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Mesoscopic devices for cavity QED

From quantum optics to quantum impurities

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CENTRE NATIONAL
DE LA RECHERCHE
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Mutual influence of small quantum systems

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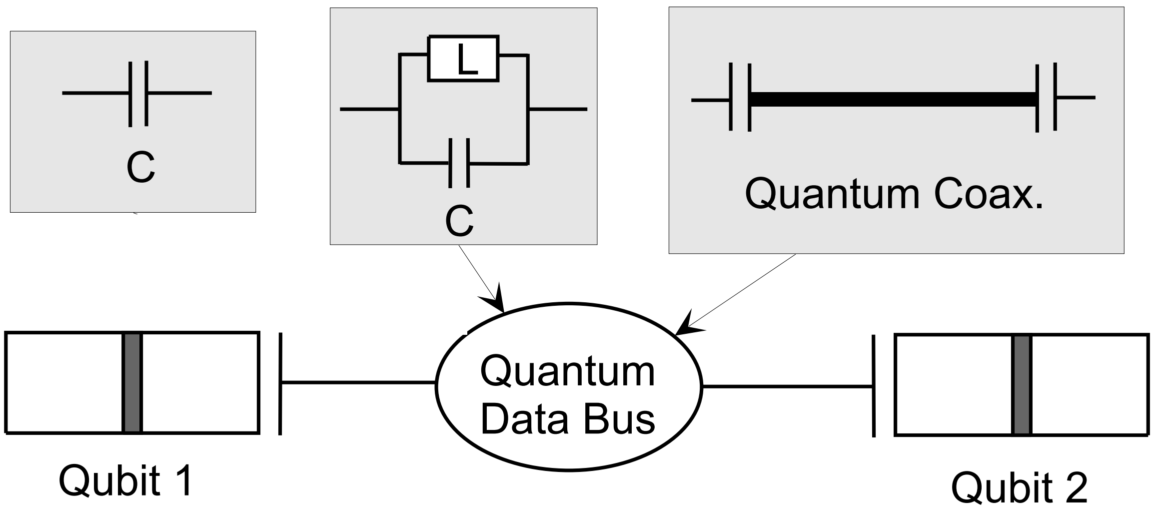
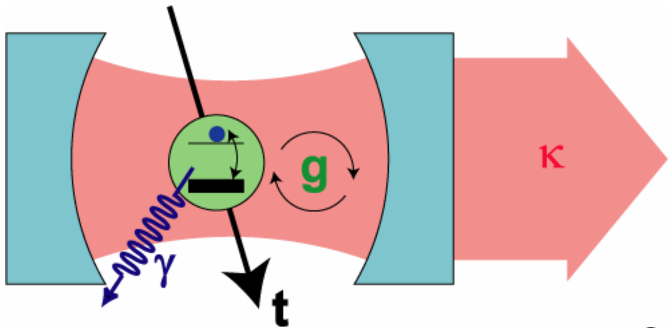
Coupling of several qubits: achieved through an intermediate system



Main problems: Mutual influence through the “quantum bus” ?

Correlations and dynamics of the two qubits ?

Cavity QED



Lowest frequency mode $\omega_0 = \frac{\pi v}{L}$

$$g \sim \frac{\pi v}{L}\beta$$

Jaynes-Cummings hamiltonian: valid for $g \ll \omega_0, \omega_{eg}$, i.e. for $\beta \ll 1$

$$\hbar^{-1} H_{JC} = \omega_0(a^\dagger a + 1/2) + \frac{\omega_{eg}}{2}\sigma^z + \frac{g}{2}(a^\dagger \sigma^- + a\sigma^+)$$

Quantized mode 2-level system dipolar coupling

Master equation for the qubit + cavity system:

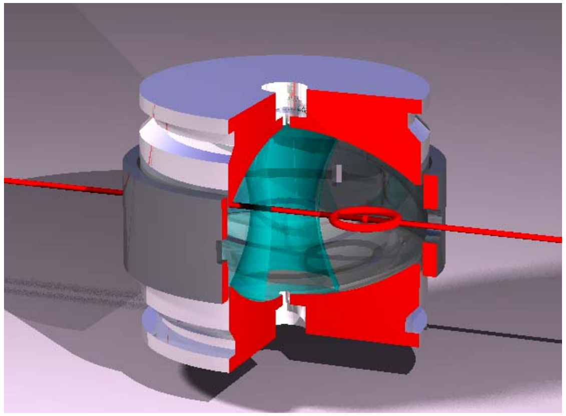
$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H_{JC}, \rho] + \boxed{\sum_a \mathcal{L}(\rho)}$$

Dissipation terms (Markovian)

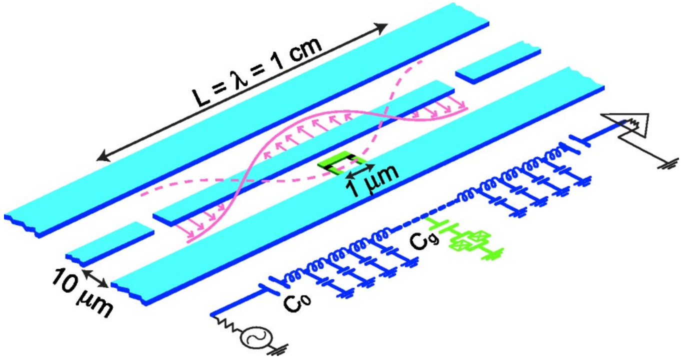
Strong coupling: $g \gg \kappa, \gamma_r$

Atom / mode interaction dominates dissipative processes.

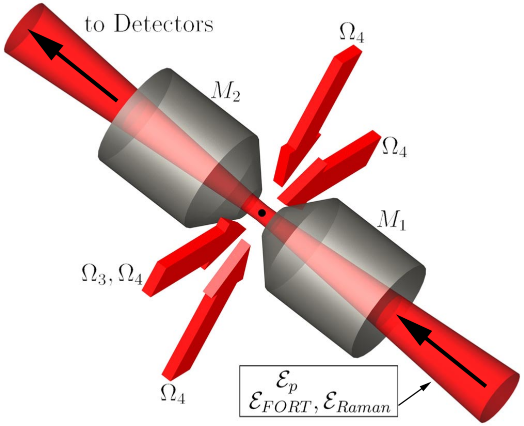
Experimental realizations of cQED



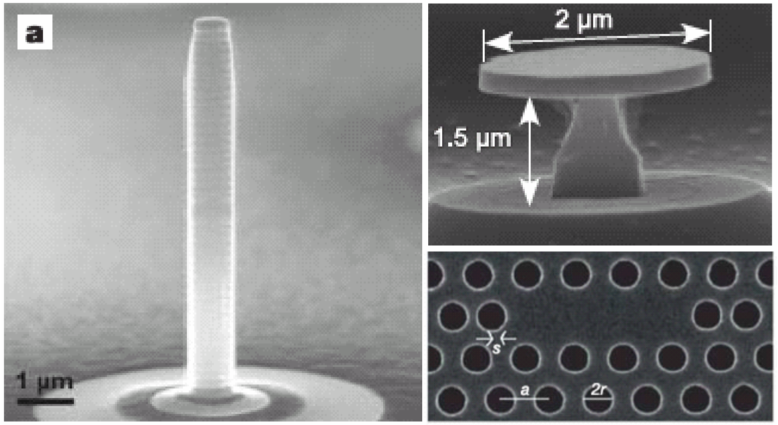
Rydberg atoms in superconducting cavities



Josephson qubits in superconducting resonators



Single atoms in optical cavities



Micropillars and photonic bandgap cavities

Beyond quantum optics cQED

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Quantum optics

Higher modes can usually be neglected

Main interest: quantum state preparation, evolution and measurement

Notable exception: non linear quantum optics

Phys. Rev. Lett. **95**, 140504 (2005)

Condensed matter physics

Qubit / cavity optimization: leads to higher values of the coupling !

Nanomechanics: achieves the super-strong coupling regime

Europhys. Lett. **78**, 60002 (2007)

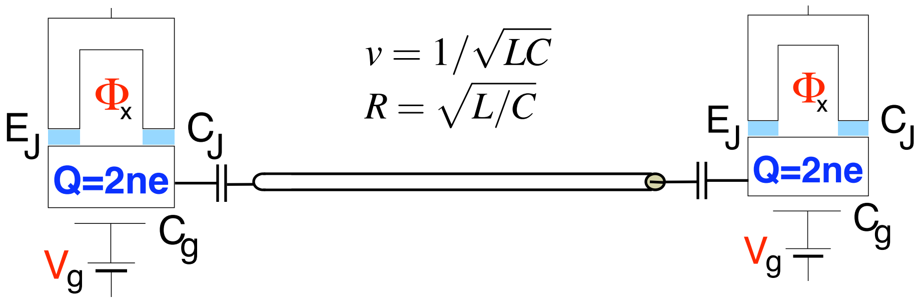
Our work: other systems also correspond to the original cQED problem (1D cavity + two level system), mapping on a double quantum impurity problem.

Europhys. Lett. **68**, 37-43 (2004)

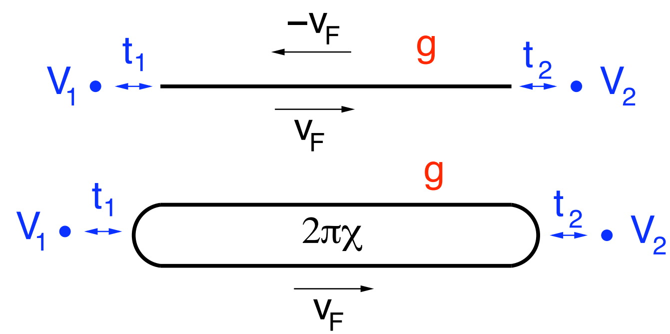
with S. Camalet, J. Schriefl and F. Delduc

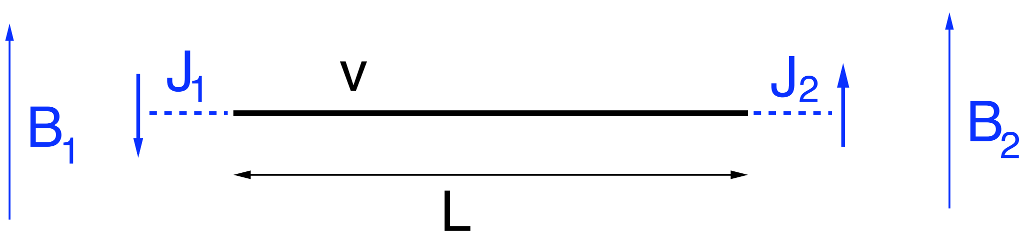
Three different mesoscopic systems...

Two qubits coupled to a transmission line



Wires and dots system





Double one channel, spin 1/2 Kondo problem

are described by the double Kondo model

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Bosonic field on a finite line

$$H_B = \frac{v}{2}\int_{-L}^{0}\left(\hbar^{-1}\Pi^2 + \hbar(\partial_x\Phi)^2\right)dx$$

Circuit QED: double spin-boson

$$H_j^{\mathrm{qb}} = -\frac{B_j^z}{2}\sigma_j^z - \frac{B_j^x}{2}\sigma_j^x \qquad\qquad H_j^{\mathrm{int}} = \frac{v\beta_j}{2}\Pi(x_j).\sigma_j^z$$

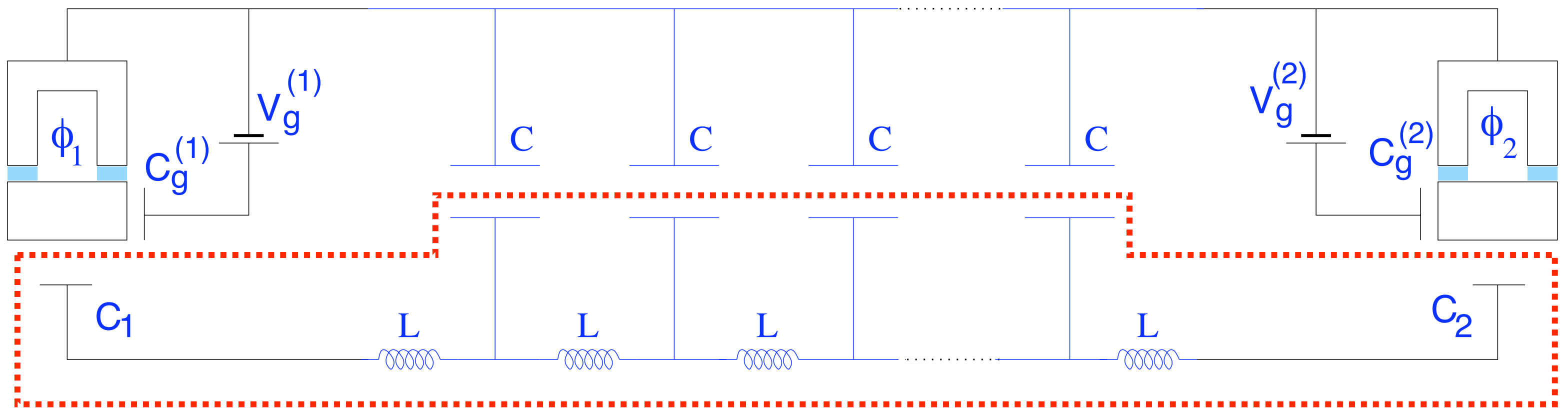
After polaronic transformation: double Kondo

$$H_K = H_B - \frac{1}{2}\sum_{j=-L,0} B_j^x\left(e^{i\beta\Phi(x_j)}\sigma_j^+ + e^{-i\beta\Phi(x_j)}\sigma_j^-\right) - \frac{1}{2}\sum_{j=-L,0} B_j^z\sigma_j^z$$

The coupling constant determines the dimension of the boundary operator.

Fixed charge condition

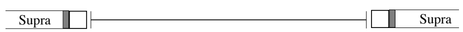
The charge stored in the red dashed box is conserved



$$\frac{1}{\beta}\int_{-L}^{0}\hbar^{-1}\Pi(x)\,dx-\frac{1}{2}\left(\sigma_0^z+\sigma_{-L}^z\right)=n_0^G+n_{-L}^G-1\,.$$

From devices to double Kondo

Qubits



$$v = 1/\sqrt{LC}$$

$$\beta_j = \frac{2C_j}{C_j + C_j^{J+g}} \sqrt{\frac{R}{\hbar/e^2}}$$

$$H_j^{\mathrm{qb}} = -\frac{B_j^z}{2}\sigma_j^z - \frac{B_j^x}{2}\sigma_j^x$$

Resonant levels

$$v = v_F$$

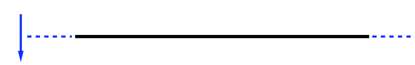
$$\beta^2/\pi = 1/g_{\mathrm{Lutt}}$$

$$B^x \propto t/\sqrt{a}$$

$$B^z = \mu$$

$$\frac{1}{2}(1-\sigma^z) = d^\dagger d$$

Kondo



$$v = v_{\mathrm{spin}}$$

$$\frac{\beta_j^2}{2\pi} = 1 + \frac{1}{2\pi}\frac{J_j^z}{\hbar v_s}$$

$$B^x = J_\perp/2\pi a$$

$$B^z = \mu_0 h^z$$

$$\sigma^z/2 = S^Z$$

$\beta \lesssim 0.3$ — Circuit QED devices

$\beta = \sqrt{\pi}$ — Fermi liquid

$\beta = \sqrt{2\pi}$ — Isotropic Kondo

Strategy

Non perturbative approaches

TBA & Destri - De Vega eqs can be used to compute thermodynamical quantities.

Perturbation theory $\beta^2 < \pi$

No Ir divergences (finite size)
UV convergence: $\Delta_\partial(e^{i\beta\Phi}) = \beta^2/\pi$

$\beta \lesssim 0.3$ $\qquad$ $\beta = \sqrt{\pi}$ $\qquad$ $\beta = \sqrt{2\pi}$

Jaynes Cummings model

Coupling to a single mode & RWA

Add dissipation: cavity relaxation, qubit relaxation & dephasing

Phys. Rev. A **74**, 033802 (2006)
Preprint **arXiv:0704.3489**

Toulouse point

Fermionic solution can be used to compute thermodynamics and dynamics.

Europhys. Lett. **68**, 37-43 (2004).

Reflexion matrices and influence lengths

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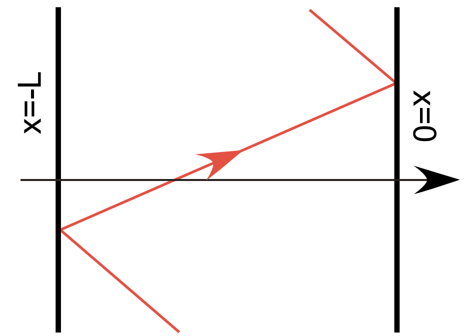
Fermionic Fabry-Perot cavity

Stationary wave condition for bouncing fermionic excitations

$$1 + e^{2i(kL-\pi\chi)} R_0(k) R_{-L}(k) = 0$$

Reflexion matrices: $$R_j(k) = \frac{k - k_j - i\alpha_j}{k - k_j + i\alpha_j}$$

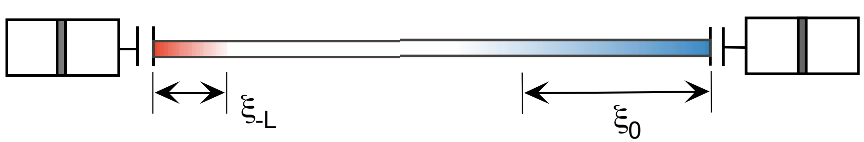


$$\xi_j \simeq (\hbar v)^2 / a E_J^2$$

$$k_j = B_j^z / \hbar v$$

The Kondo energy scale

Each quantum impurity introduces an energy scale at the boundary which gives a length scale $\xi_j = 1/\alpha_j$



Represents the influence zone of each quantum impurity (Kondo cloud).

Ground state energy from boundary field theory

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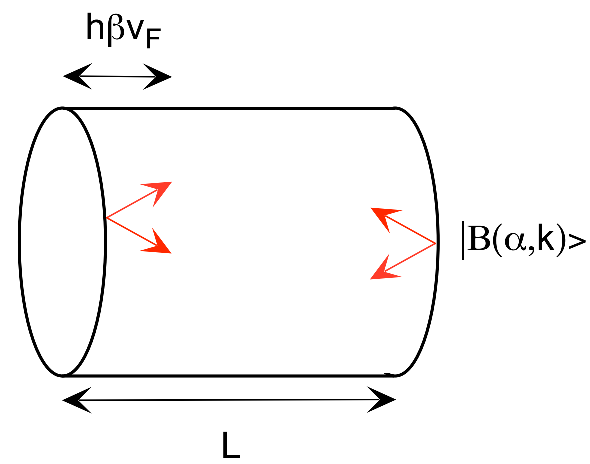
Boundary field theory



Each boundary created correlated pairs of excitations (Cooper pairs).

For $L \gg \hbar v/k_B T$, Cooper pairs do not overlap: the two qubits do not influence each other.

Free energy at T=0 (ground state energy)

Computation of the scalar product of boundary states (Caux et al, Phys. Rev. Lett. **88** (2002) 106402).

Free fermion point: Chatterjee contour integral method (Mod. Phys. Lett. **A 10** (1995), 973).

Ground state energy from boundary field theory

General structure at vanishing temperature:

$$\log\left(Z(\beta)\right) \simeq -\beta_T\left(LE_{\mathrm{bulk}} + E_\partial + \frac{h_0[(\alpha_j, k_j)_j]}{L}\right) \simeq -\beta_T E_0$$

Explicit result:

$$E_0 = \frac{\hbar v}{\pi}\int_{-\infty}^{0}\left(L + \sum_j \frac{\alpha_j}{\alpha_j^2 + (k-k_j)^2}\right) k\,dk - \frac{\hbar v}{2\pi}\int_0^{+\infty} \log\left(\left|1 + X(ik)\right|^2\right) dk$$

Bulk contribution | Boundary contribution | **Universal contribution**

where $X(k) = e^{2i(kL - \pi\chi)} R_0(k) R_{-L}(k)$

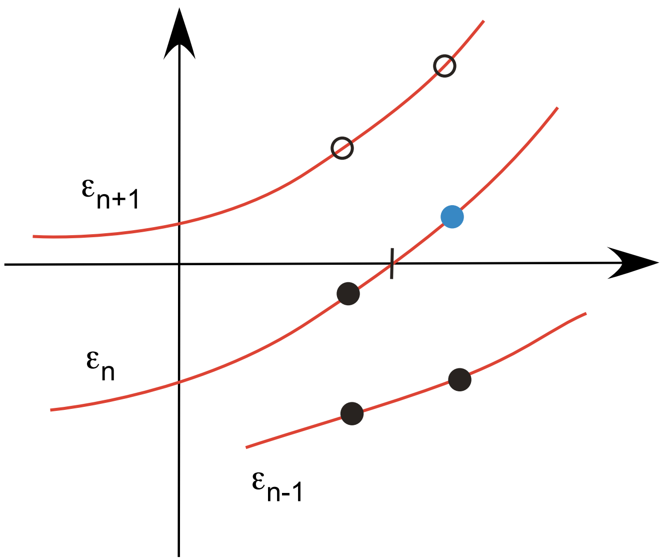
Mutual influence effects are contained in the logarithmic integral.

Analyticity issues

The logarithmic formula computes the ground state energy in the grand canonical ensemble whereas the fixed charge condition means that we are working in the canonical ensemble.

Grand canonical ensemble: change of vacuum when a one-particle energy level crosses zero.

Canonical ensemble: we musk keep the same one-particle energy levels occupied.



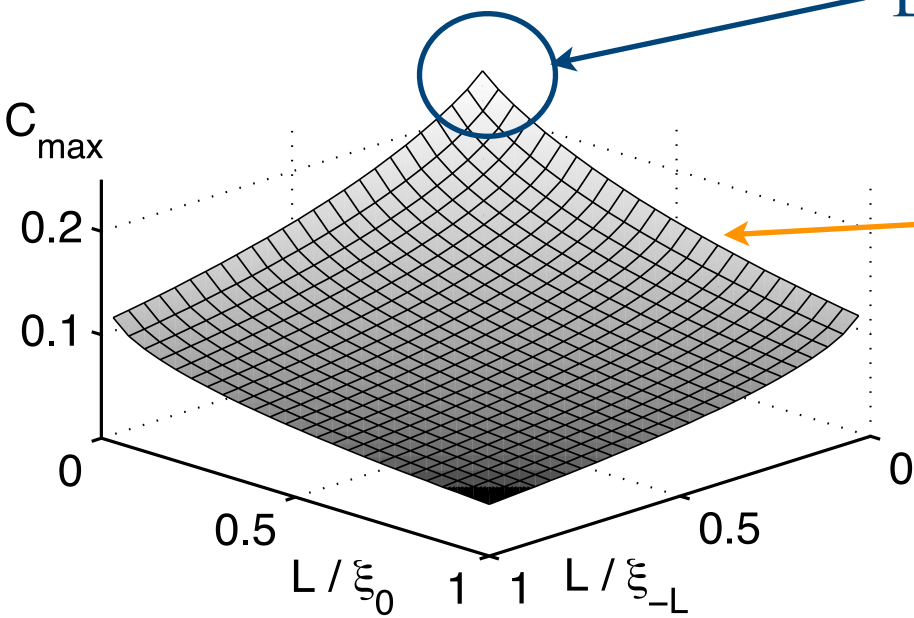
Solution: the logarithmic formula should be supplemented by an analiticity requirement.

Caux, Saleur and Siano, PRL **88** (2002), 106402
Nucl. Phys. **B 672** (2003), 411.

Maximum correlation

At **fixed Kondo lengths**, look for the maximum correlation by varying other parameters

$$C = \langle n_0 \rangle \langle n_{-L} \rangle - \langle n_0 n_{-L} \rangle$$



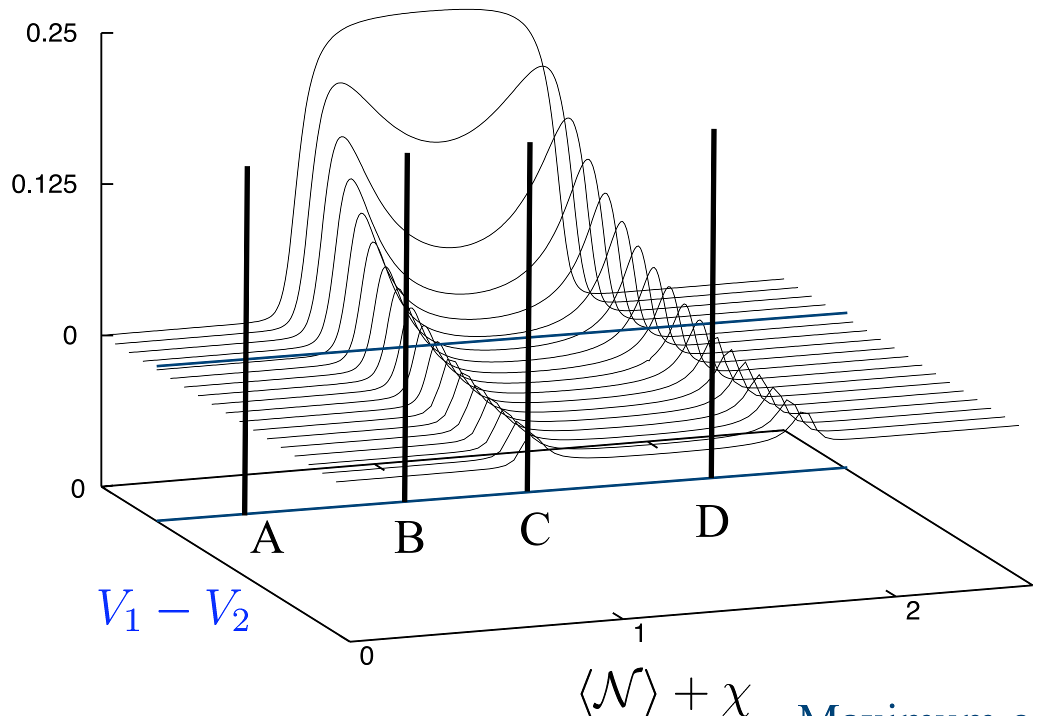
For $\xi_{-L} \gg L$

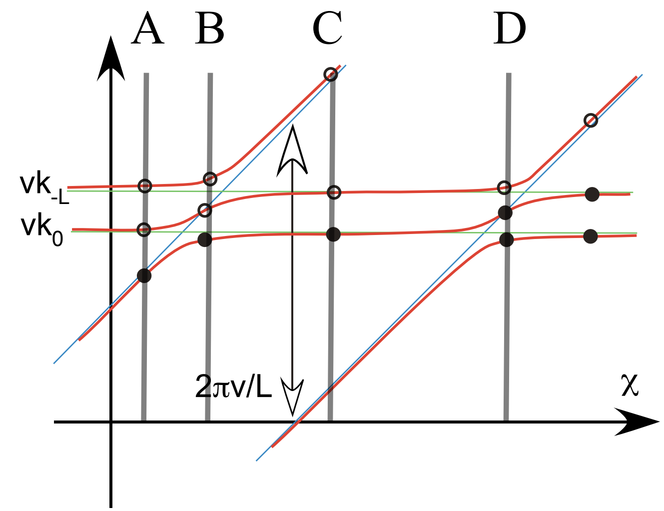
$$C_{\max} = \frac{1}{4(1 + L/\xi_0)}$$

Question: when are the correlations maximal in term of other parameters ?

Populations and correlations

Large Kondo clouds $\xi_0 = \xi_{-L} = 200\,L$





Fermionic one particle energy levels

$$2e\beta(\langle \mathcal{N} \rangle + \chi) = \sum_j C_g^{(j)} V_g^{(j)}$$

$$V_2 L = 1.2\,\hbar v$$

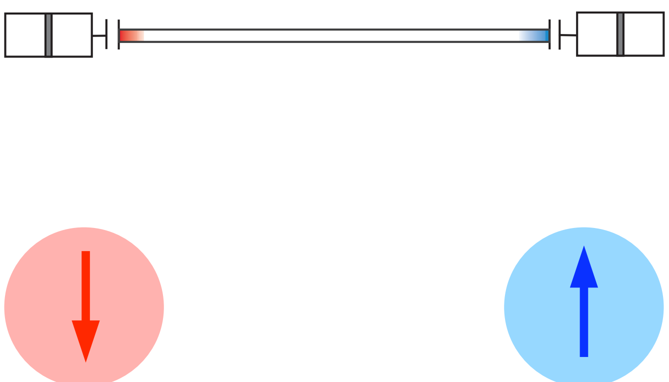
Maximum correlation correspond to maximal hybridization betweeen the line and the impurities.

Exhaustion criterion of Nozières.

Physical picture

Small Kondo clouds

$\xi_i \ll L$

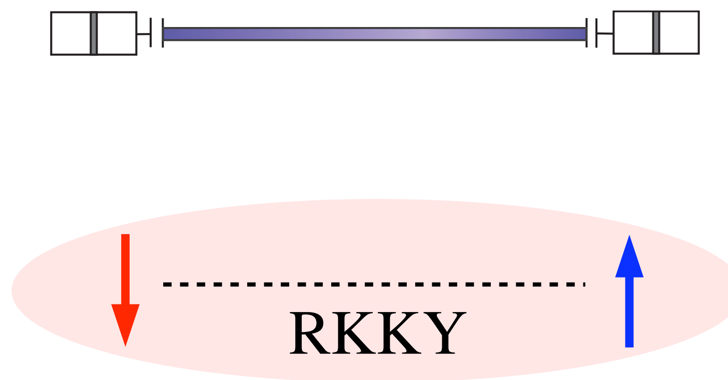


Very weak correlations between the quantum impurities.

Efficient screening.

Large Kondo clouds

$\xi_i \gg L$



Correlations emerge when the same degrees of freedom of the line participate to the Kondo cloud (exhaustion criterion)

This picture should be valid generally.

Conclusions and perspectives

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Conclusion

Description of three mesoscopic double quantum impurity problems by the double Kondo model.

Exact solution at its Toulouse point (non trivial analyticity issues).

Identification of two different regimes (Kondo and cavity regimes).
Role of Kondo lengths scales.

Perspectives

Exact solution at other points (TBA, Destri-DeVéga eqs.).

Introduction of dissipation at the Toulouse point.

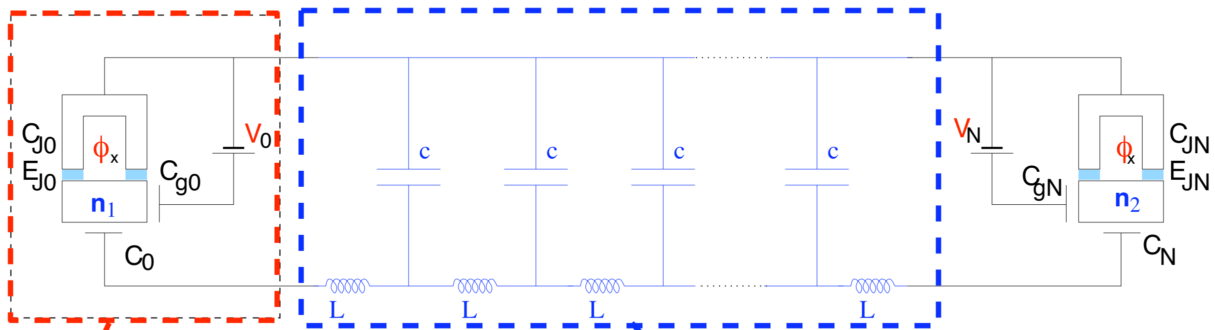
Real time dynamics at the Toulouse point (forced mesoscopic systems).

The end.

Thanks a lot for your attention.

Circuit QED with two end qubits

Two qubits coupled to a transmission line $v = 1/\sqrt{LC}$ $R = \sqrt{L/C}$



$$H_j^{\mathrm{qb}} = -\frac{B_j^z}{2}\sigma_j^z - \frac{B_j^x}{2}\sigma_j^x \qquad H_B = \frac{v}{2}\int_{-L}^{0}\left(\hbar^{-1}\Pi^2 + \hbar(\partial_x\Phi)^2\right)dx$$

Capacitive coupling $$H_j^{\mathrm{int}} = \frac{v\beta_j}{2}\Pi(x_j).\sigma_j^z \qquad \beta_j = \frac{2C_j}{C_j + C_j^{J+g}}\sqrt{\frac{R}{\hbar/e^2}}$$

Double spin boson model with transmission line

A mesoscopic Kondo problem

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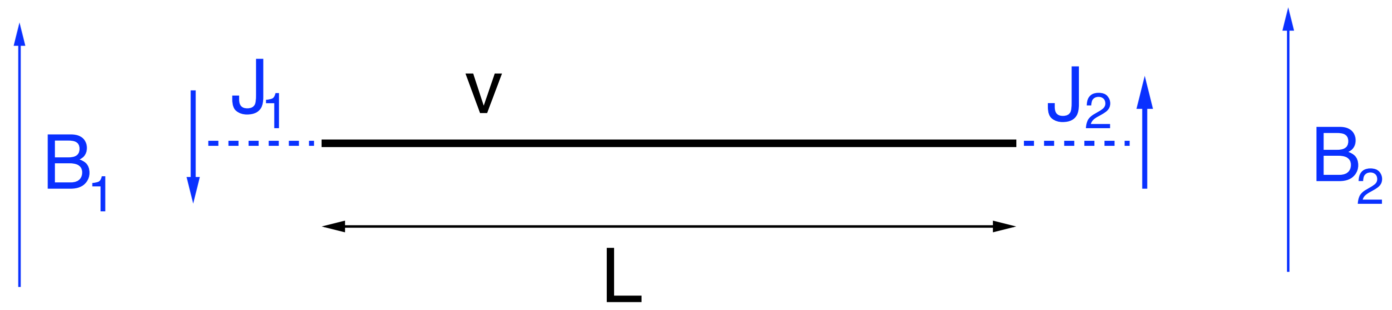

Magnetic impurities: two spin 1/2 coupled to local fields along z

Quantum wire of finite length with spin 1/2 electrons (gapless spin degrees of freedom).

Anisotropic exchange interaction between electrons and impurities

$$H_j = J_j^{\perp}\{S_j^x \mathcal{S}^x(j) + S_j^y \mathcal{S}^y(j)\} + J_j^z S_j^z \mathcal{S}^z(j)$$

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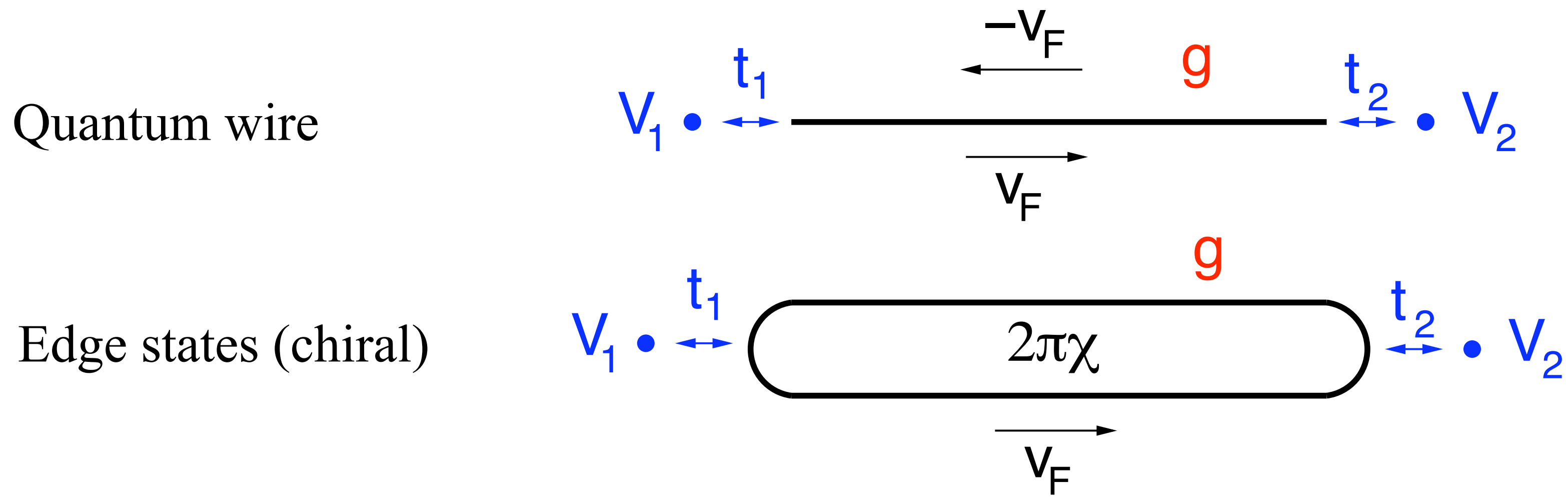
Tunneling between a quantum wire and two resonant levels



Luttinger liquid for spinless electrons (Haldane 1981, Fabrizio & Gogolin 1995)

Interaction parameter	g_{Lutt}
Renormalized Fermi velocity	v_F

Tunneling between and infinite wire and one resonant level
Furusaki and Matveev, Phys. Rev. Lett. **88** (2002), 226404.

Populations and correlations

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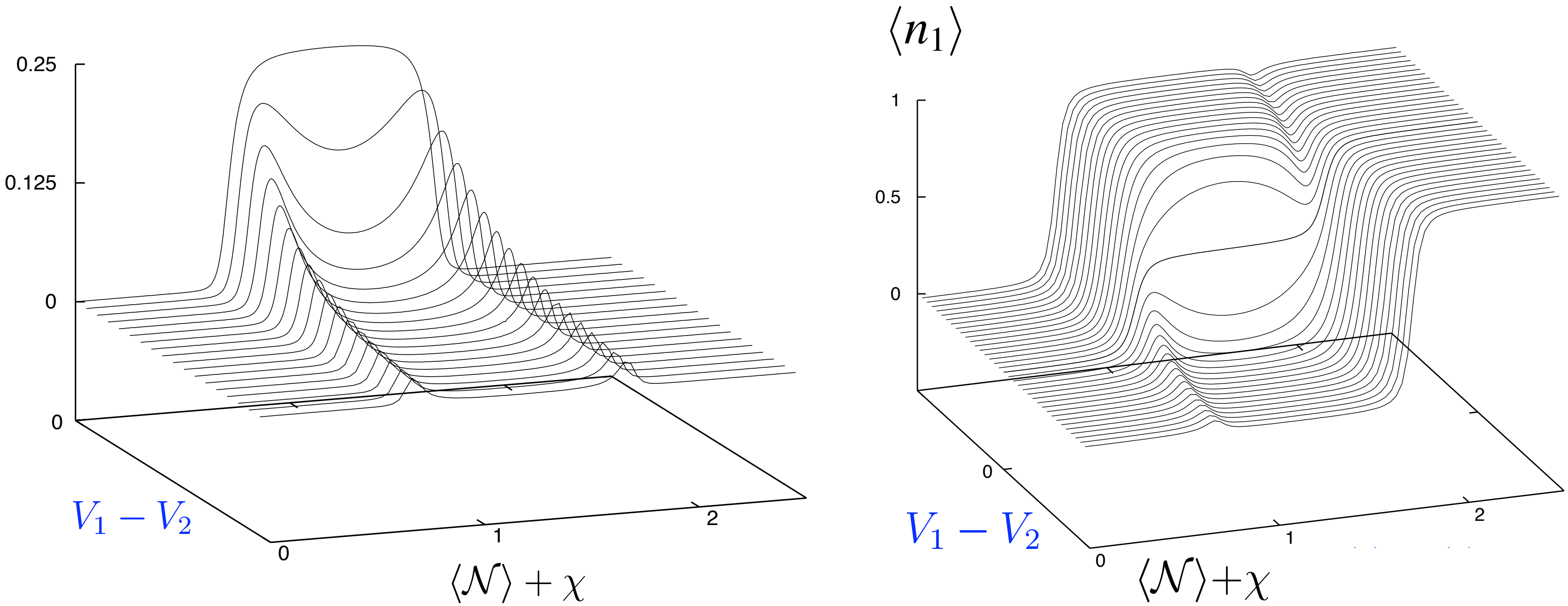
Large Kondo clouds $\xi_0 = \xi_{-L} = 200\,L$



$V_2 L = 1.2\,\hbar v$