverse qubit splitting, and slowly compared to the inverse gap Δ^{-1}.

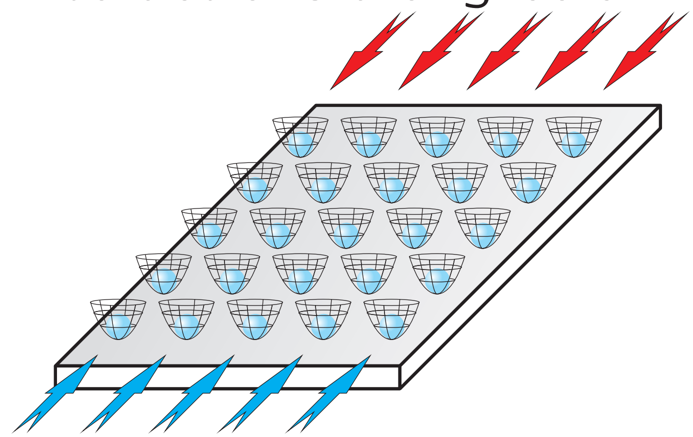
The two states are now macroscopically distinct.

Implementation with trapped ions

Interactions along rows



Interactions along columns



with T. Coudreau, P. Milman, and L. Ioffe

Getting large coherence time with trapped ions I

VOLUME 92, NUMBER 22 PHYSICAL REVIEW LETTERS week ending 4 JUNE 2004

Bell States of Atoms with Ultralong Lifetimes and Their Tomographic State Analysis

C. F. Roos, G. P. T. Lancaster, M. Riebe, H. Häffner, W. Hänsel, S. Gulde, C. Becher, J. Eschner, F. Schmidt-Kaler, and R. Blatt

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(Received 28 July 2003; published 3 June 2004)

Arbitrary atomic Bell states with two trapped ions are generated in a deterministic and preprogrammed way. The resulting entanglement is quantitatively analyzed using various measures of entanglement. For this, we reconstruct the density matrix using single qubit rotations and subsequent measurements with near-unity detection efficiency. This procedure represents the basic building block for future process tomography of quantum computations. As a first application, the temporal decay of entanglement is investigated in detail. We observe ultralong lifetimes for the Bell states $\Psi_{\pm}$, close to the fundamental limit set by the spontaneous emission from the metastable upper qubit level and longer than all reported values by 3 orders of magnitude.

Coherence time (1.05 s) is limited by finite lifetime (1.17 s) of metastable $D_{5/2}$ level of $^{40}Ca^{+}$ ions.

Getting large coherence time with trapped ions II

PRL **95**, 060502 (2005) PHYSICAL REVIEW LETTERS week ending 5 AUGUST 2005

Long-Lived Qubit Memory Using Atomic Ions

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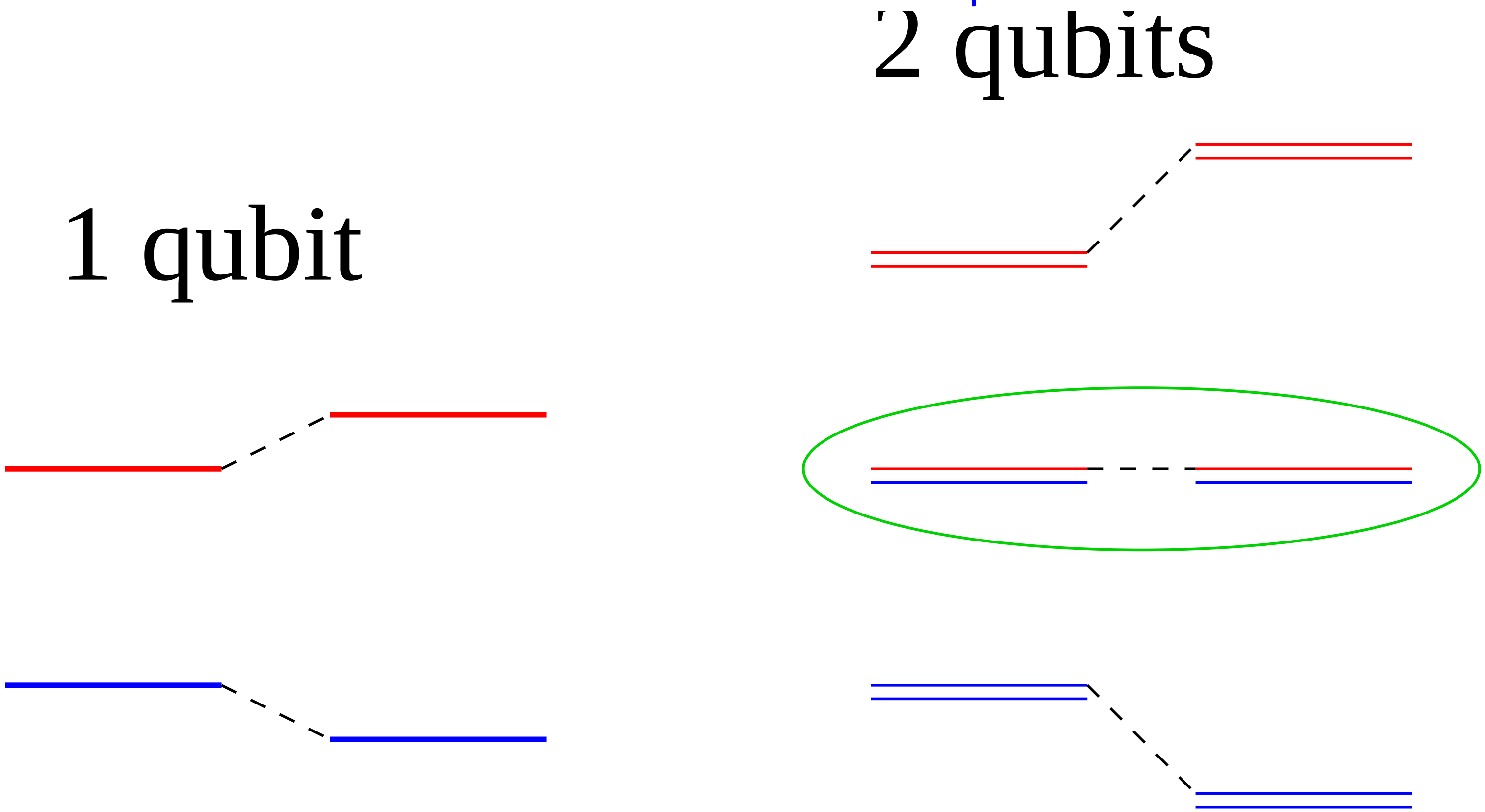
(Received 9 April 2005; published 2 August 2005)

We demonstrate experimentally a robust quantum memory using a magnetic-field-independent hyperfine transition in $^9Be^+$ atomic ion qubits at a magnetic field $B \simeq 0.01194$ T. We observe that the single physical qubit memory coherence time is greater than 10 s, an improvement of approximately 5 orders of magnitude from previous experiments with $^9Be^+$. We also observe long coherence times of decoherence-free subspace logical qubits comprising two entangled physical qubits and discuss the merits of each type of qubit.

Decoherence-free subspaces



Phonon-mediated long-ranged spin coupling

The Sørensen-Mølmer process

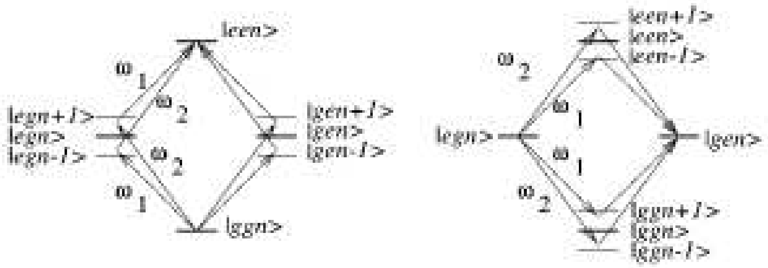


FIG. 2. Level scheme for a pair of ions sharing an oscillator degree of freedom. Left: By application of laser light with frequencies $\omega_{eg} \pm \delta$, where δ is somewhat smaller than the vibrational frequency ν, we identify four transition paths between the states $|gg\rangle|n\rangle$ and $|ee\rangle|n\rangle$, which interfere as described in the text. Right: Four similar transition paths are identified between states $|eg\rangle|n\rangle$ and $|ge\rangle|n\rangle$, yielding the same effective coupling among these states as between the states in the left panel.

Effective interaction

$$J_{\text{eff}} = (\eta\Omega)^2/|\nu - \delta|$$

Ω: Light intensity (Rabi frequency)

η: Photon energy/recoil energy

ν: Phonon frequency

δ: detuning of the main transition

Constraints for pratical implementation

Wish to maximize J_{eff}, because energy gap has to be larger than main source of noise, likely to be due to laser frequency noise, typically $\delta f \sim 500\text{Hz}$

Weak coupling: $\eta\Omega < |\nu - \delta|$, so $J_{\text{eff}} < \eta\Omega$, but:

One has to couple only to one phonon mode: $\eta\Omega < \Delta\nu$

In one dimension: $\Delta\nu = (\sqrt{3} - 1)\nu$

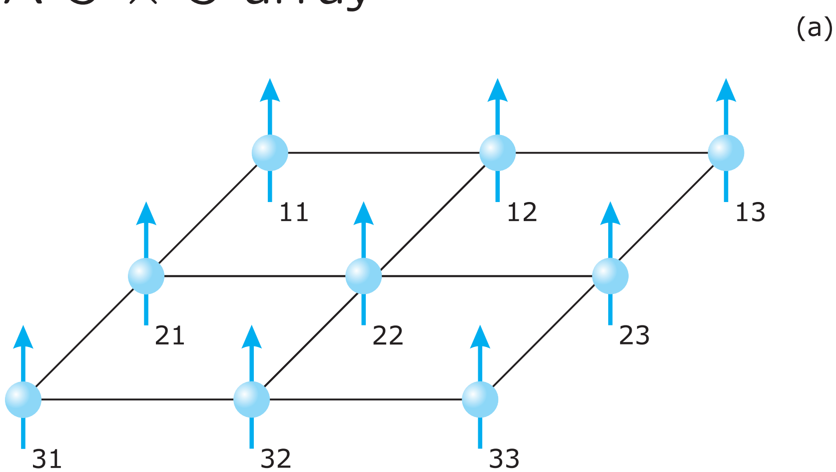
In two dimensions: $(5 \times 5$ array): $\Delta\nu \simeq 0.1\nu$

Increasing ν decreases the distance between ions

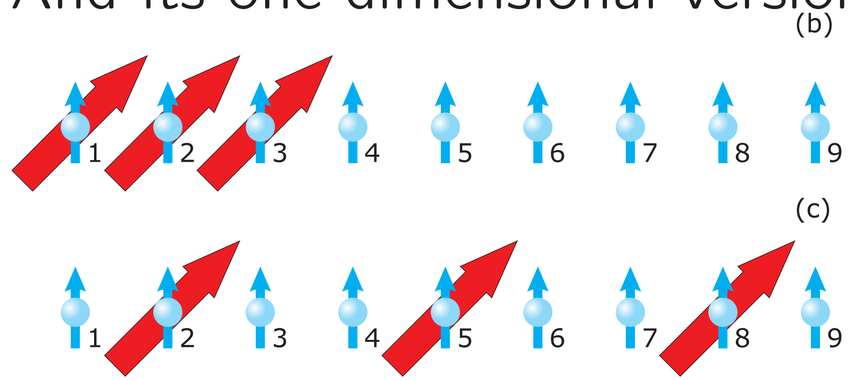
Optimal size seems to be $N \leq 3$ (1D) or $N \leq 5$ (2D)

Folding a square onto a line

A 3×3 array



And its one-dimensional version



Long range interactions help, because they induce larger gaps!

	2×2	3×3	4×4	5×5
SRI	0.84	0.58	0.32	0.20
LRI	0.84	0.96	0.92	0.80

Estimates for decoherence time

	4 ions	9 ions	5 $\times$ 5 ions
Γ_{eff} (Hz)	$1.5 \cdot 10^{-3}$	$7.5 \cdot 10^{-5}$	$1.9 \cdot 10^{-11}$
$\tau(s)$	$6.6 \cdot 10^{2}$	$1.3 \cdot 10^{4}$	$5.3 \cdot 10^{10}$

Conclusions

1) Ground-state degeneracies occur in the presence of non-local symmetries and are protected against local noise terms to high order $\sim L$ in perturbation theory.
2) Experimental realization of small Josephson junction arrays with such non-local symmetries has just started, at Rutgers university and Grenoble.
3) Possible implementation in ion traps.

Related subjects

1) Possibility to simulate lattice-gauge theories based on finite groups G using Josephson junction arrays.
2) Low energy excitations in these models are anyons, whose braiding properties may be used in designing quantum gates.